Control of a separating boundary layer with Lorentz force actuators

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The effect of time-periodic spanwise Lorentz forces on flow separation in a weakly conductive fluid is examined. An array of alternately arranged electrodes and permanent magnets constituted an electromagnetohydrodynamic actuator, which was flush mounted on a hydrofoil model with a flap that was inclined at 20° to the freestream flow in a water tunnel. Above a certain amplitude of electromagnetohydrodynamic forcing, complete reattachment of flow over the flap was seen from measured velocity fields by using particle image velocimetry. It is proposed that introduction of streamwise vorticity generated by the spanwise Lorentz forces into the flow field over the actuator surface is responsible for the reattachment of flow. © 2008 American Institute of Physics.

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I. INTRODUCTION

Magnetohydrodynamic actuation has been shown to be effective in flow control applications involving highly conductive fluids, e.g., liquid metals, and has been studied for quite some time. The use of Lorentz force for flow control in weakly conductive fluids such as sea water has been pursued for the last decade.^{1,2} Several actuator designs have been utilized to create streamwise,^{3,4} spanwise,⁵⁻⁷ and wall-normal^{1,8-10} forcing for drag reduction purposes. Among the various schemes used, oscillatory spanwise forcing, either in terms of a traveling wave or spatially uniform, has shown significant drag reduction, up to 40%, in direct numerical simulations.^{5,11} In a recent experimental study, spanwise Lorentz forcing was shown to produce over 40% reduction in turbulent skin-friction drag.¹²

Separation of boundary layers is routinely encountered in both external and internal flows of engineering interest. The undesirable outcomes of separation include (form) drag increase, lift decrease, and reduced pressure recovery in diffusers. Manipulation of adverse pressure gradients causing separation has been pursued by both passive control methods such as vane-type vortex generators and active separation control techniques such as blowing, suction, wall movement,¹³ oscillatory pneumatic forcing,¹⁴ and synthetic jets.¹⁵

The application of Lorentz force for separation control has been examined in a limited number of studies so far. It is well known that streamwise body forces (or favorable pressure gradients) delay flow separation. Recent experiments have shown that the separation point on a circular cylinder at low Reynolds numbers can be controlled by Lorentz forces applied along the tangential direction on the cylinder surface.^{16,17} Moreover, separated flow over hydrofoils can be completely eliminated by steady streamwise Lorentz forces.^{18,19} However, separation control using steady stream-

wise Lorentz forcing was shown to be too inefficient for practical systems.¹⁸

Periodic Lorentz forcing in the spanwise direction can be an effective means for the control of boundary layer separation. The motivation for this arrangement stems from the well-documented effectiveness of vortex generators^{20,21} for separation control. While streamwise Lorentz forces directly act against the adverse pressure gradient that leads to separation, spanwise forcing is believed to delay flow separation by generating streamwise vortices. The gradient of the Lorentz force in the direction normal to the actuator surface generates streamwise vorticity. Streamwise vortices are capable of transporting fluid from regions with higher kinetic energy to those with lower energy, and vice versa, as is the case with vortex generators and, thus, delay flow separation.

The present work utilizes this idea to perform separation control in a weakly electroconductive liquid. Velocity fields measured by using particle image velocimetry (PIV) were used to evaluate the performance of an electromagnetohydrodynamic (EMHD) actuator, which generated time-periodic spanwise Lorentz force upstream of the separated region of a flap on a hydrofoil model. It is contended that introduction of streamwise vorticity into the separating boundary layer is the mechanism by which the applied spanwise Lorentz force affects suppression of flow separation. To verify this, PIV measurements were also carried out in a cross-stream plane from which the streamwise vorticity field was extracted. The experimental procedure is described in Sec. II and the results are presented in Sec. IV. The mechanism responsible for separation and the forcing effectiveness are discussed in Secs. V and VI, respectively.

