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School mathematics in work and life: what we know and how we can learn more

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Abstract

Underlying current K-12 math-education reform initiatives are two premises: First, math education is necessary and should aim for success in the adult world of work and everyday life. Second, aligning the methods and content of K-12 math programs with those of real-world adult math use will facilitate learning. The existing research, however, challenges these premises. Studies of the actual demands of everyday adult practices reveal that most occupations involve only a low level of mathematical content and expose the disparate natures of everyday and school mathematics. Several limitations of existing mathematics-in-practice research, however, make it problematic to draw clear implications for school programs. In this article, I summarize the current research and identify some of its shortcomings for guiding educational content and pedagogy, including its focus on occupations and practices known to involve little math, its use of stereotypical conceptions of math, and its reliance on surveys and worker self-report. I propose a conceptual framework for future investigations of math in practice that would better inform K-12 education.

1. Introduction: the need to understand the mathematical demands of adult life

There are many ways to justify teaching K-12 mathematics. For the past 20 years, one justification has dominated policy talk and motivated large-scale curricular reforms: that math education is necessary and should aim for success in the adult world of work and everyday life. Today, this “utilitarian” philosophy undergirds virtually every document and initiative calling for improvements in math education. A passage from the National Research Council [NRC] report, *Everybody Counts* [1], is typical of this rhetoric:

Mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides the knowledge to compete in a technological community. To participate fully in the world of the future, America must tap the power of mathematics. [1] p. 1.

As this utilitarian justification has taken hold, a parallel pedagogical idea has emerged: that greater alignment of the methods and content of K-12 math programs with those of real-world adult math use will facilitate students’ math learning. The National Council of Teachers of Mathematics (NCTM) [2] and countless other math education groups now urge curricular connections between school math and real-world applications. For example, the NRC emphasizes:

Students need to experience mathematical ideas in the context in which they naturally arise – from simple counting and measurement to applications in business and science...The significant criteria for the suitability of an application is whether it has the potential to engage students’ interests and stimulate their mathematical thinking. [3] p. 38.

Textbook publishers, video producers, software writers, professional developers, and funding organizations have answered the call with abundant materials and training for teaching math through practical applications.

While these two ideas may appear to converge into one work-oriented movement, it should be noted that they are distinct and not necessarily dependent. The first proposes a goal: that the outcome of math education should be the productive adult and nation. The second posits a pedagogical theory about how to get there. Despite their similar theme, neither idea automatically follows from the other. The “New Math” reform, which began in late 50s, was arguably motivated by a utilitarian – certainly economic – goal: developing more and better American scientists and engineers in order to regain the international supremacy we feared we had lost in the space race. [4] Yet the pedagogical theory underlying this reform was hardly work-based. Instead, the federally funded School Mathematics Study Group and other university-based groups, mainly comprising academic mathematicians, took pedagogical guidance from the discipline itself, analyzing the internal, logical structure of mathematics and using this as the basis of a new curriculum. [5] The reverse is also logically possible. Some educators believe that more closely aligning the mathematical practices of students with those of working adults improves learning, for example by stimulating student interest, but feel that math should be learned for its own sake, as a paragon of cultural achievement.

Nevertheless, both ideas have an obvious, face-value appeal. It is hard to dispute that the world has become increasingly mathematized. Today, people are surrounded by numbers, charts, graphs, and symbols as never before. More and more aspects of our daily lives are controlled by mathematical models, statistics, and computer programs. It is natural to assume that the mathematical demands on individual citizens have likewise increased. Further

convincing the American public of the importance of math education is the perception that, despite our mathematized environment, both the average American adult and child are losing ground mathematically compared to earlier generations. Employers complain of workers who can't make change at the cash register, parents decry their children's dependence on calculators for basic operations, and the media trumpet our students' falling test scores and low international ranking. Some research supports this perception: national and international studies (e.g., the National Assessment of Educational Progress [NAEP], the International Adult Literacy Survey [IALS], and the Third International Mathematics and Science Study [TIMSS]) show that US adults and children are poorly able to apply basic mathematical skills in order to understand, reason with, generate, or evaluate the quantities that surround them.¹ [6, 7] To much of the public (and apparently to most politicians), this spells a full-blown crisis that threatens our children's futures and our national competitiveness. And improved math education in schools is almost always seen as the solution. [9] Indeed, the link between math education and the economy is a key justification for the Bush Administration's new Mathematics and Science Initiative, a five-year program to improve US math and science education. This link is evident in an Initiative concept paper from the Secretary's 2003 Summit on Mathematics [10]:

[T]he public must realize that advances in technology and productivity, necessary for the US to remain competitive in the global economy, depend on all students learning more mathematics and science than is currently required, but also on increasing the number of students who extend their mathematical knowledge beyond algebra so they may proceed to more advanced scientific and technical subjects.

¹ Interestingly, between 1973 and 1999, the period covered by the long-term trend component of the NAEP, US children showed slight gains in mathematics scores, at all three age levels tested. [8]

The idea that math learning can be enhanced by pedagogical methods that illustrate and even replicate mathematical practices from everyday adult life and work has been evolving since – and in reaction to – the New Math era, when school math stressed abstract, general concepts. [11] Among the criticisms of New Math was its students’ inability to apply those abstract, school-taught, math concepts. Responding to this criticism, educators, over the decades, have increasingly emphasized real-world connections in the classroom, convinced that teaching through applications facilitates the learning and transfer of mathematical concepts. [11] But this evolution has a political side as well. For decades, a large portion of students have despised, feared, and/or dismissed school mathematics, and its lack of relevance is a predominant complaint. Many math teachers, therefore, embrace real-world connections for their promise to motivate or appease students and to justify teaching the subject. [11]

