Characteristic Scales of Photospheric Flows and Their Magnetic and Temperature Markers

J. K. LAWRENCE AND A. C. CADAVID

Department of Physics and Astronomy, California State University, Northridge, Northridge, CA 91330-8268;
jlawrenc@galileo.csun.edu

AND

A. A. RUZMAIKIN

Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109

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ABSTRACT

We study the characteristic scales of quiet-Sun photospheric velocity fields along with their temperature and magnetic markers in Doppler images from the Michelson Doppler Imager aboard the *SOHO* satellite (*SOHO*/MDI) in simultaneous, Doppler, magnetic, and intensity images from the San Fernando Observatory and in full-disk magnetograms and an intensity image from National Solar Observatory (Kitt Peak). Wavelet flatness spectra show that velocity fluctuations are normally distributed (Gaussian). This is often assumed in stochastic models of turbulence but had not yet been verified observationally for the Sun. Temperature fluctuations also are Gaussian distributed, but magnetic fields are intermittent and are gathered into patterns related to flow structures. Wavelet basis functions designed to detect characteristic convection cell-flow topologies in acoustically filtered *SOHO*/MDI Doppler images reveal granulation scales of 0.7–2.2 Mm and supergranulation scales of 28–40 Mm. Mesogranular flows are weakly but significantly detected in the range 4–8 Mm. The systematic flows account for only 30% of the image variances at granular and supergranular scales and much less in between. The main flows for the intermediate range of 2–15 Mm are self-similar, i.e., chaotic or turbulent.

Subject headings: Sun: granulation—Sun: magnetic fields—turbulence

§1. INTRODUCTION

This work explores the properties—including sizes, strengths, statistical distributions, and scaling symmetry—of structures in solar surface fields. We focus on the nonacoustic convective flow fields, but we also include such markers as the scalar temperature and vector magnetic fields. We examine both high-resolution, narrow-field, and low-resolution full-disk Doppler images from the Michelson Doppler Imager aboard the *SOHO* satellite (*SOHO*/MDI). In addition, we examine simultaneous coregistered Doppler, magnetic, and intensity images from the San Fernando Observatory (SFO) Video Spectra-Spectroheliograph system, as well as full-disk magnetograms and an intensity image from the National Solar Observatory at Kitt Peak (NSO/KP).

The physical problems we address are twofold. The first concerns the importance in the photospheric velocity field of coherent structures such as granules, supergranules, or mesogranules, compared with chaotic or turbulent flows. A corollary issue involves the widths and strengths of the granular, mesogranular, and supergranular peaks in the velocity spectrum. Second, we address the statistical distribution of fluctuations in the flow fields. Many theories of turbulence (Monin & Yaglom 1971, 1975; Avellaneda & Majda 1992a, 1992b) assume a priori that fluid flows are stochastic with normally distributed fluctuations. We determine observationally that this is the case for spatial-fluctuation photospheric flows.

Such familiar spectral approaches as Fourier transforms are best for determining the spatially averaged power in a field or image (essentially variance per unit frequency k) at a given scale $\lambda=2\pi/k$. Applications of these methods are limited, however, because the relevant basis functions are infinite in extent. Thus, for example, their interpretation depends on the assumption of statistical stationarity of time signals or of spatial homogeneity of images.

Wavelet basis functions are simultaneously localized in spatial position and in spatial scale. This opens new avenues for analysis, and we will follow two. One is to study properties of the statistical distributions of the solar fields beyond their variances. Wavelets permit us to choose a spatial scale and then determine the statistical moments of fluctuations of a field at that scale over a localized spatial domain. We can then average over an entire image, or over any subset, which may consist of disconnected parts such as active regions in a magnetogram. Both the numerical values of the moments and their variation with scale have physical meaning. The second avenue is to use wavelets tailored to respond to flows with particular patterns in Doppler images. Structures of interest will be granules manifested by isolated upflows surrounded by a network of downflow, and supergranular cells with horizontal center-to-edge outflows. We will examine both the statistical distributions and the characteristic scales of these structures.

The simplest measure of intermittency of a distribution is its flatness Φ , namely, the fourth moment normalized by the square of the variance. For a Gaussian distribution, $\Phi=3$. Frisch (1995) has defined the flatness of a time signal in terms of the distribution of its high-pass-filtered Fourier amplitudes, but again the Fourier spectral method relies on stationarity of the signal. We are interested in the two-dimensional spatial analog of this procedure. Although we find solar Doppler and intensity images to be statistically homogeneous with $\Phi\approx 3$, the magnetic images are intermittent ($\Phi>3$), and Φ is a strong function of the spatial scale of the fluctuations and of spatial location. Maximum intermittency occurs at different scales in active-Sun areas than in quiet-Sun (QS) areas.

The spatial localization property of the wavelets is ideal to handle this inhomogeneity. We have previously applied this technique to solar activity time series such as the Wolf sunspot number ([Lawrence, Cadavid, & Ruzmaikin 1995b](#)).