II. EXPERIMENTAL SETUP

Electrodes and permanent magnets of about the same thickness were placed in alternate rows to form an actuator array. Successive electrodes were oppositely charged, while successive magnet rows were of opposite polarity. The coated titanium electrodes and neodymium permanent magnets of nominal surface magnetic induction of 0.4 T were

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FIG. 1. Schematic of the Lorentz force actuator. The electric and magnetic fields and the resulting Lorentz force are indicated.

aligned in the spanwise direction, perpendicular to the freestream flow in a water tunnel. The magnets were 0.125 in. thick and 2 in. long, and the spacing between successive rows of magnets was about 0.135 in. The direction of the magnetic and electric fields near the actuator surface is indicated in Fig. 1. The Lorentz force resulting from the interaction of these two fields was perpendicular to both of them, as in Fig. 1, and acted in the spanwise direction. Since the magnetic induction decayed as one moved away from the surface of the magnet, the magnitude of the Lorentz force too exponentially decreased with distance from the actuator surface. This gradient of the nonconservative Lorentz force acting on the current carrying ions in the electroconductive fluid generated vorticity perpendicular to the electrode/magnet direction. In the present experiments, this vorticity was in the streamwise direction. The relationship between Lorentz force and streamwise vorticity is shown in the Appendix.

As shown in Fig. 2, the actuator was flush mounted on a two-dimensional hydrofoil model that was placed in a water tunnel. The model spanned the entire width of the tunnel. In order to make the water electrically conducting, sodium nitrite (NaNO_2) salt was added to the water in the tunnel to a conductivity of $\sigma \approx 1.8 \text{ S/m}$. In order to reduce corrosion in

salt water, the magnets were coated with a thin layer of polyurethane. The hydrofoil model had an elliptical leading edge with a 5:1 length-thickness ratio and a trailing flap inclined at 20° with respect to the freestream, causing the flow to fully separate at the ramp in the absence of actuation. A schematic of the hydrofoil and the actuator is shown in Fig. 2.

In order to quantify the extent of flow reattachment due to spanwise Lorentz force actuation, PIV was used to measure velocity fields in a window approximately 4.2×2.8 in. in size below the trailing flap in a vertical plane. This interrogation window is shown in Fig. 2. Silver coated hollow glass spheres with a mean diameter of $40 \mu\text{m}$ were used as seeding particles. A CCD camera with a 960×640 pixel spatial resolution acquired 15 image pairs/s, which were processed by using 32×32 pixel interrogation windows²² by a cross-correlation scheme. The resulting vector spacing was $\approx 1.8 \text{ mm}$. An uncertainty of 1 mm/s in velocity measurements is estimated.²³

A unipolar square wave with 50% duty cycle generated by a function generator was amplified by a power amplifier and used to power the actuator. Although bipolar forcing was also tried initially, it did not result in any improvements over

FIG. 2. Schematic of the hydrofoil and actuator. Note the sharp angle of 20° made by the trailing flap with respect to the freestream flow. The rectangle below the trailing flap is the interrogation region over which PIV measurements were taken. Dimensions shown are in inches. The spanwise extent of the hydrofoil was 2 ft.

the baseline flow to be discussed in Sec. IV A and was not pursued further. The velocity fields measured with PIV were used to evaluate actuator performance at various frequencies and amplitudes of forcing. The forcing parameters are discussed in the next section.

III. FLOW PARAMETERS

The performance of the actuator was studied at two freestream tunnel speeds $U=0.1$ and 0.2 m/s. For both speeds, it was of interest to characterize the flowfield upstream of the point of separation at the flap vertex. Dye visualizations indicated that for both tunnel freestream speeds, the boundary layer over the actuator remained laminar in the absence of forcing. The Reynolds numbers Re_x based on the freestream velocity U and the distance x from the hydrofoil leading edge to the ramp vertex were 23 000 and 46 000 for the lower and higher tunnel speeds, respectively. As is common in low Reynolds number experiments, the laminar boundary layer over the actuator underwent transition at the ramp vertex where it began separating, resulting in a turbulent wake.

