Schwinger-boson studies of the t - J model: A self-consistent systematic expansion

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A self-consistent systematic expansion is developed for the t - J model in the Schwinger-boson approach. We treat the fermion part with use of a systematic loop expansion technique and study the effects of magnetic fluctuations on the normal-state properties. It is found that the hole motion is strongly renormalized and incoherent bands are produced by magnetic fluctuations. The conductivity and the electron density of states are calculated and shown to be anomalous. The antiferromagnetic nature of fluctuations is found to be essential for these unusual properties. The relevance of the obtained results to oxide superconductors is also discussed.

I. INTRODUCTION

Following Anderson's suggestion,¹ the strongly coupling limit of the Hubbard model and the related t - J model have been extensively studied in order to understand the unusual properties of oxide superconductors. The main issue here is to impose the single-occupancy constraint on each lattice site. In numerical simulations the doubly occupied states can be projected out, but it is much more difficult to implement in analytical calculations. One possible way is to introduce "slave" particles for electrons $c_{i\sigma} = a_i^\dagger b_{i\sigma}$, where $a_i^\dagger$ is the "slave" boson and $b_{i\sigma}$ is the fermion, or vice versa. In this way the nonholonomic constraint

$$\sum_{\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} \leq 1$$

is converted into a holonomic one

$$a_i^\dagger a_i + \sum_{\sigma} b_{i,\sigma}^\dagger b_{i,\sigma} = 1.$$

The slave-boson approach (when a_i is a boson) has been widely studied. Within this approach the ground state of a doped system turned out to be the uniform resonating-valence-bond (RVB) state suggested by Baskaran, Zou, and Anderson³ which shows Fermi-liquid-like behavior below the Bose condensation temperature. A serious difficulty of this approach is the absence of antiferromagnetic (AF) correlations. In fact, the ground state in the half-filled case does not satisfy the Marshall sign rule.⁴

On the other hand, the experiments do show the strong interplay between magnetism and superconductivity.⁵

The AF correlations survive even in the superconducting (SC) region. Theoretically, the simplest problem for this study is to consider a single hole on an AF background. Numerically, it has been studied by exact diagonalization of small clusters⁶ and truncated Hilbert space method.⁷ Analytically, variational,⁸ Green's function,^{9,10} path integral,¹¹ etc., techniques have been used to study the renormalization of hole motion due to AF fluctuations. In particular we¹⁰ have included both the spin background distortion and the dynamical spin excitations in the renormalization of hole motion. In spite of great efforts by many authors, this problem has not been solved entirely yet. The key issue here is whether the hole can still propagate in the presence of AF fluctuations, if the single occupancy constraint is properly imposed. Another nontrivial issue is how to generalize this consideration to finite doping concentrations.

It seems to us that the Schwinger-boson-slave-fermion approach, a counterpart of the slave-boson formalism, is a good candidate for small doping. Recently, this technique has been used by several authors to consider the half-filled and the doped cases.¹²⁻¹⁸ First of all, at half-filling the AF long-range order is recovered if the Bose condensation is taken into account. The ground-state energy obtained is much better than the slave-boson case. This state satisfies the Marshall sign rule.⁴ Secondly, the spin-spin correlation functions fit rather well the experimental data.^{12,14} Moreover, the density of states is rather symmetric around the Fermi surface,^{14,17,18} and the possibility of explaining the non-Fermi-liquid behavior seems to be available.^{14,17} All of these features are due to an account of the AF fluctuations at the mean-field level.

However, the phase diagram obtained in the slave-boson² and in the slave-fermion^{13,15} approaches looks quite different. According to the former approach the ground state at moderate dopings should be a uniform RVB state, while it should be a spiral AF or even ferromagnetic state following the slave-fermion approach. This shows clearly the intrinsic weakness of the mean-field approximation and the necessity to go beyond it.

Recently, there have been two attempts in this direction using the slave-boson approach. One is the gauge field theory, originally proposed by Baskaran and Anderson,¹⁹ and later developed by many others.^{20–22} In this theory, the bosons and fermions are coupled by a fictitious gauge field which to some extent materializes the reduction of the Hilbert space due to constraints. Although the constraint is still imposed only on average, this approach turned out to be quite successful in explaining some normal state properties, especially the linear temperature dependence of the resistivity in oxide superconductors. However, the AF fluctuations are not included in this approach either. An alternative approach is the large- N (N is the number of fermion components) expansion in the slave-boson formalism.²³ Although the ground state remains Fermi-liquid-like at the one-loop level, the corrections due to correlations are significant to account for some interesting properties, e.g., the change of sign for the Hall constant during doping, etc. On the other hand, the AF fluctuations are also not included explicitly in this scheme.

In this paper we study the Schwinger-boson-slave fermion formalism beyond the mean-field approximation. At small dopings, the competition between ferromagnetic and AF fluctuations gives rise to the spiral phase,^{8,15,16,24} which could be a starting point of considering the fluctuation effects. It has been noted recently that the true spiral long-range order is unstable due to frustration effects of moving holes.²⁵ A more careful analysis,²⁶ however, shows that the coherent motion of holes drives a dynamical spiral fluctuation which suppresses the spin-spin correlation in the long range, but the short-range AF correlations survive. For our purpose the important aspect is the coexistence of ferromagnetic and AF fluctuations in short range. We will use the words “spiral order” only in this sense. In order to emphasize this point we take the $T = 0^+$ limit in calculating the physical quantities, i.e., not including the true long-range order which may overshadow the fluctuation effects on the normal state properties.