The coexistence in high Reynolds number systems of turbulent, self-similar flows and of coherent flow structures is a fluid dynamics issue of long standing (see, e.g., [Holmes, Lumley, & Berkooz 1996](#)). Taking magnetic flux as a flow-field marker, [Spruit, Nordlund, & Title \(1990\)](#) noted the fractal self-similarity of photospheric magnetic field placement, which is also reflected in the relative featurelessness of the Fourier power spectra (see also [Lawrence, Ruzmaikin, & Cadavid 1993](#); [Cadavid et al. 1994](#); [Lawrence, Cadavid, & Ruzmaikin 1995a, 1996](#)). Although characteristic granular structures may be apparent in intensity images and supergranular structures in magnetograms, [Spruit et al. \(1990\)](#) questioned whether they are really present in the flow fields. At a less general level, there also has been disagreement about the frequencies, widths, and strengths of the lines in the photospheric velocity spectrum corresponding to various convective structures (see [Ginet & Simon 1992](#)).

By extracting quantitative properties of the coherent structures from the *SOHO*/MDI Doppler data, we find that although the characteristic granular and supergranular structures are present in the flow fields, they account for only a small fraction of the variance in Doppler images, even at their own characteristic scales. Consequently, their presence remains relatively obscure to standard spectral methods, which cannot exploit their particular flow patterns.

The nature of the excess flow between the granular and the supergranular scales is an obvious question. The excess cannot be ascribed to overlapping of the granular and supergranular lines in the velocity spectrum; these peaks, with maxima at 1.4 and 32 Mm, do not sufficiently overlap. Patterned flows at intermediate scales of 4–8 Mm, expected for mesogranules, are indeed detected; however, they are too weak by 1 order of magnitude to account for the observed excess. The scales of the supergranular flows we found (28–40 Mm in diameter) agree with those cited by most authors.

We propose that the excess variance arises from chaotic or turbulent flows. This interpretation is supported by time-space Fourier studies ([Cadavid et al. 1998](#)) that found Kolmogorov-like flow statistics in this scale range, and by the observation based on THEMIS data ([Bruno et al. 1998](#)) that the redshift/blueshift sign-change patterns in the flow exhibit strong scaling behavior under coarse graining in the range 2–15 Mm. We have reproduced this result with high-resolution *SOHO*/MDI data.

In § 2 we describe the wavelet analysis methods used. In § 3 comparison of San Fernando Observatory (SFO) simultaneous, coregistered Doppler, intensity, and magnetic images show that both the Doppler velocity and the temperature fields are Gaussian distributed, but that the magnetic field is intermittent. In § 4 the temperature and magnetic field results are extended to full-disk NSO/KP data. We use flatness spectra to study characteristic scales in Kitt Peak full-disk magnetograms. Section 5 focuses directly on the velocity field as shown in *SOHO*/MDI Doppler images in both full-disk and high-resolution modes. After filtering to remove effects of acoustic modes, we make use of specially designed wavelets to detect and quantify coherent flows. We determine statistical distributions and scaling properties of the flows. In § 6 we discuss results and present conclusions.

§2. IMAGE VARIANCE AND FLATNESS IN THE WAVELET REPRESENTATION

§2.1. Wavelet Bases

In two dimensions the continuous wavelet transform $W(s, \mathbf{x})$ (see, e.g., [Farge 1992](#)) of a function $f(\mathbf{r})$ is its convolution with scaled and translated versions of the basis function $\Psi(\mathbf{r})$:

$$W(s, \mathbf{x}) = \frac{1}{s} \int f(\mathbf{r}) \Psi\left(\frac{\mathbf{r} - \mathbf{x}}{s}\right) d^2 \mathbf{r} . \quad (1)$$

The basis function is localized in both scale and position, i.e., in both Fourier ($\mathbf{k}$) and configuration ($\mathbf{r}$) space, so [equation \(1\)](#) gives a strong response to structure in $f(\mathbf{r})$ of scale s at position $\mathbf{x}$, and a weak response otherwise. For computational efficiency, the convolution in [equation \(1\)](#) is carried out as a simple product in Fourier space.

In order to fulfill its function, it is necessary that the wavelet basis function integrate to zero:

$$\int \Psi(\mathbf{r}) d^2 \mathbf{r} = 0 . \quad (2)$$

In Fourier space this requirement translates to

$$\lim_{k \rightarrow 0} \tilde{\Psi}(\mathbf{k}) = 0 , \quad (3)$$

where $\tilde{\Psi}(\mathbf{k})$ is the Fourier transform of $\Psi(\mathbf{r})$. This requirement leaves a great deal of flexibility in choosing basis functions. In general, a nontrivial basis

function will have some type of oscillatory form in configuration space, although the oscillations may be of different types. We will make use of this freedom to choose wavelet bases that match photospheric flow structures we wish to detect and whose properties we wish to examine.

§2.1.1. Radial Symmetry

The most commonly used real basis function with radial symmetry is the Marr, or "Mexican hat," wavelet with Fourier transform

$$\Psi_M(k) \propto k^2 e^{-k^2/2}, \quad (4)$$

shown in cross section in [Figure 1](#). The configuration space counterpart, also shown in cross section in [Figure 1](#), consists of a positive central crown surrounded by a negative brim and modulated by a Gaussian envelope. As is apparent in [Figure 1](#), $\tilde{\Psi}_M(k)$ is large nearly all the way to $k=0$ and thus gives poor resolution in frequency or scale. In this paper we instead choose the related wavelet

$$\Psi_6(k) \propto k^6 e^{-k^2/2} \quad (5)$$

([Perrier, Philipovitch, & Basdevant 1995](#)). This wavelet, also shown in cross section in [Figure 1](#) in both Fourier and configuration spaces, better suppresses contributions from large scales and thus provides better scale resolution than the Mexican hat, albeit at the cost of degraded resolution in spatial position. We will refer to this as the Ψ_6 wavelet. It will be used in all of the spatially isotropic analyses below (all but the full-disk *SOHO*/MDI Doppler maps). In our discussion of *SOHO*/MDI high-resolution Doppler data in [§ 5.2.2](#) below, we will give a graphical comparison between the responses of the $\Psi_M \equiv \Psi_2$ and the Ψ_6 wavelets to a modeled image. In any case, for spatial isotropy we are constrained to wavelet basis functions with radial oscillations. The Gaussian envelope function is optimal because it does not give sidelobes under Fourier transformation and thus degrade scale resolution.