The EMHD forcing was characterized by two nondimensional groups, the first of which was the interaction parameter N defined as⁶

$$N = \frac{j_0 B_0 a}{\rho U^2}, \quad (1)$$

where j_0 , B_0 , ρ , and a are the current density, surface magnetic induction of the permanent magnets, the fluid density, and the electrode spacing, respectively. The Lorentz force per electrode pair can be estimated as⁶

$$F_{Lz} \sim i_0 B_0 a \approx j_0 a^2 w B_0, \quad (2)$$

where i_0 is the current flowing across an electrode pair and w is the spanwise width of an electrode. The flow dynamic pressure $\frac{1}{2}\rho U^2$ can be taken as a measure of the inertia force per unit area of the actuator. Then, ignoring the constant factor of 2, N , which is the ratio of F_{Lz} to the inertia force, takes the form of Eq. (1).

The second nondimensional group characterizing EMHD forcing was the reduced frequency f^* defined as

$$f^* = \frac{f}{f_{\text{wake}}} = \frac{f \delta_{\text{wake}}}{U} \sim \frac{fL}{U}. \quad (3)$$

Here, δ_{wake} is the size of the wake, which is estimated to be the projected dimension of the flap, $L \sin(20^\circ)$. However the factor $\sin(20^\circ)$ is dropped in Eq. (3) as it was held constant in our setup. A flow frequency characteristic of the separation occurring at the ramp was the global vortex shedding frequency f_{wake} in the wake, which scales as U/δ_{wake} . This frequency was used to scale the forcing frequency in Eq. (3).

For a given amplitude of voltage output from the power amplifier, there was some frequency-dependent attenuation of voltage at the actuator terminals, which was accounted for in the calculation of N . The interaction parameter calculated by using the nominal value of the input voltage is denoted by N_{max} . The current density j_0 in Eq. (1) was calculated as

TABLE I. Parameter ranges for which separation control experiments were performed.

Tunnel speed (U) (m/s)		0.1	0.2
Reynolds number	Re	1.14×10^4	2.3×10^4
Low current ~ 9 A	N_{max}	0.31	0.076
Medium current ~ 17 A	N_{max}	0.61	0.15
High current ~ 40 A	N_{max}	1.63	0.41
Forcing frequencies (Hz)	f	1, 2, 10, 20	2, 4, 20, 40
	f^*	1.1, 2.1, 10.7, 21.3	1.1, 2.1, 10.7, 21.3

$$j_0 = \frac{\sigma V}{a}, \quad (4)$$

where σ is the conductivity of salt water in the tunnel, V is the rms of the applied voltage, and a is the electrode spacing, which is taken here to be the thickness of a row of magnets, $a=3.3$ mm. In our experiments, the load factor⁵ $\sim (V/a)/\|\mathbf{U} \times \mathbf{B}\|$ was of the order of 10^4 . Hence, the term $\sigma(\mathbf{U} \times \mathbf{B})$ is not included in Eq. (4).

The Reynolds number Re was chosen based on the flap length L and the freestream velocity U ,

$$Re = \frac{UL}{\nu}. \quad (5)$$

The flap length L , which was representative of the size of the separated region on the flap, was deemed appropriate as a length scale to characterize the flow Reynolds number in Eq. (5).^{19,24} Moreover, this choice would allow a comparison of the Reynolds number with other studies by using different forcing methods. For the two freestream tunnel speeds explored, the Reynolds number Re and the nominal values of the forcing parameters N_{max} and f^* are listed in Table I.

For each of the experimental conditions (tunnel speed, forcing frequency, and amplitude), velocity fields were measured by using PIV over at least two realizations lasting 8 s each. The velocities so obtained were averaged at corresponding spatial locations, thereby giving a time-averaged velocity field for each forcing condition. The instantaneous velocity fields were also used to obtain other information of interest, such as vorticity and percent forward flow fields, which in turn were time averaged. Measurements taken were over the interrogation window shown in Fig. 2.

In addition, PIV measurements were performed in a vertical cross-stream plane passing through the vertex of the flap. Contours of the streamwise component of vorticity were obtained by differentiating these velocity fields.