Using a systematic expansion technique for real-time Green’s functions we have considered two-loop correction of spin-excitations on the fermion propagation. As in the case of a single hole, the fermion propagation is strongly renormalized. Apart from the modification of the coherent part, there appear additional side bands in the spectral density, coming entirely from the incoherent multiple scattering on spin excitations. Moreover, we have studied the implications of this renormalization on physical observable quantities. The frequency-dependent optical absorption consists of a central Drude peak and a side band due to incoherent scattering, which agrees well with experimental observations.⁵ Furthermore, the con-

volut density of states of a physical electron which can be measured in tunneling experiments is rather symmetric with respect to the origin and is more or less linear in voltage difference. This is not very different from the mean-field result.¹⁴ In all these results the AF fluctuations play an essential role.

The paper is organized as follows. In Sec. II we develop the formalism for a self-consistent calculation of the Green’s functions and the order parameters. In Sec. III we carry out diagrammatic calculation for boson and fermion Green’s function to implement the formalism outlined in Sec. II. In Sec. IV we calculate two physical observable quantities: the frequency-dependent optical conductivity and the tunneling density of states taking into account the AF fluctuations at the two-loop level. Finally in Sec. V, we discuss the results obtained and make some final remarks.

II. FORMALISM

Our starting point is the t - J model, which is generally represented by the Hamiltonian

$$H = -t \sum_{\langle i,j \rangle} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{H.c.}) + J \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j), \quad (1)$$

where $c_{i,\sigma}^\dagger$ ($c_{i,\sigma}$), as usual, are the electron creation (annihilation) operators at site i ; and $\mathbf{S}_i$ ($\equiv \frac{1}{2} c_{i,\alpha}^\dagger \boldsymbol{\sigma}_{\alpha,\beta} c_{i,\beta}$) and n_i ($\equiv c_{i,\alpha}^\dagger c_{i,\alpha}$) are the spin and number operators of electrons, respectively. In the Schwinger-boson approach, the electron operators are represented as $c_{i,\alpha}^\dagger = b_{i,\alpha}^\dagger f_i$, where $b_{i,\alpha}^\dagger$ are the Schwinger boson operators which describe spins and f_i are the slave-fermion operators which represent empty sites. For a doping concentration δ , there is a global constraint for f_i as $\sum_i f_i^\dagger f_i = \delta N$, N is the number of sites. In addition, we should have a local constraint for bosons and fermions, i.e., the completeness condition $\sum_\alpha b_{i,\alpha}^\dagger b_{i,\alpha} + f_i^\dagger f_i = 1$. In this representation Hamiltonian (1), together with the constraints, can be rewritten as $H = H_0 + H_1 + H_2$, where

$$H_0 = \lambda_b \sum_{i,\sigma} (b_{i,\sigma}^\dagger b_{i,\sigma} + f_i^\dagger f_i - 1) + (\lambda_f - \lambda_b) \sum_i (f_i^\dagger f_i - \delta), \quad (2)$$

$$H_1 = -t \sum_{\langle i,j \rangle} (f_i^\dagger f_j b_{j,\sigma}^\dagger b_{i,\sigma} + \text{H.c.}), \quad (3)$$

$$H_2 = J \sum_{\langle i,j \rangle} \left(b_i^\dagger \frac{\boldsymbol{\sigma}}{2} b_i \cdot b_j^\dagger \frac{\boldsymbol{\sigma}}{2} b_j - \frac{1}{4} b_i^\dagger b_i b_j^\dagger b_j \right). \quad (4)$$

Here $b_i = (b_{i,\uparrow}, b_{i,\downarrow})$, $\boldsymbol{\sigma}$ is the Pauli matrix. In (2) we have treated the completeness condition on average, so the two Lagrange multipliers λ_b and λ_f correspond to the chemical potentials of bosons and fermions, respectively.

We will make the self-consistent expansion by treating H_1 and H_2 as the perturbation Hamiltonians. We would like to remind you that this expansion is different from the usual perturbation theory in some sense. In the *self-consistent expansion*, all lines of the Feymann diagrams represent full Green's function, which should be solved self-consistently. This will lead to convergence even in the region where the usual perturbation theory fails. To illustrate this point, let us recall the one-hole problem. A hopping term expansion was also used for this problem.^{9,10} However, due to the self-consistent renormalization, the Brinkman-Rice result is reproduced for $t/J \rightarrow \infty$, where the usual perturbation certainly breaks down. We will show that for finite dopings the first-order term of this expansion exactly corresponds to the self-consistent mean-field approximation.^{15,16} Therefore, we expect that the result would be reasonable even for large t in this expansion.

Before discussing the fermion and boson Green's functions, let us first introduce the following useful order parameters:

$$\begin{aligned}\Delta_{i,\alpha} &= \left\langle \frac{1}{2} \sum_{\sigma} \sigma b_{i,\sigma} b_{i+\hat{\alpha},-\sigma} \right\rangle, \\ \chi_{i,\alpha} &= \left\langle \sum_{\sigma} b_{i,\sigma}^{\dagger} b_{i+\hat{\alpha},\sigma} \right\rangle, \\ \varphi_{i,\alpha} &= \langle f_i^{\dagger} f_{i+\hat{\alpha}} \rangle.\end{aligned}\quad (5)$$

Here Δ and χ describe short-range AF and ferromagnetic correlations, respectively; while φ is the bond charge parameter describing the motion of fermions. These parameters will be chosen to be translation invariant, so their site subscripts will be dropped henceforth. There are many ways to choose the gauge of these parameters. For the spiral phase,⁸ the gauge is chosen such that^{13,16}

$$\chi_{\alpha} = \chi, \quad \varphi_{\alpha} = \varphi, \quad \Delta_{\alpha} = -\Delta_{-\alpha} = \Delta, \quad (6)$$

with $\alpha = x, y$. The phase with different gauge choice has different energy. The spiral phase has the lowest energy at $T = 0$ for small dopings. We will use the gauge given in (6).

