ELSEVIER

29 August 1996

PHYSICS LETTERS B

Physics Letters B 383 (1996) 133–138

Nuclear correlation and finite interaction-range effects in high-energy $(e, e'p)$ nuclear transparency

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Received 28 February 1996; revised manuscript received 5 June 1996

Editor: C. Mahaux

Abstract

Nuclear transparency is calculated for high-energy, semi-inclusive $(e, e'p)$ reactions, by accounting for all orders of Glauber multiple-scattering and by using realistic finite-range pN interaction and (dynamically and statistically) correlated nuclear wave functions. The nuclear correlation effect is reduced due to the pN finite-range effect, and is only a few percent for a given one-body density, depending on details of the (presently poorly known) nuclear correlations in finite nuclei.

Perturbative quantum chromodynamics (pQCD) predicts the novel nuclear phenomenon of color transparency, in which a hadron produced by a hard scattering process could have an unusually long mean-free-path in nuclei [1]. Low-energy hadrons can have long mean-free-paths in nuclei solely due to nuclear medium effects without color transparency [2]. It is imperative to establish nuclear medium effects reliably in these high-energy reactions, so as to identify the pQCD phenomenon explicitly. The medium effect that has been most noted in high-energy reactions is nuclear correlation, but calculations vary wildly from only a few percent increase [3] to as much as a 20 to 30 percent increase [4] in proton emission in $(e, e'p)$.

In this letter we report a calculation of $(e, e'p)$ nuclear transparency in order to unambiguously establish the significance of the correlation effect. The formalism [5] used treats both the nuclear reaction mechanism and nuclear structure in a systematic way. We in-

clude all orders of Glauber multiple-scattering series, a realistic finite-range pN interaction, and realistic nuclear correlations. Numerical evaluation is done by Monte Carlo integration over all nucleon coordinates and spins. To the best of our knowledge, no previous work has been carried out with this rigor. (The only Monte Carlo calculation previously carried out [6] used schematic correlation and pN interaction.)

For the sake of clarity, we examine a fully semi-inclusive process, in which no cut is imposed on the missing energy and momentum. The nuclear transparency T that we calculate corresponds to a ratio of the proton emission rate with and without the final-state interaction in this process. We emphasize that because of the different kinematical conditions, our T is not the same as the quantity measured by the NE18 experiment [7]. We will discuss more on this point near the end of the letter.

T is given as the missing-momentum integration of the response function (after the application of clo-

sure),

$$S^\dagger = -\frac{1}{\pi} \langle 0 | O^\dagger \cdot \text{Im } G^\dagger \cdot O | 0 \rangle \quad (1)$$

for the ground-state nucleus $|0\rangle$. Here, $\text{Im } G^\dagger$ is the imaginary part of the Green's function due to proton emission, and

$$\text{Im } G^\dagger = (1 + G^\dagger V_{1,A-1}^\dagger) \text{Im } G_0 (1 + V_{1,A-1} G), \quad (2)$$

where G_0 and G are the Green's functions for the proton in free space and in the nucleus, respectively, and $V_{1,A-1}$ is the interaction between the knockout high-momentum proton (labeled as 1) and the rest of the $A-1$ nucleons [5].

We apply the Glauber approximation for the Green's functions, which consists of the eikonal approximation for the proton and the fixed-scatterer approximation for the target nucleons, and obtain

$$T = \frac{\int d\mathbf{r}_1 \cdots d\mathbf{r}_A \Pi_{j=2}^A e^{-2\text{Im} \chi_j(\mathbf{r}_1, \mathbf{r}_j)} |\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A)|^2}{\int d\mathbf{r}_1 \cdots d\mathbf{r}_A |\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A)|^2} \quad (3)$$

with

$$\chi_j(\mathbf{r}_1, \mathbf{r}_j) = -\frac{m}{p_1} \int_{z_1}^{\infty} dz_1' V_{pN}(\mathbf{r}_1' - \mathbf{r}_j)_{\mathbf{b}_1'=\mathbf{b}_1}. \quad (4)$$

Eq. (3) appeared in our previous work [8] and also in Ref. [3] as an intermediate step to Glauber multiple-scattering series [9].

