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Instabilities of spin waves in parallel-pumped easy plane ferromagnets

S. P. Lim and D. L. Huber

Department of Physics, University of Wisconsin-Madison, Madison, Wisconsin 53706

By applying the S -theory formalism of Zakharov *et al.* to a microscopic hamiltonian with uniaxial easy plane anisotropy, we arrive at a set of equations for the system that describe the spin waves and their mutual interactions. The parameters in the theory are related to the various interaction constants of the microscopic hamiltonian. Numerical studies of this system of equations indicate that the stationary states are ones where all spin-wave pair correlation functions have the same phase. The phenomenon of phase locking is universal, independent of the mode of approach to equilibrium. It is found that there is no dependence on the number of modes (up to 100) for the above behavior. This, together with the form of the equations, indicates that a similar result should hold for a macroscopic number of modes. Results for the stationary magnon population are presented. In the phase-locked regime, the approach to a stationary state is governed by a pair of coupled first-order differential equations. Linearizing these equations about the stationary points, we find that the approach to equilibrium involves purely exponential decay just above threshold, and, at higher power levels, we have damped oscillatory decay.

I. INTRODUCTION

Instabilities of spin waves in magnetic materials under both parallel and perpendicular pumping are well-known experimental facts.¹⁻⁸ The recent interest in chaos has spurred a renewed interest in this area, and recent studies show such systems to be dynamically rich.

In his pioneering paper, Suhl⁹ first gave a theoretical understanding of such instabilities for perpendicularly pumped ferromagnets. Schlömann¹⁰ *et al.* and Gottlieb and Suhl¹¹ followed up with an explanation for the parallel case. Some years later Zakharov *et al.*^{12,13} presented a microscopic theory of the parametric excitation of spin waves in parallel pumping experiments. This subsequently became known as the S theory, and various authors^{1,14,15} have investigated the equations of motion derived in Ref. 12 numerically and obtained period doubling and chaotic behavior. However, such work was done with special, ad hoc choices of parameter values and was limited to a two-mode approximation. An alternative model of instabilities in parallel pumping based on the Landau-Lifshitz equation⁶ has been proposed by Waldner *et al.*¹⁶ As for perpendicular pumping, Zhang and Suhl^{17,18} and Rezende *et al.*¹⁹ have recently analyzed it from a microscopic viewpoint, and the latter group has also investigated a two-mode model for this case—analogueous to that investigated in Refs. 14 and 15.

Although the two-mode models were successful in reproducing some of the features of the parallel pumping experiments, they remain approximations, without basis for the choice of parameters or justification for restricting the number of modes to two. As shown by Suhl and Zhang,¹⁸ in parallel pumping, an entire manifold of spin waves becomes excited above threshold. In this paper we mitigate these two objections. Starting from a microscopic hamiltonian, we obtain explicit expressions for the parameters that appear in the S theory for the model of an easy plane ferromagnet with static and oscillating magnetic fields lying in the easy plane. We also go beyond the two-mode approximation. A consi-

deration of the specific solutions to these equations for up to 100 modes leads us to infer the general solution in the neighborhood of the stationary states of a macroscopic ensemble of modes. We also characterize the approach to the stationary states and the range of pumping field strength over which such states are stable.

II. MODEL

We adopt for our study an easy plane ferromagnetic system with static and oscillating magnetic fields perpendicular to the hard (x) axis. The microscopic hamiltonian for such a system is

$$H = - \sum_{i,j} J \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{i,j} D S_i^x S_j^x - g\mu H \sum_j S_j^z + g\mu h \cos(\omega_p t) \sum_j S_j^z, \quad (1)$$

where the prime signifies a sum over nearest neighbors only and both the exchange energy J and anisotropy energy D are positive. H denotes the Zeeman field and h the oscillating field with frequency $\omega_p/2\pi$. In conventional pumping experiments, only magnons near $\mathbf{k} = 0$ are excited. On applying the S theory in this long wavelength limit, we obtain the equations of motion²⁰