The boson Green's function includes both ordinary and anomalous parts, since $\langle bb \rangle \neq 0$, as defined in (5). The boson Green's function is thus defined as

$$G(k, t - t') = -i \left\langle T \begin{pmatrix} b_{k,\uparrow}(t) b_{k,\uparrow}^{\dagger}(t') & b_{k,\uparrow}(t) b_{-k,\downarrow}(t') \\ b_{k,\downarrow}^{\dagger}(t) b_{-k,\uparrow}^{\dagger}(t') & b_{k,\downarrow}^{\dagger}(t) b_{k,\downarrow}(t') \end{pmatrix} \right\rangle, \quad (7)$$

where $\langle \dots \rangle$ is an average over the ensemble, rather than in the ground state. Equation (7) is actually the real-time Green's function at finite temperatures. For convenience we introduce the notation

$$B_k = \begin{pmatrix} b_{k,\uparrow} \\ b_{-k,\downarrow}^{\dagger} \end{pmatrix}. \quad (8)$$

Then $G(k, t - t')$ in (7) can be rewritten as

$$G(k, t - t') = -i \langle T B_k(t) B_k^{\dagger}(t') \rangle. \quad (9)$$

The fermion Green's function is, as usual, defined as

$$g(k, t - t') = -i \langle T f_k(t) f_k^{\dagger}(t') \rangle. \quad (10)$$

The zero-order Green's functions of H_0 in (2) for bosons and fermions are very simple. The inverse of them can be written as

$$G_0^{-1}(k, \omega) = \sigma_3 \omega - \lambda_b, \quad g_0^{-1}(k, \omega) = \omega - \lambda_f. \quad (11)$$

From Feymann diagrams of H_1 and H_2 , one can get (retarded) self-energies of bosons and fermions, $\Pi_r(k, \omega)$ and $\Sigma_r(k, \omega)$. Some Feymann diagrams are shown in Fig. 1.

According to Dyson equations, one can get the full retarded Green's functions of bosons and fermions as

$$G_r(k, \omega) = [\sigma_3 \omega - \lambda_b - \Pi_r(k, \omega) + i\eta]^{-1}, \quad (12)$$

$$g_r(k, \omega) = [\omega - \lambda_f - \Sigma_r(k, \omega) + i\eta]^{-1},$$

respectively. The Green's function cannot be self-consistently determined by (12) alone, since the parameters of (6) are also included in the self-energies. We must have another set of self-consistent equations determining these parameters. From the definitions of the order parameters in (5) and the constraints, and noting the gauge choice of (6), we get the following equations:

$$\begin{aligned}\chi &= 1/(2N) \sum_k \gamma_k \int \frac{d\omega}{2\pi} \text{Tr}[iG_c(k, \omega)], \\ \Delta &= 1/(2N) \sum_k \tilde{\gamma}_k \int \frac{d\omega}{2\pi} \text{Tr} \left(\frac{\sigma_1 + i\sigma_2}{2} G_c(k, \omega) \right), \\ \varphi &= 1/(2N) \sum_k \gamma_k \int \frac{d\omega}{2\pi} (-i) g^{+-}(k, \omega), \\ 2 - \delta &= 1/(2N) \sum_k \int \frac{d\omega}{2\pi} \text{Tr}[iG_c(k, \omega)], \\ \delta &= 1/N \sum_k \int \frac{d\omega}{2\pi} (-i) g^{+-}(k, \omega),\end{aligned}\quad (13)$$

with

$$\gamma_k = (\cos k_x + \cos k_y)/2$$

and

$$\tilde{\gamma}_k = (\sin k_x + \sin k_y)/2.$$

FIG. 1. The self-energy diagrams considered for the boson propagator [(a) and (b)] and the fermion propagator [(c) and (d)]. The solid line and the dashed line correspond to fermion and boson propagators, respectively.

In (13) we have used the Keldysh Green's function.²⁷ We feel that the Keldysh formalism is particularly convenient in calculating the self-energy, as we can avoid the complicated frequency summation appearing in the Matsubara Green's function techniques. The relations of the Keldysh Green's function $g^{+-}(k, \omega)$ and $G_c(k, \omega)$ with the retarded and advanced real-time Green's functions are given in Appendix A. It is noted that all equations of (12) and (13) should be self-consistently solved.

Using (12) one can get the corresponding advanced Green's functions and the spectral functions. As an example, for fermions, the advanced Green's function is

$$g_a(k, \omega) = g_r(k, \omega) |_{i\eta \rightarrow -i\eta}. \quad (14)$$

The spectral function of fermions can be obtained as

$$A(k, \omega) = i[g_r(k, \omega) - g_a(k, \omega)], \quad (15)$$

and the density of states for fermions is given by

$$D_f(\omega) = \frac{1}{2\pi N} \sum_k A(k, \omega). \quad (16)$$

$$H_1 = -\frac{zt}{N} \sum_k \sum_{k'} \sum_q \sum_{\mu=1,2} \gamma_{k'-\eta_\mu k + (1-\eta_\mu)/2q} f_{k'}^\dagger f_{k'+q} B_{k+q}^\dagger \tau_\mu B_k, \quad (17)$$

where

$$\begin{aligned} \eta_\mu &= \begin{cases} 1, & \mu = 1 \\ -1, & \mu = 2, \end{cases} \\ \tau_\mu &= \begin{cases} \tau_+, & \mu = 1 \\ \tau_-, & \mu = 2, \end{cases} \\ \tau_+ &= (1 + \sigma^z)/2, \quad \tau_- = (1 - \sigma^z)/2. \end{aligned} \quad (18)$$

The first-order correction of H_1 to the self-energy of bosons [Fig. 1(a)] can be easily obtained as

$$\Pi_1^{(1)}(k, \omega) = -\frac{zt}{N} \sum_{k'} \sum_\mu \gamma_{k'-\eta_\mu k} \int \frac{d\omega}{2\pi} (-i) g^{+-}(k', \omega) \tau_\mu, \quad (19)$$

which can be rewritten as

$$\Pi_1^{(1)}(k) = \begin{pmatrix} P_k & 0 \\ 0 & P_{-k} \end{pmatrix}, \quad (20)$$

where

$$P_k = -\frac{zt}{N} \sum_{k'} \gamma_{k'-k} \int \frac{d\omega}{2\pi} (-i) g^{+-}(k', \omega). \quad (21)$$

