Horizontal vortex ring motion in linearly stratified media

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Laminar vortex rings propagating horizontally in linearly stratified media were investigated in a water tank using the laser-induced fluorescence flow visualization technique. The vortex rings were observed to develop an asymmetry where the upper part of the ring became larger in the cross-sectional area than the lower part. The asymmetry was a result of the instability of the interface on the upper portion of the ring; the upper half of the vortex was heavier than its immediate surroundings, whereas the lower half was lighter. When the circulation-based internal Froude number was smaller than 1.3, the vortex rings retained their overall geometry and continually decreased in size by leaving significant amounts of dyed fluid behind in their wake. At Froude numbers greater than 3.2, the asymmetry was more pronounced, both in the relative position of the upper and lower cores as well as their sizes, and the overall vertical extent of the rings did not decrease until some time after the vortex formation. These observations led to a two-level classification of the collapse process. The collapse time and distance increased with Froude number at any Reynolds number. For a given Froude number and discharge time, increasing the Reynolds number delayed the vortex ring collapse. The normalized collapse time and distance for the lower Froude number vortex rings in the present study were found to scale with the composite parameter $Re Fr^{2/3}$. © 1997 American Institute of Physics. [S1070-6631(97)00109-8]

I. INTRODUCTION

The study of formation, propagation, and decay of vortex pairs and rings has been pursued for more than a century, in part due to their possible technological applications.¹⁻⁴ A number of review articles on vortex stability and interactions have appeared in the past two decades.⁵⁻⁷ The article by Shariff and Leonard⁷ provides an up-to-date survey of various developments specific to the field of vortex rings. Nearly all of the work surveyed in that review article dealt with the motion of vortex rings in homogeneous media. On the other hand, studies of vortex rings in nonhomogeneous media are relatively scarce, perhaps due to the extra complications caused by the interaction of baroclinically generated vorticity with the vortex core. The practical application of aircraft trailing vortices descending through the atmosphere, which was reviewed by Widnall,⁵ has prompted a number of studies of vortex pairs propagating in weakly stratified media.⁸⁻¹⁰ The interaction of vortex pairs and rings impinging on a density interface has been studied by Dahm *et al.*¹¹ and Linden¹² in the laminar and turbulent regimes, respectively. Other relevant studies include the vertical motions of buoyant vortex rings in neutral surroundings by Chen and Chang¹³ and in stably stratified surroundings by Turner.¹⁴ Maxworthy¹⁵ has also carried a series of preliminary experiments on turbulent rings descending into a linearly stratified tank.

In the present paper we are concerned with the motion and collapse of laminar vortex rings propagating horizontally, i.e., perpendicular to the gravity vector, in a linearly stratified medium. The motivation for this work stems from the dispersion of biologically important chemicals in the stratified regions of the oceans. Jumper and Baird¹⁶ hypothesized that deep-sea fish communicate effectively by releasing species-specific pheromones in their surroundings. The

pheromone is thought to be impulsively discharged. The pheromone release was modeled in the present study as a laminar vortex ring ejected horizontally in a linearly stratified medium. The time and distance it takes for the vortex ring to eventually collapse into a flattened patch has biological significance.

Previous research in the area of horizontal vortex rings in stratified media has been carried out by Honji and Tatsuno¹⁷ and by van Atta and Hopfinger.¹⁸ The former described the motion and structural pattern of a horizontal vortex ring propagating into a tank with a thick density interface located at the vortex ring midlevel. The top portion of the ring, which was in the lighter fluid, traveled farther than the lower portion of the ring. Their photographs revealed complicated and distorted vortex lines. The experiments of van Atta and Hopfinger were conducted in a linearly stratified tank. The rings were visualized by the shadowgraph technique, taking advantage of the index of refraction changes afforded by the density gradients. The vortex rings in their experiments slowed down while the basic ring structure was retained. van Atta and Hopfinger¹⁸ noted a "more diffuse" upper portion of the ring, in comparison with the lower half, as well as a wake behind the ring. These observations appear to contradict the earlier experiments of Honji and Tatsuno; in the experiments of Ref. 18 the ring retained its overall structure until it collapsed, whereas in Ref. 17, the ring became contorted and lost its toroidal geometry.

The experiments described here are similar to those in the study of van Atta and Hopfinger,¹⁸ with the exception of a broader Froude number range and smaller Reynolds numbers. The laser-induced fluorescence (LIF) technique was used to visualize the internal structure of the laminar vortex rings, with the hope that further insight into the collapse of vortex rings could be gained. In Sec. II we develop the dimensionless parameters of the problem while the experimen-

tal apparatus is described in Sec. III. Results are presented in Sec. IV and a discussion of the findings follows in Sec. V.

II. DIMENSIONLESS PARAMETERS

The vorticity transport equation for incompressible flow of a constant kinematic viscosity (ν) fluid in the presence of conservative forces is

$$\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \frac{1}{\rho^2} \nabla \rho \times \nabla P, \quad (1)$$

where ρ , P , $\boldsymbol{\omega}$, and $\mathbf{u}$ are density, pressure, and the vorticity and velocity vectors, respectively. The first three terms of Eq. (1) are identical to those in the expression for barotropic fluids. In the presence of stratification/inhomogeneous fluids, the last term, referred to as baroclinic torque, contributes to the creation of vorticity. The pressure gradient in Eq. (1) contains both the hydrostatic and the hydrodynamic components, even though the contributions of the latter are much smaller than the former in liquids. Separating the pressure gradient into its hydrodynamic, $\nabla p'$, and hydrostatic, $-\nabla(\rho g z)$, components results in the following baroclinic torque terms:

$$\frac{1}{\rho^2} (\nabla \rho \times \nabla p' - \rho g \nabla \rho \times \hat{k}), \quad (2)$$

where the gravity vector is taken to be along the positive z direction.