T can be cast in the well-known eikonal form expressed in terms of the one-body nuclear density, $\rho(\mathbf{r})$:

$$T = \frac{\int d\mathbf{r}_1 \exp(-2\text{Im} \chi(\mathbf{r}_1)) \rho(\mathbf{r}_1)}{\int d\mathbf{r}_1 \rho(\mathbf{r}_1)}. \quad (5)$$

Here, $2\text{Im} \chi(\mathbf{r}_1)$ is expressed in a cluster expansion as

$$\sum_{j=2}^A \int d\mathbf{r}_j (2\text{Re} \Gamma_j - |\Gamma_j|^2) \rho_2(\mathbf{r}_1, \mathbf{r}_j) / \rho(\mathbf{r}_1) + \cdots, \quad (6)$$

where $\Gamma_j = 1 - \exp(+i\chi_j(\mathbf{r}_1, \mathbf{r}_j))$ is the Glauber NN amplitude in the coordinate space.

Nuclear correlation effects appear explicitly in the leading order through the two-body nuclear density

ρ_2 , and spectator effects appear in the sub-leading order (not explicitly shown here) [3]. The commonly used eikonal expression is a consequence of invoking the zero-range approximation for $2\text{Re} \Gamma_j - |\Gamma_j|^2$ with a constant proportional to the NN cross section, σ_r : Either

$$2\text{Im} \chi(\mathbf{r}_1) \approx \sigma_r \int_{z_1}^{\infty} dz_1' g(\mathbf{r}_1 - \mathbf{r}_1') \rho(\mathbf{r}_1')_{\mathbf{b}_1'=\mathbf{b}_1} \quad (7)$$

or

$$2\text{Im} \chi(\mathbf{r}_1) \approx \sigma_r \int_{z_1}^{\infty} dz_1' \rho(\mathbf{r}_1')_{\mathbf{b}_1'=\mathbf{b}_1} \quad (8)$$

with or without the correlation, respectively. Furthermore, many authors use the total cross section, σ_{tot} , instead of the inelastic cross section, σ_r . This corresponds to discarding the $|\Gamma_j|^2$ term in Eq. (6) and is not justified for a fully semi-inclusive process [8,10] since the term is of the leading order through unitarity [8,9]. Note that the use of σ_{tot} is also inappropriate in exclusive ($e, e'p$) reactions [11].

V_{pN} in Eq. (4) is either the pp or pn potential, depending on whether the j -th nucleon is a proton or a neutron. We determine V_{pN} by applying the Abel transformation to invert the eikonal forms of the NN amplitudes [9]. The inversion is exact in the eikonal formalism for a local, spherical potential; we thus determine V_{pN} better than the frequently used Born approximation. In the several GeV/c region, V_{pN} thus determined has a much stronger real part and a little wider imaginary part. The latter is important here as seen in Eq. (3). The NN amplitudes are extracted from experimental data in Gaussian forms [12]. Because of the limited availability of pN scattering data, we also restrict the potentials to be spin-independent. The free-space pN interaction would be modified by nuclear medium corrections and by long formation lengths of excited nucleons. These are clearly of much interest, but we proceed by neglecting them in order to determine the correlation effects with minimal complication.

We now describe our findings by taking the case of $p_1 = 4.49$ GeV/c for doubly closed shell nuclei, ^{16}O and ^{40}Ca . This choice is also made to minimize effects of other matters that might complicate the issue, such

Fig. 1. $\exp(-2 \operatorname{Im} \chi(\mathbf{r}, 0))$ using the spin-independent, local V_{pp} extracted from experimentally determined pp scattering amplitude. The quantity illustrates attenuation of the knockout proton as it impinge on a proton located at the origin. The proton comes from the negative z , parallel to the z -axis.

as open nuclear shells and strong momentum dependence of the pN amplitudes.

As Eq. (3) shows, $\exp(-2 \operatorname{Im} \chi(\mathbf{r}, 0))$ contributes to T as a product with the correlated nuclear density. In Fig. 1, we illustrate $\exp(-2 \operatorname{Im} \chi(\mathbf{r}, 0))$ calculated from Eq. (4) using the V_{pp} inverted from the observed pp amplitude. Fig. 1 shows that $\exp(-2 \operatorname{Im} \chi(\mathbf{r}, 0))$ deviates from unity even for b and $z > 1$ fm. This feature is missing in the commonly used approximation of the zero-range pN interaction: The corresponding figure in the zero-range approximation would appear as a deep narrow valley following the z -axis starting from $z = 0$.

