The Axiomatic Matrix of Whitehead's *Process and Reality**

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I

Commentators on Whitehead's philosophy often mention his mathematical background as a foundation for his metaphysics. Rarely, however, do they explain just how the rigorous and technical expertise of his early work finds its relevant applications in his later cosmological framework. Nor do they explain his strenuous objections to philosophers following the pattern of mathematical method if indeed he did so himself.

Though the particular structure and procedure of *Process and Reality* appear to be quite a divergence from most philosophic exposition, Whitehead here achieves a system which requires at least a moderate understanding of the mathematician's construction. It is just this structural similarity between Whitehead's celebrated work with Bertrand Russell, *Principia Mathematica*, and his mature metaphysics developed in *Process and Reality* that I wish to investigate in this paper.

A cursory glance at *Principia Mathematica* will often lead to indigestion and perhaps discourage further interest. Indeed, the massive amalgam of symbolic logic is closer to electronic circuits of a computer than the prose of general philosophical problems. However, what is essential and more influential than the thesis itself is the method in which the authors went about their conviction. The idea that mathematics is entirely an extension of formal logic is due to Russell's earlier work *The Principles of Mathematics*. This Whitehead clearly acknowledges. But the fact that both Whitehead and Russell coalesced to produce a collaborated three-volume work extending over ten years does give some indication of the importance of the problem in Whitehead's own thought. From his earlier work *Universal Algebra* throughout the *Principia* collaboration we find that Whitehead's dominant interest was the very broad sense of mathematics as the study of pattern or relations in general; a definition easily applied to his metaphysical speculations where the patterns of relatedness express the character of anything and everything real.

The impact of *Principia Mathematica* on Whitehead's later thought centers on the issue that its method applied to a very general problem of deducing the whole of mathematics from a handful of elementary formal notions and axioms. The procedure of working from such a matrix of primitive ideas and primitive propositions

proved fundamental for a metaphysician desiring a broad basis for understanding the universe. In this regard, we may observe that the Categorical Scheme of *Process and Reality* is an axiomatic matrix of tentative generalization derived from our experience. That is, Whitehead modeled his metaphysical system on the axiomatic treatment of logical systems.

Some difficulty here arises as regards the manner in which Whitehead's system may be described as 'axiomatic'. *Principia*, like Euclid's *Elements*, develops from axioms as self-evident starting points. But clearly Whitehead does not take his metaphysical first principles as self-evident axioms from which we deduce experience. This would be a fatal error for the metaphysician. The sense in which he recognizes the first principles as tentative is that the perfection of them was the goal and not the origin of his metaphysics. This is the key to understanding Whitehead's criticism of philosophy as misled by the example of mathematics. Philosophy, in the Cartesian sense, which began with what was considered clear and indubitable premises to arrive at certainty is what Whitehead calls the "false estimate of logical procedure" (PR 8/ 11). Philosophy is not deduction from absolutely certain premises; it is rather the search for the premises.¹

The main point for Whitehead then concerns the alleged self-evidence of the axioms. How do we construe the status of the axioms? Are they absolutely certain, or are they subject to revision? How does experience play into the final outcome?

A metaphysics, Whitehead wrote in *Religion in the Making*, is a description: the metaphysician discerns in some special field of interest what he suspects to be the general character of reality; he then sets up categories from this investigation and seeks to discover whether they are exemplified in other areas of human interest (RM 76). We arrive at the categories through the primary stages of 'assemblage' which attempt to cover the infinitude of the universe by metaphysical notions of the widest extension. This provides the matrix, as a body of first principles, judged as coherent and logical depending on the manner in which each proposition requires the others in systematic interconnection.² However, as a whole, the theses of the system must be confronted with the facts of experience. The final evaluation is a rational interpretation of the metaphysics as applicable and adequate depending on its comprehensive capacity in elucidating experience—an approximation, no doubt, to the hypothetico-deductive method of scientific enquiry. What is quite clear to Whitehead is that "philosophers can never hope finally to formulate the metaphysical first principles" (PR 4/ 6) as the whole of experience forever eludes the grasp of finite judgment. However, the categorical scheme must be sought regardless of the emphasis placed on its hypothetical character. The metaphysician must progressively modify the working in his approximation to the ideal scheme.