It can be shown that P_k is expressed as (see Appendix B)

$$P_k = -zt\gamma_k \varphi. \quad (22)$$

III. DIAGRAM CALCULATIONS

In this paper we will limit ourselves to the first order for the boson (spin) part, since the spin-correlation functions obtained at this level are very good for one dimension and can well describe the neutron scattering data in the oxide superconductors. However, the second-order correction for the slave fermion is necessary for a proper description of the hole motion due to magnetic fluctuations, as for the single-hole case.^{9,10} Thus we consider the fermion diagram to the second order. All the Feynman diagrams considered here are shown in Fig. 1, where (a) and (b) are for the boson propagator, while (c) and (d) are for the fermion propagator. Diagram (d) is equivalent to that considered in Refs. 9 and 10 for the single-hole case. It is noted that the boson configurations are self-consistently determined here, in contrast with the single-hole case, where an antiferromagnetic background is assumed at the very beginning.

A. Bosonic Green's function

Now let us carry out the calculation of Feynman diagrams shown in Fig. 1. To start with, we rewrite H_1 of (3) in the momentum representation, using the notation of (8), as

We can similarly obtain the first-order contribution of H_2 to the boson self-energy [Fig. 1(b)], which is

$$\Pi_2^{(1)}(k, \omega) = \begin{pmatrix} \frac{J}{4} \chi_k & -J \Delta_k \\ -J \Delta_k^* & \frac{J}{4} \chi_k \end{pmatrix}, \quad (23)$$

where

$$\chi_k = \chi^z \gamma_k, \quad \Delta_k = \Delta^z \tilde{\gamma}_k. \quad (24)$$

From (12), (20), and (23), we get the inverse of the boson Green's function, to the first order, as

$$\begin{aligned} G^{-1}(k, \omega) &= \sigma_3 \omega - \lambda_b - \Pi_1^{(1)}(k, \omega) - \Pi_2^{(2)}(k, \omega) \\ &= \begin{pmatrix} \omega - \lambda_b + M_k & J \Delta_k \\ J \Delta_k^* & -\omega - \lambda_b + M_k \end{pmatrix}, \end{aligned} \quad (25)$$

where

$$M_k = zt\gamma_k \varphi - J/4\chi_k. \quad (26)$$

By using $\det[G^{-1}(k, \omega)] = 0$, one can readily get the spin excitation spectrum

$$\omega_k = \sqrt{(\lambda_b - M_k)^2 - (J | \Delta_k |)^2}, \quad (27)$$

which is the result of the mean-field approximation under the gauge choice of (6).^{15,16} We have thus shown that the first-order contribution is exactly the usual self-consistent mean-field approximation.

Using (25) one can easily get the boson retarded

Green's function

$G_r(k, \omega)$

with η as an infinitesimal, and the corresponding advanced Green's function is

$$= \frac{1}{-(\omega + i\eta)^2 + \omega_k^2} \begin{pmatrix} -\omega - \lambda_b + M_k & -J\Delta_k \\ -J\Delta_k^* & \omega - \lambda_b + M_k \end{pmatrix}, \quad (28)$$

$$G_a(k, \omega) = G_r(k, \omega) |_{i\eta \rightarrow -i\eta}. \quad (29)$$

From the above relation and (AI5) in Appendix A, we get $G_c(k, \omega)$ in (13) as

$$G_c(k, \omega) = (-2\pi i) \coth(\beta\omega/2) \frac{\delta(\omega - \omega_k) - \delta(\omega + \omega_k)}{2\omega_k} \begin{pmatrix} -\omega - \lambda_b + M_k & -J\Delta_k \\ -J\Delta_k^* & \omega - \lambda_b + M_k \end{pmatrix}. \quad (30)$$

B. Fermionic Green's function

The first-order contribution of H_1 in (18) to the fermion self-energy [Fig. 1(c)] is given by

$$\Sigma^{(1)}(k, \omega) = -\frac{zt}{2N} \sum_{k'} \sum_{\mu} \gamma_{k'-\eta_{\mu}k} \int \frac{d\omega}{2\pi} \{i \text{Tr}[G_c(k', \omega) \tau_{\mu}]\}. \quad (31)$$

Using (30) one can get $\Sigma^{(1)}$, which, similar to (22), can be expressed as

$$\Sigma^{(1)}(k, \omega) = -zt\chi\gamma_k. \quad (32)$$

Both analytical derivation and numerical calculation of the second-order contribution of H_1 [Fig. 1(d)] is very complicated. In order to make the actual computation possible, we replace the full fermion Green's function of Fig. 1(d) by the first-order Green's function. It seems that the standard perturbation theory is used at this step. It is noted, however, that the fermion line is substituted by the first-order propagator, rather than by the zeroth-order propagator. The first-order Green's function for fermions is, from (12) and (32), written as

$$g_r^{(1)}(\mathbf{k}, \omega) = \frac{1}{\omega - \epsilon_k + i\eta}, \quad (33)$$

with $\epsilon_k = \lambda_f - tz\chi\gamma_k$. The calculation of the second-order self-energy is tedious and will be briefly sketched in Appendix C. Here we quote the final result. The second-order retarded self-energy is