$$\frac{dn_k}{dt} = n_k \left(-\gamma + V \sin \psi_k + T \sum_{k'} \sin(\psi_k - \psi_{k'}) \right), \quad (2)$$

$$\frac{d\psi_k}{dt} = \Delta\omega + 2T \sum_{k'} n_{k'} + V \cos \psi_k + T \sum_{k'} n_{k'} \cos(\psi_k - \psi_{k'}), \quad (3)$$

and

$$V = (DSz/4) [g\mu H / (g\mu H + DSz)]^{1/2}, \quad (4)$$

$$T = -(Dz/4A) [(2g\mu H + DSz/2) / (g\mu H + DSz)], \quad (5)$$

where A is the number of spins, z is the number of nearest

FIG. 1. Upper panels: Variance in ψ , Eq. (8), vs γt for $T = -\gamma$, $\Delta\omega = 0.1\gamma$; random initial conditions. Lower panels: Total magnon population vs γt . Same parameters as in upper panel. Note difference in time scales between upper and lower panels of left figures and difference in vertical scales between left and right figures.

neighbors, and $\Delta\omega = [g\mu H(g\mu H + DSz)]^{1/2} - \omega_p/2$. We have assumed that all of the participating modes had the same frequency and decay rate γ . Here, n_k and ψ_k are related to the ensemble averages of the following correlation functions of bose operators c_k and $c_k^\dagger$ that diagonalize the zero-order hamiltonian:

$$\langle c_k^\dagger(t)c_k(t) \rangle \equiv n_k(t), \quad (6)$$

$$\langle c_k(t)c_{-k}(t) \rangle \equiv \sigma_k(t)e^{i\omega_p t} = n_k(t)e^{i(\psi_k + \omega_p)t}. \quad (7)$$

We discuss the solutions to (2) and (3) in the following section.

III. FIXED POINTS AND LINEAR STABILITY ANALYSIS

The equations of motion (2) and (3) have been investigated with parameters in the physically relevant ranges, $\Delta\omega > 0$ and $T < 0$, for a variety of initial conditions for up to one hundred modes. It is found that for $V < \gamma$ the system relaxed to the trivial fixed point where $n_i^0 = n_2^0 = \dots = 0$. When V exceeded this threshold, the existence of nontrivial (i.e., $\Sigma n_i^0 \neq 0$) fixed points become possible. In all of these calculations, we restrict $\Sigma n_i(0)$ to suitably small quantities to conform to physically realistic situations although the results still stand when this requirement is relaxed. Otherwise each $n_i(0)$ was arbitrarily chosen. Without exception it was found that both stable and unstable fixed points were ones of common phase. That is $\psi_i^0 = \psi_2^0 = \dots = \psi_0 \pmod{2\pi}$. Furthermore, in the evolution towards the stable fixed points the phases become locked before the fixed point was reached. That is, $\psi_1(t) = \psi_2(t) = \dots = \psi(t)$ as $\psi(t) \rightarrow \psi_0$.

The locking of phases is illustrated in Fig. 1 for different pumping fields with arbitrarily chosen initial conditions above threshold for a hundred-mode system. In the upper

panels, we plotted $\Delta\psi(t)$, the variance in the phases, defined by

$$\Delta\psi = \left[\left(\frac{1}{100} \sum_{i=1}^{100} \left[\psi_i(t) - \left(\sum_{j=1}^{100} \psi_j(t) \right) / 100 \right]^2 \right)^{1/2} \right], \quad (8)$$

while in the lower panels, we plotted the total magnon population $N(t)$ defined by

$$N(t) = \sum_{i=1}^{100} n_i(t). \quad (9)$$

A comparison of the upper and lower panels show that phase locking occurs before $N(t)$ reaches its stationary value.