IV. RESULTS

A. Baseline cases

Figure 3 shows results from the “baseline” case, wherein no EMHD actuation was applied, at the lower tunnel speed $U=0.1$ m/s examined. Figure 3(a) shows the time-averaged velocity vector field directly obtained from PIV measurements. The separation at the flap and the resulting reverse flow can be seen from the direction of the vectors. The vectors near the bottom of the plot in Fig. 3(a) indicate the

FIG. 3. Characterization of the “baseline” case with no EMHD actuation. The freestream tunnel speed was $U=0.1$ m/s. Dimensions shown are in cm. The quantities in (a)–(c) above are time averages. (a) Velocity field, (b) streamlines, (c) vorticity contours (s^{-1}), (d) percent forward flow, (e) contours of u'_{rms}/U , and (f) contours of $\overline{u'v'}/U^2$.

freestream velocity. This vector field was integrated to obtain the streamline pattern in Fig. 3(b), which also shows the flow reversal due to separation. The time-averaged spanwise vorticity field in Fig. 3(c) shows that most of the vorticity is concentrated in the separating shear layer, while flow outside the separated region is mostly vorticity-free. Percent forward flow is the percentage of time for which the velocity component parallel to the flap at a given point is positive; it was used as a quantitative measure to assess actuator performance throughout this study. Following the terminology of Simpson,²⁵ percent forward flow of less than 95% was identified as the onset of incipient detachment (ID) of flow, less than 75% that of intermittent transitory detachment (ITD), and less than 50% that of transitory detachment (TD). Hence, percent forward flow in excess of 95% throughout the flow field was interpreted as forward flow at all times and completely attached flow.

Figure 3(d) shows the percent forward flow contours for the baseline case at the lower tunnel speed. Consistent with boundary layer separation at the flap vertex, the percent forward flow decays from 100% in the freestream flow to nearly zero as one moves into the separated zone, and the presence of a sizable zone of transitorily detached flow can be seen. Figure 3(e) shows the rms fluctuation in u scaled by the freestream speed U , and Fig. 3(f) shows the turbulent shear stress $u'v'$ scaled by U^2 . They indicate that despite the low Reynolds number flow, the separated region is turbulent. Here, u and v are velocity components parallel and perpendicular to the flap, respectively. Similar results at the higher tunnel speed examined are shown in Fig. 4. At this tunnel speed, the baseline separated region (Fig. 4) is somewhat larger in comparison to that at the lower tunnel speed (Fig. 3).

Comparison of the time-averaged streamline patterns for various forcing conditions to the baseline patterns in Figs. 3(b) and 4(b) enabled a systematic evaluation of actuator performance in separation control. The effect of the interaction parameter N , the reduced frequency f^* , and Re on actuator performance is discussed below.

B. Effect of interaction parameter

The effect of forcing amplitude for the lower freestream tunnel speed and $f^*=1.1$ forcing frequency is discussed below. The time-averaged streamline pattern for increasing values of N is shown in Figs. 5(a)–5(d), and the corresponding percent forward flow contours are shown in Figs. 6(a)–6(d). At the low forcing amplitude in Fig. 5(b), the size of the separated region can be seen to be significantly reduced in comparison to the baseline in Fig. 5(a). Also, the point at which separation occurs is seen to be pushed downstream. Increasing the forcing amplitude completely removed the reverse flow region, as seen in Fig. 5(c). However, the distorted boundary layer on the flap was significantly thicker, as seen from the ID setting in significantly away from the flap in Fig. 6(c). Also, the existence of regions near the downstream end of the flap with ITD in Fig. 6(c) suggests that the

flow was on the verge of separation when the actuator was powered at $N=0.39$. This was the first occurrence of complete elimination of TD from the flowfield resulting from EMHD actuation. At the highest value of interaction parameter N examined in this study, there was not even ID anywhere in the flowfield; the flow remained fully reattached to the flap, and the streamlines smoothly turned around the 20° bend, as shown in Fig. 5(d).

This trend of improving separation control with forcing amplitudes was generally observed for other frequencies and the higher tunnel speed as well, although the actuator was effective to varying extents at the highest forcing amplitude examined, as will be discussed next.