In order to nondimensionalize the vorticity transport equation, appropriate length and time scales are required as well as a measure of density stratification. The natural length scale for the problem at hand is the vortex ring diameter δ . However, the vortex generator diameter D will be used as the length scale since, to the first order, the ring diameter is proportional to D . The approximation refers to the weak dependence of δ/D on the stroke length L , i.e., δ/D is equal to $1.18(L/D)^{1/3}$ for $L/D < 1$ and to $1.18(L/D)^{1/5}$ for $L/D > 1$ for tube-like generator geometries.¹⁹ The vortex ring circulation Γ divided by D will provide the velocity scale, and the corresponding time scale will be D^2/Γ . The buoyancy frequency $N = [- (g/\rho_0) d\rho/dz]^{1/2}$ will be used to make the density gradient in Eqs. (1) and (2) dimensionless; ρ_0 is a reference density. Last, the hydrodynamic pressure will be normalized by $\rho_0(\Gamma/D)^2$. The dimensionless vorticity transport equation has the following form:

$$\begin{aligned} \frac{D\tilde{\boldsymbol{\omega}}}{D\tilde{t}} = & (\tilde{\boldsymbol{\omega}} \cdot \tilde{\nabla}) \tilde{\mathbf{u}} + \left(\frac{\nu}{\Gamma} \right) \tilde{\nabla}^2 \tilde{\boldsymbol{\omega}} - \left(\frac{DN^2}{g} \right) \frac{1}{\tilde{\rho}^2} \tilde{\nabla} \tilde{\rho} \times \tilde{\nabla} \tilde{p}' \\ & + \left(\frac{N^2 D^4}{\Gamma^2} \right) \frac{1}{\tilde{\rho}} \tilde{\nabla} \tilde{\rho} \times \tilde{k}, \end{aligned} \quad (3)$$

where the parameters with overtilde denote dimensionless quantities.

In interpreting the above equation, the following dimensionless parameters emerge: (i) the circulation-based Reynolds number, $Re = \Gamma/\nu$; (ii) the ratio of stratification length scale to the flow length scale, $g/N^2 D$; and (iii) the internal Froude number, $Fr = \Gamma/ND^2$. The hydrostatic baroclinic torque term in Eq. (3) will be dominant when the Froude

number is of the order of unity and Re and $g/N^2 D$ are much greater than Fr . In this case, viscous diffusion and hydrodynamic baroclinic torque terms become smaller than the hydrostatic baroclinic torque term in Eq. (3). The flow length scale, i.e., D , is always smaller than the stratification length scale g/N^2 in practical laboratory experiments, including the present ones. Therefore, the term containing $N^2 D/g$ will not be an important factor in the present setup. A similar treatment of the vorticity equation for the case of vortex rings impinging vertically on a density interface has been carried out by Dahm *et al.*;¹¹ their dimensionless parameters are somewhat different from the present ones due to the differences in the two settings.

Though the issue of vortex core structure has not yet been mentioned, it plays a crucial role in the distribution of vorticity inside the ring. A dimensionless parameter characterizing the core vorticity distribution is the ratio of the core to the ring diameters. This parameter, in turn, depends on several parameters associated with the ejection and formation stages.²⁰ One of these is the stroke length L of the vortex generator divided by the exit diameter D . Since D was used as the length scale instead of the vortex ring diameter, the vortex generator aspect ratio L/D could be used as another relevant dimensionless parameter to represent the core-to-ring diameter ratio. Moreover, L/D can be readily determined from the geometry of the generator. Thus, in the limit of weak stratification, the structure and the interactions of a laminar vortex ring with a linearly stratified media are completely determined by the Reynolds and Froude numbers as well as the generator aspect ratio. These parameters will be utilized to classify the results.

III. EXPERIMENTAL APPARATUS

The experiments were carried out in a water tank that was 75 cm long, 30 cm wide, and 30 cm high. Optical access was available through the glass sides and bottom; the top of the tank was open.

A. Vortex ring generator

The vortex generating system consisted of three main components: a pressurized reservoir, a normally closed solenoid valve, and a conical converging nozzle. The reservoir was a clear acrylic cylinder with an internal diameter of 25 cm. The reservoir was partially filled with the vortex fluid, and the air pressure above the free surface was held constant by a regulated compressed air line. The pressure in the reservoir was set between 30 and 70 psig for different ring circulation values. A plastic pipe connected the reservoir to the solenoid valve, which had an opening time of approximately 20 ms. The opening and closing of the solenoid was controlled by a timing circuit and a solid state relay. This arrangement allowed for a repeatable period in which the valve was open. This period will be referred to as the discharge time T and was set at either 0.3 or 0.5 s.

Another plastic pipe, which pierced the free surface in the water tank, connected the solenoid valve to the nozzle. The conical nozzle had an area contraction ratio of 4 and a 2.5 cm exit diameter. The nozzle centerline was placed in the horizontal direction, approximately 15 cm above the bottom

FIG. 1. Schematic of the water tank and the vortex generator.

of the water tank. The nozzle exit was covered by a thin plate to avoid premature release of the nozzle fluid. The cover was gently removed by a fine wire prior to each run. Even though the removal of the cover caused some disturbances in the tank and the nozzle fluid, these motions did not significantly affect the vortex ring, as evidenced by the comparable rings produced in a homogeneous tank with and without the cover plate. A similar cover was used in the experiments of van Atta and Hopfinger.¹⁸ A schematic drawing of the vortex generator and the water tank is shown in Fig. 1. All vortex rings in unstratified water reached the opposite wall, which was 25 D away from the nozzle exit.