Today, one might wonder why any justification for teaching math would be needed; obvious examples of its utility should abound. Yet the opposite seems to be the case. When it comes to math education, the public exhibits a somewhat schizophrenic attitude. Adults panic over falling test scores and demand high-quality math instruction for their children (witness the booming private-tutoring and test-prep industries), suggesting they place a high value on math education. Incongruously, these same adults dismiss the importance of mathematics when they proclaim that they never use math, beyond simple arithmetic, in their own work and lives. [12] Survey studies bear out that the public, in general, sees little need for higher-level math in the workplace.² Even high-tech workers often claim to use very little math. [13] In fact, the federal Mathematics and Science Initiative includes a widespread campaign to raise public awareness about the need for more and better math education [10]:

² To some degree, academic devaluation extends beyond math. Both educators and non-educators view personal qualities as far bigger determinants of workplace success than academic preparation. [14]

[A] national campaign is needed to inform parents, students, educators, and the general public about the importance of mathematics and science learning in our changing society....Parents will learn that while they may not have needed a high-level mathematics and science background to be successful, their children will. The Initiative will enlist the aid of state and local education agencies, businesses, professional organizations, religious, and non-profit organizations to promote STEM [science, technology, engineering, and math] awareness.

Noss and Hoyles [15] attribute the public's perception of low mathematical demands to the changing nature of workplace mathematics. They explain that, "as the mathematics in working practices becomes less visible, so the formal mathematical knowledge of schools apparently becomes less applicable" [15] (p. 4). The technology boom in everyday life and the workplace has certainly reduced the visible role of math. Calculations done by hand in earlier decades are now handled by spreadsheets and calculators; specifications and formulas once memorized or sought in manuals and charts are now invisibly embedded in off-the-shelf software. Math may be all around us, as math educators (and politicians) proclaim, but apparently schools not only need to help students survive in this mathematical world, they also need to help students see the math in first place.

Not surprisingly, there is confusion at the school level. K-12 math teachers are exhorted to prepare students for the mathematical demands of adult life and to organize classroom activities that demonstrate and replicate adult mathematical practices. But many teachers wonder: what *are* the mathematical demands of adult life? How, in fact, *do* adults (other than math teachers) actually use mathematics? Amid the conflicting claims and perceptions of parents, employers, policy makers, colleges, and the professional societies of the discipline, few clear answers are available. In today's political climate, where US math education is at once

strongly emphasized and highly criticized, a better understanding of the real mathematical demands of adult life has become imperative. The recent federal No Child Left Behind Act (NCLB) and the various state policies it has spawned are transforming K-12 math education by mandating uniform (statewide) content standards and enforcing them with stringent accountability measures that carry severe consequences for children and schools. Out of respect for students and teachers, researchers and policy makers should ensure that those content requirements are justifiable – that they stand up to the rhetoric about the demands of modern occupational life.

In this spirit, I conducted the following literature review to learn what is currently known about mathematics in adult practices. The limitations I found in this research subsequently led me to develop a conceptual framework, also presented here, to shape future studies in ways that would enhance their potential to inform math education.

2. Research about adult mathematics

2.1 What do we know about math content in the world of work?

An obvious approach to discovering the nature of adults' mathematics has been to identify the school mathematics topics used in various industries or occupations. This type of research has usually been conducted by educators or members of the industry under study, expressly to inform schooling or feed directly into job-training curricula. Such studies produce lists of math topics typically required by each industry. In this section, I give findings from some major studies of this “topical” sort.

The Secretary's Commission on Achieving Necessary Skills (SCANS) [16] surveyed employees and their supervisors about the skills and competencies needed for a range of entry-

level jobs. Presented with 17 “foundation” skills, the supervisors ranked “arithmetic” 14th in importance and “mathematics” last. [17] The 1995 National Job Task Analysis, conducted by the American College Testing Corporation (ACT), extended the SCANS findings by surveying 3000 workers, at all levels, about the general skill requirements of their work. Of the 25 competencies these workers identified as most important, only one, which they ranked 14th, related to school mathematics: “perform arithmetic (addition, subtraction, multiplication, and division) as part of work activities.” [17] Mathematical requirements were then specifically investigated for a sub-sample of the workers. The ACT delineated seven levels of mathematical skills and categorized jobs accordingly. Over 90% of the jobs studied were found to require no more than Level 6 skills: “[use] negative numbers, ratios, percents, and mixed numbers...calculate multiple rates, find areas of rectangles and circles, volumes of rectangular solids, solve problems involving production rates and pricing schemes.” Three quarters of this 90% fell below even this level. [17] Apparently, the content of the most basic high school geometry and first-year algebra courses more than covers the mathematical skill requirements for the future work of the vast majority of students.

The US Departments of Education and Labor commissioned various occupational and industrial organizations to identify the skill prerequisites for entry-level jobs in 22 industries. The resulting reports corroborated the findings of the SCANS and ACT studies: in general, eighth- or ninth-grade-level mathematics courses cover the necessary content. [17]

A major work from England, *Mathematics Counts* [18], reviewed several studies of the mathematical requirements in workplaces that employ recent high school graduates. These studies again demonstrated that the mathematical content used in most work is covered early in

the secondary curriculum, but they also revealed differences in how this content is used at work and in school. Notably:

- Mental calculations are important at work but de-emphasized in school.
- Workers often use idiosyncratic calculation methods, developed by colleagues or themselves, that are mathematically unsophisticated or inefficient.
- At work, calculators are used widely but often inefficiently. (The authors propose that schools do not teach efficient calculator use.)
- When fractions are used at work, it is almost always for measurement and so involves only commonly used denominators that are readily available. The ruler itself is often used as a calculating device for operations on fractions. Other fractions (e.g., $5/7$) are rarely encountered, and calculators are used to operate on them.
- Algebra is rarely used at work. Sometimes substitution into an often-used formula is required, but formulas rarely need to be manipulated.
- Estimation is important at work but inadequately addressed in school.
- Measurement is critical at work, along with a sense of the appropriate level of precision.