[Fig. 1](#)

The meaning of the wavelet scale s may change depending on the morphology of the image feature under study. For Ψ_6 , the first zero ([Fig. 1](#)) occurs at $r=0.9$, and the first minimum occurs at $r=1.4$. We will quote and plot results using a scale equal to the wavelet width. Thus if a spectral feature appears at scale $s=2r$, then this could refer to a solid "blob" of diameter $d \sim 0.9s$ or to an empty ring of diameter $d \sim 1.4s$.

§2.1.2. Bipolar Symmetry

The radially symmetric wavelets discussed above are useful for spatially isotropic field distributions such as line-of-sight velocities near disk center. Away from disk center, however, the line-of-sight velocities arise from horizontal flows on the solar surface. Systematic supergranular outflows then appear with a bipolar symmetry with red shifts toward the solar limb and blue shifts toward disk center. We exploit this signature by means of an antisymmetric wavelet composed of two of our isotropic Ψ_6 basis functions deformed to 1:2 elliptical shape, shifted along the minor axis, and then subtracted. This basis function is shown in [Figure 2](#). When convolved with an outflow (redshift toward limb, blueshift toward disk center), this wavelet gives a positive response; over an inflow it gives a negative response. This wavelet will be employed below in the analysis of *SOHO*/MDI full-disk Doppler images.

[Fig. 2](#)

§2.2. Wavelet Spectra

§2.2.1. Energy and Variance Spectra

The space-scale energy density of an image may be expressed as

$$E(s, \mathbf{x}) = \frac{1}{s^2} |W(s, \mathbf{x})|^2 \quad (6)$$

and the global energy spectrum as

$$E(s) = \frac{1}{s^2} \int |W(s, \mathbf{x})|^2 d^2 \mathbf{x} . \quad (7)$$

This connects to the familiar Fourier energy spectrum $E(\mathbf{k}) = |\tilde{f}(\mathbf{k})|^2$ according to

$$E(s) = \int E(\mathbf{k}) |\Psi(s\mathbf{k})|^2 d^2 \mathbf{k} . \quad (8)$$

Thus the global wavelet energy spectrum is the Fourier spectrum smoothed by the wavelet basis function spectrum at each scale, and the information it carries about the image being analyzed is similar to that carried by the Fourier spectrum. It will be convenient for us to express this information in terms of the local wavelet variance $V(s, \mathbf{x}) \equiv s^2 E(s, \mathbf{x})$ and the variance spectrum $V(s) \equiv s^2 E(s)$.

§2.2.2. Flatness Spectrum

The variance (second order) spectra are measures of the amplitude of fluctuations in images. Such spectra may not effectively show structures at characteristic scales. Wavelet analysis allows one to generate higher order spectra that probe the statistical distribution of fluctuations of the image as a function of scale. The intermittence of an image distribution is measured by the flatness (fourth order) spectrum

$$\Phi(s) \equiv \frac{\langle W^4(s, \mathbf{x}) \rangle}{\langle W^2(s, \mathbf{x}) \rangle^2} . \quad (9)$$

The brackets denote spatial averaging over some area $\mathbf{x} \in A$. The flatness is normalized and thus independent of the image amplitudes; it is a measure only of the shape of their statistical distribution. If the distribution of wavelet coefficients $W(s, \mathbf{x})$ is Gaussian at scale s , then $\Phi(s) \equiv 3$. If the distribution has broad wings, and thus indicates an increased probability of bursts (intermittence), then $\Phi > 3$. If the distribution is truncated, then $\Phi < 3$; for example, a rectangular distribution gives $\Phi_{\text{rect}} \equiv 9/5$.

§3. SIMULTANEOUS VELOCITY, MAGNETIC, AND TEMPERATURE FIELDS

Figure 3 shows flatness spectra for simultaneous, coregistered images of (1) line-of-sight velocity, (2) line-of-sight magnetic field, and (3) continuum intensity fluctuations (proportional to temperature fluctuations: $\delta I \propto T^3 \delta T$) of an area of QS near disk center on 1994 September 14. These were generated by applying equation (9) to data obtained with the SFO Video Spectra-Spectroheliograph system (Chapman & Walton 1989). The pixel scale is $0''.51 \text{ pixel}^{-1}$, so the field of view is 170 Mm east-west and 80 Mm north-south. The images are 10 minute averages of data taken with 1 minute cadence, so the effects of the 5 minute oscillations are reduced. The error bars are derived from averaging spectra from a number of 128×128 pixel samples from the images. Over the range $0.7 \text{ Mm} < s < 23 \text{ Mm}$, the flatness spectra of the Doppler and intensity images are constant at $\Phi(s) = 3$ within observational error. Thus the velocity and temperature are Gaussian; that is, their strengths at different points (defined by wavelet coefficients) appear drawn from a normal distribution. The magnetic image is highly intermittent up to $s \approx 23 \text{ Mm}$. The flatness has a maximum $\Phi_{\text{max}} \approx 14$ at $s \approx 5 \text{ Mm}$, which indicates a characteristic scale for the magnetic features that is supported by visual inspection of the image. These structures appear to be the same as the small magnetic features found residing in QS areas in the NSO/KP magnetograms discussed below.