An estimate of the vortex ring circulation was required since exit velocity profiles and time histories $u_e(r, t)$ were not measured. The nozzle velocity was assumed constant across the nozzle exit area, and the velocity was presumed to be trapezoidal, i.e., increase linearly from zero to $U_{\max}$ during the valve opening time of $t_{\text{valve}} = 20$ ms, remain constant for $(T - 2t_{\text{valve}})$, and then decrease linearly to zero during a closing time of $t_{\text{valve}} = 20$ ms. The assumed velocity time history resulted in an estimate for circulation, which, based on the slug model,⁷ was solely determined by $U_{\max}$ as follows:

$$\begin{aligned} \Gamma_{\text{slug}} &\approx \frac{1}{2} \int_0^T u_e^2(t) dt \\ &= \frac{1}{6} U_{\max}^2 (3T - 4t_{\text{valve}}) = \frac{1}{2} U_{\max}^2 (T - 0.0267). \end{aligned} \quad (4)$$

The equivalent stroke length L can also be calculated from the presumed velocity time history:

$$L = \int_0^T u_e(t) dt = (T - t_{\text{valve}}) U_{\max} = (T - 0.02) U_{\max}. \quad (5)$$

In order to estimate $U_{\max}$ at each operating condition (reservoir pressure and discharge time), the total volume ejected from the nozzle in ten consecutive runs was collected and measured. The equivalent stroke length was then found by dividing the average discharge volume in these runs by the nozzle exit area. For each discharge time, $U_{\max}$ was com-

puted using Eq. (5). The slug circulation could then be calculated via Eq. (4). In the present setup, Γ_{slug} ranged from 5.8 to 9.9 cm^2/s and from 7.6 to 14.0 cm^2/s for the 0.3 and 0.5 s discharge times, respectively. The ranges of corresponding slug Reynolds numbers, Γ_{slug}/ν , are 560–950 and 730–1350.

There are two concerns that should be addressed at this point. The first involves the accuracy of the estimated vortex ring circulation, and the other pertains to the interdependence of Reynolds number and L/D . While no circulation measurements were carried out, Didden's⁴ measurements of the circulation of laminar vortex rings showed that the slug model underpredicts the actual values by a factor of $1.14 + 0.32 (L/D)^{-1}$ for $\text{Re} < 7000$. Furthermore, the difference between the predicted and measured values of circulation was found to be weakly dependent on the Reynolds number. Therefore, we expect the slug model to provide a consistent and reliable measure of vortex ring circulation, even though the actual value may be as much as 60% greater than Γ_{slug} at the smallest L/D . Additionally, either replacing the trapezoidal velocity time history with an impulsively started constant nozzle velocity or assuming valve opening times twice as long as the present value of 20 ms alters the estimated Γ_{slug} by less than 5%. Thus, in the following discussions, the stated values of circulation and Reynolds number are those calculated from the slug model. The relative accuracy of these values with respect to each other is about $\pm 5\%$, allowing a comparison of the Froude number effects in the present experiments. The larger absolute error in our circulation estimates is not very sensitive to the changes in the experimental conditions.

Regarding the interdependence of Reynolds number and L/D , for a constant nozzle diameter and discharge time, the slug length has to be enlarged to increase the vortex circulation. The two dimensionless parameters of Re and L/D will therefore increase concurrently. The variation of L/D against the Reynolds number in the present setup is plotted in Fig. 2 for the two discharge times. Applying Didden's correlation for $\Gamma_{\text{actual}}/\Gamma_{\text{slug}}$ with L/D to the present values revealed that

FIG. 2. Stroke length as a function of the Reynolds number for the two discharge times of 0.3 and 0.5 s.

the changes in the former are less than 10% for each of the two discharge times. It can then be assumed that the actual values of circulation are nearly independent of variations in L/D for a given discharge time, and the variations of any dependent parameter with the (slug) circulation are primarily Reynolds number effects. On the other hand, for comparable Reynolds numbers produced by different discharge times, any variations of the dependent parameters will reflect L/D effects. As will be seen, these variations are relatively small for most of the results.

B. Stratification

The water tank was stratified using the well-known two-tank method of Oster.²¹ Brine solutions of continuously decreasing strengths were created in a separate reservoir and were poured over a foam float that remained on the water surface as the tank filled up. The 1 cm thick float prevented splashes, reduced any mixing caused by the vertical momentum of the feeding tube, and distributed the brine evenly. Once the tank was filled, the float was removed. This technique produced a linearly stratified tank, except near the top and bottom surfaces. To verify the linearity of the stratification and to measure its strength, a conductivity probe that measured the salt concentration was utilized. Density is linearly proportional to salt concentration for the present ranges of brine strengths.

Prior to any experiment, the probe was slowly introduced into the tank at a location away from the nozzle, and measurements were taken at regular depth intervals. Plots of conductivity against depth always showed that the central 20 cm of the tank was linearly stratified. The density gradient and the density at the nozzle centerline were extracted from such plots. By setting the reference density ρ_0 in the buoyancy frequency expression equal to the value at the nozzle

centerline, N was calculated. It varied from 0.28 to 1.7 rad/s in the present experiments.

Care was taken to maintain the nozzle fluid at the same density as that in the stratified tank at the nozzle centerline. The top of the vortex ring was therefore slightly heavier than its immediate surroundings, and the bottom of the ring was slightly lighter. This density matching ensured that the mean vortex trajectory would remain horizontal in the tank. Further, the tank was refilled frequently, typically after three to four runs, since the collapse of vortex rings eroded the linearity of stratification.