Process and Reality and *Principia Mathematica* are both constructive schemes which develop from a matrix of undefined primitive notions. The goal of *Process and Reality* is to provide metaphysi-

cal generalizations by which we can interpret the whole of experience. The goal of *Principia Mathematica* is to provide a formal system which attempts to encapsulate the whole of pure mathematics in a complete axiomatic system—an enterprise which, according to the theory of Gödel, was doomed to failure. Gödel's incompleteness theorem of 1931 succeeded in proving that it is impossible, in principle, to set up a system of mathematics by logical propositions which does not imply some propositions which the system is unable either to prove or disprove. The result had quite an impact on what was thought to be the last frontier in certainty—mathematical logic. Though *Process and Reality* was written three years before Gödel's published results, the general tone of Whitehead's remarks regarding the thesis of *Principia* seems to anticipate trouble. He writes: "... even in mathematics the statement of the ultimate logical principles is beset with difficulties, as yet insuperable" (PR 8/12). The point of comparison in his footnote is the second edition of *Principia Mathematica* where Russell considers suggested improvements to the system. Here Whitehead implies that *Principia* has committed the same fallacy of overestimating the potential of logic and mathematics in the Cartesian sense of certainty. And later in *Modes of Thought* he acknowledges the incompleteness of logical systems by saying: "even Logic itself is struggling with the discovery embodied in a formal proof, that every finite set of premises must indicate notions which are excluded from its direct purview" (MT 26). But what he never doubted was the axiomatic method as a procedure for constructing systems, provided that we realize that their premises are always incomplete and subject to overthrow with the decline of the epoch in question. For this reason Whitehead insists that we keep our systems open with due attention to their limitations.

Whitehead himself, in a memoir published in *Mind*, "Indication, Classes, Number, Validation," criticizes various technical details of *Principia Mathematica*, especially the definition of number contained therein. The attempt in *Principia* to exhibit mathematical truths as logical truths fails because it requires an essential reference to the metaphysical notion of types which is founded on a level of individuals derived from the physical world. A cardinal number is defined in *Principia* as a complete class of equinumerous classes (of objects); thus every change in the number of physical objects will alter the meaning of each number. Mathematics, by this definition, is bound up with intension and history whereby "... a new litter of pigs alters the meaning of every number, and of every extension of number, employed in mathematics" (ESP 232). Whitehead's modification in the memoir is to redefine number solely in terms of extension by purely logical terms.

From the ten year course in which *Principia* was produced, Whitehead retained not only the emphasis on its procedure but also its general aim of producing a universal system of thought. This is admirably disclosed in a paper by David Harrah who investigates the logical and mathematical analogues in Whitehead's metaphysical

conclusions (ILM 420). The mathematical procedure, as distinct from the content of *Principia Mathematica*, provides an interesting insight into the workings of Whitehead's thought as creative synthesis. Compare, for example, the procedure in the formal system of building premises into conclusions which again become premises as forms of implication used in the developing theme of *Principia*. Such a method can easily be seen in the metaphysical notion of prehension in which occasions of experience become by inheriting the forms of antecedent occasions, which in turn become the forms or data for future prehensions. This analogy is helpful to the extent that it articulates the general notion of passage from many data to a novel one. But even here we must not push such parallels too far due to the existence of the infinite number of isometric forms generated in logic where the novelty is in some sense only apparent.

At most the metaphysics tacitly understood in *Principia* is that: "the universe consists of objects having various qualities and standing in various relations" (PM 43). It is not, however, committed to the actual existence of classes and relations. From this basis it is interesting to note the divergence in two radical types of pluralism developed by Whitehead and by Russell after their collaboration. One became the philosophy of organism admitting both internal and external relations as a consequence of the dominance of procedure, while the other became a logical atomism of purely external relations as a consequence of the content of *Principia Mathematica* (cf. ILM 420 and UW 152f.).

