CALIFORNIA STATE UNIVERSITY NORTHRIDGE

Targeted Professional Development to Enhance Content Specific Workgroups

A Dissertation Submitted in partial fulfillment of the requirements

For the Degree of Doctor of Education

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Dedication

I will be forever grateful to my husband John, who supported me throughout the entire program, listened to me when I was discouraged and knew just what to say to move me forward. And I am also grateful to my children, John, Christopher, Pamela Jean and Carole Anne and my grandchildren, Ashley, James, Riley, Collin and Soyer, who understood that eventually Mom or Grandma would be able to devote more time to babysitting, watching soccer and baseball tournaments, playing games or even doing more arts and crafts once her “paper” was done.

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ABSTRACT

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by

Patricia J. Pernin

Doctor of Education Degree

in Educational Leadership

The promise that traditional professional development from an outside expert would improve the instructional practices of all mathematics teachers has not been realized in many school districts. To fix the problem some districts have turned to content-specific collaborative workgroups that use a lesson study protocol. The collaborative model assumes that teachers are proficient in their subject area. This assumption was not true for the elementary school teachers' in the study. This study provided a targeted professional development (TPD), specific to the needs of a group of fifth and sixth grade teachers that increased teachers' mathematical knowledge for teaching (MKT) and impacted their instructional practices in their classroom. The five teacher case studies and one workgroup study provided the data for analyzing the growth of the participants. The findings suggested that the MKT of all of the teachers was increased and the instructional practices of some of the participants were impacted. The increased MKT developed in the TPD appeared to enhance the discussions within the workgroup sessions.

Chapter I

The promise that traditional professional development from an outside expert would improve the instructional practices of all teachers has not been realized in many schools or districts, as indicated by the continued low scores in standardized testing. In a traditional professional development setting, experts in a particular field such as mathematics present teachers with teaching strategies and/or subject specific content. After teachers receive the information, it is then the teachers' responsibility to use the provided information within their classrooms. Hamilton, Stecher, Russell, Marsh, and Miles (2008) found that when teachers work in isolation, providing traditional professional development by an expert has little effect in improving classroom practices. Hamilton, et al. (2008) said that this may be due to the lack of teacher input into the professional development process at the school level as well as the lack of teacher collaboration to discuss implementation of the new strategies. This lack of impact of traditional professional development in mathematics was evident in a study done by the Los Angeles Unified School District (LAUSD) (Newton, 2005).

Newton's research (2005) cited the lack of teacher "buy-in" when a traditional professional development format was used in mathematics and proposed that this may be a contributing factor to the lack of systemic change in the instructional practices of mathematics teachers. Although "buy-in" is necessary for systemic change in instructional practices (Schmoker, 2004), I believe that teachers must also have sufficient mathematical knowledge for teaching (Ball, 2000) in order to effectively teach mathematics. Without appropriate professional development to build the capacity of a teacher's mathematical knowledge for teaching, collaboration between mathematics teachers is futile and systemic improvement in classroom instruction cannot be realized.

Background

The initiation and implementation of traditional professional development in mathematics in LAUSD began in 2000 and continued until 2006. The traditional professional development included concept lessons. The intent of professional development was to increase the capacity of teachers to facilitate classroom discussions amongst students that was focused on deepening students' conceptual understanding of mathematics. Concept lessons were provided as a tool that the teachers would use to engage, facilitate and guide students in developing their capacity to make connections in mathematics concepts leading to a deeper conceptual understanding of mathematics (Stein, Smith, Henningsen & Silver, 2009).

The work was done in collaboration with the *Institute for Learning* (IFL), University of Pittsburgh (Institute for Learning, 2007). In particular, mathematics educators from the IFL and University of Pittsburgh worked with the District mathematics leadership to provide professional development in mathematics for sixth grade, seventh grade and algebra 1 teachers. The IFL team facilitated and led the LAUSD mathematics instructional team of coordinators, specialists and coaches through the concept lessons, pressing the team to make connections between the mathematical concepts by using numerous access strategies. During professional development sessions, the IFL team consistently modeled questioning techniques that resulted in eliciting from the participants the intended connections within the concept lesson. The mathematics team was then required to present the teachers in LAUSD with the same professional development. It was assumed that the strategies, pedagogy and content used in professional development presented by the mathematics team would be replicated by the teachers in their classroom with permanent changes in instructional practices of the teachers, resulting in improved student achievement.

In 2005, LAUSD re-examined the effectiveness of traditional professional development delivered by IFL. The impact of conceptual mathematics lessons provided to the teachers was found to be negligible (Newton, 2005). Newton suggested that teachers' resistance to implementation of the required lessons was due to a lack of teacher input into the development of the concept lessons. Although teachers were required to implement the lessons, they were not required to reflect upon the implementation of the lessons or the effectiveness of the lessons. Teachers felt disengaged in the process, as reported in the District research and planning report (Newton, 2005).

The final year with IFL, 2006, was funded by a grant. The project, PRISMA (Providing Rigorous Instruction for all Students in Mathematics), was an Algebra 1 traditional professional development model for high school Algebra I teachers. An additional component of the PRISMA project included providing time for teachers to collaborate and develop a lesson or lessons that would augment each IFL concept lesson. The teacher collaboration occurred at the school site; however, no particular protocol was provided to structure the collaborative meetings of the teachers.

As a result of the lack of systemic change in instructional practices in the classroom (Newton, 2005), an additional District level professional development plan was initiated in collaboration with Pearson Learning Teams. The pilot, Learning Teams, was offered to the same high school Algebra I teachers in the PRISMA program and the program was also offered to eight middle schools throughout the District. The Learning Teams' pilot provided a structure and research-based protocol for collaborative content-specific workgroups. The teachers in each workgroup collaboratively developed a lesson based on a student need, analyzed the resulting student work and determined the effectiveness of the instruction. Although Learning Teams was piloted in mathematics, the protocol was generic and was suitable for any content-specific workgroup. The entire process was dependent upon the knowledge of the teachers in the group. Learning Teams did not provide content expertise to build the mathematical knowledge of the teachers. The advisers only provided support for implementation of the Learning Teams protocol.

By building the mathematical knowledge for teaching (MKT) of participating teachers through the PRISMA program, the collaborative inquiry-based content-specific Learning Teams' lessons should have been enhanced. The problem appeared to be the lack of collaboration and coordination between the two programs. The programs were implemented independently. Although teachers were reluctant to use the lessons given to them in PRISMA, the lessons that were created or modified by the teachers during their collaborative workgroup sessions were implemented more readily in the classroom. The lessons that were developed by the teachers embraced some of the mathematical pedagogy and content introduced by IFL. The problem with the two programs running simultaneously was the lack of understanding by the teachers about how these two programs could complement each other rather than compete with each other. There was confusion as to how the Learning Teams protocol could be used to develop lessons required in PRISMA. In addition, the PRISMA concept lessons did not necessarily align to the student needs as determined by the teachers in the collaborative workgroup.

When the grant funding ended for the PRISMA project, the District decided not to fund PRISMA and discontinued collaborating with IFL. Instead the District expanded the Learning Teams pilot. The District concluded that the collaborative content-specific workgroups would provide the setting where teachers and administrators could discuss and improve instruction and increase student achievement. It was assumed that additional professional development to build the content capacity of the mathematics teachers was not necessary. This assumption proved to be inaccurate. Although the Learning Teams project promised increased student achievement measured by the California Standardized Test scores, in June, 2010, the District stopped funding the program centrally due to a limited budget and the lack of conclusive data indicating the success of the program.

Although collaborative content-specific workgroups may provide a setting to discuss and improve instructional practices, teachers must have sufficient mathematical knowledge to be effective. According to Deborah Ball (2000), mathematical knowledge for teaching (MKT) is fundamental to teaching mathematics effectively. Ball and Hill (2009) describe the many components of mathematical knowledge for teaching (MKT), which includes specialized content knowledge specific to teaching mathematics. The specialized content knowledge specific to teaching mathematics includes multiple pathways for solving problems. This knowledge requires that teachers have additional knowledge for solving problems that is beyond the common content knowledge that many adults possess for solving mathematics problems. Mathematical knowledge for teaching (MKT) is the specific knowledge for teaching that consists of specialized content knowledge, common content knowledge and pedagogy. MKT integrates pedagogical content knowledge and subject matter knowledge (Ball, Hill & Bass, 2000). A lack of mathematical knowledge for teaching could contribute to the inability of the teacher to effectively teach all students (Hill, Sleep, Lewis & Ball, 2007). In addition, Sowder (2007) states that professional development must not only increase content knowledge, it must also increase the pedagogical knowledge of the teacher. Sowder contends that building MKT requires that all professional development is relevant to the student curriculum that the teachers teach and relevant to the student lesson that is developed in the workgroup.

As the Administrative Coordinator of Learning Teams for LAUSD, I had the opportunity to observe mathematics workgroups and analyze the lessons developed by the workgroup. In many of the workgroup meetings there appeared to be a lack of mathematical knowledge for teaching, as demonstrated by the use of informal mathematical terminology and/or incorrect use of mathematical terminology. This lack of mathematical understanding contributed to low cognitively demanding lessons developed during the collaborative workgroup sessions. In my observations of collaborative workgroups and examination of the resulting lessons, I noted that the mathematics lessons generated in the Learning Teams workgroups were procedural and were not cognitively demanding. The majority of the student lessons that I reviewed from over eighty schools reflected the lower cognitive levels identified by Stein and Smith (1998) as Memorization Tasks or Tasks Without Connections with little or no cognitive demand.

I believe that teachers who are provided professional development focused on developing mathematical knowledge for teaching are prepared to develop and implement more effective lessons. The lack of mathematical knowledge for teaching for many of the teachers who participated in Learning Teams initiative, especially in the lower grades, inhibited the development of mathematics lessons that were cognitively demanding. In addition, a collaborative workgroup in and of itself will not necessarily provide the appropriate setting to increase teacher understanding of MKT if there is not at least one content expert in the group. I hypothesized that this problem could be eliminated through a supplemental targeted professional development that addressed the content and pedagogy necessary for teaching.

Problem Statement

The past professional development initiatives that were focused on teacher collaboration have not “fixed” the problem of classroom effectiveness because they do not address the problem of teachers’ poor mathematical knowledge for teaching (MKT). When teachers work collaboratively to determine how to provide more effective classroom instruction, teachers must have a deeper understanding of the mathematical knowledge for teaching that allows them to teach beyond the procedural level of mathematics (Ball, 2000; Stein, & Smith 1998). Content-specific teacher workgroups provide the vehicle to drive this change; however, if the teachers in the workgroup do not have the MKT expertise then the process of collaboration may not produce the intended results of cognitively demanding lessons. Collaborative content-specific workgroups are effective in changing the instructional practices of the participants only when the participants have the tools and expertise to make the changes. A supplemental targeted professional development (TPD) based on the needs of the participating teachers that builds the capacity of the participants’ MKT may provide the tools and expertise needed to enhance the effectiveness of classroom instruction. Fixing the problem means addressing the MKT deficiencies of teachers who teach mathematics. The TPD must be specific to the needs of the teachers and students as determined by the teachers. Previous professional development in LAUSD has not effectively addressed this problem.

Purpose and Significance

The purpose of this study was to provide insight into how schools might impact teachers' instructional practices in the classroom through a targeted professional development for teachers participating in a collaborative content-specific workgroup. Prior to the study, the District emphasized either a traditional professional development model consistent with IFL or a Learning Teams model consisting of content-specific collaborative workgroups. Since the intent of the District was not the merging of traditional professional development and the workgroup sessions there was no need to develop a relationship between the two professional development settings. If the relationship were initiated, the traditional professional development would presumably extend the knowledge of the participants to enhance the work in the collaborative workgroup setting and the workgroup setting would inform the direction of the traditional professional development sessions. This relationship, however, was never articulated or initiated.

I believe that the relationship between the traditional professional development and the workgroups sessions needs to be explicit; the traditional professional development needs to be specifically aligned to the workgroup sessions and the needs of the teachers and the needs of the students. In the study presented in this dissertation, I refer to professional development as targeted professional development (TPD). It is developed based on the needs of the teachers and the students as expressed by the teachers. The TPD is designed to build the capacity of the teachers' MKT in the areas determined by the teachers' needs. This study examines whether explicitly integrating targeted professional development with content-specific workgroup sessions affects MKT of teachers in the study and whether the instructional practices of the teacher are affected.