C. Flow visualization

The nozzle fluid consisted of brine solutions mixed with dye. Both ordinary and fluorescent dyes were used in the experiments. The ordinary dye was useful for the examination of the external features of the vortex ring and for the measurements of collapse distance and time. A weak solution ($\sim 10^{-6}$ molar) of disodium fluorescein, a fluorescent dye, and a laser sheet were used to visualize the internal features of the ring. The laser sheet, shown in Fig. 1, was always vertical and was aligned with the nozzle axis. The laser sheet was approximately 1 mm thick and was generated by reflecting the blue line of an argon laser beam off an oscillating mirror. The 1.2 kHz oscillation frequency of the mirror was sufficiently rapid that the beam appeared as a sheet in still and video images. The video images at 30 frames/s were used for the measurements of collapse distance and time, while the still images, acquired with a 35 mm camera at an exposure of 1/60 s, were used for the investigation of fine flow details. Since the index of refraction varies with salinity, the laser-induced fluorescence technique did not produce very clear images of the internal structure of the ring at high stratification strengths, primarily due to the distortion effects of density gradients. However, for $N \leq 0.5$, the LIF images clearly revealed the internal vortex structures.

IV. RESULTS

A. Qualitative observations

The evolution of a single vortex ring with $Fr=2.1$ and $Re=950$ is shown in Fig. 3. The camera was moved manually to keep the ring within the field of view. These LIF images reveal the vertical cross section of the vortex ring. As expected, the initial vortex ring is symmetric and appears undisturbed by the stratification. Despite the lack of structural detail within the vortex, which is due to the density gradient-induced optical distortions, several features are still readily discernible. The ring developed an asymmetry whereby the upper portion of the ring appeared somewhat larger than the lower one, as seen in Figs. 3(b) and 3(c). The maximum vertical extent of the ring decreased continually with time throughout its trajectory, from Figs. 3(a)–3(d). An important feature of the vortex ring in these images is that it retained its ring-like structure. The horizontal distance between the two arrows marking the centers of the upper and lower vortex cores in Fig. 3 does not vary appreciably with time. The relative position of the arrows indicates that the

FIG. 3. Vertical cross sections of a vortex ring propagating in a linearly stratified tank. The vortex moved from right to left; $Re=950$ and $Fr=2.1$. The ring collapsed completely just after the photograph in (e). The vertical arrows indicate the approximate centers of the upper and lower vortex cores.

upper core tends to lag behind the lower core, and in none of the images in Figs. 3(a)–3(d) can the “overtaking” of one vortex core by the other be clearly detected. Fluid was continuously left behind by the ring in its wake after the formation stage. This process persisted until the vortex collapsed completely. At this point, the vortex very nearly ceased its

forward propagation. Visual observations from the top revealed that the fluid left by the vortex in its wake, spread in the horizontal plane. During the collapse, the ring fluid was transformed to a horizontal patch of finite thickness that moved forward extremely slowly. When viewed from the top of the tank, this horizontal patch consisted primarily of a pair of weak, counter-rotating eddies. This collapse process will be referred to as type I. The present observations of the initial symmetry, appearance of the wake, and the development of asymmetry were also noted in the shadowgraph images of van Atta and Hopfinger¹⁸ for similar Froude numbers.

A different collapse sequence is revealed in Fig. 4, in contrast to that observed in Fig. 3. The vortex ring in this run had the same Reynolds number (950) as before; however, the weaker ambient stratification had resulted in a Froude number of 4.8. The center of the upper core advanced noticeably farther than the lower core, see Figs. 4(c)–4(e). Not only the displacement of the upper core with respect to the lower core in this sequence increased with time, but also it was significantly greater than that observed with type I collapse in Fig. 3. The upper core was consistently farther along than the lower core, indicating that the upper core was “overtaking” the lower one. Moreover, the vertical extent of the upper portion of the ring became considerably larger than the lower portion [Figs. 4(c)–4(e)] and notable amounts of ambient fluid were entrained into the ring. A wake emerged behind the ring, specifically after the onset of asymmetry. Just prior to the collapse, the ring structure became highly distorted [Figs. 4(d) and 4(e)], and the fluid within the core was mixed with the entrained ambient fluid. This is evidenced by the uniformity of the fluorescence intensity in Fig. 4(e). The maximum vertical extent of the ring did not decrease until after the establishment of the asymmetry, in contrast to the continual decrease of the ring extent at low Froude numbers. The reduction of the ring size is primarily due to the loss of material to the wake and partially due to the relative displacement of the upper and lower parts of the ring. This collapse sequence was termed type II.

Even though the two collapse sequences described have a number of similarities, they can be clearly distinguished by (i) the consistent and appreciable farther advancement of the upper core with respect to the lower core present in the type II and absent in the type I collapse; (ii) the significant asymmetry, both in core size and relative position of the upper and lower cores, as well as extensive distortions associated with the type II collapse, as opposed to the approximately ring-like structure of the type I collapse; and (iii) the continuous reduction of the vertical extent of the vortex ring throughout the lifetime of the type I collapse, in contrast to the initially constant size and then the reduction following the onset of asymmetry in the type II collapse. The images of the type II collapse have a number of features in common with the observations of Honji and Tatsuno,¹⁷ namely the asymmetry of the ring and the advancement of the upper part over the lower part.