A further insight into the relations of Whitehead's mathematical procedure and cosmological construction is detailed in his memoir of 1906 "On Mathematical Concepts of the Material World." In this essay, written in the middle of the *Principia* collaboration, Whitehead states his thesis as a "mathematical investigation of various possible ways of conceiving the nature of the material world" (MCMW 11). He considers five different concepts from the standard world view of classical physics to a view which closely resembles the cosmology put forth in *Process and Reality*. Each concept is considered in light of a set of definitions which apply to an 'essential relation' which defines the concept dealt with, and a set of primitive entities chosen for that view. The definition of material world, for instance, is "a set of relations and of entities which occur as forming the field of these relations" (MCMW 13)—a most curious definition when we consider Whitehead's later view that the world (ourselves included) is understandable as a coherent logical system of polyadic relations of actual occasions. Whitehead then adds to each concept a set of axioms which state relationships between the 'essential relation' and the primitive entities from which he thereby deduces theorems. In the later cosmology, such a method is applied to experience in general where relatedness is taken as primary, and the extensive properties of nature are dependent upon the function of the universe as a creative process, albeit such a method loses its deductive character when applied to the observed connectives of experience.

II

Although Whitehead in *Process and Reality* develops his cosmology in quite a different language from the symbolism which unfolds from *Principia Mathematica* and "On Mathematical Concepts of the Material World," we can discern in the cosmology the basic logical structure of the axiomatic matrix used throughout his work. This involves: (i) a given operation or function, (ii) a set of entities, (iii) explanation of the primitive ideas, and (iv) axioms as rules governing the relations of the entities. Having set the matrix at the beginning of *Process and Reality*, Whitehead then attempts to elucidate the scheme by applying it to special topics in the succeeding chapters. It also becomes quite evident how the categories require one another as a coherent system. But again it must be emphasized that, in the cosmology, Whitehead is not producing theorems from the definitions and axioms in the categoreal scheme. Rather the matrix and the special topics evolve concurrently, despite the fact that the categoreal scheme gains an air of completeness by its having been placed at the beginning of the system. There is, in fact, a strong indication that the complete categoreal scheme is the last stage of Whitehead's final product (cf. WPO 37f. and EWN 192 and ch. 9, sec. J).

The scheme is thus set out with four major categories of which three are more special of the one ultimate. For Whitehead, Creativity is the ultimate presupposition or given operation which describes the universe as a harmonizing of data into a novel unity. This is the principle of the highest generality governing the advance of everything that becomes. It is the pulse of existence which conjoins the disjunctive diversity of the antecedent world. Analogously, in *Principia Mathematica*, inference and substitution are the given operations used in the deduction of pure mathematics.³ Inference is the dropping of a true premise, the dissolution of an implication (PM 9). This is simply *modus ponens* (i.e., the first part of a theorem or axiom is dropped when we have 'P and P $\supset$ Q, implying Q'). And substitution is the exchange of appropriate expressions for variables. For example, having accepted P $\supset$ Q, Q can be substituted for P, obtaining as a theorem the formula Q $\supset$ Q. This is also expanded with new rules for quantification which come later in Part I of *Principia*. These operations are metalogical since they are understood as rules of deductive procedure which govern the valid moves in asserting propositions as resulting theorems. Just as Creativity functions to produce the novel one from the disjunctive many, inference allows the dropping of a set of premises which conjointly imply some proposition asserted as the new theorem (ILM 424).

The categories of existence in *Process and Reality*, eight in number, are the classifications of the primary entities which function in accordance with the category of the ultimate. From these, the actual occasions and eternal objects are singled out as the most fundamental ontological units by which the others (i.e., prehension, subjective

form, etc.) have an intermediate character (PR 22/ 33). Whitehead thinks that the three most concrete elements in experience are actual entities, prehensions, and nexūs. All else, in accordance with the ontological principle, are derivative abstractions. On the other hand, the theory of existence in *Principia* is modeled on the method of algebra. That is, the domain of individuals is left completely unspecified. However, the authors always refer, in formal statements, to one domain over which all apparent (bound) individual variables are to range. Thus ' ϕx ' is any propositional function with individuals for its arguments, ' $(\exists x)\phi x$ ' means 'there exists some individual in the universal domain that has the property ϕ '.⁴ At the lowest level in the theory of types there are individuals— $a, b, c, d, \dots, n$, but the precise identification of the individuals is left open since the system of logic was only to be used as a foundation for pure mathematics. In principle, the unspecified entities could be a number of things: sense data, electrons, stars, etc. They need not be explicitly defined since, for the purpose of logic, all that was needed was the symbolic representation for the calculus. Fundamental to the theory of existence and the theory of types is propositions as combinations of individuals or atomic facts. These provide a basis for the propositional and quantification logic. However, it must be noted that the individuals form the foundation for the hierarchy of classes of classes of individuals kept separate by the theory of types. Mathematics then, being defined in terms of the infinity of types, is dependent upon the lowest level of individuals interpreted as unspecified entities of the physical world.