Fig. 3

These data have the advantage that the velocity, magnetic, and intensity maps are exactly coregistered and simultaneous. Also, the availability of at least short time series affords the opportunity to roughly correct for acoustic mode effects. These last effects are most critical, of course, for the Doppler data and least critical for the magnetic. The data have the disadvantages of covering only a small portion of the solar surface and of being subject to terrestrial seeing

effects.

§4. MAGNETIC FIELD STRUCTURES

Although losing simultaneity, we overcome the lack of spatial coverage for the magnetic and intensity parts of the previous data set by examining several full-disk magnetograms of both active and quiet Sun and a full-disk intensity image of mostly quiet Sun from NSO/KP. The image scale is $\sim 1''$ pixel $^{-1}$. [Figure 4](#) shows flatness spectra for a mostly active Sun on 1991 July 11 and a mostly quiet Sun on 1988 April 25. The spectra are averaged over 100 256×256 pixel squares at various locations on the hemisphere, and each square is numerically remapped to disk center. To limit the effects of foreshortening on both the projected line-of-sight fields and the pixel sampling, we have restricted the sample centers to lie within a heliocentric angle Θ of disk center such that $\cos \Theta \geq 0.6$. As for the SFO data, the magnetic field is highly intermittent ($\Phi \gg 3$).

[Fig. 4](#)

The 1988 QS flatness spectrum, shown as open circles in [Figure 4](#), has a peak at $s \approx 5$ Mm with a drop for larger scales. This is in good agreement with the SFO result. By applying principal component analysis ([Murtagh & Heck 1987](#)) to the 100 sample areas in the 1991 active-Sun image, we find that their flatness spectra fall into two distinct classes. One class, comprising 65 of the 100 samples, is shown in [Figure 4](#) as filled squares. It also rises to a maximum at $s \approx 5$ Mm but then declines to a broad enhancement in flatness for $20 \text{ Mm} < s < 110 \text{ Mm}$. We identify this enhancement with the presence of extended unipolar areas of QS in the 1991 magnetogram that are absent from the 1988 magnetogram. The flatness spectrum of the remaining class of 35 samples is shown in [Figure 4](#) as filled circles. It rises to a high maximum flatness for scales between 15 and 22 Mm. The localization of our samples indicates that this latter, smaller class is characteristic of areas including solar active regions and associated strong network, while the larger class is characteristic of QS. By tracking the signs of wavelet responses, we also find that much of the active region response is due to empty rings of flux corresponding to cells of diameter 18–50 Mm in enhanced network. These results are typical of those of a larger sample of magnetograms made at various times.

[Figure 4](#) includes, as a line without symbols, a flatness spectrum made from a full-disk NSO/KP intensity image. The error bars indicate the variations among 16 256×256 pixel squares around disk center. The slight indication of intermittence at smaller scales is partly due to the presence of a small sunspot group in one sample. In general, however, this flatness spectrum is consistent with a Gaussian distributed photospheric temperature field at scales up to 60 Mm.

§5. COHERENT VELOCITY STRUCTURES

The abundance of Doppler images of the Sun provided by *SOHO*/MDI ([Scherrer et al. 1995](#)) offers an excellent opportunity to study the structural properties of the photospheric velocity field. The *SOHO*/MDI images are, of course, free of terrestrial seeing effects. Sufficiently long time series of images are available at sufficiently high cadence to permit filtering to thoroughly remove acoustic modes from the data analysis, and the data are plentiful enough to ensure the capture of typical flow behavior.

A complication intrinsic to Doppler data arises from projection effects. At smaller granular scales, the vertical and horizontal motions are roughly equal in magnitude, but at supergranular scales the motions are principally horizontal to the solar surface. As one moves away from disk center, motions vertical to the solar surface will appear to decrease as the cosine of the heliocentric angle, a second-order effect for small angles, and granular flow patterns are weakly affected. Supergranular motions are mostly horizontal to the surface and grow as the sine of the heliocentric angle: a stronger, first-order effect.

§5.1. Acoustic Mode Filtering

§5.1.1. High-Resolution Images

In high-resolution mode ([Scherrer et al. 1995](#)), the MDI produces Doppler images with resolution $0''.625$ pixel $^{-1}$. These are normally 1000 pixel east-west by 486 pixel north-south, slightly offset from disk center, but the sizes may vary. Our primary high-resolution data set here consists of 120 consecutive 1000 pixels east-west by 600 pixels north-south Dopplergrams with 60 s cadence made on 1997 February 5. The first step in our analysis is the separation of the observed line-of-sight velocity field into the parts due to acoustic oscillations and to photospheric flows. To achieve this separation we carry out a space-time Fourier transform of a sequence of 120 consecutive 600×600 pixel disk-centered subimages. [Figure 5](#) shows a k - ω plot of the power spectrum, which has been azimuthally averaged in k on the assumption of two-dimensional spatial isotropy. Units of ω are per minute and of k are per pixel. The dotted line in the plot corresponding to $\omega = \omega_0 = 3 + k/5$ divides the plot into two zones. The ridged structure for $\omega > \omega_0$ represents acoustic modes of the motions. The diffuse structure for $\omega < \omega_0$ represents the other (convective, turbulent) components of the motions. The dotted line itself has been chosen to run along the minimum of the valley between these two structures. To remove the acoustic modes from the Dopplergrams, we apply the filter $F(k, \omega) = 1 - (\omega/\omega_0)^2$ for $\omega < \omega_0(k)$, and $F(\omega) = 0$ for $\omega > \omega_0(k)$ to the Fourier transform, and then transform back to configuration space. The Welch form of the filter is used to reduce spurious features introduced by the inverse transform of $F(k, \omega)$.