An objection may arise regarding the initial asymmetry of the vortex ring in Fig. 4. The upper half of the ring appeared to have entrained a certain amount of ambient fluid, whereas the lower half contained relatively purer nozzle

FIG. 4. Vertical cross sections of a vortex ring with $Re=950$ and $Fr=4.8$. Note the asymmetric development of the ring where the upper portion of the ring became larger and advanced farther than the lower portion. Substantial amounts of vortex material were left in the wake only after the vortex traveled a certain distance, as in (d). The vertical arrows indicate the approximate centers of the upper and lower vortex cores.

fluid. This initial asymmetry is believed to be caused by the unstable vertical interface present at the nozzle exit just before the vortex ring was released. On the nozzle side of the interface, the density was uniform, while a density gradient existed in the tank. The nozzle density was equal to the tank

density at the nozzle centerline. The difference between the tank density and the nozzle fluid density near the top and bottom edges of the nozzle will create a disturbance across the interface after the cover is removed. This can be argued as the source of vortex ring nonuniformity during the ring formation stage. The distortions at the interface between the tank fluid and the nozzle fluid will have a time constant of the order of N^{-1} . This time is to be contrasted against the ring injection times of 0.3 and 0.5 s. For the range of N in the present experiments, the interface distortion time varied from 0.6 s at the largest N to 3.6 s at the lowest N . Thus, the injection times are shorter than the time associated with interface distortions, especially for the higher Froude number rings with smaller N values. Moreover, the observed development of the ring following the formation stage was not greatly dependent on the initial vortex formation. Therefore, we suggest that the described collapse sequences are robust in spite of the minor differences in the formation stage.

The experiments revealed that for the same Reynolds number, two different collapse sequences could be observed, depending on the Froude number of the ring. The specifics of the present observations agree closely with the findings of van Atta and Hopfinger¹⁸ at the low Froude numbers and with those of Honji and Tatsuno¹⁷ at higher Froude numbers. On the basis of the present observations, the collapse sequences were classified into two categories according to the differentiating features described earlier. Namely, the type II collapse was identified by the consistent advancement of the upper core with respect to the lower core as well as the distortions and significant asymmetry of the upper and lower core sizes. Type II rings did not sustain a reduction in their vertical extent until after the asymmetry was established. On the other hand, the type I collapse was associated with vortex rings whose cores retained their relative position with respect to each other (i.e., an approximately ring-like structure) as well as the continuous reduction in the vertical extent of the ring following the formation stage. Although some rings appeared to have characteristics in common with both categories, they were only observed over a limited range of parameters.

A classification diagram of the collapse type in terms of the Froude and Reynolds numbers is shown in Fig. 5. The “mixed type” rings had Froude numbers of about 2.5 and are not shown in the diagram. Within the present Reynolds number range of 560–1350, the boundary between the two regimes is at $1.3 < Fr < 3.0$ near the lowest Reynolds numbers and at $2.3 < Fr < 3.2$ near the largest Reynolds numbers. Thus, the maximum Froude number below which only type I collapses were observed increased with Reynolds number from 1.3 to 2.3. On the other hand, the minimum Froude number beyond which only type II collapses could be detected increased from 3.0 to 3.2. It has to be emphasized that the Froude and Reynolds numbers are based on estimates of the slug circulation for each injection time. Therefore, the mean boundary between the two regimes shifts to higher Froude numbers with increasing Reynolds numbers. Of course, vortex rings become turbulent at high Reynolds numbers, and the collapse may become completely different.

Further support for the boundary between the two col-

FIG. 5. The Re-Fr diagram marking the regions where the two collapse types could clearly be identified. The transition region between the two types had collapse characteristics common to both types.

lapse types comes from the descriptions of van Atta and Hopfinger.¹⁸ They observed only features similar to the type I collapse at Reynolds numbers up to 2000 and Froude numbers extending to 2. A maximum Reynolds number of about 1400 in the Honji and Tatsuno¹⁷ setup can be deduced from their maximum velocity estimate of 9 cm/s. Froude numbers could not be estimated, for the latter experiments were not in a linear density stratification. However, the observations of farther advancement of the upper core with respect to the lower core and highly distorted vortex lines in Ref. 17 place their experiments within the type II collapse regime. Then, the two previous studies of vortex rings moving horizontally in stratified media can be interpreted as each producing only one kind of collapse, i.e., type I in Ref. 18 and type II in Ref. 17. In Sec. V, a possible mechanism for the two collapse sequences is proposed.

B. Collapse distance and time

In order to quantify the distance and time needed for the complete collapse of a vortex ring, an objective, readily measurable definition for the collapsed ring is necessary. It was observed that when the vortex fluid was reduced to a patch of about 5 mm vertical thickness ($\sim 20\%$ of the initial maximum extent), the forward progression of the patch was nearly halted, and little vertical motions existed beyond this point. Therefore, the 5 mm vertical extent of the collapsed patch was used as the criterion for a completely collapsed ring. The distance S from the nozzle exit and the time τ from the flow initiation (valve opening) to the collapsed state were measured from the video recordings. The uncertainty of the collapse distance and time were estimated to be within 2.5 cm and 2 s, respectively.

The normalized collapse distance as a function of Froude number is shown in Fig. 6 for several Reynolds numbers. At

FIG. 6. The dependence of the collapse distance on Froude and Reynolds numbers. The filled and open symbols denote the two discharge times of $T = 0.5$ and 0.3 s, respectively.

any Re, the collapse distance increased as the Froude number became larger. Moreover, the rate of increase of S/D decreased as Fr was increased. When Fr is less than about 1.5, S/D increased with Fr at nearly the same slope for various Re. The $Fr < 1.5$ regime is associated approximately with the type I collapse, where the continuous decrease in the vertical extent of the vortex ring resulted in shorter collapse distances. Beyond this regime, the rate of increase of S/D with Fr decreases. This is argued to be due to the changing of the collapse types, i.e., switching from the type I to II collapse. At the higher Reynolds numbers, the change in the slope of S/D occurs at larger Froude numbers, e.g. at about Fr of 3 for the largest Reynolds number. This is not surprising in light of the fact that type II collapses were only observed at Froude numbers greater than about 3.2. The minimum Froude number, beyond which changes in the S/D slope are detected, corresponds roughly with the boundary of type I collapses. Thus, the observed behavior regarding the changes in the S/D slopes can be approximately related to the change in the ring collapse sequence from type I to type II.