Research Questions

The purpose of the study was to determine whether and how supplemental targeted professional development (TPD) designed to build mathematical knowledge for teaching (MKT) influenced classroom instructional practices of fifth and sixth grade teachers who were participating in a content-specific workgroup. The intended intervention, TPD, was aligned to the lesson the teachers developed during the workgroup sessions and the needs of the teachers in the workgroup. The TPD was intended to increase the MKT of the work group participants. The research study addressed the following question:

> What is the impact of TPD sessions on teachers' understanding of and fluency with mathematical concepts and on the teachers' instructional practices in the classroom?

Intervention: Targeted Professional Development (TPD)

In order to increase the effectiveness the instructional practices of mathematics teachers, I developed a targeted professional development (TPD) that was intended to increase the participants' mathematical knowledge for teaching (MKT). Integrating the appropriate MKT into the collaborative setting required that the targeted professional development be relevant to the workgroup setting and specific to the needs of the teacher.

The TPD that I designed and facilitated provided the teachers the opportunity to develop a deeper level of MKT used in their classroom instruction. The teachers determined the focus area for the TPD would be fractions since they felt that fractions were the most difficult area in the curriculum to teach to their students.

The targeted professional development (TPD) sessions I presented were focused on questioning techniques, making connections in mathematics and increasing MKT. The TPD was based on the fifth and sixth grade mathematics curriculum being taught by the teachers during the study. Since the teachers determined that the most difficult area for their students to understand was fractions, the TPD was based upon building MKT of fractions through appropriate concept lessons and the released questions from the University of Michigan, Ann Arbor (2008).

Overview of Methodology

In order to address the research question, a case study approach was used. I chose the case study in order describe the development of the teachers over time. The case study also allowed me to describe the TPD within the school environment making the situations relevant to the existing real-life settings of the participants. I used the five individual case studies to describe the journey of each teacher in the study and one case study to describe the journey of the workgroup as a whole.

The participants were fifth and sixth grade teachers from a year-round newly reconfigured kindergarten through sixth grade elementary school. Data collection for this study included interviews of workgroup participants, observations of targeted professional development sessions, observations of content-specific collaborative workgroup sessions, classroom observations and analysis of the workgroup’s collaboratively developed lesson and the associated student work. Prior to and after the implementation of the collaboratively developed lesson, observations of each classroom and in-depth interviews with each participant were conducted.

Analysis included categorizing the questions that were asked during the observations as to the level of rigor based on the Level of Questions Guide developed by Boaler and Brodie (2004). I analyzed the level of cognitive demand of the tasks developed by the teachers in their workgroup sessions as well as the connections that the teachers made between mathematical concepts during their discussions of the tasks presented in the TPD and workgroups sessions. Since the three TPD sessions were interspersed between the seven collaborative workgroup sessions and the TPD sessions sometimes continued into a workgroup meeting, dialogue during the workgroup meetings was analyzed over time. The intent was to analyze both the TPD and the workgroup meetings to determine any changes. Classroom observations and interviews were analyzed to determine the teacher's perceptions of effective professional development and to determine if there were changes in instructional practices due to the TPD. In addition the observations were analyzed using that Task Analysis Guide (Smith & Stein, 2009) and the Level of Questions Guide (Boaler & Brodie, 2004).

Finally the analysis of the lessons developed by the workgroup provided insight into the impact of the TPD on the development of lessons. In addition, the lessons provided an opportunity to evaluate the fidelity of implementation of the lessons developed by the group and the resulting student work. During some of the workgroup sessions, the participants' analyzed the implementation of lessons. The transcripts from these meetings provided teacher reactions to the lesson, changes in the implementation and any insights the teachers may have learned from the student work and implementation of the lesson.

Limitations and Delimitations

Limitations to this study were due to serving all three roles at once during the research: the role of principal researcher, active participant in the workgroup and as the provider and developer of the targeted professional development. These three roles may make the findings difficult to replicate since I was the facilitator of the workgroup sessions, the expert provider of the TPD and the researcher who gathered and interpreted the data. The insights, however, may have implications for further study and may also have implications for further research in the area of increasing teacher capacity in classroom instruction and increasing the effectiveness of the collaborative workgroup setting through a targeted professional development. According to Yin (2009) a case study can be used for generalizations even though the case study does not utilize a formal "sample."

The specific professional development was tailored to the teachers' ability or special needs and may not be relevant to other schools with different demographics. Replication of the study would require a researcher that is knowledgeable in MKT and one who could provide TPD for any part of the curriculum for a specific grade level or levels.

Organization for the Dissertation

Chapter II discusses relevant research and synthesizes the empirical literature relating to the available theory and research needed to develop the targeted professional development designed to increase knowledge of the teachers in this study and impact their classroom instruction. The review of literature answers the following questions: 1) Why teacher workgroups and what should they do? 2) What kind of content knowledge do teachers need for teaching? 3) What do we know about elementary teachers' level of mathematical knowledge for teaching? 4) What are the components of an effective mathematics professional development? Chapter III provides the rationale for the utilization of the case study format and describes the research setting, participants, data collection and analysis methods. Chapter IV presents the results and includes the presentation of relevant qualitative data. Chapter V interprets and discusses the results in relation to the research questions, literature review, and conceptual framework, and concludes with recommendations for policy and practice.

Chapter II

This literature review provides the background information identifying the limitations of the collaborative content-specific workgroup setting, the mathematical knowledge necessary for teaching mathematics and the components of effective professional development. The available theory and research in the literature review was used to develop a targeted professional development (TPD) designed to increase teachers' mathematical knowledge. The review of literature addressed the following questions: 1) Why teacher workgroups and what should they do? 2) What kind of content knowledge do teachers need for teaching mathematics? 3) What do we know about elementary teachers' level of mathematical knowledge for teaching? and 4) What are the components of an effective mathematics professional development? The chapter ends with a summary of the literature and implications for this study.

Why Teacher Workgroups and What Should They Do?

Teachers have a large degree of autonomy with little or no collaboration inherent in their work (Stecher, et al., 2008). Stecher, et al., contend that the autonomy may inhibit changing a teacher's instructional practices within the classroom. Schmoker (2004) states that if there are established organizational structures within a larger system such as school and these smaller structures are collaborative teams, the opportunity to change practices may be possible. One such collaborative structure is the collaborative content-specific teacher workgroup that promotes an environment where teachers examine various teaching strategies, content and pedagogy through a collaboratively developed lesson (Alvine et al., 2007; Judson, Schein & Yoshida, 2007; Stewart & Brendefur, 2005). Flecknoe (2005) emphasizes that for teachers to improve in their practice, there must be opportunities for teachers to collaborate. The collaborative content-specific workgroup provides teachers the opportunity to study in-depth their teaching practices through a collaboratively developed lesson by analyzing the resulting student work (Gallimore, Ermeling, Saunders, & Goldenberg, 2009).

Goldenberg and Sullivan (1994) identified four elements of their School Change/Getting Results model that led to improved teaching and learning for all collaborative groups within a school: participants agree on shared goals; indicators to measure success are established; assistance is provided for a workgroup by capable others; and school leadership provides both support and pressure. Saunders, Obrien, Hasenstab, Marcelletti, Saldivar and Goldenberg (2001) further describe how these four basic elements for change are operationalized within a school. One of the professional development settings in the study by Saunders, et al. (2001) was the teacher work group[1]. This teacher work group model in the study helped teachers reach the student learning goals that the teachers determined while participating in the work group. The protocol for the workgroup session was an inquiry-based structure that guided teachers through developing a lesson based on a student need, analyzing the student work and determining next steps. The teachers studied and worked collaboratively to implement effective instructional practices.

In the collaborative workgroup model (Saunders, et al., 2001) the teacher workgroup was responsible for eliciting help from experts, referred to as “capable others,” to provide specific teaching methods when the group’s knowledge was not sufficient. The emphasis of the workgroup discussion focused on developing methods of teaching.

[1] In the study by Saunders, et al. (2001), the term workgroup was intentionally separated indicating teachers “work” and get things done

Powell, Goldenberg and Cano (1995) said that the effectiveness of the workgroups described by Saunders, et al. (2001) was dependent upon the expertise available, whether within the group or from an expert outside of the group. As stipulated, the capable others provided a variety of teaching methods such as reading comprehension strategies. The capable others were not expected to build the specific content knowledge of the participants. If the participants lacked content knowledge, this could contribute to a weakness in the model for workgroups. In Saunders, et al. (2001) collaborative workgroup model study, there was an underlying assumption that the participants were knowledgeable in the content area.

At Harvard University, graduate students in mathematics, who were also undergraduate instructors of mathematics, participated in a collaborative lesson study group (Alvine, 2007). Alvine noted that there was a positive impact on undergraduate mathematics. The emphasis upon teaching and self-reflective learning by the undergraduate instructors was evident in the increased effectiveness of their teaching practices. Although not highlighted, the participants' in-depth content knowledge when using a lesson study model focused the participants on their instructional practices related to the content they were teaching. The results of this study suggest that the collaborative mathematics workgroup may provide a setting to examine instructional practices leading to greater effectiveness of classroom instruction.

What Kind of Content Knowledge Do Teachers Need for Teaching Mathematics?

Hiebert, Morris, Beck and Jansen (2007) maintain that teachers should be proficient in analyzing their teaching through the analysis of their students' work related to the lesson. Hiebert, et al. further believe that pre-service teachers should learn from their teaching. In addition, teachers need to treat their lesson as an experiment that is analyzed and refined (Morris, Hiebert & Spitzer, 2009). If teachers are to analyze lessons either in a collaborative workgroup or individually, they must develop what Shulman (1986) has described as pedagogical knowledge for teaching (PCK) and the knowledge later expanded by Ball and Hill (2009) known as mathematical knowledge for teaching (MKT). The development of the idea of PCK as well as the in-depth analysis by experts in the field of teacher education of the knowledge necessary for teaching mathematics has been ongoing for over 25 years.

Pedagogical knowledge for teaching.

The introduction of pedagogical content knowledge (PCK) by Shulman in 1986 continues to influence professional development of teachers. He believes that pedagogy and content should not be separate practices but instead should be intertwined to support effective instruction. He claims that PCK includes elements of general pedagogical knowledge, broad principles and strategies for classroom management and organization. The elements of PCK are beyond the delivery of pure subject matter. Shulman proposes that PCK also includes the teacher's knowledge about the learners. He proposes that the knowledge of educational contexts that range from classroom workings to the character of the community and cultures of the learner or student should be understood by the teacher. In addition to knowing the content, he believes that PCK should include teacher knowledge of the curriculum, materials and programs and that this knowledge should serve as "tools of the trade" for teaching the content (Shulman, 1987). Pedagogical content knowledge, according to Shulman is "that special amalgam of content and pedagogy that is uniquely the province of the teachers, their own special form of professional understanding" (Shulman, 1987, p.8). Shulman consistently references the need to study content and the role that pedagogy plays in teaching since the connection between the two domains is apparent (Shulman, 1986).

Shulman (1986) defined the characteristics of PCK as:

> The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others…Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

The idea of pedagogical content knowledge has taken root in education, as indicated by the increased research in mathematics (Ball &Hill, 2009; Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008; Thames & Ball, 2010). Over the past 24 years, the explicit delineation of the specific attributes of PCK and the implications in teacher education continue to be studied and expanded upon.