C. Effect of frequency

The time-averaged streamlines for increasing values of the reduced frequency f^* are shown in Figs. 7(a)–7(d). Corresponding contours of percent forward flow are shown in Figs. 8(a)–8(d). As noted in Sec. III, even though the applied voltage amplitude at the source was the same in the cases in Figs. 7(a)–7(d), there was some frequency-dependent attenuation of voltage at the electrodes, which is accounted for in the N values shown in the figure captions. This was the highest voltage amplitude examined in this study.

When the actuator was forced at $f^*=1.1$ and 2.1 [Figs. 7(a), 7(b), 8(a), and 8(b)], the reverse flow present in the baseline [Fig. 3(b)] was completely absent, and full reattachment of the flow was seen. Close to the surface of the flap, the streamlines seem to curve upward. At the higher f^* values of 10.7 and 21.3 [Figs. 7(c), 7(d), 8(c), and 8(d)], reattachment was seen too, although the distorted boundary layer on the flap was thicker compared to the $f^*=1.1$ case, as suggested by the location of the onset of ID in Figs. 8(c) and 8(d). Figures 9 and 10 show, respectively, the streamlines and percent forward flow contours for the “medium” amplitude forcing. The values of the interaction parameter N and reduced frequency f^* are shown in the figure captions.

Although the flow is seen to reattach at all frequencies examined, on comparing Figs. 8(a) and 10(a) to Figs. 8(b) and 10(b), respectively, it can be seen that the case $f^*=1.1$ produced the highest overall percent forward flow near the flap at the lowest measured value of N . Hence, it was deemed to be the frequency at which the most favorable actuation resulted.

D. Effect of Reynolds number

The performance of the actuator in separation control had a definite dependence on the flow Reynolds number. Figures 11 and 12 show the streamlines and percent forward flow contours, respectively, for forcing at the two different Reynolds numbers Re but similar values of N and f^* . Comparison of the percent forward flow contours in Figs. 12(a) and 12(b) clearly shows improved actuator performance at the higher Reynolds number for nearly the same value of f^* , in spite of the interaction parameter N being slightly lower in

FIG. 4. Characterization of the “baseline” case at the freestream tunnel speed of $U=0.2$ m/s. Dimensions shown are in cm. The quantities in (a)–(c) above are time averages. (a) Velocity field, (b) streamlines, (c) vorticity contours (s^{-1}), (d) percent forward flow, (e) contours of u'_{rms}/U , and (f) contours of $\overline{u'v'}/U^2$.

FIG. 5. Time-averaged streamlines at the lower tunnel speed for increasing values of the interaction parameter N . The reduced frequency f^* in (b)–(d) above is 1.1. (a) $N=0$, (b) $N=0.19$, (c) $N=0.39$, and (d) $N=1.03$.

FIG. 6. Percent forward flow contours at the lower tunnel speed for increasing values of the interaction parameter N . The reduced frequency f^* in (b)–(d) above is 1.1. (a) $N=0$, (b) $N=0.19$, (c) $N=0.39$, and (d) $N=1.03$.

FIG. 7. Time-averaged streamlines at the lower tunnel speed for increasing values of f^* at the same applied voltage amplitude. (a) $f^*=1.1$, $N=1.03$; (b) $f^*=2.1$, $N=1.25$; (c) $f^*=10.7$, $N=1.33$; (d) $f^*=21.3$, $N=1.33$.

FIG. 8. Percent forward flow contours at the lower tunnel speed for increasing values of f^* at the same applied voltage amplitude. (a) $f^*=1.1$, $N=1.03$; (b) $f^*=2.1$, $N=1.25$; (c) $f^*=10.7$, $N=1.33$; (d) $f^*=21.3$, $N=1.33$.

FIG. 9. Time-averaged streamlines at the lower tunnel speed for increasing values of f^* at the same applied voltage amplitude. (a) $f^*=1.1$, $N=0.39$; (b) $f^*=2.1$, $N=0.47$; (c) $f^*=10.7$, $N=0.5$; (d) $f^*=21.3$, $N=0.5$.

the latter case. This shows that there are at least three parameters, *viz.*, N , f^* and Re , that characterize separation control using Lorentz force actuators, at least for the range of parameters studied.