In order to evaluate T , we require the A -body nuclear density that is accurate in the length scale of several tenths of fm or smaller. Since such densities are not readily available for various nuclei, we construct them by following a simple scheme and compare with a variational Monte Carlo calculation of ^{16}O [13]. We use a Jastrow-form of the wave function: a product of antisymmetrized Hartree-Fock (HF) single-particle wave function (for Skyrme force with Z_σ parameters [14]) and of pair wave functions for nuclear matter at various densities (for the AV14 plus UVII NN potential model [15]). The wave function thus includes both of the statistical and dynamical correlations. In terms of various pair wave functions f 's,

Fig. 2. The two-particle distributions of ^{16}O , ρ_{pn} and ρ_{pp} , calculated using the wave function of Eq. (9) with and without pair correlation functions, shown by full curves and open circles, respectively.

the A -body density $|\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A)|^2$ is written as

$$|A\Phi|_s^2 \cdot \prod_{i < j} [(f_c^2 + 3f_\sigma^2 + 3f_\tau^2 + 9f_{\sigma\tau}^2 + 6f_t^2 + 18f_{t\tau}^2) \pm 2(f_c f_\tau + 3f_\sigma f_{\sigma\tau} + 6f_t f_{t\tau} - f_\tau^2 - 3f_{\sigma\tau}^2 - 6f_{t\tau}^2)] \quad (9)$$

for a pn pair ($-$) and a pp or nn pair ($+$). Invoking the local density approximation, we evaluate the f 's as functions of $|\mathbf{r}_i - \mathbf{r}_j|$, for the (one-body) nuclear density that is generated from $|A\Phi|_s^2$ (spin-summed) at $|\mathbf{r}_i + \mathbf{r}_j|/2$.

Here, we include the dynamical correlations only between nucleon pairs, neglecting correlation operator symmetrization and off-diagonal contributions. The terms depending linearly on the spin-dependent operators are discarded, and those depending quadratically are averaged. This simple scheme should be reasonable for closed-shell nuclei, especially because all non-central f 's are much smaller than the central f_c . The tensor correlations are important in the momentum distribution above the Fermi momentum [16], but this aspect is of little significance in our calculation using Eq. (3).

With these quantities, T is computed by Monte Carlo integration of Eq. (3). The integrations also sample the weight, i.e., the nuclear density. Fig. 2 illustrates the two-particle distribution functions,

Table 1

Nuclear transparency of closed-shell nuclei at the knockout proton momentum 4.49 GeV/c in various approximations and models: With or without pN finite-range interaction (FRI), dynamical correlation (DC), and statistical correlation (SC) in the eikonal [Eq. (3)] and Glauber multiple-scattering (GMS) [Eq. (5)] formalism. The numbers in parentheses denote additional equations used. (See text for details.) Single-digit values in parentheses are statistical uncertainties (in the last digits) in Monte Carlo integrations

	FRI	DC	SC	Comments	^{16}O	^{40}Ca
a)				eikonal [(8); σ_{tot}]	0.448	0.339
b)				eikonal [(7); σ_{tot}, g]	0.533	0.409
1)	zero	no	no	eikonal [(8); σ_{inel}]	0.551	0.437
2)	zero	yes	no	eikonal [(7); σ_{inel}, g]	0.626	0.504
3)	finite	no	no	GMS [POD; (9)]	0.568(2)	0.442(3)
4)	finite	yes	yes	GMS [(9); nospect]	0.582(2)	0.450(3)
5)	finite	yes	yes	GMS [(9); full]	0.577(2)	0.447(3)

$\rho_{pn}(|\mathbf{r}|)$ and $\rho_{pp}(|\mathbf{r}|)$ for ^{16}O . For comparison, we also show the distribution functions obtained using the $A\Phi$ without the f 's. Our ρ_{pn} and ρ_{pp} have the same shapes as those by the variational Monte Carlo calculation [13] with the peak being at the same location. But our peak heights are lower than theirs by about 40%, and our ρ_{pn} and ρ_{pp} calculated from $A\Phi$ alone are also lower than their mean-field results by a similar amount. Our one-body density changes a little by the application of f 's (from 2.65 fm to 2.63 fm for the ^{16}O RMS radius) and remains close to observation (2.62 fm), while the one-body density by the variational calculation changes substantially (from 2.60 fm to 2.43 fm) [13].