[Fig. 5](#)

§5.1.2. Full-Disk Images

For the full-disk images we use temporal averages of images to suppress acoustic mode effects. A temporal average over time span T minutes corresponds to suppressing image power in [Figure 5](#) with $\omega > 1 + 120/T$. Inspection of this figure shows that an average over 30 minutes (i.e., a cutoff for $\omega > 5 \text{ minute}^{-1}$) removes the effects of the acoustic modes represented by the ridged structures; however, it also cuts off much of the higher spatial frequency part of the nonacoustic flow signal at large ω . Longer temporal averages cut off more of the convective signal without improving the acoustic filtering. On the other hand, an average over only 10 minutes, corresponding to a cutoff for $\omega > 13 \text{ minute}^{-1}$, seems to just escape the acoustic modes while keeping most of the background motions.

§5.2. Vertical Flow Patterns

[Figure 6](#) shows a magnified portion of a filtered image of the convective motions from the middle of the high-resolution time sequence. Disk center is at the image center. In this image, white corresponds to redshifts (downflows) and black to blueshifts (upflows).

[Fig. 6](#)

§5.2.1. Flatness Spectra

[Figure 7](#) shows flatness spectra constructed with the spatially isotropic Ψ_θ wavelet from 11 filtered images at minutes 10, 20, ..., 110 of the 120 minute high-resolution sequence. The 10 minute intervals were chosen to encompass significant granular evolution. The beginning and end of the temporal sequence were avoided because of end effects of the Fourier filtering. Error bars are derived from the scatter among the 11 images used. The filled circles represent the flatness for the convective motions. Here $\mathcal{F}(s) \approx 3$ up to about 10 Mm, and there is some intermittence structure near 30 Mm. This appears to reflect the supergranular motions visible in [Figure 6](#) away from disk center. (These light and dark structures seen by projected horizontal flows denote halves of supergranular cells.) In fact, if the flatness is calculated only for the central 90×90 Mm square (*open circles*), the Gaussian range is extended to ~ 20 Mm. All this indicates that the wavelet coefficients of the pure granular flows are Gaussian distributed with respect to the isotropic Ψ_θ wavelet, but that the supergranular flows in projection appear intermittent.

[Fig. 7](#)

§5.2.2. Upflow Structures

The fine details in [Figure 6](#) show that the small-scale flows are composed of isolated upflows (dark spots in the Doppler image) surrounded by a connected web of downflows (light in the image). A Ψ_θ wavelet kernel centered on a granule should thus give a negative response. One can then generate a spectrum by calculating that part of the variance spectrum produced by negative wavelet coefficients and then subtracting from it the part of the variance spectrum produced by positive wavelet coefficients. [Figure 8](#) shows the total variance spectrum consisting of the sum of the positive and negative contributions and the difference spectrum corresponding to the difference between the negative and positive parts of the variance. The latter indicates the contribution due to the excess of isolated upflows surrounded by a downflow network over the reverse configuration. The primary contribution, due to granules, lies between 0.7 and 2.2 Mm, with a maximum at 1.4 Mm. A weak but significant shoulder of the granular peak seen between 4 and 8 Mm indicates mesogranular flows. The still weaker structures near 15 and 30 Mm appear to match the weak intermittence at these scales seen in [Figure 7](#). They probably are related to supergranular flows seen in projection.

[Fig. 8](#)

Even at its maximum value, the difference spectrum appears to be a factor of 10 below the total spectrum. However, even given an array of pure upflow cells, not all wavelet responses at all locations will be negative, so the total spectrum must always be larger than the differential. To estimate the magnitude of this effect, we constructed a model square array of upflows in a downflow background with upflow sizes and separations similar to those in the data. The dashed lines with peaks near 1.4 Mm represent the total and differential variance spectra of this model. The secondary peaks near 0.5 Mm in the two model spectra are an artifact of the interaction of the wavelet structure with the model's regular array of flow cells. This feature disappears when the array lattice symmetry is destroyed. The model differential spectrum peak lies a factor of 2.7 below the peak of the model total spectrum.

Thus we find that half an order of magnitude (a factor of ~ 3) in the data variance at the peak remains unaccounted for by the regular cellular flows. At larger scales some 1.5 orders of magnitude are not accounted for by the regular flows.