V. MECHANISM OF BOUNDARY LAYER REATTACHMENT

The 20° angle made by the flap with the freestream flow ensured boundary layer separation at its vertex even for low Reynolds numbers, as confirmed from the baseline results in Figs. 3 and 4. The application of the time-periodic spanwise Lorentz force above certain forcing amplitudes caused the separated boundary layer to fully reattach and to smoothly negotiate the 20° turn to the flap. Instead of directly acting against the adverse pressure gradient leading to separation, the spanwise Lorentz forces are believed to have generated streamwise vorticity to bring high momentum fluid parcels away from the hydrofoil surface into the boundary layer, thus replenishing the separating boundary layer with kinetic energy from the freestream. We believe that the mechanism is similar to that of vortex generators^{20,21} that delay stall on various applications.

Cross-stream PIV measurements enabled the measurement of streamwise vorticity generation. Figure 13 shows the time-averaged vorticity fields in the cross-stream plane discussed in Sec. III for increasing values of the interaction parameter N at the lower tunnel speed and reduced frequency

f^* of 2.1. The vorticity seems to be concentrated in pockets roughly 10 cm apart, which approximately equals two magnet lengths. Since the magnetic induction of the permanent magnets peaks at its ends, so does the Lorentz force over an actuator comprising these magnets.

In the absence of forcing, the time-averaged vorticity remained largely spread out over the separating boundary layer and at a much lower amplitude. The highest contour levels in the baseline cases are approximately $\frac{1}{2}$ of the lowest contour level in the forced cases. When the actuator was powered on at $N=0.47$, two vortex pairs separated by roughly two magnet lengths appeared near the actuator surface, as seen in Fig. 13(b). Figure 10(b) shows the performance of the actuator in separation control under these conditions. The spatial extent of the vortices increased when the interaction parameter N was increased to $N=1.25$, as shown in Fig. 13(c), enabling larger parcels of fluid from the freestream to be convected into the boundary layer. The resulting improvement in reattachment is evident from a comparison of Figs. 10(b) and 8(b). The circulation Γ around these vortices was evaluated as

$$\Gamma = \oint \mathbf{u} \cdot d\mathbf{l}, \quad (6)$$

where the integration was done along a closed vorticity contour; l represents the length along the path of integration. For each of the prominent vortices, starting from the vortex core

FIG. 10. Percent forward flow contours at the lower tunnel speed for increasing values of f^* at the same applied voltage amplitude. (a) $f^*=1.1$, $N=0.39$; (b) $f^*=2.1$, $N=0.47$; (c) $f^*=10.7$, $N=0.5$; (d) $f^*=21.3$, $N=0.5$.

and moving outward, Γ was evaluated along various vorticity contours by using Eq. (6), thereby enabling the construction of a plot of circulation versus vorticity. This relatively linear plot was extrapolated to obtain the circulation at zero vorticity or the boundary of the vortex. The highest extrapolated Γ among different vortices in a given vorticity contour plot is

reported here as the peak circulation. The peak circulations around the vortices in Figs. 13(b) and 13(c) were 12 and 14.8 cm^2/s , respectively.

A similar result holds for EMHD actuation at the higher freestream speed $U=0.2$ m/s. Figure 14 shows the stream-wise vorticity fields for this tunnel speed at increasing values

FIG. 11. Time-averaged streamlines for two different Reynolds numbers at similar values of N and f^* . (a) $Re=1.14 \times 10^4$, $N=0.39$, $f^*=1.1$; (b) $Re=2.3 \times 10^4$, $N=0.31$, $f^*=1.1$.

FIG. 12. Percent forward flow contours for two different Reynolds numbers at similar values of N and f^* . (a) $Re = 1.14 \times 10^4$, $N = 0.39$, $f^* = 1.1$; (b) $Re = 2.3 \times 10^4$, $N = 0.31$, $f^* = 1.1$.

of the interaction parameter N . Here, too, increased spatial extent of vortices can be seen at higher N by comparing Figs. 14(b) and 14(c), wherein the peak circulations around the vortices are 5.6 and 13.7 cm^2/s , respectively. The increasing strength of streamwise vortices with N directly impacted the observed improvement in actuator performance. In order to help understand the influence of various forcing parameters on actuator performance, expressions for the vorticity generated by the Lorentz force and the resultant circulation are developed in the Appendix. Consistent with the circulation values presented above and the vorticity contour plots in Figs. 13 and 14, Eq. (A6) predicts stronger vorticity generation and higher circulation with increasing forcing amplitudes.