$$\Sigma_r^{(2)}(\mathbf{k}, \omega) = -(tz)^2 \frac{1}{N} \sum_{\mathbf{q}} \frac{1}{N} \sum_{\mathbf{k}'} \frac{1}{4\omega_{\mathbf{k}'}\omega_{\mathbf{k}'+\mathbf{q}}} \sum_{\beta_1, \beta_2=\pm} [(\Theta_{1,k,k',q}^{\beta_1, \beta_2} + \Theta_{2,k,k',q}^{-\beta_1, -\beta_2}) X_{\mathbf{k}\mathbf{k}'\mathbf{q}}(\beta_1\omega'_k, \beta_2\omega_{k'+q}) Y_k(\omega, \beta_1\omega'_k, \beta_2\omega_{k'+q})], \quad (34)$$

where

$$\Theta_{1,k,k',q}^{\beta_1, \beta_2} = f(\epsilon_{k+q}) N_{\beta_1, k'} N_{\beta_2, k'+q}, \quad \Theta_{2,k,k',q}^{\beta_1, \beta_2} = [1 - f(\epsilon_{k+q})] N_{\beta_1, k'} N_{\beta_2, k'+q}, \quad N_{+,k} = N(\omega_k), \quad N_{-,k} = 1 + N(\omega_k),$$

$$X_{\mathbf{k}\mathbf{k}'\mathbf{q}}(\omega_{\mathbf{k}'}, \omega_{\mathbf{k}'+\mathbf{q}}) = \gamma_{\mathbf{k}-\mathbf{k}'}^2 (\omega_{\mathbf{k}'} - \lambda_b + \alpha\gamma_{\mathbf{k}'}) (\omega_{\mathbf{k}'+\mathbf{q}} + \lambda_b - \alpha\gamma_{\mathbf{k}'+\mathbf{q}}) + \gamma_{\mathbf{k}+\mathbf{k}'+\mathbf{q}}^2 (\omega_{\mathbf{k}'} + \lambda_b - \alpha\gamma_{\mathbf{k}'}) (\omega_{\mathbf{k}'+\mathbf{q}} - \lambda_b + \alpha\gamma_{\mathbf{k}'+\mathbf{q}}) - \gamma_{\mathbf{k}-\mathbf{k}'} \gamma_{\mathbf{k}+\mathbf{k}'+\mathbf{q}} 2(Jz\Delta)^2 \tilde{\gamma}_{\mathbf{k}'} \tilde{\gamma}_{\mathbf{k}'+\mathbf{q}}, \quad (35)$$

$$Y_k(\omega, \omega'_k, \omega_{k'+q}) = [\omega - (\epsilon_{k+q} - \omega_{\mathbf{k}'} - \omega_{\mathbf{k}'+\mathbf{q}}) + i\eta]^{-1}.$$

Here $N(\omega)$ and $f(\omega)$ are, respectively, the boson and fermion distribution functions, and the k summation is carried out over the entire Brillouin zone. We have also introduced the notation $\alpha = zt\varphi$. From (12) and (15) and the above self-energy expression, one can obtain the fermion Green's function and the spectral function with two-loop corrections.

C. Self-consistent equations for order parameters

Using boson and fermion Green's functions obtained above, the self-consistent equations of (13) can be expressed as follows:

$$\begin{aligned}\chi &= 1/N \sum_k \gamma_k (\lambda_b - M_k) / \omega_k \coth(\beta\omega_k/2), \\ \Delta &= 1/N \sum_k \tilde{\gamma}_k (J\Delta_k) / (2\omega_k) \coth(\beta\omega_k/2), \\ \varphi &= 1/N \sum_k \gamma_k \int d\omega / (2\pi) A(k, \omega) f(\omega), \\ 2 - \delta &= 1/N \sum_k (\lambda_b - M_k) / \omega_k \coth(\beta\omega_k/2), \\ \delta &= 1/N \sum_k \int d\omega / (2\pi) A(k, \omega) f(\omega),\end{aligned}\quad (36)$$

where M_k and ω_k are given by (26) and (27), respectively. All these equations are similar to those obtained by Kane *et al.*,¹⁵ since the boson part is approximated to the same level. The fluctuation effects on the motion of fermions are included in the spectral function $A(k, \omega)$. The completely self-consistent calculation of these equations is rather complicated and will be left to a future study. In this paper we will concentrate on the fluctuation effects on the physical quantities, assuming the mean-field (first-order approximation) values for the order parameters in calculating the spectral function $A(k, \omega)$ and physical quantities. However, the chemical potential of fermions must be solved self-consistently so that the sum rule of fermions [the last equation of (36)] is satisfied.

IV. PHYSICAL QUANTITIES

In order to investigate the normal state properties of the system we take the limit $T = 0^+$, i.e., there is no Bose condensation and the long-range order is absent. In this case we set the boson distribution $N(\omega_k) = 0$ in the expression of $\Sigma_r(k, \omega)$ of (34), which is not expected to change the normal state properties qualitatively. We would like to remind the reader that the real boson number is not equal to zero, but rather to $\sum_k v_k^2$, where v_k is the Bogoliubov transformation coefficient [which can be obtained from the diagonalization of (25)]. It can be easily seen, from $\Sigma_r^{(2)}(k, \omega)$ and ω_k of (27), that if we keep only the ferromagnetic correlations via χ and neglect the AF correlations by setting $\Delta = 0$, $\Sigma_r^{(2)}$ is equal to zero. This means that the fluctuation effects discussed here are mainly of an AF nature.

All the physical quantities of the system should be gauge invariant under the following local transformation:

FIG. 2. The fermion density of states in arbitrary units for $\delta = 0.2$ and $t/J = 2$ including magnetic fluctuations (solid curve) and at the mean-field level (dashed curve).

$$b_{j,\sigma} \rightarrow b_{j,\sigma} e^{i\theta(\mathbf{r}_j, \tau)}, \quad f_j \rightarrow f_j e^{i\theta(\mathbf{r}_j, \tau)}, \quad (37)$$

since the Hamiltonian (2)–(4) is invariant under this transformation. Before discussing the gauge-invariant quantities, let us first consider the fermion density of states (DOS). Although it is not a gauge-invariant quantity, it shows clearly the strong interaction between bosons and fermions and all its features will have observable implications on the physical quantities.