§5.2.3. Scaling Properties

The disk center Doppler data are amenable to analysis by means of a scaling parameter called the "cancellation exponent" (Ott et al. 1992; Cadavid et al. 1994). This parameter describes how patches of different sign in a signed measure (here upflows and downflows) cancel when observed at different coarse-graining scales. Let $\mu(s)$ represent a signed measure; in this case, the line-of-sight velocity in a high-resolution Doppler image. We suppose that the measure has been partitioned into small cells of size s and averaged over each cell. All sign changes on scales smaller than s are thus canceled within each cell. Then we calculate the sum over all partitioned cells of the absolute value of the measure:

$$\chi(s) = \sum_i |\mu_i(s)| \propto s^{-\kappa}, \quad (10)$$

where κ is called the cancellation exponent. If the sign-change pattern is scale invariant over some range of s , then κ is constant over that range. If the measure μ is all of the same sign, then the sum is independent of the partitioning and the cancellation exponent vanishes exactly ($\kappa=0$). If, however, at the finest level of partitioning the measure has the form of zero-centered Gaussian white noise, then there is cancellation within all coarser partitionings such that $\kappa=1$; the scale invariance follows in this case from the central limit theorem. Figure 9 shows a plot of this sum versus coarse-graining scale s . We find that over the range $2.2 \text{ Mm} \leq s \leq 15 \text{ Mm}$, the sum shows strong scaling behavior with $\kappa=0.635 \pm 0.008$. The linear fit in this range explains a fraction $R^2=0.9992$ of the variation of $\chi(s)$. Such scaling behavior has previously been found also in Doppler images from the French-Italian THEMIS telescope (Bruno et al. 1998).

Fig. 9

This result is a strong indication that the excess image variance between the granular and supergranular scales is scale invariant and thus chaotic or turbulent in nature.

§5.3. Horizontal Flow Patterns

Full-disk *SOHO*/MDI images at $2''$ pixel $^{-1}$ (Scherrer et al. 1995) are used to examine the projected horizontal surface flows away from disk center. Here we have worked with a 120 minute sequence of full-disk images made on 1997 February 14, and a 60 minute sequence made on 1997 January 3, both at 1 minute cadence. Acoustic modes are suppressed by temporal averaging of images over various time spans from 9 minutes to 2 hr. As we did for the NSO/KP full-disk magnetograms, we randomly sample a large number (usually 100, thus an oversample) of image subsets that are mapped to disk center and rotated to orient the limb direction upward. We restrict the image samples to 186×186 Mm squares with centers randomly chosen on the hemisphere but limited to heliocentric angle cosines between $0.6 \leq \cos \theta \leq 0.8$. This de-emphasizes the relatively weak vertical velocities while avoiding excessive foreshortening effects near the limb. The samples are then remapped to disk center. Figure 10 shows a 372×372 Mm remapped image. We wavelet transform the entire image, but only the central 186×186 Mm portion is used in computing the spectra. This eliminates edge effects and wrap-around errors in fast Fourier transform calculations.

Fig. 10

§5.3.1. Flatness Spectra

As in the case of the high-resolution MDI images, the convective background velocity fields include coherent structures, mainly supergranules. These consist of convection cells that rise in the center, flow outward to the boundary, and then flow downward in the intercellular network. Because of the cellular aspect ratio, the horizontal flows are much greater than the vertical. The Doppler image in Figure 10 responds to the horizontal outflows seen in projection with redshifts (*light areas*) on the sides of the cells toward the limb and blueshifts (*dark areas*) on the sides of the cells away from the limb. This image has been remapped to disk center so the features do not appear foreshortened. We exploit the flow asymmetry by means of the bipolar wavelet basis illustrated in Figure 2. Figure 11 shows the flatness spectra for various temporal averaging spans calculated with the bipolar wavelet over 100 samples. The result is Gaussian: $\mathcal{D}(s) \approx 3$ over the range $9 \text{ Mm} < s < 140 \text{ Mm}$.

Fig. 11

§5.3.2. Outflow Structures

When convolved with an outflow (redshift toward limb, blueshift toward disk center), the bipolar wavelet in [Figure 2](#) gives a positive response; over an inflow it gives a negative response. [Figure 12](#) includes the total variance spectra for various temporal averaging spans, which include both the positive and negative responses. The spatial scales are calibrated by application of the wavelet to modeled flow structures. We see that for scales larger than 32 Mm the variance is not significantly affected by the length of the interval of temporal averaging. For smaller scales, the variance is more suppressed the longer the averaging interval. This is consistent with our discussion above that indicates that the shortest averaging time, with the higher small-scale variance, gives the most accurate results.

[Fig. 12](#)

Also shown are the corresponding differential spectra composed of that part of the variance produced by outflows minus that part produced by inflows. These indicate the variance due to the coherent outflow structures. We note first a clear maximum at a cell diameter of 32 Mm with width $28 \text{ Mm} < d < 40 \text{ Mm}$. Second, we note that the shapes of these spectra are essentially invariant under changes of temporal averaging span except for the shortest (9 minutes) span at small scales (less than 12 Mm). Thus the supergranular structures appear to reside near the $\omega=0$ axis in [Figure 5](#) (i.e., live longer than 2 hr) and are not affected by the temporal frequency cutoffs used. Third, there is a shoulder in the differential spectrum indicating some structure near 12 Mm in diameter. This may represent mesogranulation.

To estimate the amount of excess variance in the total spectrum that is not explained by the cell flows, we have extracted from the data some idealized cellular images for comparison. To do this we convolve an image with a modeled outflow structure at a range of diameters and look for localized maxima in the x - y scale space of the response. Then a "clean" image is constructed as a sum of the modeled flows at the corresponding scales and amplitudes. The result matching the [Figure 10](#) data is given in [Figure 13](#). Shown in [Figure 12](#) as heavy solid lines with filled circles and error bars are the total and differential variance spectra for an average of seven such cleaned images. They have been adjusted vertically to match the differential spectra of the full data. This indicates that about half an order of magnitude of the total spectrum is unexplained by the purely cellular flows at the peak. At $d=10 \text{ Mm}$, the cellular flows are 1 order of magnitude below the 9 minute total spectrum, which we have argued is most representative at smaller spatial scales.