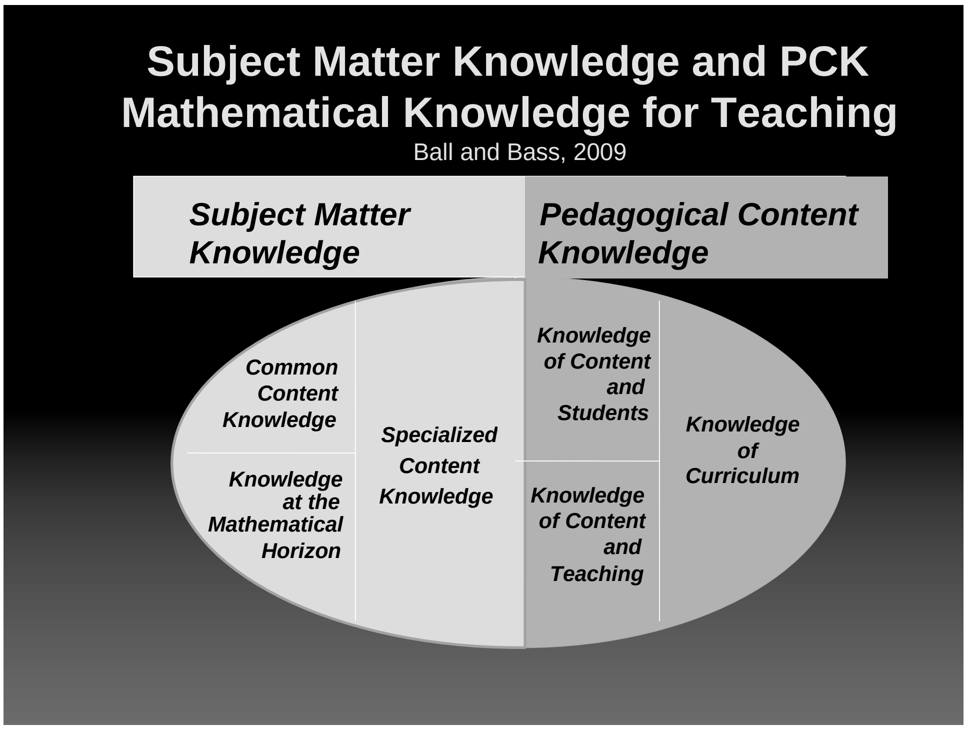
Mathematical knowledge for teaching.

For over 20 years researchers have continued to expand on the ideas of pedagogical content knowledge (PCK) and the implications for teacher education and professional development. Defining the knowledge necessary for teaching may prove to be a monumental task; however, the result could be professional development that would align with the needs of teachers. As Ball and Hall (2009) continue to delve into the refinement of Shulman's PCK, their work provides a framework that influences the training of pre-service to skilled mathematics teachers for grades K through 12.

As early as 1988, Shulman's influence on Deborah Ball led her in pursuit of defining the elements of teaching mathematics. Ball's exploration of the knowledge needed for teaching developed over 15 years of work and includes a set of testable hypotheses about the mathematical knowledge for teaching and a survey that measures of content knowledge for teaching mathematics (Ball, Thames & Phelps, 2008). In 1999 Ma also tried to measure the mathematical understanding of elementary teachers in China and the United States by using questions that required in-depth understanding of teaching mathematics as well as questions that measured a teachers' ability to answer basic elementary level mathematics problems. The work of researchers such as Ball, Hill, and Bass (Ball, 2000; Ball, & Forzani, 2009; Ball, & Hill, 2009; Ball, Hill, Bass, 2005; Ball, & Hill, 2009; Ball, D., Thames, M., Phelps, 2008; Hill, & Ball, 2004; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Hill, Schilling, & Ball, 2004; Hill, Sleep, Lewis, & Ball, 2007) helped to refine and define with more detail the pedagogical content knowledge introduced by Shulman in the late 1980's. Ball and other researchers' work continues to delve deeply into the multidimensional aspects of teaching mathematics (Ball & Hill, 2009; Ball, Hill, & Bass, 2005; Ball, Thames & Phelps, 2008).

In Figure 2.1, Ball and Hill (2009) identify two main domains of mathematical knowledge for teaching (MKT): subject matter knowledge (SMK) and pedagogical content knowledge (PCK). The pedagogical content knowledge domain was originally identified by Shulman (1998) with further delineations introduced by Ball & Hill (2009). Ball, Thames and Phelps (2008) found that there was a significant positive association between the teacher's levels of MKT and the quality of mathematical instruction. The areas of knowledge associated with MKT, specialized mathematics knowledge (SMK) and common content knowledge (CCK), are both important to developing a high level of MKT. Basically, teachers must be able to do the mathematics that the students are being asked to do. This knowledge is called the common content knowledge. The common content knowledge (CCK) is not unique to teaching. It is knowledge that other professions may use. The other component of MKT is specialized content knowledge (SCK). This knowledge is unique to teaching and is equally important for teachers to develop for effective mathematics teaching.

Figure 2.1. Subject Matter Knowledge and PCK, Mathematical Knowledge for Teaching



SCK (specialized content knowledge) includes the ability to look for student errors, understand different methods of solutions and make decisions about the appropriate problems to use for a particular operation or concept, and choose appropriate examples and representations that best exemplify the concept being taught. Ball and Hill (2009) found that this type of knowledge is unique to teaching. It is this type of mathematical knowledge that sets apart the teaching of mathematics versus the mathematics used in other professions. This type of mathematical knowledge specific to teaching should be an integral part of professional development designed for teachers (Ball & Hill, 2009).

Not only must teachers' knowledge include mathematical knowledge, it must also include the knowledge of the content and the students (Ball &Hill, 2009). This knowledge requires knowing what students will do with the work that is given to them. Teachers must anticipate student work products, misconceptions and atypical responses. This is a key component that teachers bring to the classroom and requires that teachers know the content and the possible student responses.

The final area of knowledge that effective teachers must bring to the classroom is knowledge of content and teaching (Ball & Hill, 2009). Knowing the content is not sufficient without the appropriate pedagogy to enhance the effectiveness of the teaching. Shulman (1986) reiterated this idea when he said: "Mere content knowledge is likely to be as useless pedagogically as a content-free skill" (p.8). This area requires teachers to be able to make decisions that meld together the content and pedagogical strategies resulting in accomplishing the intent of the lesson.

The intent of clarifying and delineating these areas of teacher knowledge is not to abandon PCK, but rather to define and categorize the many intricacies of teaching (Ball, 2000). Through a deeper understanding of knowledge that is needed in effective teaching, appropriate professional development to build these areas of knowledge is possible.

What Do We Know about Elementary Teachers' Level of Mathematical Knowledge for Teaching?

It is necessary to consider the content knowledge of elementary school teachers to include not only the procedural skills inherent in mathematics but the conceptual skills as well. Ensuring that all teachers are prepared to teach elementary mathematics has been a concern of educators and researchers over the last twenty years (Hill, & Ball, 2004). In the 1990's Stoddard, Connell, Sofflet and Peck (1993) found that pre-service teachers demonstrated only a 5-10% accuracy in conceptual skills and a 37-93% accuracy in procedural skills for elementary school mathematics. Ma's study (1999) found that Chinese teachers have a much better grasp of mathematics than American teachers in the area of conceptual understanding for elementary mathematics. Both conceptual understanding of mathematics and procedural skills for mathematics are necessary components of MKT.

Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, and Ball (2008) found that the quality of mathematical lessons seemed to be a function of the mathematical knowledge of the teachers. Hill et al. (2008) further state that there is a need to improve teacher knowledge in the conbtent area. They define the content area as mathematical knowledge for teaching that includes common mathematical knowledge and the specialized mathematical knowledge for teaching. Some of the topics included in the specialized knowledge for teaching were the why and how a procedure works and types of student errors. In this study, Hill, et al. (2008) found a strong relationship between "what a teacher knows, how she knows it, and what she can do in the context of teaching" (p. 496). In addition Hill, et al. (2008) noted the wide variation in mathematical quality of the lessons, which the researchers attribute to the level of mathematical knowledge that the teacher had. Again, the mathematical knowledge was not just the content knowledge but included the specialized knowledge for mathematics teaching.

In 2001, The Conference Board of Mathematical Sciences (CBMS) recommended at least three courses specific to teaching mathematics for elementary pre-service teachers. The US Department of Education (2002) followed with a statement that content knowledge was absolutely necessary for teaching students mathematics; however, defining the mathematics content necessary was not defined. Tsao (2005) suggests that pre-service elementary school teachers' knowledge is poor in the areas of number sense. Flores, Patterson and Shippen (2010) found that over half of the pre-service and credentialed elementary and special education teachers in their study had difficulty with fractions and solving word problems involving multiple steps. In addition, Welder and Simonsen (2011) found that there are gaps in mathematical knowledge of elementary teachers and suggested that the math knowledge of pre-service and in-service math teachers may be insufficient. The implications of all of these studies are that the pre-service curriculum may be lacking the appropriate mathematics training for elementary school teachers.

All of these studies pointed to the necessity of appropriate training for elementary teachers in mathematics. Although the Conference Board of Mathematical Sciences (CBMS) report of 2001 (Conference Board of the Mathematical Sciences, 2001) recommended that pre-service elementary teachers have at least three courses of fundamental elementary mathematics, the National Council on Teacher Quality (NCTQ) found in 2008 that there was a lack of consistency in the courses that pre-service programs provided as well as the number of courses required in the credentialing programs. Due to the lack of consistency in training for elementary pre-service teachers, districts should consider professional development in mathematics to increase teachers' knowledge of fundamental mathematics content. If teachers have not been provided sufficient course work in mathematics prior to teaching, instruction in mathematics will be negatively affected.

Studies by Goldenberg (1992), Gallimore et al. (2009) and Saunders et al. (2001) suggest that simply having teachers meet in content-specific workgroups will not be sufficient if the knowledge base of the participants is lacking unless there are experts in the content area available to support the workgroup. Newton (2005) found that merely providing professional development was also not effective. Newton suggested that providing professional development did not provide long-term change in classroom instructional practices of the teacher. Research would suggest that merging a targeted professional development that is based on MKT with a collaborative workgroup setting where teachers may use the knowledge learned in the targeted professional development may allow the effectiveness of both settings to be maximized.

What are the Components of an Effective Mathematics Professional Development?

Professional development in mathematics has been spurred on by the accountability demanded by No Child Left Behind Act of 2001 (NCLB). To increase the effectiveness and capacity of teachers it is necessary to proved effective professional development to address the needs of the teachers (Ball & Cohen, 1999). Darling-Hammond and Sykes (1999) stated that the quality of professional development must improve beyond what had previously been done in mathematics allowing for teachers to improve their skills and to become “capable of far more sophisticated forms of practice” (Darling-Hammond & Sykes, 1999, p.153).

Effective professional development provides opportunities for professional growth to support learning for all students, does not isolate learning strategies and includes growth in practice (Ball & Cohen, 1999). Ball and Cohen further emphasize that professional development must address all of the complexities of mathematical knowledge for teaching (MKT) due to the interactions of all of the components that affect or influence how teachers teach.

Sowder (2008) discusses six goals that need to be taken into consideration when designing professional development: developing a shared vision; developing mathematical content knowledge; developing an understanding of how students think about and learn mathematics; developing pedagogical content knowledge; developing the role of equity in school mathematics; and developing a sense of self as a teacher of mathematics. Similar to Sowder, Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) focused on four types of goals for professional development that included: student learning, teacher learning, teaching practice, and organization. The focus in this section is on teacher learning and practice in the classroom and is organized according to Sowder's goals for teacher learning and professional development with other literature interjected as necessary. Sowder's goals provided the structure for the development and implementation of the teachers' targeted professional development (TPD) in this study.

Goal 1: Developing a shared vision.

The first goal of professional development and learning is to develop a shared vision. Professional learning requires time and more importantly requires building a learning community that includes a commitment to a vision. DuFour, Eaker and Dufour (2005) stated that when goals are determined it is also necessary to describe what the expected outcomes would look like. This is the first step in developing professional development that empowers teachers with increased opportunities to learn.

Although a shared vision is important, Fullan (2008) believes that a vision can be a result of action. He believes that adopting a “ready, fire, aim” (p.31) attitude may produce more effective results. The implication for the professional development model is designing professional development with a purpose, implementing the professional development and using the outcomes to clarify and solidify a shared vision. Schmoker (2002) believes that specifically targeting subjects that are lowest scoring and targeting specific standards should drive learning goals. For example, in LAUSD, the district where this study took place, data indicated that achievement in mathematics declined dramatically in fourth grade and continued to decline for each higher grade. Focusing on the particular mathematics standards that align with the initial decease would translate into providing targeted professional development around the key standards. Focusing on specific student goals for learning then drives professional development and learning for teachers. Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) allege that teacher learning and teacher practice goals must flow out of the student learning goals, knowledge and skills of the teacher addressing the student learning goals and standards that are dictated by the state. Teacher professional development in mathematics should specifically target the instructional strategies and content that impact student learning.