Next are the categories of explanation in the cosmology; here Whitehead explains how the categories of existence interact in the fluid and dynamic nature of process. Generally, the twenty-seven categories of explanation describe the creative activity of actual occasions in terms of the functions of prehension and concrescence. Category xii, for instance, explains the two species of prehension as positive, absorbing antecedent data, and negative, eliminating incompatible data with the present actual occasion. Throughout these categories we find the descriptions of the various types of entity becoming more definite as the categories build to more complex explanations which involve the previous ones. In *Principia*, what are termed the 'primitive ideas' correspond roughly to these categories of explanation in *Process and Reality* except that they are very few by comparison. They serve the purpose of exposing the undefined notions of the system. Here Whitehead and Russell emphasize "the primitive ideas are by means of descriptions intended to point out to the reader what is meant; but the explanations do not constitute definitions, because they really involve the ideas they explain" (PM 91). Aside from the symbolization of elementary propositions— p, q, r —the fundamental notions explained are: assertion $\vdash p$, negation $\sim p$, and disjunction $p \vee q$, by which implication, conjunction and equivalence are later defined—

*1.01. $p \supset q = \sim p \vee q$ Df. (PM 94),

*3.01. $p \cdot q = \sim(\sim p \vee \sim q)$ Df. (PM 109),

*4.01. $p \equiv q = .p \supset q \cdot q \supset p$ Df. (PM 115).

We now come to comparative analysis of the axioms in the two systems as concise statements governing the relations between the various entities. In *Process and Reality* the categoreal obligations perform the function of axioms expressing the necessary conditions for which actual occasions must obtain in order to achieve a fully determinate end as a unit of feeling. What we find in these remaining nine categories is a set of refined laws which state how actual occasions are obligated to become in accordance with the other three categories of the matrix, i.e., Creativity, Existence, and Explanation. These nine categories involve the concrescent process inside the actual occasion explaining the interplay between the mental and physical poles. Categoreal Obligations iv-viii, for example, explain how subjective aim could come into existence. One point, parenthetically, confirms Whitehead's remarks about the perfection of the final generalities as his goal, and not the origin, of his metaphysics; that is, axiom v, the category of conceptual reversion, was later abolished (PR 249f./ 381f.). This category became unnecessary once Whitehead discovered how novelty could be derived from God (cf. PS 8:147 and EWM 212, 222). Throughout the various stages in the composition of *Process and Reality*, new axioms were admitted in the evolving thesis replacing those of the original matrix. Such moves often appear as ad hoc procedure in Whitehead's metaphysics, but the real issue at stake is the coherence and adequacy of the entire system. Likewise, in *Principia Mathematica*, the axiom of reducibility was introduced to relax the excessive restrictions of the ramified theory of types, thus enabling theorems for all functions of numbers. Systems thinking, in general, must allow for such internal development until the whole can be disclosed—a concession to an Idealist logic of a Bradlean sort. The laws adopted as formal axioms in *Principia* are called 'primitive propositions.' The sense in which they are 'primitive' is that they must be assumed without proof since all subsequent inference develops from what has been previously asserted. What we find here is the beginning of the forms of implication as self-evident axioms used by Whitehead and Russell to deduce hundreds of proved theorems by displaying the logical connections between the primitive axioms and the theorems. There are five primitive propositions adopted as formal axioms—

*1.2. $\vdash: p \vee p. \supset p$ Tautology,

*1.3. $\vdash: q. \supset p \vee q$ Addition,

*1.4. $\vdash: p \vee q. \supset q \vee p$ Permutation,

*1.5. $\vdash: p \vee (q \vee r). \supset q \vee (p \vee r)$ Associative Principle,

*1.6. $\vdash: q \supset r. \supset: p \vee q. \supset p \vee r$ Principle of Summation.
(PM 96f.)