[Fig. 13](#)

§6. DISCUSSION AND CONCLUSIONS

By means of the flatness spectra in [Figures 3, 7, and 11](#), we have observationally verified for the solar photosphere the common assumption of theories of turbulence that the velocity fields are Gaussian distributed. We have indeed found this for the solar photospheric flows in SFO images and in *SOHO*/MDI images in both resolution modes and for vertical and horizontal flows. This also serves to validate the Gaussian assumption we have made previously in an analysis of the turbulent aspects of these flows ([Cadavid et al. 1998](#)) in terms of the Avellaneda & Majda (1992a, 1992b) model. Hydrodynamic turbulence is known in fact to be non-Gaussian. However, high-velocity bursts (evidence of intermittency) appear only at very small scales (see [Frisch 1995](#)). Scales resolved on the Sun apparently are still too large to reveal any such intermittency, so the velocity distribution appears Gaussian.

The results of the comparison of flatness spectra of simultaneous Doppler, intensity, and magnetic images from SFO, magnetic and intensity images from NSO/KP, and *SOHO*/MDI velocity images can be reconciled by fundamental arguments. First, the velocity field is Gaussian distributed. The intensity fluctuations represent temperature fluctuations: if $T=T_0+\delta T$, then $I \propto T^4$ and $\delta I \sim 4T_0^3 \delta T$. Temperature is a scalar "contaminant" of the flow and so is simply advected by the velocity, and, for a wide class of flows, its advective derivative vanishes: $D\delta T=0$. Extensive theoretical studies (for a review see [Zeldovich et al. 1988](#)) indicate that if the velocity is Gaussian distributed, then asymptotically the temperature and intensity fluctuations must be as well, as confirmed by the SFO and NSO/KP data. In contrast, the frozen-in vector magnetic field B_i is stretched by stochastic flows: $DB_i = (B_j \partial_j) V_i dt$. Then, similar arguments indicate that if, for example, $\partial_j V_i = \delta_{ij} \eta$ is an isotropic, Gaussian, random variable, then asymptotically $B_i(t) \sim e^\eta$ will have a lognormal distribution and be highly intermittent. The actual form of the magnetic field distribution in general random flows is still an unsolved problem. However, it is clear that this distribution cannot be Gaussian, and we have shown this to be the case here. The intermittent magnetic fields in the SFO QS data are organized into concentrations of scale $\sim 5 \text{ Mm}$ by the coherent flow structures. These features appear to represent the locations of sinks in the supergranular flow field ([Smithson 1973](#); [Simon, Title, & Weiss 1995](#)). It is worth emphasizing that these structures do not characterize flux tubes in magnetohydrostatic equilibrium, but rather are consequences of aspects of the photospheric flows, in this case sinks.

We have found these same small magnetic features in NSO/KP magnetograms (and in others we have analyzed) to be widespread across the Sun in areas away from active regions, again apparently marking sinks in the supergranular flow fields. In the more active areas of the solar surface, with magnetic flux in plentiful supply and with more rapid field evolution, the wavelet analysis indicated, in addition to large-scale flux features, the presence of supergranular cells (up to diameter $d \approx 50 \text{ Mm}$) at least partly outlined by flux. This range of supergranular scales is in good agreement with that found in the full-disk *SOHO*/MDI Doppler images.

We have focused more directly on the coherent structures of the flows in the *SOHO*/MDI data. [Figure 8](#) is generated from acoustic-mode-filtered MDI high-resolution images using a wavelet approach designed to detect the particular flow pattern expected for convection cells: isolated upflows (blueshifted) embedded in a network of downflows (redshifted). It shows a strong peak sharply contained between $s \approx 0.7 \text{ Mm}$ and 2.2 Mm , with a maximum at $s=1.4 \text{ Mm}$. This represents granular cells and corresponds well to the distribution of granular sizes found from intensity images by [Berrilli et al. \(1997\)](#). The weak shoulder near 7 Mm indicates mesogranulation. After the appropriate corrections described above, the granular peak in the differential variance spectrum accounts for only about a third of the corresponding value of the full variance spectrum. Away from the peak the discrepancy is a full order of magnitude or

more. Although coherent flow structures may be readily apparent to the eye in images, even at their characteristic scales the bulk of the image energy is in the random field.

A similar result holds for spectra of the horizontal flow fields in *SOHO*/MDI full-disk Doppler images as shown in [Figure 10](#). As in the high-resolution case, the total variance is about half an order of magnitude greater than that due to coherent structures at the supergranular peak and greater still away from the peak. The size scales we find for supergranular flows ($28 \text{ Mm} < s < 40 \text{ Mm}$) match the supergranular scales originally found by [Simon & Leighton \(1964\)](#) from autocorrelations of Doppler images $d \approx 32 \text{ Mm}$ and reproduced by numerous workers since.

Although clearly detected at its own characteristic scale, the mesogranular feature at $\sim 7 \text{ Mm}$ is about 1.5 orders of magnitude weaker than the granular peak in the high-resolution/vertical flow difference spectrum. Given the easy detectability via our method of the granular and supergranular flows, the mesogranular weakness is notable.