The consistent message of this body of topical research is that a surprisingly low level of mathematical content is generally required in today's workplace and that much of the standard high-school curriculum far exceeds this level. Yet many workers are not sufficiently competent even in these basic math skills. The ACT studies found gaps between worker competence and required mathematical skill at all levels. For example, about half of the jobs studied by ACT required at least Level 5 skills, characterized by the ability to determine the information and calculations needed to convert within or between systems of measurement and to calculate with mixed units, like hours and minutes. Only a third of the workers, however, were competent at

this level. [17] Packer [17] concluded: “Today’s mathematics classrooms stress skills that few students will use while neglecting skills that employers really need.” (p. 141).

Why have these topical reports had so little impact on school curricula? Some studies (e.g., NRC, 1998 [19]) begin with a set of conventional school-math topics and seek their applications in industry. Because the topics are taken as initial conditions, such studies can only reinforce, not challenge, the standard curriculum. Alternately, studies that start with a clean slate and truly attempt to discover the skill requirements of an industry may be victims of their own unwelcome results. Their finding – that most work requires only basic mathematical content – might warrant de-emphasizing the more advanced topics in the high-school curriculum or relaxing course requirements. Such actions would threaten certain sectors of the math-education industry and confound popular initiatives to promote more and higher levels of math instruction. An unwillingness to consider curricular alternatives to the standard march towards calculus may also reflect a notion disparaged by reformers but societally prevalent: that math education should be a “filter.” That is, a major reason to teach high-level math is to stratify students and weed out the less able. [20]

2.2 *What do we know about the mathematical behavior of adults?*

Some scholars (e.g., Harris, 1991; Mathews, 1991) [21, 22] have criticized the studies cited in *Mathematics Counts* [18] and similar research for presenting limited or inaccurate pictures of what workers actually do. They charge that these topical studies miss many examples of workplace mathematics because their interview and survey questions focus too narrowly, mainly on arithmetic calculations and measuring. Topical studies are also faulted for relying on workers to identify their own math use, which, critics argue, can be distorted by the workers’

stereotypical conceptions of math. A more valid and sophisticated picture of adult math use would come from direct observations of adults' actual behavior in real settings.

Such direct-observation studies fall under a larger genre of ethnographic research about thinking, problem solving, and learning that takes a “situated” perspective. Cognition is viewed as a form of social practice, integral to and deriving its meaning from the context of a particular community or culture. [23] Many situated studies of adults' mathematical or problem-solving behavior have been conducted outside of education, and rarely has school improvement been their direct (or even indirect) purpose. Instead, these studies have aimed, variously, to:

- increase the sophistication of our understanding of non-mathematicians' mathematical knowledge and use of math in everyday settings (e.g., Lave, Smith, and Butler, 1998; Hall, 1999; Hall and Stevens, 1995; Stevens and Hall, 1998; Lave, Murtaugh, and de la Rocha, 1984; Hutchins, 1995; Scribner, 1984) [20, 24 - 29]
- explore the role of cultural artifacts and tools in embedding and socially distributing knowledge (e.g., Hutchins, 1995; Vincenti, 1990; Pea, 1993) [28, 30, 31]
- improve technologies that support work (e.g., Nardi, 1996) [32]
- develop more accurate histories of knowledge development (e.g., Vincenti, 1990) [30]
- improve the performance of workers in particular industries (e.g., Noss and Hoyles, 1996) [15]
- uncover the sources of workplace errors (e.g., Scarsellatta, 1997) [33].

Despite their nominal distance from the center of math education research, these ethnographic studies provide information that could and arguably should inform K-12 math education. Below I discuss some key ethnographic studies.

2.2.1 *Mathematical behavior in everyday life*

Ethnographic studies of child and adult math use in everyday life have generally all come to the same conclusion: there is a wide gap between school-taught mathematical methods and the math people use outside of school. Context plays a central role in everyday problem solving. Studying farmers, carpenters, and fishermen in Brazil, Nunes, Schliemann, and Carraher [34] showed that these adults relied on contextual aspects of the quantitative problems of their work to help them calculate in ways unlike school-taught algorithms. While school arithmetic is considered rule-based and divorced from meaning, these adults' "street arithmetic" preserved situational meaning at each calculation step. In their study of grocery shoppers, Lave, Murtaugh, and de la Rocha [27] demonstrated that the setting of the store (the layout of the aisles, the price labels, the appearance of products, etc.) was inextricably linked to the shoppers' arithmetic decision making, with the setting both constraining and supporting their solutions. The shoppers' arithmetic performance in shopping was far better than on a test of mathematically analogous, school-type operations. The shoppers' school backgrounds correlated only with their scores on the school-type test and not with their performance on, or willingness to use, arithmetic in shopping. This study also illuminated a key feature of real-life problem solving: *gap closing*. Typically, the shoppers would reshape their shopping calculation problems while simultaneously shaping the solutions, thereby moving problem and solution towards each other – an impossible strategy for most school math problems.

People avoid even well known school-taught algorithms when solving everyday problems. Asked to find $\frac{3}{4}$ of a $\frac{2}{3}$ -cup serving of cottage cheese, with the actual cheese and measuring cups available, adult dieters in de la Rocha's study [35] ignored the standard

algorithm for multiplying fractions and instead invented strategies that often involved manipulating the physical objects.

After reviewing this body of research, Lave, Smith, and Butler concluded:

In everyday settings, people look efficacious as they deal with problems of number and space....People do not stop to perform canonical, school-taught mathematics procedures and then resume activity. In the supermarket and kitchen, jpfs [just plain folks] have more than sufficient resources of mathematical knowledge to meet the exigencies of their activities. [20] p. 67.