The fermion density of states in (16) is calculated and is shown in Fig. 2. The DOS in the mean-field approximation (dashed line) consists of the central part only, which comes from the noninteracting particles. After including the fluctuation effects (solid line) the central (coherent) part is renormalized and, in addition, two side bands appear due to the multiple scattering on spin fluctuations. This part is incoherent and will lead to observable effects. These features agree qualitatively with small cluster calculations.⁶ The small wiggles appearing there are, probably, due to the finite-size effect.

A. Optical conductivity

One unusual and interesting quantity in oxide superconductors is the optical conductivity. We now consider this quantity in our approach. In the Schwinger-boson technique, an electron is represented by the product of a boson and a fermion. The external field can only be coupled to one of them, say, fermion. According to Ioffe and Larkin,²⁰ one can separately calculate the contributions of bosons and fermions, $\sigma_F(\omega)$ and $\sigma_B(\omega)$. The total conductivity is obtained by the relation

$$\sigma^{-1}(\omega) = \sigma_F^{-1}(\omega) + \sigma_B^{-1}(\omega). \quad (38)$$

The current density of fermions is defined as

$$j_\alpha = 2\rho\chi/N \sum_k \sin(\mathbf{k} \cdot \mathbf{e}_\alpha) f_k^\dagger f_k, \quad (39)$$

where $\mathbf{e}_\alpha$ is the unit vector along the direction α , $\rho = e/a$ with e as electron charge and a as lattice constant. In the lowest polarization (one-loop) approximation, the fermion conductivity can be easily shown to be given by the standard formula²⁸

$$\sigma_F(\omega) = \frac{2(\rho\chi)^2}{N\omega} \sum_{\mathbf{p}} (\sin^2 p_x + \sin^2 p_y) \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} A(\mathbf{p}, \varepsilon) A(\mathbf{p}, \varepsilon + \omega) [f(\varepsilon) - f(\varepsilon + \omega)], \quad (40)$$

where $A(k, \omega)$ is given by (15). Similarly, the boson part is given by

$$\sigma_B(\omega) = 2(\rho\varphi)^2/(N\omega) \sum_{\mathbf{p}} (\sin^2 p_x + \sin^2 p_y) (Jz\Delta\gamma_p/\omega_p)^2 \delta(\omega - 2\omega_p). \quad (41)$$

We have used the boson Green's function of (28) in deriving σ_B above. Figure 3 shows the fermion (solid line), the boson (long dashed line) contributions, and the resulting conductivity (short dashed line). At low frequencies the conductivity is limited by the scattering of fermions, while the high frequency part is cut off by twice the boson bandwidth. For the resulting conductivity, in addition to the Drude peak, there is a broad band at higher frequencies, which is consistent with experiments on oxide superconductors.⁵ All of these features mainly come from the fermion part. Comparing (40) and the fermion DOS (Fig. 2) we find that the Drude peak results from the renormalized coherent motion while the broad band is due to the convolution of the central part with the incoherent side band. Similar results have been obtained in cluster calculations.²⁹ Again, the small wiggles appearing

there are due to the finite-size effect. We believe this is the first time that such results are obtained for finite-hole concentrations in the thermodynamic limit. As emphasized earlier, these properties are all due to AF fluctuations. In calculating $\sigma(\omega)$ we have neglected the vertex correction which was important for the gauge-field fluctuations,^{21,22} but not here.

B. Electron density of states

Another interesting quantity is the density of states of real electrons which can be measured directly from the tunneling experiments.³⁰ The gauge-invariant electron DOS is obtained by convoluting the boson and fermion Green's functions and can be written as

$$\mathcal{D}_\sigma(\omega) = \frac{1}{N} \sum_k [D_k(\omega_k - \omega)(f(\omega_k - \omega) + N(\omega_k))(\lambda_b - \alpha\gamma_k + \omega_k) + D_k(-\omega_k - \omega)(1 - f(-\omega_k - \omega) + N(\omega_k))(\lambda_b - \alpha\gamma_k - \omega_k)]/\omega_k, \quad (42)$$

where $D_k(\omega) = 1/(2\pi N) \sum_q A(k - q, \omega)$. Equation (42) is spin-orientation invariant. $\mathcal{D}_\sigma(\omega)$ is shown in Fig. 4. The dashed curve is the result by taking $D_k(\omega)$ as the mean-field value. The solid curve corresponds to the case with two-loop corrections. We note that $\mathcal{D}_\sigma(\omega) \sim |\omega|$ at

the scale of J on both sides of $\omega = 0$. Moreover, the DOS has two parts, the $\omega < 0$ part comes from the second factor of (42) including $1 - f$, so the area below $\omega = 0$ is proportional to $1 - \delta$, i.e., the number of electrons, as it should be. Thus we see a possibility of converting the small Fermi surface ($\sim \delta$) to a large Fermi surface ($\sim 1 - \delta$). We should emphasize that the symmetric distribution of DOS around zero frequency in the Schwinger-

FIG. 3. The frequency-dependent conductivity in units of $(10\rho)^2$. The solid and the dashed curves are the fermion and the boson contributions, respectively; the dotted curve is the resulting conductivity.