Goal 2: Developing mathematical content knowledge.

Sowder’s second goal is to develop mathematical content knowledge. Ma (1999) showed that teachers need more mathematics preparation as well as a different preparation in mathematics. Taking college mathematics is not sufficient for teaching mathematics at the elementary level. As noted earlier, Ball, Hill and Bass (2005) said that the mathematics knowledge is specific to teaching of mathematics and utilizes information that may not be used in other professions. Elmore (2001) further noted that pre-service teachers many times know the rules and procedures and lack the knowledge of concepts and reasoning. Professional development must provide teachers opportunities to learn more about mathematics through the lens of student thinking, curriculum and classroom events (Sowder, 2008).

Goal 3: Assessing students' conceptual understanding.

Although professional development may provide an opportunity for increasing the depth of understanding of mathematics for teachers, teachers must have an understanding of how their students think about mathematics and how students learn mathematics.

Sowder's third goal of professional development includes Schifter's (2001) contention that it is important to listen and assess the dialogue of the students to determine their conceptual understanding of mathematics and to use that knowledge in designing professional development. This professional development goal of determining the knowledge of students and using that knowledge in professional development is in alignment with the Ball's (2000) contention that teachers' knowledge of content and students is important in the classroom. This is similar to cognitively guided instruction (CGI) (Carey, Fennema, Carpenter & Franke, 1995) that is often used to elicit this understanding of student thinking through listening, understanding, and teaching based on the information collected in the classroom (Sowder, 2007). Cheng (2010) used cognitively guided instruction in his work with a group of pre-service teachers to analyze high school students' daily progress and to adjust their instruction accordingly. In

Cheng's Responsive Teaching Cycle (RTC), student teachers had an opportunity to apply theoretical principles learned in their courses and to reflect on their practices. In addition the teachers were developing an understanding of student thinking by examining their own students' work. The mentor teachers were also part of the professional development collaboration and actively participated in the daily lesson planning with the pre-service teachers. Cheng (2011) suggests that the pre-service and mentor teachers experienced growth in their teaching. Cheng's findings further suggest that the collaborative nature of the teacher and mentors provided an opportunity for growth in teaching practices and through analysis of student work the development of teacher understanding of how students think about and learn mathematics occurred (Sowder, 2007). If teachers are to grow in their instructional practices it is essential to provide an opportunity for teachers to study the work of their students to gain greater understanding of how their students' think and learn mathematics.

Goal 4: Development of PCK and MKT.

Sowder's fourth goal of professional development requires the development of pedagogical content knowledge of teachers. The mathematical knowledge must be directly tied to pedagogy. The mathematical knowledge of the teachers must be sufficient to provide teachers the opportunity to make connections in the mathematics and to allow teachers to make adjustments to the instruction based upon the student needs. Change will not occur in the classroom instruction unless professional development addresses all of the components of MKT (Ball, Hill & Bass, 2005; Hill & Ball, 2004; Hill, Sleep, Lewis & Ball (2007). Simon (1997) included the elements indicated by Ball and also included the process that teachers should use continuously to refine their practice:

the reflective, refining and recursive process of teaching. Simon's reflective, refining and recursive process of teaching was evident in the collaborative content-specific workgroups that used the Learning Teams reflective cycle implemented in over eighty schools in LAUSD. What was not evident in the LAUSD implementation in many mathematics workgroups was MKT. The literature suggests that if collaborative content-specific mathematics workgroups are to be effective then there must be knowledge of MKT or a professional development model that includes building the MKT capacity of teachers based on the needs of the students and the teachers. Gallimore, et al. (2009) referred to individuals who supported the collaborative workgroup as capable others; however, the capable other was not the content expert. In the Gallimore, et al. (2009) study, the participants in each workgroup were expected to possess sufficient content knowledge.

Goal 5: Developing an understanding of the role of equity in school mathematics.

Sowder's (2007) fifth goal focuses on equity in school mathematics and includes providing culturally responsive education. Research indicates that when students focus on making meaning of their mathematics rather than emphasizing skills and procedures, they learn more mathematics and have a better understanding of math (Boaler, 2002; Silver, Smith, & Nelson, 1995). For example, the work of the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project initiated in middle schools, located in six school districts from different regions of the country all serving low-income urban communities, demonstrated that when teachers provided an opportunity for students to work collaboratively and develop the conceptual understanding of mathematics, students gained more understanding than when teachers focused on teaching skills and procedures in isolation (Silver, Smith & Nelson,1995). Professional development for teachers must provide teachers the opportunity to explore how to make meaning of mathematics and how to translate this understanding into the classroom.

Goal 6: Developing a sense of self.

Sowder's (2007) last goal requires that mathematics teachers develop a sense of self as a teacher of mathematics. Professional development should allow teachers to experience professional development as a learner through exploration of learning to develop their confidence to build their knowledge of mathematical content (Nelson & Hammerman, 1996). The professional development then must be in a collaborative setting (Cohen & Hill, 2001) that provides teachers an opportunity to delve into learning about students' thinking in mathematics leading to a change in the teachers' conceptual understanding of teaching. Key to this goal is providing teachers the opportunity to experience math as a learner. This goal allows the teacher to be an active participant in his/her leaning and to gain the confidence to continue learning. This is especially true for elementary school teachers. By providing elementary teachers an opportunity to understand mathematics in greater depth, they will be able to increase their understanding of how to teach mathematics and will increase confidence in teaching mathematics.

Tools for professional development.

The introduction of mathematical tasks and exploration of these tasks can provide a vehicle to increase MKT. Tasks used in the QUASAR project (Silver, Smith & Nelson, 1995) engaged teachers in thinking and reasoning about important mathematical ideas.

Silver, et al. imply that choosing a high cognitively demanding task for students increases the cognitive level of classroom discussions as long as the teacher maintains the high cognitive demand. In order for this to occur, the researchers found it was necessary that teachers understand the mathematical task and are able to coordinate the discussions around the concepts being addressed.

Stein and Smith (1998) also used *The Task Analysis Guide* to allow teachers to analyze their lessons and to develop cognitively demanding tasks. *The Task Analysis Guide* provides descriptions of four levels of classroom tasks ranging from low-level cognitive demand to high-level cognitive demand. The lowest cognitively demanding level tasks are Memorization Tasks. Memorization Tasks use previously learned information or committing rules and facts to memory and require the exact reproduction of previously learned information. These tasks provide no connections to underlying concepts. Procedures without Connections tasks focus on producing the correct answer and are algorithmic requiring limited cognitive demand. Procedures with Connections focus attention on using procedures to deepen the level of understanding in mathematics. Procedures with Connections are broad and may apply to underlying concepts. Normally there are multiple access routes to the solution of the problems and the ability to make connections is essential to developing meaning. These tasks are more cognitively demanding than Procedures with Connections and Memorization. Doing Mathematics Tasks require exploration and understanding of the underlying mathematical concepts. The tasks are non-algorithmic and require complex thinking. The tasks require a high level of cognitive thinking. Using *The Task Analysis Guide* (Stein, Smith, Henningsen & Silver, 2009) as a tool by teachers allows for analysis of classroom tasks. To develop tasks of varying cognitive demand it is necessary that there is professional development for teachers that will build their capacity to understand the differences in the tasks. High cognitively demanding tasks can increase the mathematical capacity of the teachers if the tasks are analyzed and discussed by the participants within the professional development. The tasks provide an opportunity for teachers to practice making connections in mathematics as well as increase pedagogy and mathematical knowledge.

Another tool that has been used for analysis during teacher observations and may provide insight into teacher professional development is the Categories of Questions (Table 4) developed by Boaler and Brodie (2004). The table identifies nine types of questions: Gathering information; leading student through a method; Inserting terminology; Exploring mathematical meanings and/or relationships; Probing, getting students to explain their thinking; Generating discussion; Linking and applying; Extending thinking; Orienting and focusing; and Establishing context. During classroom observations, Boaler and Brodie coded the questions asked by teachers. The study suggests that teacher questions asked in the classroom can influence the cognitive demand in the classroom (Boaler & Brodie, 2004) as well as student interaction. In particular, when the teachers ask conceptual questions then the students start to ask more conceptual questions. When appropriate questioning is used by the facilitator in professional development, the ability of teachers to pose a higher level of cognitively demanding questions in the classroom may occur. This process of asking cognitively demanding questions will also provide an opportunity to increase MKT.

How Is Mathematical Knowledge for Teaching Assessed?

Understanding Sowder’s goals of professional development is the first step in developing effective targeted professional development for teachers of mathematics. Effective professional development fosters collaboration. After determining the goals for professional development, professional development planning must assess the needs of the teacher and students. Designing professional development based on real data allows the plan to be relevant to the needs of both students and teachers. Data include assessing teachers' mathematical knowledge (Hill, Sleep, Lewis & Ball, 2007) and planning professional development accordingly.

There is considerable pressure to determine how professional development influences student learning and in particular what mathematical knowledge is necessary for the teaching profession (Hill, et al., 2007). In the late 1800's and early 1900's mathematics teachers were assessed on mathematical computations that required content knowledge. The mathematical knowledge was not tied to the actual teaching of mathematics. The theory of teaching was tested separately and included questions such as "When may the algebraic equation be used to advantage in arithmetic?" (Michigan Department of Public Instruction, 1901, p.70). The characteristics of a good teacher and what content or methods are effective for teacher training are still being studied by educational researchers.

The *Teacher Education and Learning to Teach* study (Kennedy, Ball, & McDiarmid, 1993) began to evaluate teachers by providing questions that tested a situation teachers might encounter in teaching. This was one of the first attempts to test the knowledge necessary for teaching. Researchers have continued to develop assessments that analyze teacher knowledge (Ball, Thames & Phelps, 2008; Hill, Shilling & Ball, 2004; Hill, Sleep, Lewis & Ball, 2007). Assessing teacher knowledge provides information that informs teacher education programs and informs teacher professional development. The following assessments centering on mathematical knowledge for teaching are discussed at length by Hill's research team (Hill, Sleep, Lewis & Ball, 2007). The following three assessments of different constructs measure the knowledge of teachers for teaching of mathematics.

The Study of Instructional Improvement (SII)/Learning Mathematics for Teaching (LMT, 2006b) is a multiple choice test covering three domains of teacher knowledge: content knowledge, knowledge of content and students, and knowledge of content and teaching (Shechtman, Roschelle, Knudsen, Vahey, Rafanan, Haertel, et al., 2006). The second assessment is the Knowledge for Algebra Teaching (KAT) measures algebra knowledge (Knowing Mathematics for Teaching Algebra Project, 2006). The assessment is built on a three dimensional matrix covering: two algebra topics of expression, equations and inequalities and functions and properties; knowledge of school algebra and advanced knowledge of algebra to assure that the material being taught ties appropriately to higher level mathematics; and four domains of mathematics knowledge consisting of core knowledge, representation of math knowledge, application and reasoning and proofs. The third assessment is the Diagnostic Teacher Assessment in Mathematics and Science (DTAMS) (Bush, Ronau, Brown & Myers, 2006). DTAMS is a diagnostic test and measures the status of teachers' knowledge. DTAMS tracks teacher learning and student achievement as a function of teacher knowledge and also measures teacher knowledge that teachers must teach to students.

Although all of these instruments address the need to assess teacher knowledge brought about by the push for accountability driven by NCLB, there is still a need to refine these assessments as more information emerges about the knowledge that teachers need for teaching (Hill, et al., 2007). The assessments are multiple choice and/or short answer, and they attempt to assess many areas of teaching; however, due to the complexity of teaching, not all areas are tested. Since teaching requires numerous judgments resulting from many different situations in the classroom, it is difficult to put the situations in a test format that requires a single correct answer or to test for all situations. Although there are significant drawbacks in using these assessments, the instruments could provide some insights into designing effective teacher professional development. If used to provide a basis for professional development and not as an evaluative measure, the actual professional development may better relate to the needs of the teachers.

Effective professional development requires assessing the needs of the teachers in professional development. According to Loucks-Horsley et al.(2003) when designing professional development for teachers it is essential to determine what the teachers know, to provide an opportunity for teachers to construct the learning for themselves and to understand that acquiring the knowledge is a process and requires various learning opportunities. Although these elements have been used to develop classroom lessons, Loucks-Horsley, et al. (2003) believe that these elements should also be applied to the design of professional development for teachers.