2.2.2 Mathematical behavior in the workplace

Several ethnographic studies have sought to characterize mathematical or problem-solving behavior in occupational practices. Like the everyday mathematics studies, these frequently take a situated perspective, bringing to the fore the social and cultural nature of workplace mathematics.

Hutchins's extensive "cognitive ethnography" of the practice of navigation on a Navy ship [28] illuminated several features of "cognition in the wild." Navigation, Hutchins showed, is accomplished through a complicated, interconnected system of human tasks, methods of representation and communication, instruments, tools, charts, tables, codes, techniques, and traditions. The knowledge required to dock a huge ship is distributed among these system elements, too great for any one person or tool to possess entirely. The cognitive demands on any single crewmember are actually quite mundane and technological tools have supplanted most calculations. According to Hutchins, tools do not amplify the cognitive abilities of individuals in the system, as is often claimed, but replace their more difficult tasks (like calculating) with easier ones (like chart reading). Stressing the cultural diffusion of knowledge, Hutchins warned

researchers not to infer the cognitive requirements of an individual from observations of the overall activity in which he participates. Hutchins's interest, in fact, was not the individual but the "environment for thinking" that the culture of navigation has developed over its history.

According to Hutchins:

The real power of human cognition lies in our ability to flexibly construct functional systems that accomplish our goals by bringing bits of structure into coordination....[A] proper understanding of human cognition must acknowledge the continual dynamic interconnectivity of functional elements inside with functional elements outside the boundary of the skin. [28] p. 316.

Hutchins's warning challenges the educational-policy rhetoric that our highly mathematized world imposes high-level mathematical demands on each citizen. Today's technologies may indeed have replaced many of our mathematical tasks with button pushing and chart reading, thereby reducing our need to personally possess sophisticated calculation skills.

Analyzing the mathematical behavior of architects and structural engineers jointly engaged in a design session, Hall [24] found that quantitative reasoning was prevalent but mostly involved only simple arithmetic with routine quantities. More interestingly, Hall showed how forms of mathematical representation are socially negotiated, both historically and on the spot. The diverse perspectives and accountabilities of the architects and engineers shaped their mathematical descriptions, models, and uses of quantity. Hall's portrayal of these workers' mathematical behavior – culturally and socially dependent, flexible, parochial, and open for reinvention and interpretation – clashes with the traditional image of mathematics as a static, objective, universal language that exists virtually independently of people, untainted by subjective perceptions and motivations.

Hoyles, Noss, and Pozzi [12] investigated experienced nurses' use of proportional reasoning in calculating drug doses. Though trained to use a single computational routine for this purpose, in practice the nurses employed a wide range of effective proportional-reasoning strategies. Hoyles et al. believe their findings confirm the existence of a large gap between school math and the math of the workplace, where abstract rules “fade into the background” [12] (p. 23), and culturally held and artifact-reliant procedures serve just as well or better. While rejecting the notion that the nurses used general, school-taught algorithms at work, Hoyles et al. proposed that the nurses had themselves “abstracted” the theoretical concept of concentration (the proportion of drug to solvent) in the course of their years of practice.

3. Summary of Research and Further Questions

The research reviewed above paints a fairly consistent picture of adults' everyday and workplace mathematics: culturally determined, socially distributed, technology-reliant and technologically embedded, contextual, local, personally invented, and basic. It bears little resemblance to the idealized view of mathematics on which traditional K-12 math instruction is based. Not surprisingly, some scholars (e.g., Dowling [5, 36] and Lave [20, 37]) raise the possibility that schooling may have little impact on adult mathematical behavior. Limitations of the research, however, prevent a clear understanding of its educational implications. For example:

- The research may reveal what *is* but does not tell us what is *possible*. Arguably, the average adult's everyday mathematical behavior offers an impoverished vision for education, especially if it were the case that schooling played a negligible role in shaping that behavior. Normative, not just descriptive, research is necessary. [31]

- Ethnographic studies provide far more detailed descriptions than topical studies, but usually the ethnographers' purpose is to demonstrate the social and distributed nature of mathematical activity. Thus, ethnographic studies generally take the culture or setting as the unit of analysis – a strategy that yields important information for educators but also masks the requirements for the individual, the unit of greatest concern to schools.
- The findings that the mathematical content of most adult tasks is low level and that formal procedures are rarely employed may be artifacts of the particular settings and practices studied. Few math-in-practice researchers have targeted occupations presumed to involve higher-level math or formal mathematical theory.³ Those that do still typically focus on the low-level mathematical behavior in these fields, perhaps due to the researchers' limited mathematical expertise or the dearth of appropriate research instruments. A different picture might emerge from investigations of higher-level math activity in more math-intensive occupations.

These limitations indicate the need for ethnographic studies that explore the use of higher-level mathematics and the role of formal mathematical theory in the workplace. To be most relevant for educators, these studies should examine the mathematical behavior of individual workers, including atypical practitioners who are especially adept users of math. In the remainder of this article, I present a conceptual framework for future studies of workplace mathematics that I believe holds the most promise of informing K-12 math education.

³ A notable exception is an existing body of research that characterizes the mathematical behavior of academic mathematicians. By definition, these “practitioners” use formal, school-type math, and in many ways they drive the design of the K-12 curriculum. I regard this exceptional and tiny group of “workers” as *makers* of math and exclude them as practitioners who *use* math to accomplish the tasks of their work.

4. A conceptual framework for future workplace-mathematics research

Complicated theoretical issues underlie any ethnographic inquiry into workplace mathematics. A study's position on these issues largely delineates its conceptual framework and significantly influences its form, findings, and usefulness. In this section, I discuss some of these issues and the way a study aiming to inform K-12 math education should address them.