VI. ACTUATION EFFECTIVENESS

The effect of actuation and flow parameters on a separating boundary layer was discussed in Sec. IV. The extent of reattachment as quantified by the minimum percent forward flow in the flowfield was influenced by the interaction parameter N , frequency f^* , and flow Reynolds number Re . An overall performance map for the actuator capturing these effects is shown in Figs. 15(a) and 15(b) at the lower and higher tunnel speeds, respectively, was constructed in the following manner. For each forcing frequency, the *minimum* value of percent forward flow in the flowfield at a given forcing amplitude was extracted, e.g., the minimum percent forward flow in Fig. 6(c) is 71%. Curve fitting the plot of percent forward flow versus N enabled the estimation of the interaction parameters required to overcome ID and TD throughout the flowfield. The map was constructed by plotting these N values versus forcing frequency at the two tunnel speeds with polynomial curvefits to the data. The ID and TD curves divide the N versus f^* plane into areas based on the effectiveness of the actuator. The areas above the ID lines, marked “fully attached” in Figs. 15(a) and 15(b), rep-

resent the conditions with the best performance of the actuator in separation control, while the areas below the TD line, marked “separated” in Figs. 15(a) and 15(b), represent conditions under which the actuator was ineffective.

The two dashed curves indicate that higher forcing amplitudes are needed to overcome ID with increasing frequency. While complete reattachment of the flow was seen at the lowest applied frequencies, regions of reverse flow continued to exist in the flowfield even at the maximum value of the interaction parameter N when the actuator was forced at the higher frequencies. In fact, at the higher frequencies, the curvefits to percent forward flow versus N asymptoted to values less than 95, due to which the ID curves in Fig. 15 (the dotted lines) are restricted to the lower frequencies examined. At very high frequencies, one would expect, consistent with Fig. 15, the flow to be unable to keep up with the actuation, preventing the latter from being effective in separation control. In this limit, the circulation Γ , whose role was clarified in the previous section, vanishes, as seen from Eq. (A7).

At high forcing amplitudes, the electrolysis of water produced intense bubbling at the actuator surface due to the formation of hydrogen gas, which was continuously swept away by the freestream flow. To verify that the observed flow reattachment over the 20° flap was indeed due to EMHD actuation, the electrodes were powered and PIV measurements taken when the magnets on the actuator were replaced with blank stripes of plastic. Doing so still resulted in intense bubble production, but Lorentz forces and the consequent streamwise vorticity were absent. The PIV measurements indicated that there was no flow reattachment in this case wherein flow and actuation conditions were identical to those for which full reattachment was seen with Lorentz forces present. This confirmed that the observed reattachment was a consequence of EMHD forcing.

In light of the mechanism of reattachment proposed in Sec. V, the interesting question of whether the entire actuator

FIG. 13. Contours of streamwise vorticity (s^{-1}) in a vertical cross-stream plane passing through the flap vertex at the lower tunnel speed. The reduced frequency of forcing in (b) and (c) was $f^*=2.1$. Dimensions shown are in cm. (a) $N=0$, (b) $N=0.47$, and (c) $N=1.25$.

surface was necessary to achieve the observed complete reattachment arose. To address it, only the electrode pair furthest downstream, out of a total of eight pairs, was powered in a set of experiments at the lower tunnel speed. Though the forcing voltage amplitude was twice that in Fig. 6(d), full reattachment was seen at half the power consumption. This showed that it was adequate to concentrate the forcing near the point of separation instead of spreading it over a large area, thereby resulting in a more economical operation of the actuator.

These results are promising, and future studies should focus on identifying methods to further reduce the power consumed by actuation. More detailed exploration of frequency effects on reattachment of flow and broadening the scope of this work to include higher Reynolds number flows could render the results more readily applicable to practical devices.