One method of designing effective professional development can be found in the work of Vygotsky (1978) regarding the Zone of Proximal Development (ZPD). ZPD is the difference between what a learner is able to do without assistance and what the learner can do with assistance. The theory of ZPD may be applied to increasing a teacher's capacity in MKT. Through data analysis of the teachers' level of expertise, planning professional development necessary to meet the goals should include taking teachers from what they can do without assistance to what they can do with assistance, thereby employing the ZPD theory of learning.

Professional development is driven by a well-defined model of an effective classroom, is research based and addresses the needs of the adult learner. According to Ball (2000) professional development in mathematics should include a significant amount of mathematics content and pedagogical content. The marriage of these components can be difficult (Ball, 2000). Thompson and Zeuli (1999) believe that teachers must be engaged in the learning resulting in a "transformative" shift caused by a cognitive conflict that challenges both beliefs and practices. Inherent in the professional development model is a reflective cycle of learning resulting in a culture of continuous learning.

Summary

Research indicates that teacher guided collaborative workgroups that use an inquiry based protocol is a form of professional development that may impact classroom instruction (Goldenberg, 1992; McDougall, Saunders & Goldenberg, 2007; Gallimore, Ermeling, Saunders, & Goldenberg, 2009). Although collaborative workgroups can be effective, Saunders (2007) said that a capable other and expertise in the content area is needed in a collaborative workgroup setting. Since a capable other may not be available within the collaborative group and the content expertise is not always available within a group, a targeted professional development (TPD) would provide the expertise to support the collaborative setting. Other research indicates that if the workgroup members do not have a deep understanding of their content and the teaching of the content, then the collaborative workgroup may not effectively improve practice in the classroom (Arbaugh, 2003). Teacher inadequacies in MKT were particularly evident in the research studies about the knowledge of elementary school teachers (Flores, Patterson & Shippen, 2010; Hill, Shilling, & Ball, 2004; Tsao, 2005). As indicated by the CBSM, the lack of pre-service mathematics courses for elementary school teachers is a symptom of poor preparation for mathematics teaching. Even when there are courses, the curriculum included in the courses is not standardized at the elementary level. According to the research, the teaching of mathematics is hampered at the elementary level due to a lack of teacher knowledge in mathematics (Ma, 2009; Hill, Shilling & Ball, 2004; Tsao, 2005). The research indicates that elementary teachers are not properly prepared for teaching mathematics and that they are not well prepared in MKT. This problem in elementary schools can be addressed through appropriate targeted professional development.

The TPD in this study is intended to address the problem of inadequate MKT of elementary school teachers by increasing the capacity of the teachers' mathematical knowledge specific to the needs of the teachers and their students. As indicated by the literature, elementary teachers tend not to have a deep knowledge of mathematics for teaching. An intervention in the form of a targeted professional development that is developed specific to the mathematical areas where the teachers are deficient may increase the instructional effectiveness of elementary teachers in mathematics.

Chapter III provides the rationale for positioning this study in a case study format, describes the research setting and participants, and describes the data collection and analysis methods as well as the tools that were used in this study.

Chapter III

The review of the literature indicated that teachers need mathematical knowledge for teaching if they are to be effective classroom teachers and that elementary teachers may not be equipped with this knowledge. My study was conducted to determine if the intervention, TPD, impacted the teachers' understanding of and fluency with mathematical concepts and if the TPD changed their classroom practices.

This chapter describes the research design for my study, the setting and context of the study, the participants and data sources, the characteristics of the participants, instruments and procedures used to gather the data. In addition the chapter includes the justification for the instruments used, detailed description of the targeted professional development design, data collection methods, data analysis procedures, and my role as the principal researcher with explicit reference to assumptions, beliefs and biases.

Research Design

This study was an interpretive analysis of the knowledge and behavior of teachers who participated in the study. The intervention, TPD, was intended to increase the Mathematical Knowledge for Teaching (MKT) of each participant. The TPD was specific to the mathematics curriculum that the teachers followed. The case study format allowed for in-depth analysis of the participants' interactions and dialogues within the settings.

The bounded case studies (Miles & Huberman, 1994) of the teachers in this study focused on the workgroup sessions, targeted professional development sessions, classroom instruction and the initial and final interviews. Teacher discussions that occurred in various settings including the workgroup meetings, classroom instruction, targeted professional development (TPD) and initial and final interviews were analyzed. The case studies relied on descriptions of the observations in the classroom and included teacher utterances in the form of questions and categorized according to the work of Boaler and Brodie (2004) in Table 3.4. The tasks used in the lessons were rated according to *The Task Analysis Task Guide* (Smith & Stein, 1998) found in Table 3.5.

As the researcher, I examined the experiences of the teachers participating in a content-specific mathematics workgroup and the effect that the TPD had on the teachers' instructional practices in the classroom. I was an active member and facilitator of the workgroup and I provided the TPD. Since I was an active participant in all sessions of TPD and the workgroups, I audio taped and transcribed all of the sessions. As the researcher, it was necessary to create an environment of trust and support within the workgroup. Such a position required the suspension of judgments and preconceived notions about the experiences of the participants and their mathematics knowledge until all data were gathered and analyzed. This study required me to change between the roles of the workgroup facilitator, participant and TPD provider. The audio taping provided me the opportunity to exist in each appropriate role and to review the data after the various settings had occurred.

This study also examined the teachers' perception of the effect of the TPD upon their own understandings of and fluency with the mathematical procedures and the effect of TPD sessions on their classroom practices. Through initial and final in-depth interviews of each participant, the participants' perceptions of the effect of professional development were examined. It was necessary for me to understand how the participants viewed professional development and the effect it may have had on their practice to determine if there were inconsistencies in their perceptions and my classroom observations. It was also necessary for me to understand the relationship between the professional development and the participant's perceived experience with the professional development models, TPD and the collaborative workgroup, and the effect on classroom instructional practices.

A case study approach was used for the study. Yin (2009) defined the case study as an "empirical inquiry that investigates a contemporary phenomenon in depth and within a real-life context, especially when the boundaries between the phenomenon and context are not clearly defined" (p. 18). In conjunction with the case study format the collection of data required multiple data collection techniques, with a logical design that included many variables that did not equate to the same number of data points (Yin, 2009). In this study the cases allowed me to "describe an intervention and the real-life context in which it occurred" (Yin, 2009, p. 20).

The case studies used multiple sources of data that included the study of the teacher questioning within the inquiry based collaborative workgroup settings, the TPD settings and the classroom setting. As with many case studies, the use of in-depth interviews was instrumental to understanding and reporting on the experience of the participants with professional development. Rubin and Rubin (1995) said that interviews need to be flexible and allow the researcher to ask questions that will provide greater information based on the first line of questioning. As a case study to determine the impact of the TPD, analysis of the teachers' questioning over time in the workgroup and in the TPD provided data to determine if there were significant changes in the workgroup conversations that include more in-depth mathematical references and if the teachers were making connections within the mathematics. The initial and final classroom observations provided an opportunity to note changes in teacher actions and to note any impact that the targeted professional development may have had upon the teachers' classroom practices.

Research Setting and Context

This study provided an opportunity to examine a TPD setting intended to enhance the participants' mathematical knowledge for teaching and effect upon classroom instruction. The study was conducted in the second largest urban school district in the United States, Los Angeles Unified School District (LAUSD). LAUSD consists of eight smaller divisions called Local Districts. The school where the study was conducted was a year-round school consisting of four individual tracks. I purposefully chose a school that had lower CST scores in mathematics since it was a representative of the Local District within the larger District.

The workgroup consisted of fifth and sixth grade teachers. Since the work was in mathematics, I chose a school that had lower scores in mathematics as measured by the California Standardized Test (CST) at the fifth grade level Table 3.1. The school was demographically representative of many of the schools in the Local District with over 90 percent of the students Hispanic, approximately 59 percent designated as English Learners and over 78 percent of the students classified as low socioeconomic status (students on free and reduced lunch) as indicated by the 2009 – 2010 CST data. Although the school's API (Academic Performance Index) was met for 2009-2010, closer examination of the CST scores in mathematics revealed a dramatic decrease in the test scores in mathematics for fifth grade students as well as all of their subgroups. This appeared to follow the District's decrease in test scores for grades five and six (Table 3.1). Since the school changed to a K-6 configuration from a K-5 configuration in the fall of 2010, CST data were unavailable for the sixth grade students coming into the school prior to the study; however, indications from the entire District showed a decrease in proficiency for sixth grade mathematics across the District. In addition the Annual Measurable Objectives (AMOs) for the school were not met school wide in mathematics or within each subgroup.

Table 3.1

Percent of Students Proficient and Above in Mathematics on CST 2010*

Grade Level	2	3	4	5	6
District Wide Performance	54%	59%	63%	55%	39%
Research School Performance	48%	50%	63%	40%	NA

*California Standardized Test

The elementary school had participated in some type of collaborative work. Although the school indicated that there had been collaborative workgroups, the workgroups were focused on operational aspects of the school and were not collaborative content-specific workgroups. Since the participants had not previously participated in a collaborative content-specific workgroup, the responsibility of organizing the workgroup and leading them through a reflective cycle was mine.

Intervention

The intervention was a targeted professional development (TPD) consisting of professional development that was specific to the content that the teachers were teaching and provided teachers the opportunity to increase their Mathematical Knowledge for

Teaching (MKT). The following research provided the basis for the TPD structure that I designed:

- *Implementing Standards-based Mathematics Instruction* (Stein, Smith, Henningsen & Silver, 2009)
- *Connecting Mathematical Ideas* (Boaler & Humphreys, 2005)
- Mathematical tasks (Boaler & Humphreys, 2005; Stein, Smith, Henningsen & Silver, 2009).
  - *Mathematical Tasks Analysis Guide* (Stein, Smith, Henningsen & Silver, 2009)
  - *Mathematical Tasks Framework* (Stein, Smith, Henningsen & Silver, 2009)
  - Categories of Questions (Boaler & Brodie, 2004)
  - LAUSD Mathematical Tasks developed in collaboration with University of Pittsburg, Institute for Learning
- *Mathematical Knowledge for Teaching (MKT) Measures, Mathematics Released Items, 2008* (Hill, Schilling, & Ball, 2004)
- *Getting into Mathematical Conversations* (Elliott & Garnett, 2008)

The TPD was aligned with the lessons and curriculum for each grade level as noted in District's Mathematics Instructional Guide. The TPD consisted of developing MKT through the examination of tasks. The Mathematics Task Framework (Stein, Smith, Henningsen & Silver, 2009) provided a framework for these tasks and the subsequent teacher developed lessons. As suggested by Sowder (2007), the teachers participated actively in their learning.

The teachers determined the area that was difficult to teach their students was fractions. The TPD was designed to address fractions by increasing the MKT specifically for fractions and other number sense concepts. The TPD used tasks designed to enhance the teachers' understanding of fractions and were designed to provide opportunities for the teachers to make connections between mathematical representations of fractions. The categories of questioning (Table 3.4) that I used and modeled in the TPD reflected Boaler's and Brodie's (2004) categories with an emphasis on: Exploring Meanings and Relationships; Probing, Getting Students to Explain their Thinking; Generating Discussions; and Extending Thinking. I decided to emphasize these four areas of questioning to model how to deepen student understanding in the classroom and to also provide the teachers an opportunity to deepen their understanding of the mathematics that they taught. I consistently modeled the process of questioning and emphasized how this type of questioning could be used in the classroom to deepen students' understanding.

As noted above, the inquiry-based workgroup was comprised of four fifth grade teachers and one sixth grade teacher. I facilitated the workgroup meetings since the teachers had no experience in a reflective cycle that focused on developing a collaborative lesson that would be implemented in the classroom. The protocol used for the workgroup was: goal setting and planning based on student data, developing a lesson to meet the students' needs, implementing the lesson, discussing student work from the lesson, reflecting on their implementation of the lesson and then determining the next steps. The group was able to complete one full cycle and begin a second cycle over the seven workgroup sessions. The TPD supported the workgroup sessions by focusing on the student need determined by the workgroup, fractions, and building the capacity of the teachers by providing multiple access strategies for the identified need. The intervention, TPD, provided support for the teachers in the workgroup sessions and the workgroups sessions informed the TPD sessions about teacher instructional needs.