4.1 *"Finding the math" in practice*

Studies of mathematics in the workplace or daily life generally take one of two approaches to "finding the math" in practice:

- a) The researcher identifies the mathematical procedures or thinking actually carried out by the practitioner to accomplish an overarching nonmathematical task. An example is de le Roche's [35] observation of the dieter who found $\frac{3}{4}$ of $\frac{2}{3}$ of a cup of cottage cheese by literally patting $\frac{2}{3}$ of a cup of cheese into a disk shape and removing a 90° wedge.
- b) The researcher theoretically reframes nonmathematical activity in terms of mathematical concepts. For example, scholars from the University of Bath [38] reconceptualized in terms of set theory the task of sorting articles into groups for packaging.

The first approach seeks actual behavior. The second seeks a conceptual structure but makes no claims about the necessity or even advantage of this mathematization for an individual performing the overarching task. In some cases, a theoretical reframing "defrosts" the actual mathematical thinking that someone – an innovator, years earlier – actually carried out, before the task became routinized. [36] For example, many rules of thumb enable today's engineer to size structural elements and avoid the lengthy calculations that earlier generations of engineers used to develop those rules. A theoretical reframing could offer a new mathematization that

might lead to improved methods of accomplishing the task. Or a theoretical reframing could add a conceptual layer that has never been, and will never be, useful to anyone carrying out the task – the case with reframing the packaging of items in terms of set theory. Another impractical reframing appears in Stevens’s study [39] of the mathematics of architects. Describing an episode in which an architect figures the required number of toilets for a particular building area, Stevens notes:

[An] important feature of this excerpt is how some scholastically familiar mathematics is embodied in a somewhat hard to recognize form. [The architect] articulates a pair of linked mathematical inequalities...that are embodied in talk but not in familiar symbolic form (e.g., if $O_B(A) \leq 15$, then $T = 1$. If $O_B(A) > 15$, then $T = 2$, where O_B is building occupancy, T is toilets per room, and A is area). [39] p. 112.

Few would argue that the ability to express symbolically the toilet-occupancy relationship is necessary or even helpful for the architect in this situation (though such a representation could possibly be useful when formally documenting calculations). Clearly, the information gained via the empirical approach to finding math holds greater relevance for educators aiming to replicate or prepare students for workplace mathematics. Studies hoping to inform K-12 education should seek to discover the actual mathematical activity of real individuals.

4.2 *Defining mathematical behavior*

To investigate mathematical behavior in practice, researchers must first determine what kinds of behavior “counts.” Throughout the literature, mathematical behavior has been construed in various ways. At one extreme, *doing mathematics* is reserved to describe the

activity of academic or research mathematicians, i.e., the creation of new mathematics.

Schoenfeld takes this stance when he characterizes the “mathematical enterprise” as:

...a community of trained practitioners (mathematical scientists) engaged in the science of patterns – systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically (“pure mathematics”) or models of systems abstracted from real world objects (“applied mathematics”). The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. [40] p. 335.

Apparently, Schoenfeld excludes from the enterprise of mathematics the *application* of math by non-academicians in solving real problems.⁴ Schoenfeld would probably contend that workplace-math researchers have not actually been observing workers *doing* mathematics (which is logically possible to find but highly unlikely), only using or applying it.

Bishop [41] admits a far broader range of people and activity into the doing of mathematics. He identifies three mathematical subcultures, only one of which – the *technical* subculture – maps onto Schoenfeld’s mathematicians. Technical mathematicians work on mathematical problems, generating specialized math concepts and techniques and thus contributing to the growth of mathematical knowledge. They also critique mathematics but do so “in the abstract, as theoretical issues, as philosophical problems, rather than as affecting practical problems and real situations.” [41] (p. 86). But people who use math also qualify as doers of math, and Bishop locates them in two other subcultures. The *formal* subculture is populated by engineers, architects, economists, and others who deliberately and explicitly value and use symbolism and mathematical conceptualization to solve nonmathematical problems.

⁴ This view of mathematics compels Vincenti [30] to lobby for recognizing engineering as a discipline in its own right, rather than merely an application of the disciplines of math and science, as is often presumed. Vincenti bases his argument on these disciplines’ distinct ends: engineering uses math to design real, specific products, while math and science aim to produce new, general knowledge.

The *informal* subculture includes the everyday quantitative problem-solving behaviors and “street math” of “just plain folks,” but it extends to the cognitively lowest-level uses of vaguely quantitative concepts, for example, employing words like always, never, and equals in ordinary conversation.

For educational research, neither extreme end of the continuum is an appropriate boundary for defining mathematical behavior. Unless we plan to study only academic mathematicians, we must count instances of math *use*, for purposes other than creating more mathematical knowledge, as mathematical behavior; otherwise, we would find virtually no mathematical behavior in adult practices. At the other extreme, analyzing every quantity-related utterance would be a waste of time; there is little mystery about whether and how people in highly mathematized fields use numbers and terms like never or equal. More importantly, studies of the lowest cognitive levels of mathematical behavior would lack any significance for schools, since such behavior is probably learned, and should be learned, almost entirely outside of school. For a study to be relevant to education, the mathematical activity counted for observation and analysis should include:

- that which falls, ostensibly, within the domain of school math programs
- and/or that whose occurrence in everyday practice is questioned in the literature
- or that which replaces school-taught methods in real-life problem solving.

4.3 *The gap*

The gap between formal (or school-type) and informal (or everyday) math has become an accepted paradigm in ethnographic studies of mathematical behavior. And students, adult workers, employers, and educators are aware of – and troubled by – a mathematical

disconnectedness that is certainly real. Nevertheless, as a frame for research, the concept of a gap is highly problematic.