Figs. 1 and 2 show that $\text{Im } V_{pp}$ reduces $\exp(-2\text{Im } \chi(\mathbf{r}, 0))$ even beyond 1 fm, that is, over the range of the short-range repulsive correlation. Since T is the mean of a product of quantities similar to those in Figs. 1 and 2, we see that the finite interaction-range effect reduces the nuclear correlation effect. This is our main finding and will be quantitatively demonstrated below. Note that Nikolaev et al. [3] reached a conclusion similar to ours based on a schematic treatment of the correlation and finite-range effects with a truncated Glauber multiple-scattering series (see also [17]). Also note that a similar conclusion has been found by schematic calculations on relativistic heavy-ion collisions [18].

In Table 1, we tabulate T computed under various assumptions by applying two formalisms, the eikonal and Glauber multiple-scattering (GMS) formalisms. The eikonal calculations are done by the use of Eq. (5) with Eqs. (7) and (8) (together with the observed

$\sigma_{\text{tot}} = 42.2$ mb) and of the one-body density generated from Eq. (9). The GMS calculations use Eq. (3) with the many-body density of Eq. (9) either directly or as a product of the one-body densities (POD) generated from it.

In Table 1, for the purpose of comparison, we first show T obtained by popular eikonal calculations by the use of σ_{tot} a) with and b) without the pair-distribution function $g(r) = 1 - \exp(-(r/a)^2)$ ($a = 0.7$ fm). From a) to b) T is seen to increase by about 30% and becomes accidentally rather close to that of our full result of 5) below, but this hides the essential physics that we are addressing.

1) in Table 1 is the eikonal result by the use of σ_{inel} . This corresponds to the leading-order [Eq. (6)] calculation of our full expression [Eq. (3)] under the zero-range approximation without correlation [8,3,10]. 2) is the eikonal result with σ_{inel} and $g(r)$. The difference between 1) and 2) is an estimate of the dynamical correlation effect with the zero-range interaction, but note that $g(r)$ used is only a schematic representation of the pair distribution (see below.)

The difference between 1) and 3) gives the finite interaction-range effect, and the difference between 3) and 5) gives the nuclear correlation effect when the finite interaction-range effect is included. 5) is of our full calculation. 3) and 5) are calculated for the same one-body density distribution, and their difference is less than 2%. This establishes our main finding. Note that the net effect becomes less in the heavier nucleus.

Our formalism permits us to examine the spectator effect [3] by removing the nuclear correlations between non-knocked-out nucleons. 4) in Table 1 is

computed in this way, and its comparison with 5) shows that the spectator effect of Ref. [3] persists in the full GMS series. Our nuclear density of Eq. (9) also allows us to remove the dynamical correlations but keep the statistical correlations: For ^{16}O , we find T of 3) and 5) to become 0.570(2) and 0.579(3), respectively, when we use the HF density $|A\Phi|_s^2$ instead of the full Eq. (9) density. The deviations from 3) and 5) are quite small.

We now discuss the dependence on the form of the dynamical correlation taking the case of ^{16}O . First, we artificially enhanced our f 's so that they yield the ρ_{pn} and ρ_{pp} by the variational calculation [13]. We observed the RMS radius of the one-body density became as small (from 2.63 fm to 2.48 fm) as in Ref. [13], and T of 5) in Table 1 decreased to 0.553 ± 0.003 , making the nucleus more opaque. Note that the enhanced f_c lifts the ρ_{pn} and ρ_{pp} peaks around 1–2 fm above the mean field results more than our original f_c , emphasizes more the intermediate-range NN attraction [15], and increases the one-body density. Second, we calculated T by removing the correlation but keeping the one-body density the same as that with the enhanced f 's. T became 0.535 ± 0.02 , a 3% decrease, which is a little more than the decrease from 5) to 3). This T is smaller than 5), basically reflecting the smaller, more dense nuclear distribution. We conclude that the correlation effect is of a few percent for a given one-body density and its precise amount depends on details of the presently poorly known NN correlations in finite nuclei.

We make here a relevant note: The short-range repulsive and intermediate-range attractive NN correlations affect the one-body distribution in opposite ways, and the common practice of examining only the former effect on T must be treated with care. For example, we find that a schematic f_c , $1 - \exp(-(r/a)^2)$ ($a = 0.7$ fm), representing only the repulsive correlation, together with other zero f 's increases the RMS radius (to 2.71 fm from the HF value of 2.65 fm) and T (to 0.605 ± 0.003 from 0.577 ± 0.03). In this case, the nucleus is merely enlarged and becomes quite transparent.