FIG. 4. The electron density of states in arbitrary units for $\delta = 0.2$ and $t/J = 2$ at the mean-field level (long dashed curve) and with two-loop corrections (solid curve). The corresponding quantity in the slave-boson approach is also shown schematically by the short dashed curve around $\omega = 0$.

boson approach is due to the nonvanishing value of $\langle bb \rangle$ which gives rise to two branches of spin excitations, $\pm\omega_q$, reflecting the AF nature, i.e., the zero-point fluctuation effects. Otherwise, if $\langle bb \rangle \sim \Delta = 0$, only the positive (ferromagnetic) branch will survive, so the symmetric behavior will not be obtained, as for the slave-boson case. For comparison we have drawn a schematic diagram for the DOS in the slave-boson formalism (the dashed curve).²¹ In that approach, the spin fluctuations are described by fermions with positive energies, and the DOS is asymmetric with respect to zero frequency. In this calculation the electron density of states obtained is nearly zero at $\omega = 0$, because we have simply set $N(\omega_k) = 0$ at $T = 0^+$, in order to see fluctuation effects in the normal state and avoid too many complications. For any finite temperature, however, $N(\omega_k)$ should be self-consistently determined. As a test we have taken a small constant $N(\omega_k)$ in calculating (42) and found that the density of states does not vanish at $\omega = 0$, but all features described above still remain. A more self-consistent calculation is clearly needed at finite temperatures. The DOS at the mean-field level was calculated earlier in Ref. 14, using the Schwinger-boson technique. It is obvious that the main features of the DOS are robust with respect to further accounts of magnetic fluctuations.

We would like to mention that the linear tunneling conductance has recently been interpreted in terms of inelastic scattering by Kirtley and Scalapino.³¹ Their approach looks very different from ours, but the basic physical process involved is rather similar — scattering due to AF fluctuations. They included these effects into tunneling matrix elements, whereas we consider them as renormalization of DOS.

V. DISCUSSIONS AND CONCLUSIONS

In this paper we have developed a systematic expansion for the t - J model using the Schwinger-boson approach. The first-order approximation of this formalism is shown to correspond to the self-consistent mean-field theory. The second-order approximation takes further account of magnetic fluctuations which renormalize the hole motion. Therefore our work here, on one hand, is a first step to go beyond the mean-field approximation, and, on the other hand, is a natural extension of the single-hole case to the finite doping using the self-consistent Green's function approach. It can be readily shown that the second-order approximation in this expansion is similar to the consideration of Gaussian fluctuations in the large- N expansion.²³ However, our formalism can be solved in a self-consistent way so that the fluctuation effects on the ground state are also included on an equal footing. The present work is, of course, far from being completely self-consistent. A fully self-consistent calculation of the phase diagram with magnetic fluctuations included up to the second order is currently in progress.

In this paper we have shown the important renormalization effects of fermion motion due to magnetic fluctuations in the Schwinger-boson approach. The AF nature of fluctuations has explicit implications for observable quantities

like frequency-dependent conductivity and electron density of states. They are consistent with experiments. We do not claim that the present calculation explains fully the experimentally observed normal-state properties in oxide superconductors, but the results obtained within this approximation scheme are encouraging.

The mean-field results of the slave boson and the Schwinger boson look very different, but inclusion of higher-order fluctuations may lead to more converging results. Our study is also a step in this direction. The large- N expansion in the slave boson approach²³ yields results similar to the consideration of gauge-field fluctuations.²¹ Those papers and the present study suggest that the systematic expansion may be an efficient way to study the normal-state properties of strongly correlated electron systems. The gauge field approach emphasizes the single-occupancy constraints, while the loop expansion takes account of AF fluctuations. It seems that these two aspects are complementary to each other. How to combine these two aspects together would be an interesting question for further study.

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APPENDIX A

In the Keldysh representation the following Green's function are introduced:

$$G^{-+}(k, t - t') = -i\langle B_k(t)B_k^\dagger(t') \rangle, \quad (A1)$$

$$G^{+-}(k, t - t') = -i\langle B_k^\dagger(t')B_k(t) \rangle,$$

for bosons and

$$g^{-+}(k, t - t') = -i\langle f_k(t)f_k^\dagger(t') \rangle, \quad (A2)$$

$$g^{+-}(k, t - t') = i\langle f_k^\dagger(t')f_k(t) \rangle,$$

for fermions. The Keldysh Green's functions defined above have, in the frequency space, the following relations with the usual retarded and advanced Green's functions of bosons (fermions), G_r (g_r) and G_a (g_a):

$$G^{-+}(k, \omega) = [1 + N(\omega)](G_r(k, \omega) - G_a(k, \omega)), \quad \sum_{k'} \xi(k'_x) g^{+-}(k', \omega) = \sum_{k'} \xi(k'_y) g^{+-}(k', \omega), \quad (\text{B1})$$

$$\begin{aligned} G^{+-}(k, \omega) &= N(\omega)(G_r(k, \omega) - G_a(k, \omega)), \\ g^{-+}(k, \omega) &= [1 - f(\omega)](g_r(k, \omega) - g_a(k, \omega)), \end{aligned} \quad (\text{A3})$$

$$g^{+-}(k, \omega) = -f(\omega)(g_r(k, \omega) - g_a(k, \omega)).$$

The correlation Green's functions are defined as

$$\begin{aligned} G_c(k, t - t') &= G^{-+}(k, t - t') + G^{+-}(k, t - t'), \\ g_c(k, t - t') &= g^{-+}(k, t - t') + g^{+-}(k, t - t'). \end{aligned} \quad (\text{A4})$$

From (A3) one has

$$\begin{aligned} G_c(k, \omega) &= [1 + 2N(\omega)](G_r(k, \omega) - G_a(k, \omega)), \\ g_c(k, \omega) &= [1 - 2f(\omega)](g_r(k, \omega) - g_a(k, \omega)). \end{aligned} \quad (\text{A5})$$

APPENDIX B

Let us now show Eq. (22) of the main text. From the symmetry we can assume

where $\xi(x) = \cos x$ or $\sin x$. Equation (21) of the main text can then be rewritten as

$$\begin{aligned} P_k &= -\frac{zt}{N} \gamma_k \sum_{k'} \gamma_{k'} \int \frac{d\omega}{2\pi} (-i) g^{+-}(k', \omega) \\ &\quad - \frac{zt}{N} \tilde{\gamma}_k \sum_{k'} \tilde{\gamma}_{k'} \int \frac{d\omega}{2\pi} (-i) g^{+-}(k', \omega) \\ &\equiv -zt\gamma_k A - zt\tilde{\gamma}_k B. \end{aligned} \quad (\text{B2})$$

For the gauge chosen in (6) and the definition of order parameters in (5) one can readily show that A in (B2) is nothing but the parameter φ of (6) and $B = 0$. We have tried other gauges in which both A and B are not equal to zero. We have found, however, that the phase with gauge (6) still has the lowest energy. Thus, we take $B = 0$ in this paper.