Participants and Data Sources

The District's analysis of the mathematics CST scores indicated that in fifth and sixth grade mathematics, the District's CST scores dropped dramatically prior to the study (Table 3.1). My decision to work with fifth and sixth grade teachers was due to the drop in CST mathematics scores. The selection of a K-6 elementary school was due to my previous work in mathematics and the research indicating that teachers in elementary school have limited mathematics background (Flores, Patterson & Shippen, 2010; Tsao, 2005). In California, the elementary multiple-subject credential does not require any undergraduate work in mathematics. Elementary school teachers are required to have a baccalaureate degree and passage of the California Subject Examination for Teachers (CSET). The CSET is a Multiple Subjects test is based on a set of content specifications specific to the subject areas taught in multiple subjects. The Multiple-Subject Credential for elementary education teachers in California requires only one methods course in mathematics and not necessarily a mathematics content course (California Commission on Teacher Credentialing, 2011).

I began the process of determining the school site in late spring of 2010 when I met with the principals from a Local District within LAUSD. At the meeting I presented my intended research study. Principals were given the opportunity to ask questions and volunteer their school site for the study. It was at that time that the principal, Ms. Jones[2], from an elementary school volunteered her school as a possible site for the study.

In the beginning of September, prior to beginning of the research, a meeting was arranged with the Ms. Jones. Since the school was a year-round multi-track school[3] with four tracks, Ms. Jones and her coordinator, Ms. Smith, decided to ask only those teachers on Tracks A and C to participate in the study. The two tracks that were chosen seemed to align more closely than other tracks for both the curriculum that the participants were teaching and their work calendar. The calendars were important since teachers from both tracks would be teaching the same material at the same time. During the meeting we calendared an informational meeting with the fifth and sixth grade teachers as well as all TPD and workgroup meetings. Ms. Jones was confident that the teachers would be receptive.

Ms. Jones, Ms. Smith and I discussed professional development and collaboration activities that the school had previously implemented. This information, in addition to the test scores pulled from the California Standards Test, was used to inform the curriculum and structure of the TPD. Although a combined fifth and sixth grade group, the focus of the TPD on MKT for both grade levels remained on common curriculum that both fifth and sixth grade teachers taught. Throughout the study the emphasis remained on developing content and pedagogy.

The informational meeting about the study with the teachers was held in mid-September with fifth and sixth grade teachers on Tracks A and C. All of the teachers at

[2] All names used in the report, other than the name of the district, are pseudonyms.
[3] A four track multi-track school runs concurrently two different groups (tracks) of children simultaneously. The tracks do not share the same school calendar or faculty. Each track represents a small school within a large school.

the meeting agreed to be part of the study and signed the Informed Consent Form (Appendix A). The teachers in this study were all Multi-Subject Credentialed teachers with varying years of classroom experience (Table 3.2). The variance in the years of teaching and the grade level teaching assignments allowed for diversity within the group. The school matched demographically with many of the schools in the Local District, which may allow for some generalizations and applications to other cases. The school, previously structured as K-5, was a newly re-structured K-6 elementary school. Due to budget constraints, there was very little District sponsored professional development offered to schools during this study. Originally there were two sixth grade teachers who were included in the workgroup. One sixth grade teacher dropped out of the group due to a change in his assignment. Although there was only one sixth grade teacher, the TPD

Table 3.2

Teacher Information

Participant	Ms. Guzman	Ms. Becker	Mr. Palmer	Ms. Talika	Mr. Juarez
Grade Level	5	5	6	5	5
Years Teaching	5	5	9	4	7
Years at School	5	5	1st year	11	7
Previous grades/ subject taught	Fourth Grade	Fifth Grade	Sixth Grade English, ESL, History		Third and Fourth Grades

addressed the California standards and strands for both the fifth and sixth grade levels. This did not present any problems since the standards were similar and were tied together incrementally.

The principal secured a stable setting for all meetings with the teachers. All fifth and sixth grade teachers on track A and C were invited to the initial meeting and all of the fifth and sixth grade teachers on A and C tracks agreed to participate in this study. Originally there were two sixth grade teachers; however, due to a change in the teaching assignment of one of the sixth grade teachers, one teacher dropped out of this study. Each participant attended the majority of meetings and only missed due to either an absence from school or a conflict with another meeting they were required to attend (Table 3.3).

Table 3.3

Teacher Participation

Participant	Ms. Guzman	Ms. Becker	Mr. Palmer	Ms. Talika	Mr. Juarez
Initial Observation	11/05/10	10/01/10	10/11/10	9/29/10	11/04/10
Initial Interview	11/3&6 /10	10/10/10	10/11/10	9/29/10	11/04/10
TPD1: 9/23/10	X	X	X	X	
WRK 1: 9/27/10		X	X	X	
WRK 2: 10/11/10	X	X	X	X	X
WRK 3: 10/18/10	X	X	X		X
TPD 2: 10/20/10		X	X	X	X
WRK 4: 10/25/10	X		X	X	X
WRK 5: 11/15/10	X	X	X	X	X
WRK 6: 11/29/10	X	X	X		X
TPD 3: 12/01/10	X	X	X	X	X
WRK 7: 12/13/10	X	X	X	X	
Final Observation	01/06/11	12/10/10	12/10/10	12/13/10	01/03/11
Final interview	01/10/11	12/16/10	12/10/10	12/10/10	01/04/11

TPD: Targeted Professional Development
WRK: Workgroup Sessions
X: Attendance

Data sources included interviews, audio tapes of the TPD and the content-specific collaborative workgroup, classroom observations and analysis of the student lesson generated by the workgroup. I conducted interviews to understand the perceived relationship between the intervention or targeted professional development, the teacher discussions in the workgroup and the teacher-generated questioning in the classroom (Schram, 2006). Audio taping the TPD and the collaborative workgroup meetings aided in analyzing the discussions in each setting. Through analysis of the settings, I looked for the types of teacher-generated questions in the classroom and the relationship to the questioning I modeled in the TPD and workgroup sessions. Interviews and observations provided data to determine if the mathematical connections were evident in their classroom practices.

Mathematical understanding, multiple representations and in-depth utterances in the form of teacher generated questions were analyzed in each setting to look for occurrences over multiple settings. Initial and final observations of the classrooms focused on the teacher questioning in the classroom, teacher-student interactions and the level of the tasks required of the students. The final interview with each teacher included debriefing of the observed lesson and also questions on the participant's perceived influence of the TPD on his/her instructional practices (Appendix C and D).

Instruments and Procedures

In order to gather the data I decided to use instruments that were directly tied to the intervention provided. This allowed for consistency when analyzing the questions used in the workgroup meetings, TPDs and the classroom observations. Since the tasks for the TPD were developed according to specific guidelines for judging cognitive demand of the task, I decided to also use the same guidelines for the classroom observations. These tools were incorporated into the observation protocol for each classroom observation (Appendix B).

Classroom observation.

For the data collection in the classroom observations, I focused on the categories of questions used in the lesson (Boaler & Brodie, 2004) as noted in Table 3.4. The levels of cognitive demand for the classroom tasks were based on the Task Analysis Guide (Stein, Smith, Henningsen & Silver, 2009) noted in Table 3.5. All field notes were transcribed after the observation to ensure accuracy. The classroom observations provided a springboard for the initial and final interviews. In addition I scripted questions asked by the teacher during the lesson. Only questions that were related to the mathematics lesson were scripted.

The protocol used for the observations was adapted from the 2007-08 Observation Protocol for Model Lesson Study, based on components from the LAUSD Science Observation Protocol (Appendix B). The main categories of the protocol were Teacher Activity, Student Activity, Direction of Discourse and Talk / Questioning using the Observation protocol (Appendix B). I recorded my observations in ten-minute intervals by recording the predominant activities during each ten-minute period of time. The types of questions (Table 3.4) were categorized and recorded based on Boaler's and Brodie's work (2004) on the Observation Protocol. Questions asked by the teacher were also recorded on the observation form. The Task Analysis Guide (Table 3.5) was used to rate the tasks that were observed in the classroom and recorded on the observation form. The four levels of cognitive demand for tasks were Memorization, Procedures without

Connections, Procedures with Connections and Doing Mathematics (Stein & Smith, 1999).

Interviews.

I conducted the initial interviews prior to the intervention when possible. Due to teacher conflicts and scheduling difficulties some of the initial interviews occurred after the intervention began (Table 3.6). The initial interviews included a debriefing of the first classroom observation.

Only in one case I found it necessary to start the interview prior to the observation and then continue the interview after the classroom observation to include the debriefing of the lesson.

I conducted all final interviews after the intervention and after the final classroom observation: thereby, highlighting any possible effects of the TPD. The interview was semi-structured to elicit responses that would indicate the participants' perceptions of the TPD and any changes in their MKT and classroom practices. The protocol for the final interview allowed me to focus on my area of study without limiting my questioning to only use those questions indicated in the interview protocol. Listening carefully to the responses allowed me to follow with questions that could lead to more insights. For the initial and final interviews, it was necessary to ask probing questions thereby eliciting in-depth insights in a friendly manner while keeping my research questions in mind (Yin, 2009.

Table 3.4

Categories of Questions

Question Type	Key	Description	Example
1. Gathering information, leading student through a method	GI	Requires immediate answer Rehearses known facts/procedures Enables student to state facts/procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	IT	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3. Exploring mathematical meanings and/or relationships	EM	Points to underlying mathematical relationships and meaning. Makes links between mathematical ideas and representations	Where is this x on the diagram? What does probability mean?
4. Probing, getting students to explain their thinking	P	Ask students to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5. Generating Discussion	GD	Solicits contributions from other members of class	Is there another opinion about this? What did you say Justin?
6. Linking and applying	LA	Points to relationship among mathematical ideas and mathematics and other areas for study/life	In what other situations could you apply this? Where have we previously used this?
7. Extending Thinking	ET	Extends the situation under discussion to other situations where similar ideas may be used	Would this work with other numbers?
8. Orienting and focusing	OF	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
9. Establishing context	EC	Talks about issues outside of math in order to enable links to be make with mathematics	What is the lottery? How old do you have to be to play the lottery?

** Adapted from categories of teacher questions from Boaler, J. & Brodie, K. (2004). The importance, nature and impact of teacher questions. In D. McDougall & J. Ross (Eds.), *Proceedings of the 26th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 773-781). Toronto: Ontario Institute of Studies in Education/University of Toronto. Added Coding.

Table 3.5

*The Task Analysis Guide**

Memorization	Procedures With Connections Tasks
• Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedural because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated • Have no connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced	• Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections between multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Task	**Doing Mathematics Tasks**
• Are algorithms. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connections to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding . • Require no explanation, or explanations that focus solely on describing the procedure that was used.	• Require complex and non-algorithmic thinking (i.e., there is not a predictable, well rehearsed approach or pathway explicitly suggested by the task, task instrument, or a worked-out examples) • Require students to explore and understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

* Based on the characteristics of mathematical tasks at each level of the four levels of cognitive demand (Stein & Smith, 1998) Smith, M. S. & Stein, M. K. (1998). Selecting and creating mathematical takes: From research to practice. *Teaching Mathematics in the Middle School*, 3(5), 344-350.

The protocol for the initial or first semi-structured interview included questions about effective professional development models and collaborative work they had experienced (Appendix C). The initial interview protocol was structured to elicit responses that would indicate the teachers' perceptions of what an effective professional development should include to understand their perspective of an effective professional development and to determine if the teachers had participated in any collaborative content-specific workgroup sessions. Knowing the perspectives of the participants provided me insight into what they were accustomed to experiencing in professional development and what was expected of them as participants. For example, one participant was very unhappy with professional development that she received since she was unable to use the information in her classroom. The initial interviews were to occur prior to the workgroup meetings and TPD; however, this was not possible due to scheduling difficulties.

During the final interview (Appendix D) participants were asked to reflect on their lesson that I observed and what, if any, effect the workgroup and TPD meetings had on their classroom practice. The final interview was structured to elicit the teachers' perception of the TPD and if the TPD had an impact upon their classroom instruction. The scripted questions for the final interview began with questions focused on the classroom observation. I asked probing questions based upon their responses to the scripted questions for the final interview, thereby eliciting their perceived insights into the effectiveness of the TPD and workgroup meetings. The final interviews occurred after all TPD and workgroup sessions ended.