Our principal result in this regard is that regular flows of any kind account for only a small fraction of the Doppler image variances at scales intermediate between granules and supergranules. Although the acoustic modes are strong at these scales, they have been removed. Furthermore, to the extent that the granular and supergranular spectral peaks refer to regular cellular flows, they do not overlap with anywhere near sufficient strength to account for the excess. The scaling symmetry between 2.2 and 15 Mm indicates that the flows in this range are mostly chaotic or turbulent. This also is consistent with time-space Fourier spectral studies by the authors ([Ruzmaikin et al. 1996](#); [Cadavid et al. 1998](#)) that indicated Kolmogorov-like scaling properties of photospheric flows in this range. Finally, this result implies that such statistical properties of the photospheric motions as eddy diffusivity or eddy viscosity ([Simon & Weiss 1989, 1997](#)) must depend on chaotic motions in addition to simple flow structures, at least at some scales.

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FIGURES

[Full image \(52kb\)](#) | [Discussion in text](#)

FIG. 1.—Cross sections of Mexican hat and sixth-order wavelet kernels in configuration space (*dashed lines*) and in wavenumber space (*dotted lines*). The Gaussian envelope, not an admissible wavelet, is the same in both spaces and is shown as a solid line.

[Full image \(43kb\)](#) | [Discussion in text](#)

FIG. 2.—Image of the two-dimensional bipolar wavelet basis function used to analyze projected horizontal supergranular flows.

[Full image \(69kb\)](#) | [Discussion in text](#)

FIG. 3.—Flatness vs. scale in megameters for simultaneous coregistered San Fernando Observatory Video Spectra-Spectroheliograph images of line-of-sight magnetic field, Doppler velocity, and continuum intensity for an area of QS near disk center.

[Full image \(81kb\)](#) | [Discussion in text](#)

FIG. 4.—Flatness vs. scale in megameters for line-of-sight magnetic field from NSO/KP full-disk magnetograms for QS on 1988 May 25 (*open circles*), active-Sun areas on 1991 July 11 (*filled circles*), and QS areas on 1991 July 11 (*squares*). Also, flatness for near disk-center samples of NSO/KP intensity image on 1993 February 3.

[Full image \(172kb\)](#) | [Discussion in text](#)

FIG. 5.—Space-time k - ω Fourier power spectrum for a 120 minute sequence of high-resolution *SOHO*/MDI Doppler images at 1 minute cadence. Units for ω are minute⁻¹ and for k are pixel⁻¹. Ridged structures below the dotted line represent acoustic modes. The diffuse structure above the dotted line represents the convective flows, cellular and turbulent. Images are filtered by suppressing the signal on one side or the other of the dotted line.

[Full image \(226kb\)](#) | [Discussion in text](#)

FIG. 6.—Magnified central portion of high-resolution *SOHO*/MDI Doppler image with acoustic modes removed by filtering. Units are in megameters. Disk center is at coordinates (0, 0). Dark shades are upflows; light shades are downflows.

[Full image \(49kb\)](#) | [Discussion in text](#)

FIG. 7.—Flatness vs. scale in megameters for an average of 11 *SOHO*/MDI high-resolution Doppler images spaced 10 minutes apart in time. These include the entire images (*filled circles*) and the central 95×95 Mm portions thereof (*open circles*).

[Full image \(63kb\)](#) | [Discussion in text](#)

FIG. 8.—Variance vs. scale in megameters for an average of 11 high-resolution *SOHO*/MDI Doppler images with acoustic modes removed and spaced 10 minutes apart. The lower filled circles represent that part of the variance due to isolated upflows surrounded by downflows; the upper filled squares represent the total variance spectrum. The dashed lines represent the equivalent spectra for a perfectly square array of upflows in a downflow background.

[Full image \(38kb\)](#) | [Discussion in text](#)

FIG. 9.—Sum of partitioned measure vs. scale in megameters for an average of five filtered high-resolution *SOHO*/MDI Doppler images with acoustic modes removed and spaced 10 minutes apart. The slope in the scaling range gives the cancellation exponent.

[Full image \(215kb\)](#) | [Discussion in text](#)

FIG. 10.—186×186 Mm off-disk-center sample, remapped to disk center, of a 60 minute average of full-disk MDI Doppler images made on 1997 January 3 at 1 minute cadence. Note the characteristic cell pattern with bright redshifts toward the limb (*up*) and dark blueshifts away from the limb (*down*).

[Full image \(49kb\)](#) | [Discussion in text](#)

FIG. 11.—Flatness vs. scale in megameters for 100 186×186 Mm off-disk-center samples like that in [Fig. 10](#) for various temporal averagings: 9 minutes (*diamonds*), 1/2 hr (*filled circles* and *filled squares*), 1 hr (*open circles*), and 2 hr (*open squares*).

[Full image \(73kb\)](#) | [Discussion in text](#)

FIG. 12.—Total and difference variance spectra vs. scale in megameters for 100 186×186 Mm off-disk-center samples like that in [Fig. 10](#) for various temporal averagings: 9 minutes (*diamonds*), 1/2 hr (*open circles*), and 2 hr (*open triangles*). The filled circles represent an average of seven similar images cleaned as described in the text.

[Full image \(192kb\)](#) | [Discussion in text](#)

FIG. 13.—186×186 Mm off-disk-center sample corresponding to [Fig. 10](#), which has been cleaned as described in the text. This displays identifiable outflow cells in the image.