The phase-locked fixed points can be obtained from the equations of motion by setting the left-hand sides to zero and taking $\psi_k \equiv \psi_0$. We then have

$$\sin \psi_0 = \gamma/V, \quad (10)$$

$$N_0 = \sum_k n_k^0 = (-\Delta\omega - V \cos \psi_0)/(3T). \quad (11)$$

Since the physically meaningful states must have $N_0 > 0$ and as $T < 0$ in an easy plane ferromagnet, with $\Delta\omega > 0$, one can have stationary states with

$$N_0 = [\Delta\omega + (V^2 - \gamma^2)^{1/2}]/(3|T|) \quad (12)$$

for all $V > \gamma$. In addition there are also physically accessible stationary states where

$$N_0 = [\Delta\omega - (V^2 - \gamma^2)^{1/2}]/(3|T|) \quad (13)$$

for $\gamma < V < (\gamma^2 + \Delta\omega^2)^{1/2}$. As will be shown below, these latter states are unstable with respect to infinitesimal perturbations whereas the former are stable.

Finally we mention that were $\Delta\omega < 0$, one would have physically meaningful stationary states with

$$N_0 = [(V^2 - \gamma^2)^{1/2} - |\Delta\omega|]/(3|T|) \quad (14)$$

for $V > (\gamma^2 + \Delta\omega^2)^{1/2}$ and like (12) above, they are stable.

The analysis leading to Eqs. (10) and (11) does not provide information on the stability of these fixed points. We can obtain such information by linearizing the equations of motion about the fixed points. In doing this we take phase locking into account explicitly by setting $\psi_k = \psi_k = \psi$. Summing (2) over k and linearizing with respect to $\delta N = N - N_0$ and $\delta\psi = \psi - \psi_0$, we find the solutions for δN and $\delta\psi$ to be sums of two exponential functions with arguments given by

$$S_{\pm} = -\gamma \pm (\gamma^2 + 12N_0VT \cos \psi_0)^{1/2}. \quad (15)$$

From this, it is quite evident that δN and $\delta\psi$ will decay to zero provided $12N_0VT \cos \psi_0 < 0$. With the physical requirement that $N_0 > 0$ and $T < 0$, the accessible stable stationary states are ones where $V \cos \psi_0 > 0$. That is $V \cos \psi_0 = + (V^2 - \gamma^2)^{1/2}$. Otherwise, for $V \cos \psi_0 = - (V^2 - \gamma^2)^{1/2}$, the states are unstable. Note that for $12N_0VT \cos \psi_0 < -\gamma^2$ the roots $S_{\pm}$ are complex leading to damped oscillatory decay for $\delta N(t)$ (cf. right half of Fig. 1). On the other hand, when $12N_0VT \cos \psi_0 + \gamma^2 \approx 0$ the two roots become $S_{\pm} \approx 6N_0VT \cos \psi_0/\gamma$ and $S_{-} \approx -2\gamma$. Hence after a time $\gg \gamma^{-1}$, $\delta N(t)$ decays asymptotically as

$$\delta N(t) = \delta N(0) \exp[-6N_0|T|(V^2/\gamma^2 - 1)^{1/2}t], \quad (16)$$

which is the behavior displayed in the left half of Fig. 1. This behavior is characterized as the adiabatic limit.²²

IV. DISCUSSION

The two important points that came out of our investigation are (1) the stationary states are ones of common phase and (2) the phases lock before the system arrives at the final fixed point. The presence of phase locking reduces the macroscopic set of equations (2) and (3) to two:

$$\frac{dN}{dt} = N(-\gamma + V \sin \psi), \quad (17)$$

$$\frac{d\psi}{dt} = \Delta\omega + 3TN + V \cos \psi. \quad (18)$$

We see that as long as phase locking occurs, the results are the same whether we are investigating two modes, a hundred modes or a macroscopic number of modes. Note that the quantity that is relevant experimentally is N , not the magnitude of each individual occupation number. The fixed point in our result actually resides on an $(A - 1)$ dimensional hy-

perplane since it is the sum of the occupation number that equals a constant, the individual components being dependent on the initial conditions.