APPENDIX C

Here we describe the main steps to derive the second-order self-energy of fermions. Following the Keldysh Green's function representation given in Appendix A, the second-order self-energy can be expressed as

$$\Sigma_{-+}^{(2)}(k, \omega) = -(tz)^2/N \sum_q \int \frac{d\omega'}{2\pi} F_k^{-+}(q, \omega') g^{-+}(k+q, \omega+\omega'), \quad (\text{C1})$$

$$\Sigma_{+-}^{(2)}(k, \omega) = -(tz)^2/N \sum_q \int \frac{d\omega'}{2\pi} F_k^{+-}(q, \omega') g^{+-}(k+q, \omega+\omega'),$$

where

$$\begin{aligned} F_k^{-+}(q, \omega) &= 1/N \sum_{k'} \int \frac{d\omega'}{2\pi} \sum_{\mu\mu'} \Gamma_{kk'q}^{\mu\mu'} \text{Tr}[G^{-+}(k', \omega') \tau_\mu G^{+-}(k'+q, \omega+\omega') \tau_{\mu'}], \\ \Gamma_{kk'q}^{\mu\mu'} &= \gamma_{k-\eta_\mu k'+(1-\eta_\mu)/2q} \gamma_{k+q-\eta'_\mu(k'+q)-(1-\eta'_\mu)/2q}. \end{aligned} \quad (\text{C2})$$

$F_k^{-+}(q, \omega)$, as well as $F_k^{+-}(q, \omega)$, can be obtained by using (28) and (A3), which are expressed as

$$\begin{aligned} F_k^{-+}(q, \omega) &= \frac{2\pi}{N} \sum_{\mathbf{k}'} \frac{1}{4\omega_{\mathbf{k}'}\omega_{\mathbf{k}'+\mathbf{q}}} \sum_{\beta_1, \beta_2=\pm} \beta_2(1+N(\beta_1\omega_{\mathbf{k}'}))N(\omega+\beta_1\omega_{\mathbf{k}'}) \\ &\quad \times \delta(\omega+\beta_1\omega_{\mathbf{k}'}-\beta_2\omega_{\mathbf{k}'+\mathbf{q}}) X_{\mathbf{k}\mathbf{k}'\mathbf{q}}(-\beta_1\omega_{\mathbf{k}'}, \omega+\beta_1\omega_{\mathbf{k}'}), \\ F_k^{+-}(q, \omega) &= \frac{2\pi}{1+N} \sum_{\mathbf{k}'} \frac{1}{4\omega_{\mathbf{k}'}\omega_{\mathbf{k}'+\mathbf{q}}} \sum_{\beta_1, \beta_2=\pm} \beta_2 N(\beta_1\omega_{\mathbf{k}'})[1+N(\omega+\beta_1\omega_{\mathbf{k}'})] \\ &\quad \times \delta(\omega+\beta_1\omega_{\mathbf{k}'}-\beta_2\omega_{\mathbf{k}'+\mathbf{q}}) X_{\mathbf{k}\mathbf{k}'\mathbf{q}}(-\beta_1\omega_{\mathbf{k}'}, \omega+\beta_1\omega_{\mathbf{k}'}), \end{aligned} \quad (\text{C3})$$

where $X_{kk'q}$ is given in (35). Substituting (33) and (C3) into (C1), one can get $\Sigma_{-+}^{(2)}(k, \omega)$ and $\Sigma_{+-}^{(2)}(k, \omega)$ as

$$\Sigma_{-+}^{(2)}(k, \omega) = (tz)^2 \frac{2\pi i}{N^2} \sum_{qk'} \frac{1}{4\omega_{k'}\omega_{k'+q}} \sum_{\beta_1, \beta_2=\pm} \beta_1\beta_2(1 - f(\epsilon_{k+q}))(1 + N(\beta_1\omega_{k'}))N(\beta_2\omega_{k'}) \\ \times \delta(\epsilon_{k+q} - \omega + \beta_1\omega_{k'} - \beta_2\omega_{k'+q})X_{kk'q}(-\beta_1\omega_{k'}, \epsilon_{k+q} - \omega + \beta_1\omega_{k'}), \quad (C4)$$

$$\Sigma_{+-}^{(2)}(k, \omega) = (tz)^2 \frac{2\pi i}{N^2} \sum_{qk'} \frac{1}{4\omega_{k'}\omega_{k'+q}} \sum_{\beta_1, \beta_2=\pm} \beta_1\beta_2(-f(\epsilon_{k+q}))N(\beta_1\omega_{k'})[1 + N(\beta_2\omega_{k'})] \\ \times \delta(\epsilon_{k+q} - \omega + \beta_1\omega_{k'} - \beta_2\omega_{k'+q})X_{kk'q}(-\beta_1\omega_{k'}, \epsilon_{k+q} - \omega + \beta_1\omega_{k'}).$$

From the spectral representation of $\Sigma_r^{(2)}$

$$\Sigma_r^{(2)}(k, \omega) = \int \frac{d\omega'}{2\pi i} \frac{\Sigma_{-+}^{(2)}(k, \omega') - \Sigma_{+-}^{(2)}(k, \omega')}{\omega' - \omega - i\eta}, \quad (C5)$$

and (C4), we have $\Sigma_r^{(2)}(k, \omega)$ given in (34).

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