All of the individual interviews ranged from 30 to 45 minutes in length, and were conducted at the convenience of the teacher. In some cases the interviews occurred during lunch and students were present. The teachers indicated that they were fine with the students being present. I audio taped all interviews and I transcribed the interviews shortly after the interviewed occurred to eliminate any loss of understanding. I included any field notes I had taken and I wrote clarifications within the transcribed interviews where the transcript was unclear.

Targeted professional development (TPD) sessions.

The three two-hour TPD meetings interspersed between the workgroup sessions (Table 3.6) were audio taped with some parts video taped. The TPD sessions were interspersed between the workgroup meetings during the first three months of this study. Since I provided the TPD, and was therefore unable to take field notes, the video taping and audio taping helped to capture in greater detail the interaction of the participants and the utterances of the conversations. When appropriate, I changed the TPD due to the changing needs of the teachers. There were instances when the TPD was infused into the workgroup meetings, especially during the development of the lessons.

Workgroup sessions.

All workgroup meetings were audio taped and some were video taped. There were seven meetings over a period of four months (Table 3.6). The meetings normally occurred twice a month for one hour; however, there were times when the group members voluntarily stayed longer if the meetings were after school. Video taping was used periodically with audio taping in order to capture teacher reactions through body language that would not have been evident on the audio tapes. Since I was also a participating workgroup member and facilitator, taking notes as an observer was not possible. Audio and video taping of the workgroup meetings allowed for rich description of the activities and teacher interactions of the meeting. The materials that we used in the TPD and workgroup sessions provided additional information about the sessions. I inserted additional descriptions into the transcriptions to provide further clarification of the sessions and the group discussions.

Table 3.6
Schedule of Data Collection

Month	Data collection	Time/incidence	Number of Participants	Method
9/29 – 10/ 5	Initial Observations	55 – 70 minutes	5: one per Participant	Field notes Transcription
9/ 29-11/ 11	Initial Interviews	20 – 30 minutes	5: one per Participant	Audio taping/ Transcription
9/23	Targeted PD 1	2 hours	All participants	Audio taping / transcription
9/ 27	Workgroup meeting	1 hour	All Participants	Audio taping / transcription
10/ 11	Workgroup meeting	1 hour	All Participants	Audio taping / transcription
10/18	Workgroup meeting	1 hour	All Participants	Audio taping / transcription
10/20	Targeted PD 2	2 hours	All Participants	Audio taping / transcription
10/25	Workgroup meetings	1 hour	All Participants	Audio taping / transcription
11/15	Workgroup meetings	1 hour	All Participants:	Audio taping / transcription
11/ 29	Workgroup meetings	1 hour	All Participants	Audio taping / transcription
12/1	Targeted PD 3	2 hours	All Participants	Audio taping / transcription
12/ 13	Workgroup meetings	1 hour	All Participants	Audio taping / transcription
12/ 10 -1/ 6	Final Observations	55 – 70 minutes	5: one per Participant	Field notes Transcription
12/10 – 1/10	Final Interviews	20 – 30 minutes	5: one per Participant	Audio taping/ Transcription

Data Analysis

In order to analyze the data, I read the transcriptions of the workgroup and TPD sessions multiple times, wrote summaries of the classroom observations, added clarification to the transcripts and continually organized and chunked data when possible (Rossman & Rallis, 2003). This was an ongoing process and required multiple reviews of all of the data. Flexibility when analyzing the data for possible insights was necessary. As Yin (2009) suggested, I started to "play" with the data (p. 129).

Table 3.6

Observations analysis.

For each classroom observation I categorized the activities that I observed based on the observation protocol (Appendix B). For each ten-minute interval, I recorded the predominant activity for each category. I analyzed the predominant activities by tabulating and organizing the frequency of the dominant activity per category. The most frequent activities during each ten-minute interval were recorded. I then determined the cumulative occurrences of each activity and recorded occurrences that were evident in at least 50% of the ten-minute segments for each category for each observation (Table 4.1). In addition, I evaluated the tasks for each observation and recorded the level of cognitive demand based upon the Task Analysis Guide (Table 3.5) and recorded the information in Table 4.2. I analyzed the changes in cognitive demand and used that information to inform the individual cases of each teacher. I also wrote a short summary of the observations that included the scripted questions asked by the teachers related to the mathematics instruction as well as a description of the lesson. Statements that asked the students to explain their reasoning were also scripted.

Interview analysis.

The interviews were transcribed and provided additional information about the lesson that was observed. After reading numerous times, I highlighted the comments in the last interview that directly related to the teachers' learning and the evidence of change in the last classroom observation as well as his/her learning in the TPD sessions and workgroup sessions. The first interview was used to determine their expectations for their learning. This information also informed the individual case study of each person and provided data for the baseline discussed in Chapter IV. After reviewing all of the data I initiated a data reduction process that resulted in six cases (Appendix E).

Targeted professional development analysis.

While examining the data obtained from the TPD sessions, I had to read the TPD transcripts numerous times, highlighting in different colors the individual teacher comments and responses that reflected the question categories, student interaction categories and task analyses guide that were used for the observations. The individual teacher comments that indicated greater understanding were noted and then connected to the classroom observation to determine if the TPD had transferred to the classroom instruction. The data were used in the case study and provided concrete examples of the journey of each teacher over the term of the study.

Workgroup sessions analysis.

The workgroup sessions transcriptions were also read numerous times and I highlighted each according to the categories evident in the teacher observation protocol. Since the TPD sessions sometimes overlapped into the workgroup sessions it was necessary to include the information under the workgroup session material as TPD material. The comments made by the teachers during the workgroup sessions provided some insights into their learning over the duration of the study. Additional notes were recorded on the transcriptions when necessary, such as references to problems that were being solved by the participants when the transcription would not provide the specific information.

The individual case study.

Miles and Huberman (1994) describe that frequencies of occurrences of different events may be used when analyzing data for a case study. After noting any changes that occurred between the initial and final classroom observation determined by the frequency of the activities for each category in the observation protocol, I proceeded to read all of the transcripts and the additional field notes again to find data that would support the changes noted.

After analyzing all of the data sources, including Table 4.1 and Table 4.2, I reduced the data and wrote the journey of each teacher over the duration of the study. This required me to review all data for each individual person separately and to record only pertinent information for each teacher in separate stories. When I worked on a case study for one teacher, I looked for data that only related to that individual. Since the TPD and workgroup sessions were ongoing, each case study was developed in a time ordered sequence. The case studies were similar to a story of a teacher's experiences and included information specific to the teacher from the various settings. The tables that were developed from the observations provided me insight into any changes that may have occurred in the teacher's practices. I then looked for teacher comments and responses from the workgroup and TPD sessions as well as classroom observations and teacher interviews that may have supported the findings observed in the final observation.

The workgroup case study.

The workgroup case study grew out of the necessity to see how the group developed over the span of the study. This case study was focused on the group as a whole. I recorded the interactive learning of the workgroup during the TPD sessions and the workgroups sessions. As I reviewed the data my attention was not directed toward individual learning but the learning of the group as a whole. The observation categories (Appendix B) provided the basis of the information that I was examining and included the mathematical connections that the teachers were making during each of the settings. The workgroup case highlighted the ability of the participants acting as a group versus individually. Arbaugh (2003) also used the case study of a geometry workgroup to study the interactions of the members of the workgroup and to study the participants' growth in understanding of mathematics over time.

Analysis of all case studies.

As Yin (2009) said, the case study does not need to be confined to a single case but can be multiple-case studies where the information is then linked over the multiple cases. The first five cases highlighted the journey of each individual teacher through the interviews, intervention, workgroup meetings and observations. Each chronological case study provided a reduction in the data for the individual teacher within the context of the process he/she underwent during the study. The sixth case study was the journey of the group of teachers as they experienced the TPD and the content-specific workgroup sessions. The group case study allowed me to view the individuals as an interactive group. The case study included the intervention process of TPD and content-specific workgroup sessions. The group case study allowed for analysis of any changes that occurred as the group moved through the TPD and the collaborative workgroup sessions and also captured the instances when the TPD flowed into the workgroup session. The six case studies resulted in a data reduction providing me the opportunity to look across the cases to determine if there were any trends or themes that evolved. The case studies provided more clarity into the process that all participants went through during this study (Appendices E, F, G, H, I).

Once all case studies had been written, I looked at each case study and noted any common occurrences across individual case studies first. The main categories of common occurrences that I found were the mathematical ability of all of the teachers to make connections to alternative representations of the mathematics, a teacher awareness of the need to and how to connect the “why” to student learning, and changes in the classroom instructional practice of the five teachers that included questioning and cognitive demands of classroom tasks.

As Yin (1994) emphasizes, the case study lends itself to real-life interventions, descriptions of real life context in which the interventions occur and descriptions of the interventions. In my study, the case study format lent itself to the holistic in-depth method of analysis of the intervention and provided an opportunity to highlight some of the changes that I noted in the observations. The categorization allowed description of segments of the data collected and permitted a focus for each individual case study (Appendices E, F, G, H and I). This process provided the opportunity to determine similarities and differences of the individual changes over time relative to the intervention. Holistically, the connections to the data in actual context provided insight into the complexity of the workgroup meetings, professional development and classroom interactions and how the effect of these interactions may affect each teacher differently. The holistic approach of the contextual data resulted in a narrative picture of the intervention studied. The process to analyze the data provided an opportunity for triangulation of the data by using multiple aspects of collection (Miles & Huberman, 1994): audio taping of workgroups and TPD, in-depth initial and final interviews, classroom observations and the collaboratively developed lesson.

Role of Researcher

I had three roles in this study: active participant and facilitator in the collaborative workgroup setting, the expert delivering the targeted professional development; and the principal researcher when interviewing and observing the participants in the workgroup. My role was supported by my in-depth understanding of collaborative workgroup and mathematics instruction. My mathematics knowledge stems from a degree in mathematics, over thirty years of teaching courses ranging from middle school mathematics through calculus, administration of professional development to mathematics teachers and almost 5 years of work with the content-specific collaborative workgroup initiative in the district where this study took place.

My mathematical background was an asset for analyzing changes in classroom instruction without evaluating the strategies or the participants. However, a mathematics background provided some challenges. In particular I needed to suspend pre-judgment of teacher effectiveness and base all of my findings on the data from the transcripts of all workgroup and TPD sessions. Peshkin (1988) came to a similar finding when he realized that he went into the classroom wanting to “fix” the teaching, thereby losing some of his objectivity.

My background in mathematics and inquiry-based collaborative workgroups allowed for the careful delineation of the two professional development models and provided expert insight into the attributes of each model. My background in teaching mathematics helped me to interpret and analyze what the teachers learned and how they increased their capacity. To assist in maintaining objectivity, the in-depth interviews with the workgroup members, workgroup meetings and targeted professional development sessions were audio taped and transcribed.

Summary

The data from this study provided an understanding of a very complex setting involving numerous sessions that include three targeted professional development and seven content-specific workgroup sessions. The qualitative study allowed me to investigate the relationship between the TPD and classroom instruction. The six individual case studies allowed the reduction of data providing a means to look at data across all of the case studies. The findings of the study are presented in Chapter IV that include: an increase in the understanding of and fluency with mathematical concepts with an increase in the ability of all of the teachers to make connections to alternative representations of the mathematics; an increased awareness that connecting the “why” to student learning is necessary for most of the teachers; and changes in the classroom instructional practice of some of the teachers.

Chapter IV

The purpose of this study was to determine if targeted professional development (TPD) designed to build mathematical knowledge for teaching (MKT) impacted the mathematical knowledge of fifth and sixth teachers and if the TPD impacted classroom practices. The TPD was determined by the needs of teacher and the lesson that the teachers developed during this study. The TPD was intended to build the Mathematical Knowledge for Teaching (MKT) of the participants in the work group. The research study addressed the following question:

> What is the impact of TPD sessions on teachers' understanding of and fluency with the mathematical concepts and on teachers’ instructional practices in the classroom?