As an interrelated structure, the matrix of *Principia* is, of course, more precise in its development of immediate consequences. This is the great merit of the calculus and the formal language developed therein. But what it gains in logical clarity it loses in philosophical depth with regard to certainty. Whitehead's metaphysical system, on the other hand, utilizes the axiomatic structure to form a coherent matrix of categories, yet, like Plato's *Timaeus*, offers an explanation of the limitations inherent in any cosmological endeavor. The first principles (inclusive of the axioms) are treated as hypotheses judged as applicable to the whole of experience beyond their nascent conceptions. What Whitehead seems always to have held is that the relatedness within nature provides a logical order for our interpretation, and that this gives rise to our ability to form a cosmos from a chaos. Our interpretation, however, is an expression of value in defining such an order and will always depend on our purpose in mind. This is admirably expressed in the conclusion of the first chapter of *Science and the Modern World*, when Whitehead says:

... this system includes the harmony of logical rationality, and the harmony of aesthetic achievement: to know that, while the harmony of logic lies upon the universe as an iron necessity, the aesthetic harmony stands before it as a living ideal moulding the general flux in its broken progress toward finer, subtler issues. (SMW 24)

His solution to this necessity involves a set of entities derived from our experience, which form a field of relations by the complex function of Creativity, operating in accordance with the explanations and obligations of the system.

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NOTES

* I have profited greatly from critical comments on the original draft of this paper by Lewis Ford, Forrest Wood, Jr., and John R. Baker.

¹ Some logicians may here object that the very meaning of the term 'axiomatic' is unforgivingly deductive and that Whitehead's metaphysical system can hardly satisfy this requirement. But to this the sufficient reply is that although it is clear that Whitehead is not proceeding deductively, his setting out a definite statement of his first principles at the outset of his system indicates that he held the axiomatic method to be an ideal form in which one should strive to organize thought into a system.

² "Systematic interconnection" or "coherence" here does not mean that the fundamental proposition or axioms are definable in terms of one another. Indeed the coherence of the axioms seems to require a certain independence of the axioms as well. As Whitehead puts the point, coherence "means that what is indefinable in one such notion cannot be abstracted from its relevance to the other notions" (PR 3/ 5).

³ The logical calculus is formulated first in terms of propositions and propositional functions and is soon expanded into a formal theory of classes and relations until the topics gradually become more specific to the point of a purely logical theory of cardinal and ordinal numbers.

⁴ Curiously enough Quine complains that "there is an inconspicuous detail that embodies the germ of a Platonic ontology of universals" in *Principia*. Φ and Ψ are allowed to occur in quantifiers, which allows a theory of attributes in the quantification logic. It seems that some form of the doctrine of eternal objects was recognized as part of the theory of existence in *Principia* (WRL 144f.).

Whiteheadian Philosophy and Prolog Computer Programming

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Whitehead came to his mature philosophical position in *Process and Reality* after many years of wrestling with problems in the foundations of logic and mathematics. His doctrine of eternal objects in both his earlier and later philosophy can be understood as a description of the ontological nature of pure logic and mathematics (EWP 14-28). Eternal objects as patterned after mathematical structures found in *Science and the Modern World* are understood primarily in a logical guise coming from *Principia Mathematica*. In *Process and Reality* Whitehead developed a philosophy of actual entities that are comprised of and established by prehensions. In this same work eternal objects as potential structures for actuality do not have an evident form as structures for prehension but maintain their original logical and mathematical interpretation. This does not exclude, however, the possible existence of a logical or mathematical structure for prehensions. Indeed we would expect that Whitehead, had he been able to continue his logical and mathematical development and especially had he known of contemporary computer developments, would have expanded the boundaries of the logical structure of eternal objects to include a direct description of the structure of prehension. At present, however, the contemporary understanding of Whitehead's philosophy of organism involves eternal objects resting on the intuitions of logic and mathematics that contrast with concreting actual entities united through prehension. The abstract logical structure of prehensions is not represented naturally by the familiar form of Whiteheadian eternal objects. Does there exist a form for representing eternal objects that can also show in a clear manner the prehensive structure of actual entities?

An Imaginative Leap Via Microcomputers

We believe that we have found a set of structures that allows descriptions of prehensions as well as providing a symbolic understanding of Whiteheadian eternal objects and propositions. Surprisingly these structures are to be found in the form of a modern computer programming language—Prolog. We have been struck by the similarity of the development of logic programming into Prolog with that of Whitehead's historical philosophical development. Whitehead as mathematician and logician desired to make logic applicable to actual experience in the real world. The fruits of this desire are found