Increasing the Reynolds number (for any constant Froude number) extended the collapse distance monotonically, for each of the two discharge times of 0.3 and 0.5 s. This implies that once the stratification effects are fixed (relative to the inertia), the viscous effects also become important in determining the collapse distance. This is expected from the terms in Eq. (3). As the Reynolds number increases, the viscous effects become smaller and the viscous diffusion of vorticity takes a longer time. Consequently, the collapse distance increases. This behavior is similar to that of vortex rings in homogeneous surroundings, where the distance traveled between the source and the final decay of vortex ring increases with the Reynolds number.

There also appears to be an irregularity between the datasets from the two discharge times of 0.3 and 0.5 s. The

FIG. 7. The dependence of the normalized collapse time on Froude and Reynolds numbers. The filled and open symbols denote the two discharge times of $T=0.5$ and 0.3 s, respectively.

data at the Reynolds number of 1020 with the larger discharge time fall below their smaller discharge time counterparts at $Re=950$. The difference between the S/D data of $Re=1020$, $T=0.5$ s and $Re=950$, $T=0.3$ s remained constant at about $1-1.5 D$ as Froude number increased from 1 to greater than 5. Had the irregularity been due to stratification related effects, the significant variations in N , i.e., from 1.6 at the lowest Fr to 0.3 at the largest Fr , would have had to alter the difference between the two datasets as the Fr increased. Therefore, the irregularity is a result of L/D variations, which cause the actual values of Γ to be larger than Γ_{slug} for small L/D values associated with the shorter discharge time, as stated in Sec. III. In any case, the observed variations due to the discharge time (or L/D) are relatively small and do not alter the general trends in the data.

The collapse time τ was normalized with the time scale D^2/Γ proposed in Sec. II, and is plotted in Fig. 7 as a function of the Froude number for several Reynolds numbers. The collapse distance and time data were extracted from the same experiments. As expected from the collapse distance data, $(\Gamma/D^2)\tau$ increased with the Froude number for all Reynolds numbers. However, the rate of increase becomes smaller at higher Fr . Furthermore, the initial slopes of $(\Gamma/D^2)\tau$ curves increased noticeably with Reynolds number. This steepening will be explained in terms of the expression developed in the next paragraph for correlating the normalized collapse times with various initial conditions. The normalized collapse time for each discharge time also increased with Re at any Froude number. The reason for this increase is the same as that when the collapse distance was considered. Increasing the Reynolds number for fixed Fr weakens the effects of viscosity relative to inertia and, therefore, the viscous diffusion of vorticity takes a longer time. As before, an irregularity between $Re=950$, $T=0.3$ s and $Re=1020$, $T=0.5$ s exists in the collapse time data, where the latter fall

below the former. The irregularity, which is relatively small, is a consequence of different L/D parameters resulting from the two different discharge times utilized. This was explained in the previous paragraph.

To develop a parameter capable of correlating the collapse distance and time data with different initial Reynolds and Froude numbers in Figs. 6 and 7, the collapse was presumed to occur when the kinetic energy of the vortex is unable to overcome the potential energy needed to overturn fluid parcels around the vortex ring. In other words, as the ring loses vorticity to its wake, circulation associated with the ring gets reduced to the point that it cannot transport fluid parcels over the ring. The potential energy for overturning can be estimated by $(d\rho/dz)gD^2/4$, where the vertical displacement and the density difference were taken to be the ring radius $D/2$ and $(d\rho/dz)D/2$, respectively. The kinetic energy $\rho_0 V^2/2$ is that associated with the translation of the ring. Thus, vortex rings in stratified media collapse when $\frac{1}{2}\rho_0 V^2 \sim (d\rho/dz)g(D^2/4)$, or when $V \sim ND$. If we assume that the ring translation velocity V under stratification is qualitatively equal to that for unstratified laminar rings with the same initial circulation and diameter, Saffman's 1970 model²² can provide an expression for $V(t)$. In that model,

$$V \approx C_1 \frac{\Gamma}{D} \left(1 + C_2 \frac{\nu t}{D^2} \right)^{-3/2}, \quad (6)$$

where C_1 and C_2 are dimensionless constants. By setting the above expression proportional to ND at collapse time, the collapse time in terms of initial conditions follows:

$$1 + C_2 \frac{\nu \tau}{D^2} \sim \left(\frac{\Gamma}{ND^2} \right)^{2/3}.$$

Neglecting 1 in comparison with $C_2 \nu \tau / D^2$, results in

$$\frac{\Gamma}{D^2} \tau \sim \left(\frac{\Gamma}{\nu} \right) \left(\frac{\Gamma}{ND^2} \right)^{2/3} = Re Fr^{2/3}. \quad (7)$$

Thus, $Re Fr^{2/3}$ should be able to consolidate the collapse times if the above arguments are viable.

If the ring translation velocity in Eq. (6) is integrated with respect to time, an approximate expression for the temporal evolution of the vortex core can be obtained. Substituting the collapse time scaling from Eq. (7) in that expression would result in $S/D \sim Re (1 - Fr^{-1/3})$. Obviously, this expression is invalid for Froude numbers less than unity. However, for the range of Froude numbers below 3, $(1 - Fr^{-1/3})$ varies nearly linearly with $Fr^{2/3}$. Therefore, the collapse distance should also scale with $Re Fr^{2/3}$ for the lower Froude number rings.