VII. CONCLUSIONS

The performance of an EMHD actuator in a separation control experiment has been quantified by using PIV measurements. Time-periodic spanwise Lorentz forcing resulted in the complete reattachment of a boundary layer, otherwise separating at the vertex of a 20° flap of a hydrofoil at both freestream velocities examined. Contours of streamwise vorticity established that the action of transporting high momentum fluid from the freestream flow into the momentum starved boundary layer by the vorticity generated by the non-conservative Lorentz forces on the actuator surface is the mechanism through which reattachment is effected.

For the range of parameters investigated, interaction parameters ~ 0.5 – 1.0 and scaled frequencies of order 1 resulted in the full attachment of the flow. The forcing amplitude needed to overcome separation appears to be Reynolds

FIG. 14. Contours of streamwise vorticity (s^{-1}) at the higher tunnel speed in a vertical cross-stream plane passing through the flap vertex. The reduced frequency of forcing in (b) and (c) was $f^* = 2.1$. Dimensions shown are in cm. (a) $N=0$, (b) $N=0.12$, and (c) $N=0.32$.

number dependent, with reduced forcing amplitude at the higher Reynolds number. Future work should examine the performance of spanwise EMHD forcing at Reynolds numbers beyond our range of $\sim 10^4$.

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APPENDIX: LORENTZ FORCES

The Navier–Stokes equations governing an incompressible fluid in the presence of the Lorentz body force are

$$\nabla \cdot \mathbf{u} = 0, \quad (\text{A1})$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho} \mathbf{J} \times \mathbf{B}, \quad (\text{A2})$$

where p , ρ , and ν are the pressure, density, and kinematic viscosity, respectively. $\mathbf{u}$, $\mathbf{J}$, and $\mathbf{B}$ are the velocity, current density, and magnetic induction vectors, respectively. Assuming constant density and viscosity, the curl of Eq. (A2) gives the vorticity equation

$$\frac{D\omega}{Dt} = \omega \cdot \nabla \mathbf{u} + \nu \nabla^2 \omega + \frac{1}{\rho} \nabla \times (\mathbf{J} \times \mathbf{B}). \quad (\text{A3})$$

For the case where the electrodes are oriented in the spanwise direction, the actuator creates a spanwise force F_z , which is nearly uniform in the streamwise (x) and spanwise (z) directions,^{4,5}

FIG. 15. Overall performance map of the actuator at the (a) lower and (b) higher tunnel speeds. The interaction parameter needed to overcome ID and TD throughout the flowfield is shown.

$$F_z \approx j_0 B_0 \exp\left(-\frac{\pi y}{a}\right), \quad (\text{A4})$$

where y is the perpendicular distance from the actuator surface and a is the electrode spacing. Equation (A3) reveals that a spanwise Lorentz force is a source of streamwise vorticity as

$$\rho \frac{d\omega_x}{dt} \mathbf{i} \approx \nabla \times (F_z \mathbf{k}) = -\frac{\pi j_0 B_0}{a} \exp\left(-\frac{\pi y}{a}\right) \mathbf{i}. \quad (\text{A5})$$

The derivative of F_z provides an estimate for the magnitude of streamwise vorticity generated per unit time. If the actuator is driven by a unipolar square wavetrain of frequency f and duty cycle α , the magnitude of the resulting streamwise vorticity generated per cycle (ignoring sign) would be

$$\omega_x \sim \frac{\pi j_0 B_0}{\rho a f} \alpha \exp\left(-\frac{\pi y}{a}\right). \quad (\text{A6})$$

Applying Stokes' theorem to Eq. (6) and integrating Eq. (A6) over the entire flowfield gives the circulation Γ_z generated per unit spanwise distance z as

$$\Gamma_z = \frac{\pi j_0 B_0 \alpha}{\rho a f} \int_0^\infty \exp\left(-\frac{\pi y}{a}\right) dy = \frac{j_0 B_0 \alpha}{\rho f}. \quad (\text{A7})$$

This expression indicates that the strength of streamwise vortices increases with the amplitude of Lorentz forcing and decreases with frequency.

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