The analysis outlined in the preceding sections characterizes the nonequilibrium stationary states of the system. As discussed in Ref. 8, with increasing power the stationary state in the easy plane ferromagnet $(\text{CH}_3\text{NH}_3)\text{CuCl}_4$ is followed by a regime where the spin-wave population oscillates in time and, ultimately, to a chaotic regime. An extensive analysis of (2) and (3) fails to reveal any periodic or chaotic behavior at high-power levels. The failure to locate any periodic or chaotic behavior appears to invalidate the applicability of the S theory in describing these phenomena on easy plane ferromagnets with dominant single ion or exchange anisotropy.

- ¹S. M. Rezende, F. M. de Aguiar, and O. F. de Alcantara Bonfim, *J. Magn. Magn. Mater.* **54-57**, 1127 (1986).
- ²G. Gibson and C. Jeffries, *Phys. Rev. A* **29**, 811 (1984).
- ³F. M. de Aguiar and S. M. Rezende, *Phys. Rev. Lett.* **56**, 1070 (1986).
- ⁴M. Mino and H. Yamazaki, *J. Phys. Soc. Jpn.* **55**, 4158 (1986). See also V. L. Grankin, V. S. L'vov, V. I. Motorin and S. L. Musher, *Zh. Eksp. Teor. Fiz.* **81**, 757 (1981) [*Sov. Phys. JETP* **54**, 405 (1981)].
- ⁵H. Yamazaki, *J. Phys. Soc. Jpn.* **53**, 1155 (1984).
- ⁶F. Waldner, R. Badii, D. R. Barberis, G. Broggi, W. Floeder, P. F. Meier, R. Stoop, M. Warden, and H. Yamazaki, *J. Magn. Magn. Mater.* **54-57**, 1135 (1986).
- ⁷H. Yamazaki, M. Mino, H. Nagashima, and M. Warden, *J. Phys. Soc. Jpn.* **56**, 742 (1987).
- ⁸H. Yamazaki and M. Warden, *J. Phys. Soc. Jpn.* **55**, 4477 (1986).
- ⁹H. Suhl, *J. Phys. Chem. Solids* **1**, 209 (1957).
- ¹⁰E. Schlömann, J. J. Green, and U. Milano, *J. Appl. Phys.* **31**, 386S (1960).
- ¹¹P. Gottlieb and H. Suhl, *J. Appl. Phys.* **33**, 1508 (1968).
- ¹²V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, *Zh. Eksp. Teor. Fiz.* **59**, 1200 (1970) [*Sov. Phys. JETP* **32**, 656 (1971)].
- ¹³V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, *Usp. Fiz. Nauk.* **114**, 609 (1974) [*Sov. Phys.-Usp.* **17**, 896 (1975)].
- ¹⁴K. Nakamura, S. Ohta, and K. Kawasaki, *J. Phys. C* **15**, L143 (1982).
- ¹⁵S. Ohta and K. Nakamura, *J. Phys. C* **16**, L605 (1983).
- ¹⁶F. Waldner, D. R. Barberis, and H. Yamazaki, *Phys. Rev. A* **31**, 420 (1985). See also G. Broggi, P. F. Meier, R. Stoop, and R. Badii, *Phys. Rev. A* **35**, 365 (1987).
- ¹⁷X. Y. Zhang and H. Suhl, *Phys. Rev. A* **32**, 2530 (1985).
- ¹⁸H. Suhl and X. Y. Zhang, *Phys. Rev. Lett.* **57**, 1480 (1986).
- ¹⁹S. M. Rezende, O. F. de Alcantara Bonfim, and F. M. de Aguiar, *Phys. Rev. B* **33**, 5153 (1986).
- ²⁰S. P. Lim and D. L. Huber (to be published).
- ²¹C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1964), Chap. 4.
- ²²V. E. Zakharov, V. S. L'vov, and S. L. Musher, *Fiz. Tverd. Tela* **14**, 832 (1972) [*Sov. Phys.—Solid State* **14**, 710 (1972)].