One problem is its definitional circularity. The behaviors presumed to exist on either side of the gap are closely associated with – in fact, virtually defined by – the contexts in which they are enacted and the role of the enactors. Most gap-finding reports offer only a few general criteria by which to identify these two behavior types. Informal math might be briefly described as invented or context-dependent, but the indicators primarily used to identify it are that it takes place in non-school settings and is carried out by “just plain folks.” Formal or school-type math might be described as rule-based or general, but, again, researchers are more apt to avoid explicit descriptions and implicitly appeal to their readers’ stereotypical notions of what students do in math class. Tellingly, the gap-finding research never presents instances of school-type mathematics occurring outside of school (though it does find evidence of informal mathematics in school (e.g., Lave, Smith, and Butler, 1998 [20]). Rather than taking this asymmetry as proof that formal math is never used outside of school, I see it as a byproduct of the goals of this research strand: to expose the *dissimilarities* between school and real-life math and to challenge common assumptions about the utility of school mathematics.

Location-dependent definitions of formal and informal mathematics, however, conceptually limit the research that can be done to determine their natures and the conditions that enable and constrain them. Moreover, location-dependent definitions deliver a self-fulfilling prophecy. If what people naturally do outside of school is automatically labeled “out-of-school” behavior (and likewise with their behavior in school), then there will always necessarily be two non-overlapping categories. Taking these categories *a priori* makes it impossible to test whether this presumed polarity corresponds to a real behavioral dichotomy, whether the mathematical

behavior within either location might itself include significant variation or gaps, or whether behaviors across the locations might overlap.

Harris [21] presents the rare attempt to delineate criteria for the purpose of research analysis (Table 1). Still, what should we make of the fact that the second column is called School Mathematics? Must behavior be observed in a school to qualify for this column? Where should we place apparent anomalies like “explicit” or “named” mathematical processes in the workplace, such as “quantity take-offs” in structural engineering, or the “tidily presented” data found in the drawings and codebooks of the building trades?

[Insert Table 1 about here]

Also troublesome is the characterization of school-type math as “decontextualized.” Others (e.g., Ueno, 1998 [42] and Boaler and Greeno, 2000 [43]) argue that no mathematics – indeed, no human activity – is ever decontextualized; schools and classrooms are as validly construed as contexts for math-doing as are workplaces. Harris [21] contrasts a worker’s “functional” motivation for using math on the job with a student’s “intrinsic” motivation for doing math in school, but it is easy to view the student’s motivation as “functional” as well: just like the worker, the student performs culturally expected behaviors to reap the rewards or avoid the punishments of the institution. Harris’s list, while valuable for other purposes, is problematic as a set of criteria for distinguishing two discrete cognitive phenomena.

A second shortcoming of the concept of the gap is its dichotomous form. Nothing in the research precludes there being three, ten, or a hundred kinds of mathematical behavior. A framework with only two kinds of behavior implies two internally homogeneous categories: *the*

way people use math to solve problems every day versus *the* way they do in school. It seems more reasonable to presume that many kinds of mathematical behavior, displaying various degrees of formality, generality, and precision, are not only exhibited within single settings but by single practitioners in response to varying conditions. The dichotomous concept of a gap may spawn unintended interpretations that unduly constrain further research. Carraher (1991) makes this point:

[O]ut-of-school mathematics is not a clear type, that is, a sufficiently unified and coherent class to merit the referent 'it'. Similarly, while [studies] suggest that there are indeed differences between maths knowledge in and out of school, there remains the question of conceptual or empirical overlap. People tend to employ many of the same representational tools in and out of school, such as linguistic expressions, the actual words for numbers and numerical operations. [44] p. 170.

Carraher worries that efforts to debunk the conventional wisdom about school-math utility will be misread as claims that school math has no preparatory value for out-of-school problem solving:

[T]he role of schooling is not a simple one. If the 'street maths' and 'written and oral maths' studies were to be taken in isolation, it might appear that the mathematics taught in schools is irrelevant or even detrimental to solving maths problems in real life (even though this was not concluded by the authors). For the great majority of mankind, it is likely that practices, representations, and procedures used in school will have some bearing on how mathematical problems are handled outside of school. [44] p. 194.

In fact, Carraher found that school-math training had several “payoffs” for Brazilian workers’ everyday problem solving, including decreased error rates, the use of abstract qualities for grouping, the use of abstract quantities, and the generation of multiple solutions. [44]

Historically speaking, the math-gap paradigm has contributed significantly to our current understanding of how people use math and the role schools can and do play. As a dichotomous frame, however, the gap has serious limitations for characterizing what is probably a broad range of behaviors. Rather than automatically incorporating this dichotomy into their conceptual framework, researchers should allow new descriptive categories to grow from their observations and analyses. Certainly if educators are to benefit, further research must move beyond the defeating notion that workplace and school math are inherently and irrevocably separate forms of behavior.

4.4 Technology, the environment, and the unit of analysis

The ubiquity of technological tools in our lives and workplaces and their controversial role in education oblige researchers of math in practice to address the relationships between technology and mathematical behavior. Several ethnographers (e.g., Hall, Stevens, Hutchins, Lave [24, 25, 28, 37, 39]) have demonstrated the heavy reliance of everyday mathematical behavior on tools and technologies ranging from pencil and paper to sophisticated software. Given this “massive environmental support” (Perkins) [45], some scholars argue that knowledge should not be considered the property of an individual, located completely inside the head, but that the boundary of cognition should be expanded to include the various external resources that aid thinking. Implicit in this view is notion that the unit of analysis for cognitive research should be the “person-plus” (Perkins) [45], which comprises the individual, her environmental support, and the ways she uses it, rather than the isolated mind of the “person-solo.”⁵

⁵ Another implication of this view is that schools should abandon their “persistent campaign to make a person-plus a person-solo” (Perkins) [45], acknowledge the resources for thinking that are available outside of school (Pea) [31], and teach students how to take advantage of them – what Perkins calls the “art of distributing cognition” [45].