The parameter $Re Fr^{2/3}$ in Eq. (7) is more appropriate for the type I collapse than type II because the significant asymmetry and the advancement of the upper portion of the ring with respect to the lower portion of it may invalidate the use of the translation velocity in Eq. (6) with type II rings. On the other hand, since type I rings retain their toroidal geometry, the parameter in Eq. (7) may be suitable for scaling type I collapse data. The normalized collapse time and distance are plotted against $Re Fr^{2/3}$ in Figs. 8 and 9, respectively. The type II collapse data have been excluded from these plots. The two datasets are well correlated by the $Re Fr^{2/3}$ param-

FIG. 8. The collapse time as a function of the composite parameter $\text{Re Fr}^{2/3}$ for vortex rings with type I collapse. The linear regression fit to the data set is denoted by the line.

eter and the correlation coefficients are better than 0.95 in both cases. Inclusion of type II data in Figs. 8 and 9 would result in considerably smaller correlation coefficients and much larger scatter around the regression line. The increasing initial slope of $(\Gamma/D^2)\tau$ curves with Re in Fig. 7 can now be simply explained in terms of the derivative of $\text{Re Fr}^{2/3}$ with respect to Fr . At low Froude numbers, where type I collapses are observed, $\partial[(\Gamma/D^2)\tau]/\partial\text{Fr}$ is proportional to $\text{Re Fr}^{-1/3}$. Consequently, the aforementioned slopes should increase linearly with Re .

FIG. 9. The collapse distance as a function of the composite parameter $\text{Re Fr}^{2/3}$ for vortex rings with type I collapse. The line marks the linear regression fit to the data.

V. DISCUSSION

It is perhaps surprising that the more extreme form of collapse, type II, is associated with the vortex rings having larger initial Froude numbers. One would expect that the lower Froude number rings to suffer greater deformations since the stratification effects are relatively stronger. The arguments in this section are intended to shed light on this counterintuitive behavior and to describe possible mechanisms for the two collapse sequences.

The primary influence of stratification on the vortex ring evolution is through the generation of baroclinic vorticity and the consequent mixing. Throughout this section, the vortex ring, following the formation stage, is assumed to have a uniform density equal to that in the ambient fluid at the midlevel of the ring. The top portion of the ring will then be in a lighter environment, and the bottom portion will be in a heavier environment. The density gradient across the ring perimeter gradually increases from zero at the front stagnation point to its maximum value at the top (or bottom) of the ring. Since the gravity vector is always vertical, the baroclinic vorticity generation will be largest somewhere between the front stagnation point and the top (or bottom) of the ring. The baroclinic vorticity over the front half of the ring has the same sign as that of primary vorticity in the core for both the top and bottom portions. Conversely, the baroclinic vorticity is of opposite sign to the core vorticity over the back portion of the vortex. Thus, the core vorticity is reinforced by the baroclinic vorticity over the front half of the ring, while it is opposed over the back half.

The baroclinic vorticity is envisioned to form a layer of mixed fluid over the perimeter of the ring. This layer of mixed fluid, which will be referred to as the “mixed layer” henceforth, can be observed around the ring in Figs. 3(b) and 3(c) and 4(b)–4(d), specifically around the top of the ring. The mixed layer is analogous to the viscous diffusive layer around laminar vortex rings in homogenous surroundings in the model of Maxworthy.³ The thickness of the mixed layer at the back of the ring can be estimated by the effective gravitational acceleration around the ring multiplied by the square of the time it takes a fluid parcel to travel from the front stagnation point to the back of the ring. Taking the effective gravitational acceleration and the travel time to be proportional to $N^2 D$ and D^2/Γ , respectively, the mixed layer thickness at the back of the ring will scale with D/Fr^2 . Thus, the layer thickness will rapidly decrease with the initial Froude number.

The Rayleigh criterion for the stability of rotating fluids was extended to heterogeneous fluids by Synge,²³ who proved that a necessary and sufficient condition for the stability of variable density fluids experiencing centrifugal forces is $d(\rho u_{\theta}^2/r^2)/dr > 0$. On the upper portion of the ring in the present experiments, the density gradient $d\rho/dr$ is negative. On the other hand, the density gradient is positive around the lower portion of the ring. Thus, if the circulation ($u_{\theta}r$) distribution is similar over the top and bottom portions of the ring and is confined to the dyed part of the vortex, the upper portion will be unstable, whereas the lower portion will be stable according to Synge’s criterion. Since the

FIG. 10. The schematics of vortex ring development in stratified media, (a) low Fr, type I and (b) high Fr, type II collapse processes. The shaded areas mark the baroclinically mixed fluid, while the small circular arrows indicate the sign of baroclinic vorticity. Vortex rings are moving from right to left.

mixed layer over the top of the ring is more unstable than the one on the bottom, the growth of the top mixed layer is therefore enhanced by this instability. The difference between the top and the bottom mixed layer growth rates is presumed to be responsible for the observed asymmetry.

The same generalized form of Rayleigh criterion was used by van Atta and Hopfinger¹⁸ to explain the instability and collapse of horizontally propagating vortex rings in stratified media. They argued that baroclinic vorticity erodes circulation around the perimeter of the ring, and since the radial density gradient decreases in the upper portion of the ring, it is more unstable. Although this instability mechanism is consistent with the majority of the observations, it does not explain the details of how the wake is formed at the lower Froude numbers. Moreover, the sum of baroclinically generated vorticity over the front and back of the ring has to be zero for a symmetric ring. Thus, there can be no loss of the overall circulation of the ring due to the baroclinic vorticity, and the only possible loss mechanism is through the material left behind in the wake. To provide more detail into the asymmetry and the wake formation process, we present a model of the flow based on the dynamics of parcels near the back of the vortex ring.