In this work, we have examined the fully semi-inclusive reaction by applying closure (no missing energy cut) and simplified kinematics (no missing momentum cut.) The NE18 experiment [7] that would be relevant to our calculation, however, was carried out

with finite cuts in both, which prevents us from making a direct comparison. For the purpose of the comparison, a Monte Carlo simulation is underway to examine the closure application [19]. Furthermore, our formalism can actually allow a finite missing momentum cut, though it makes Eq. (3) more complicated by involving one-particle off-diagonal nuclear density and the real part of V_{pN} . The feasibility of such Monte Carlo integration is under consideration together with its application to *inclusive* (e, e'). We will describe a detailed account of the present work and these considerations in forthcoming publications.

In conclusion, we find that 1) the finite-range pN interaction reduces the effect of nuclear correlation, 2) the net effect increases transparency by a small (a few percent) amount for a given one-body density, and is smaller in heavier nuclei, and 3) its precise strength depends on details of the dynamical correlations in finite nuclei, which are presently known rather poorly.

We thank S. Pieper and R. Wiringa for the use of their variational wave functions and for instructive discussions on nuclear correlations, and T. O'Neil for the use of his compilation of the pN data. R.S. and T.S. acknowledge stimulating and instructive discussions with members of the Kellogg Radiation Laboratory. The work was completed while one of us (R.S.) was at the Argonne National Laboratory. He thanks R. Wiringa and members of Theory Group for their kind hospitality. This work is supported by the U.S. DOE at CSUN (DE-FG03-87ER40347) and by the U.S. NSF at Caltech (PHY91-15574 and PHY94-20470); and by a Grand-Aid of the Japanese MESC (No. 02640216), and Fellowships of JSPS for Junior Scientists, at U. Tokyo.

References

- [1] S.J. Brodsky and A.H. Mueller, Phys. Lett. B 206 (1988) 685, and references therein.
- [2] V.R. Pandharipande and S.C. Pieper, Phys. Rev. C 45 (1992) 791.
- [3] N.N. Nikolaev, et al., Phys. Lett. B 317 (1993) 281.
- [4] O. Benhar et al., Phys. Rev. Lett. 69 (1992) 881.
- [5] O. Morimatsu and K. Yazaki, Prog. Nucl. Part. Phys. 33 (1994) 679.
- [6] S. Frankel, M. Frati and N.R. Walet, Nucl. Phys. A 580 (1994) 595.

- [7] N.C.R. Makins et al., Phys. Rev. Lett. 72 (1994) 1986; T.G. O'Neill et al., Phys. Lett. B 351 (1995) 87.
- [8] A. Kohama, K. Yazaki and R. Seki, Nucl. Phys. A 536 (1992) 716; A 551 (1993) 687.
- [9] R.J. Glauber, Lectures in Theoretical Physics, eds. W.E. Brittin and D.G. Dunham (Interscience, N.Y., 1959) Vol. 1, p. 315.
- [10] A.S. Rinat and B.K. Jennings, Nucl. Phys. A 568 (1994) 873; A.S. Rinat and M.F. Taragin, Phys. Rev. C 52 (1995) R28.
- [11] L. Frankfurt, E. Moniz, M. Sargsyan and M. Strikman, a preprint (1995).
- [12] Compiled by Thomas G. O'Neill (private communication).
- [13] S.C. Pieper, R.B. Wiringa and V.R. Padharipande, Phys. Rev. C 46 (1992) 141.
- [14] P.-G. Reinhard Computational Nuclear Physics, eds. K. Langanke, J. Maruhn and S. Koonin (Springer-Verlag, Berlinh 1991) Chapter 2, and references therein.
- [15] R.B. Wiringa, V. Fiks and A. Fabrocini, Phys. Rev. C 38 (1988) 1010, and private communication with S. Pieper and R. Wiringa.
- [16] O. Benhar, V.R. Pandharipande and S.C. Pieper, Rev. Mod. Phys. 65 (1993) 817, and references therein.
- [17] A.S. Rinat and M.F. Taragin, Phys. Rev. C 52 (1995) R28.
- [18] G. Baym et al., Phys. Rev. C 52 (1995) 1604.
- [19] T.G. O'Neill, B.W. Filippone and R. Seki (in preparation).