Thus, how to study the role of technology is part of a larger debate about the boundaries of human cognition and the usefulness, even the validity, of the idea of individually held knowledge that is free of social interaction and other environmental resources. Pea [31] epitomizes one side of the debate, claiming that a “clean, pure, solo intelligence of the independent and capable thinker...is a theoretician’s fantasy” (p. 80). Others advocate a more balanced approach. Salomon [46] notes that cognition is not always distributed, and some of it, like the meta-cognitive “executive functions” used to organize and evaluate problem solving, may not be distributable at all. While acknowledging the importance of the context of cognitive activity, Nickerson [47] argues, “it does not follow that what individuals bring, in their heads, to the problems on which they work is of no consequence, or that there are no questions worth asking about the cognitive capabilities that an individual possesses” (p. 259).

Schools prepare individuals, not groups (evinced by the virtually exclusive use of individual assessment).⁶ Therefore, studies that aim to have educational implications must presume that an individual’s cognitive behavior can be meaningfully isolated for investigation and analysis. Even so, as scholars on both sides of the debate agree, an individual’s behavior should be interpreted within its culture and context, and her interaction with environmental resources must be a central consideration.

Interestingly, regardless of where they draw the cognitive boundary, most ethnographers describe the individual’s relationship with technology and the rest of the environment in positive and active terms, portraying a problem solver who cedes labor to, controls, and manipulates environmental resources in support of her task. I find little discussion of a downside, but it

⁶ Schools do and should try to teach the skills necessary for team membership, collaboration, and social interaction. Nevertheless, this effort represents the preparation of individual students. It is quite different from training a specific, permanent cohort, such as a ship’s crew, sports team, or platoon, for group performance, where the growth or expertise of any single member is irrelevant.

stands to reason that environmental factors also constrain and hinder our problem solving and cognition, often in ways we cannot control. A dieter in her own kitchen is free to spread cottage cheese on the table, but if an engineering firm has purchased AutoCAD⁷ and AutoCAD drawings are the firm's normal form of representation, its employees may not select a different tool for a particular problem, even if it would serve better. Further, when discussing the ways people control, exploit, or create environmental resources for solving, researchers rarely specify whether "people" refers to the individual or society. Yet this distinction is critical. An individual who works with tools she can freely select, modify, use in innovative ways, personalize, or invent has a vastly different relationship with those technologies from the person whose tools are selected for her, can only be altered by the publisher, were designed to accommodate thousands of workers, not just her or her particular problem, and whose methods of use may be determined by her employer or profession. Studies of cognition in practice should explicate the degree of environmental control individuals actually have, the societal level at which relevant technological decisions and modifications are made, and how the environment constrains problem solving as well as supports it. A better understanding of workers' agency over their environment would help schools clarify their missions with respect to teaching with and about technology tools; similarly, it could illuminate appropriate classroom roles for other environmental supports, for example, interactions with other students.

4.5 The role of abstract concepts and theory

The math gap, to the extent that it exists, calls into question the role of generalized, abstract rules and concepts in the workplace. Abstraction embodies the power of mathematics, and teaching abstract, general rules and concepts has long been the assumed mission of school

⁷ AutoCAD is a CAD (computer-aided design) drawing software product of Autodesk, Inc.

math. But much of the research reviewed in this article indicates that these school-taught, abstract rules, powerful though they may be, are the remotest form of mathematics from everyday problem solving. A related paradox plays out in university-level pre-professional education, where the general, abstract principles of a discipline, referred to as the “theory,” are juxtaposed to the knowledge gained in “practice.” The emphasis that pre-professional programs have traditionally placed on theory reflects an institutional conviction that a deep understanding of the fundamental concepts underlying practical methods is crucial to effectiveness in the field. Yet university educators, recognizing their students’ inability to apply theory to real-world problems, have increasingly integrated practical experiences into their courses. A 1989 MIT study of engineering education [48] concluded:

The postwar evolution of the engineering curriculum in the direction of engineering science was both inevitable and desirable; theory and practice are each essential components of modern engineering. But by now the pendulum has probably swung too far from real-world problem solving, especially as it relates to industrial production...engineering students are taught to analyze systems but not really to design them.

This report recommended exposing students to problems “that go beyond the idealized abstractions that have dominated text and homework since the 1950’s.” [48] Many programs go farther than that: a current trend in engineering education is to incorporate design projects in every year of the undergraduate program. More recently, education reformers have raised the bar even higher by calling for mathematical modeling in K-12 and university classrooms. Not only should students be able to merely apply general mathematical rules and concepts; reformers now expect them to mathematize real situations and create mathematical models. [2, 11]

Given society’s enduring (if not innocent) faith in the power of mathematical theory, it is no surprise that school math is general even though everyday math is situation specific. Many

educators believe a mastery of abstract concepts is precisely the kind of mental preparation that most effectively facilitates the widespread application of math to specific problems. The question, then, is not whether a mismatch exists between the natures of school and everyday math but whether people can and do import general, school-taught techniques and theory to non-school contexts to solve problems there, or whether people's everyday problem-solving strategies have other origins (e.g., are generated or learned on the job).

Some scholars are extremely skeptical about people's use of general theories and concepts. Dowling writes:

We might expect school mathematics – at least in its rhetorical form – to emphasize “understanding” rather than rote learning; [but] the latter would be entirely effective and appropriate in the workplace; it is, after all, unnecessary for the machine operator to grasp the engineering principles of the machine that he or she operates; this applies to a surgeon using laser technology and to a pilot using information technology as much as to a factory worker employing less exotic hardware.” [36] p. 100.

Similarly, Lave, Smith, and Butler [20] claim, “[M]athematical practice in work settings is primarily a matter of interaction with instruments and procedures in which relations of quantity and their transformations are stored...[T]here is no open field for the employment of mathematical problem-solving skills” (p. 76). In this view, people avoid mathematical problem solving in favor of the less cognitively taxing *selection* from a limited repertoire of already-known routines.