As the fluid parcels in the mixed layers approach the back of the vortex ring, they can either be entrained into the ring or shed into the wake. The fate of the mixed parcels approaching the rear stagnation point is determined by a local Richardson number defined by $Ri_m = g'a/w^2$, where a is the ring radius, w is the local velocity, and $g' = g|\rho_m - \rho_a|/\rho_a$. Here ρ_m and ρ_a represent the mixed and ambient fluid densities. After the vortex formation stage, this local Richardson number can be related to the inverse of the initial Froude number, i.e., $Ri_m \sim Fr^{-2}$, because $a \sim D$, $w \sim \Gamma/D$, and $g' \sim N^2 D$. As the ring evolves, Ri_m will increase due to the reduction in w . It is postulated that when Ri_m is greater

than a critical value Ri^* , a significant portion of the fluid in the mixed layers will be left behind in the wake. Otherwise, the mixed parcels are distributed according to the dynamics of laminar vortex rings in the homogeneous case. All of the mixed parcels will be deposited into the wake when $Ri_m \gg Ri^*$. Similar criteria have been used to describe the conditions under which a wake is formed behind buoyancy reversing thermals.²⁴

As a vortex ring slows down, its local Richardson number increases. At some point Ri_m becomes large enough ($> Ri^*$) to force the shedding of a substantial amount of the mixed layer parcels. Since the parcels within the bottom mixed layer have a larger density than those of the top mixed layer, more of the bottom layer is expected to be left behind. As the vortex sheds material and vorticity, it slows down further. Eventually, Ri_m becomes much greater than Ri^* , and all of the parcels within the mixed layers get left behind. When the majority of the ring material and vorticity are deposited into the wake, the ring will collapse because the kinetic energy of the remaining ring is not able to transport fluid parcels across the different densities between the centerline and top (or bottom) of the vortex.

To better illustrate the response of horizontal vortex rings to stratification in our model, schematics of the flow at low and high initial Froude numbers are shown in Fig. 10. The vortex rings are assumed to have a uniform initial density of ρ_0 and initial circulation Γ . The small circular arrows in Fig. 10 denote the location and sense of the baroclinic vorticity. In the case of small initial Froude numbers [Fig. 10(a)], the mixed layers are relatively thick, and we suspect that the upper one is somewhat thicker than the lower one. After the vortex formation stage, most of the mixed parcels in the upper and lower layers are left behind since Ri_m is greater than Ri^* . Some of the upper layer parcels may also be retained. As the ring velocity decreases, Ri_m increases

further, and nearly all of the mixed parcels get left behind in the wake. This is the case of the continuously shrinking vortex ring, corresponding to the images in Fig. 3.

The case of initial Froude numbers greater than 3 is depicted in Fig. 10(b). Initially, the local Richardson at the back of the vortex is small enough so that the mixed parcels are primarily entrained into the vortex. The asymmetry starts to develop because the upper layer is somewhat thicker than the lower one. This asymmetry causes a difference in the circulation of the top and the bottom portions, and the top portion advances farther than the lower portion. Eventually the vortex ring slows down by the vorticity diffusion mechanism and the vorticity loss to the wake, such that Ri_m increases, and all of the lower layer fluid and a portion of the upper layer fluid are left behind in the wake. The asymmetry then becomes quite pronounced, considering the lower portion of the ring is losing fluid while the upper portion is retaining some of the mixed fluid. This scenario illustrates the type II collapse.

When the initial Froude number is even greater, the vortex ring has to travel a longer distance prior to the point where a notable wake appears. In this case, the asymmetry will be present and will continue to grow. Ultimately the vortex slows down, leaves an appreciable portion of its fluid in the wake, and finally collapses. At very high initial Froude numbers, the influence of baroclinic vorticity becomes quite small; the mixed layers are quite thin. The primary effect of the Reynolds number in the above processes is in terms of the diffusion of vorticity from the ring and the rate of vortex ring deceleration.

It is interesting to speculate on how the above mechanisms would be modified in the case of a vertically descending vortex ring. First, the asymmetry would not be present, and the sign of baroclinic vorticity would be opposite that of the core vorticity if the ring is lighter than the surroundings. A Froude number behavior comparable to that in horizontal rings is expected. At low Froude numbers, the rings should continuously shed their material, whereas at higher Froude numbers the rings have to travel a certain distance before they slow down and significant shedding begins. Maxworthy¹⁵ has observed some of these features, namely the continuous shedding in turbulent vortex rings descending into a stratified tank.

VI. CONCLUSIONS

The evolution of horizontally propagating laminar vortex rings in linearly stratified surroundings has been investigated via flow visualization. The important parameters are the circulation-based Froude and Reynolds numbers. The vortex rings developed an asymmetry where the upper portion of the ring became larger in the cross-sectional area than the lower part. Significant amounts of ring fluid were also left behind in the wake at lower Froude numbers. Generally, the collapse of the ring could be divided into two regimes. At low initial Froude numbers, $Fr < 1.3$, the ring retained its overall geometry, its diameter decreased continually, and a wake was present immediately following the formation stage. At higher Froude numbers, $Fr > 3.2$, the vortex rings were quite asymmetric, both in the relative position of the

upper and lower cores and their respective sizes. The overall vertical extent of the ring did not decrease until some distance from the nozzle. These observations were explained in terms of mixed layers that are created by the baroclinic vorticity present around the vortex ring perimeter. The layer on top of the ring is more unstable than the lower one due to the extended Rayleigh instability criterion. A local Richardson number in the back of the vortex ring is hypothesized to determine the fate of mixed fluid and the loss of vorticity and material to the wake. Both the collapse distance and time increased with the Froude number for all Reynolds numbers. For a given initial Froude number and discharge time, increasing the Reynolds number delayed the collapse due to the relatively weaker viscous effects. For type I collapses, the collapse time and distance data of vortex rings with various initial conditions can be scaled by the composite parameter $Re Fr^{2/3}$.

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