A less pessimistic view holds that people *construct* mathematical strategies to solve problems. According to Carraher and Schliemann (2002) [49]: “[S]chools encourage memorization and repetitive practice, whereas the street market setting encouraged sellers to

solve problems through mental computation, using flexible strategies that they developed and efficiently applied...” (p. 135). Despite that contrast, Carraher and Schliemann believe that one source for these flexible strategies is school math, adapted or *recontextualized* for “street” use:

Much of the work in developing flexible mathematical knowledge depends on our ability to recontextualize problems – to see them from diverse and fresh points of view and to draw upon our former experience, including formal mathematical learning. Mathematization is not to be opposed to contextualization, since it always involves thinking in contexts....(It is ironic that the mechanical following of algorithms characterizes the approaches of both highly successful and highly unsuccessful mathematical thinkers.)

[49] p. 147.

Lave, Carraher, Schliemann, and most other workplace researchers focused on arithmetic skills in everyday activities. Generalized methods, abstract theory, and mathematical modeling might be expected to play a more significant role in highly mathematized fields, but the literature here is thin. Two exceptional reports offer somewhat contrasting pictures. The Mathematical Association of America (MAA) [50] convened engineering educators to articulate the necessary mathematical preparation for various fields. Independently, all four working groups – mechanical, civil, electrical, and chemical engineering – emphasized constructing mathematical models as a central requirement of their field. The Society for Industrial and Applied Mathematics (SIAM) [51] surveyed managers of masters- and PhD-level mathematicians in industry jobs. The managers reported that they valued their mathematician-employees primarily for their ability to create mathematical models, a skill that distinguished these employees from those trained in science or engineering. This implies that mathematical modeling is an important strategy for high-tech companies but that it requires high-level mathematical training and is generally not accomplished by engineers or other non-mathematicians. Companies without

mathematicians on staff – for instance, the typical engineering firm – may lack the capacity to generate mathematical models for solving problems. If true, this would challenge the claim of the engineering faculty in the MAA proceedings. (It also suggests that the math education community might underestimate the difficulty of teaching modeling to K-12 students.)⁸

This discussion leaves us with at least three possible (and non-exclusive) pictures of the role of mathematical theory and models in the workplace:

- 1) Workers actively create unique mathematical models for solving real problems.
- 2) Workers actively select or adapt established, formal mathematical theory, tools, and models and apply them to solving real problems.
- 3) Workers use situation-specific procedures or routines (culturally determined or personally preferred) that allow them to avoid mathematical theory and difficult decisions about models.

Understanding which of these descriptions best represent work in mathematized occupations would hold significant value for education, to justify (or not) the call for modeling in the math classroom and to help educators teach through authentic uses of theoretical tools.

One caveat should be noted. Observations of actual workplace behavior do not automatically imply the human potential for applying formal mathematics, using theory, and creating models. This caveat probably explains the perseverance of teachers. Even if research were someday to supply incontrovertible evidence that most people use clumsy, situation-specific methods, with little understanding of the underlying mathematical concepts, and often become so overwhelmed by the calculations that they simplify or abandon the problem,

⁸ A further implication of the SIAM report is that companies lacking in-house mathematicians may appear to function adequately, but their productivity may actually fall short of what they could achieve had they the capacity to generate their own mathematical models. This would be a serious situation, as it would apply to most companies. A mitigating factor, however, is that SIAM only studied firms that employed mathematicians. These might constitute an unusual category of workplaces with special needs for mathematical models, explaining their hiring of trained mathematicians in the first place.

educators would still hold out hope that things could be otherwise. Rather than restrict our samples to the “typical” worker, some research should identify and examine practitioners who expertly use mathematically sophisticated tools and models to solve everyday problems, in order for us to understand the potential for human mathematical behavior.

5. Summary

The federal Mathematics and Science Initiative and other education reform programs demand improved and expanded math education as an engine for economic productivity and a means of enhancing individuals’ occupational prospects. Bringing the world of work into the math classroom is promoted not only as a motivational strategy but also a pedagogical one. Unfortunately, our current knowledge about the actual mathematical requirements of today’s workplace is far from complete. Policy makers lack sufficient information to validate their newly mandated content requirements on the basis of their potential to prepare students for actual occupations. Educators are uncertain what kind of classroom activities and curricula would authentically illustrate and replicate adult mathematical practices. And we have hardly begun to investigate how (and whether) learning math in school contributes to adult problem-solving proficiency. Further investigations of mathematical behavior in the workplace are critical. To be most beneficial to educators, new research should aim to:

- reveal the actual mathematical and problem-solving activities of workers in their real, everyday practices and settings
- emphasize individual behavior and cognition rather than distributed activity
- illuminate the relationships between worker behavior and its contexts (e.g., cultural, situational, technological, interpersonal)

- focus on a level of mathematical activity relevant to school math programs (or on problem-solving behavior that workers substitute for such mathematical activity)
- examine fields of practice presumed to be highly mathematical and seek instances of modeling activity and the use of theoretical tools
- avoid *a priori*, dichotomous, or location-dependent behavior-categorization systems
- investigate workers who are particularly adept at the use of mathematics in everyday work.⁹

⁹ I used this theoretical framework as the basis for my own study of the mathematical behavior of structural engineers. [52]

Table 1. Informal vs. School Math (Harris, 1991, p. 129) [21]

Informal Mathematics	School Mathematics
Embedded in task	Decontextualized
Motivation is functional	Motivation is intrinsic
Objects of activity are concrete	Objects of activity are abstract
Processes are not explicit	Processes are named and are the objects of study
Data is ill-defined and 'noisy'	Data is well defined and presented tidily
Tasks are particularistic	Tasks are aimed at generalization
Accuracy is defined by situation	Accuracy is assumed or given
Numbers are messy	Numbers are arranged to work out well
Work is collaborative, social	Work is individualistic
Correctness is negotiable	Answers are right or wrong
Language is imprecise and responsive to setting	Language is precise and carefully differentiated

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