From the research data, which consists of TPD transcripts, interviews, classroom observations and workgroup transcripts, I developed five individual teacher case studies that examined the journey of each teacher and one case study of the collective workgroup. The individual case study was used to analyze each participating teacher’s growth or change over the course of this study and provided an opportunity to analyze if their participation in the TPD impacted their classroom practices. The workgroup case study was designed to study collectively the teacher workgroup through the three TPD sessions and the additional seven content specific workgroup sessions and to note any changes related to the teachers’ understanding and fluency with mathematical concepts.

In order to understand how the TPD impacted teachers’ classroom practices, it was necessary to determine a baseline for the teachers’ classroom practices and their mathematical knowledge for teaching (MKT). The tools used included an observation tool (Appendix B) adapted from the LAUSD Science Observation Protocol that includes Boaler's and Brodie's Categories of Questions (2004) in Table 4.1, *Mathematical Task Analysis* in Table 4.2 developed by Smith and Stein, (1998), transcriptions of all TPD sessions and workgroup sessions and the six resulting case studies. After reviewing all data, I determined a baseline of the teachers' mathematical knowledge and their classroom practices using Table 4.1, Table 4.2, initial interviews and the first TPD and the beginning workgroup sessions. This baseline was used to note any changes that occurred as a result of the Targeted Professional Development (TPD).

Baseline Analysis

Analysis of the data indicated that all of the teachers began this study with a procedural understanding of mathematics. Their knowledge could be described as knowing how to do the problems but, as indicated by Ma (1999), not sufficient knowledge for teaching well. Ball, Thames and Phelps (2008) would categorize this knowledge as common content knowledge, or knowledge and skill used in settings other than teaching. This knowledge was demonstrated in the first classroom observations where all of the participants predominantly *modeled* (Table 4.1) a process for their students to learn.

For example, Ms. Becker, who was in her fifth year of teaching fifth grade, taught her students how to "move the decimal point" to the right when multiplying by a 10, 100 or 1000 and move the decimal to the left when multiplying a number by .1, .01 or .001. The places to move for multiplication was determined by counting the number of zeros

TABLE 4.1

Dominant Codes per Category for Each Teacher Observation

	Ms. Guzman		Ms. Becker		Mr. Palmer		Ms. Talika		Mr. Juarez	
	Obs. 1	Obs. 2	Obs. 1	Obs. 2	Obs.1	Obs.2	Obs. 1	Obs. 2	Obs. 1	Obs. 2
Teacher Activity	M, MGW	M, GD, MGD	M	GD	M	M, GD	M	M	M, GD	M. GD
Student Activity	IP, NT	GW, IP, NT	IP, GW, NT	GW, GD	IP	GW, GD	NT	NT, IP	IP	NT, GW
Direction of Discourse	T-S	T-S, S-S	T-Only with some T-S	T-S, S-S	T-only	T-only T-S, S-S	T-S	T-S	T-only and T-S	T-S
Talk/ Questioning*	GI, IT, P	GI, IT, EM, P, GD	GI, IT	GI, IT, EM, P, GD	GI	GI, IT, EM	GI	GI, IT	GI	I, IT, P, GD

Teacher Activity

M	Modeling		
GD	Group Discussion		
MGW	Monitor Group/Student Work		
IP	Independent Practice		

Student Activity

IP	Independent Practice
GW	Group Work/ whole and small
NT	Note Taking

Direction of Discourse

T-Only	Teacher does all the talking/presenting of ideas with little to no participation of students	S-S	Conversations are primarily student to student
T-S	Conversations are primarily teacher to student	NA	Instruction does not involve oral discourse

***Talk/Questioning:*

GI	Gathering Information, leading students through a method	OR	Orienting and focusing
IT	Inserting Terminology	EC	Establishing Context
EM	Exploring Mathematical Meanings and/or relationships	P	Probing, getting kids to explain their thinking
GD	Generating Discussion	LA	Linking and Applying
ET	Extending Thinking		

* This table represents a cumulative occurrence of activities recorded in at least 50% of the ten-minute time segments during each observation.
**Adapted from 2007-08 OBSERVATION PROTOCOL FOR MODEL LESSON STUDY, Based on components from WCER/SCALE protocol, 8th grade CCOP 3/14/08

when multiplying by 10, 100 or 1000 and then moving the decimal point to the right the number of zeros counted. The left movement was determined by the number of digits after the decimal point and moving the decimal point to the left and then using zeros as “place holders.” This method of teaching was procedural using Memorization tasks (Stein & Smith, 1998).

Analysis of the questions the teachers asked their students during the first observation suggested that all teachers were Gathering Information (Boaler & Brodie, 1998) and guiding students through a method that is identified as procedural (Table 4.1). The Gathering of Information was evident in the low cognitively demanding level of questioning. The majority of questions posed by the teachers during the first observation were not open-ended and required quick, single word answers. Mr. Palmer’s questions during the first observation were indicative of the majority of questions asked by all of the teachers: “… when we add, subtract or compare fractions, you usually do what first,” “you have to have what first?”, “Is that also true with multiplication and division?”

Most of the lessons during the first observations required the students to repeat the teacher led directions and included specific steps students should follow to arrive at the correct answer. Mr. Juarez provided insight during his first interview into his use of a procedural method of teaching rather than providing an opportunity for students to explore and build conceptual understadning. “I think that every kid can get just about any skill. It’s just that time and effort has to go into teaching those particular skills. Over and over and over and over. It’s drill and repetition that makes them stick in there [referring to the brain].” The procedural method of teaching mathematics was evident in varying degrees among all of the teachers, as noted in all initial teacher observations (Table 4.1).

Three of the five teachers provided tasks that involved a low cognitive demand and could be categorized as *Memorization* (Smith & Stein, 1998) as indicated in Table 4.2. The tasks involved reproducing previously learned facts or rules and any additional problems given to the students for practice required "exact reproduction of previously seen material" (Smith & Stein, 1998).

The teachers made no connections, during the lessons, among the mathematical concepts or skills nor did they provide the students opportunities to make connections. The lessons tended to be rule driven. For example, during the first lesson I observed, Ms. Talika provided her students lower-level cognitive demand tasks that may be categorized as a *Memorization* task (Table 4.2). The focus of her lesson was on the multiplication of two digit numbers, 1.5 x 1.7. Ms.Talika used the Smart Board to model the multiplication process. The modeling lacked interaction and the Smart Board was used as a tool similar to using a PowerPoint. All of the steps were written out prior to the lesson and displayed on the Smart Board. Mathematical terminology was limited and tended to be informal. Since the problems were displayed on the Smart Board, Ms. Talika pointed to each level of multiplication and led the students through the process step by step. She included a set of steps, written on the Smart Board, that the students were to follow when they were doing their problems. Ms. Talika made no reference to the underlying concepts of multiplication. All the steps were procedural.

Although three of the teachers in the first observation used Memorization Tasks, two of the teachers used Procedures Without Connections tasks that were algorithmic and

Table 4.2

Mathematical Task Analysis per Teacher per Lesson Observation

	Ms. Guzman		Ms. Becker		Mr. Palmer		Ms. Talika		Mr. Juarez	
	Obs. 1	Obs. 2	Obs. 1	Obs. 2	Obs. 1	Obs. 2	Obs. 1	Obs. 2	Obs. 1	Obs. 2
Memorization Tasks *Lower-Level Demands*			X		X		X			X
Procedures Without Connections Tasks *Lower-Level Demands*	X							X	X	
Procedures With Connections Tasks *Higher-Level Demands*		X		X		X				
Doing Mathematics Tasks *Higher-Level Demands*										

used procedures based on prior instruction. The tasks required limited cognitive demand for successful completion (Smith & Stein, 1998) (Table 3.5). The emphasis was on finding the correct answer and not mathematical understanding. Ms. Guzman's first observed lesson was indicative of this lower-level demand that was slightly higher than *Memorization* (Table 4.1). During the first lesson observation, she asked her students why they obtained the answers; however, the student answers only included the procedures they used and never addressed why the procedures were used or why the procedures were valid methods. Ms. Guzman consistently accepted the lower level responses from the students and did not press the students for more in-depth answers.

Findings suggest that all five teachers were aware of their procedural knowledge of mathematics and their ability to solve the problems the students were required to solve; however, they did seem to feel that just knowing how to solve a problem in mathematics may not be the only type of knowledge necessary for teaching mathematics. Content knowledge, although procedural, is necessary if there is to be collaboration that leads to in-depth knowledge building for teaching; however, it is not sufficient for teaching mathematics effectively (Ball, 2000). For example, although Ms. Becker often indicated, during the workgroup and TPD sessions and during her initial interview, that she was "good in math," she also realized that teaching math is more than knowing how to do the math.

> I'm really good in math, not as good at teaching it. And that's odd because I thought because I know math so well that I thought that would be my strong subject to teach, and when I started doing this group stuff [teacher workgroup] and we started talking, oh how am I going to teach that? How am I going to teach that? Oh, wait. Okay, maybe being good. (Ms. Becker, Initial Interview)

Mr. Palmer also felt that he was able to do the math; however, he felt that his method of doing math was limited to only one approach and that he needed to change his method of teaching.

> You know, I learned mathematics in a…way, I mean, very mechanical and when you stop and think about it you realize there's a little bit different perspectives you can look at it. (Mr. Palmer, Final Interview)

As the discussions continued in the TPD, workgroup sessions, and interviews, findings suggest all of the teachers in this study were not confident explaining the underlying reasons behind the mathematics. For example, during Ms. Guzman's initial interview she expressed frustration that she did not know how to explain the reasoning behind the mathematics: "When we're doing our [math] concepts I don't know 'why', you just do it …What I am doing right now is I'm teaching them but not explaining why." Ms. Becker further indicated during her initial interview that she did not know "how to explain why we move the decimal," referring to the first lesson that I observed. Ms. Talika during the first TPD indicated that her knowing why the procedures work was also important to her work: "Maybe for me knowing really why, because I still really don't know why, when you multiply decimals why you don't align [the decimals points as in addition]."

The baseline for analysis indicated that the teachers in this study had a procedural understanding of mathematics and a lack of understanding of the "why" behind the mathematics. The TPD sessions were designed to address the teachers' limited depth in mathematical understanding by increasing the ability of the teachers to access the mathematics through multiple solution methods or paths and thereby increase their Mathematical Knowledge for Teaching (MKT) and to note if the increase in MKT affected their classroom instructional practices.

Impact of Targeted Professional Development

The intent of the TPD was to build the MKT of the teachers involved in this study and to increase the mathematical knowledge of the teachers and determine the impact on the teachers' classroom practices. There appeared to be an impact of the TPD in the following areas: an increase in the understanding of and fluency with mathematical concepts with an increase in the ability of all of the teachers to make connections to alternative representations of the mathematics; for most of the teachers an increased awareness that connecting the "why" to student learning is necessary; and changes in the classroom instructional practice for three of the five teachers as indicated by Table 4.1 and the level of cognitive demands of tasks as indicated in Table 4.2 .

Increase in Mathematical Understanding

The first Targeted Professional Development (TPD) session was designed to cover fractions that the teachers would be teaching during the fall semester. Four out of five teachers attended the first and third session and all attended the second session (Table 3.3). The first TPD session began with a discussion of released items from the Mathematical Knowledge for Teaching (MKT) Measures, University of Michigan at Ann Arbor, 2008. The problems were specifically tied to the topics the teachers taught during the study and were intended to build the mathematical knowledge of the teachers.

The first topic of discussion focused on understanding the meaning of zero. The first problem in the TPD was used to determine the knowledge base of the teachers and was used to increase their knowledge base as the discussion included zero, the use of zero and even and odd numbers in general (Figure 4.1).

Figure 4.1 Elementary Content Knowledge Item: Learning Mathematics for Teaching Released Items, University of Michigan, Ann Arbor, 2008

1. Ms. Dominguez was working with a new textbookand she noticed that it gave more attention to the number 0 that her old book. She came across a page that aksed students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not Sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

Most of the teachers referred to a zero after a decimal number (2.80) as having no meaning. The teachers referred to zero's value as "nothing" and indicated that zero was not an even number. One teacher, Mr. Palmer, indicated that zero was an even number, but he did not know why it was an even number or how to justify hi. Finally, after the discussion of zero during the first TPD session, he was able to expand and explain his method of reasoning to the group to verify zero was an even number to another teacher.

> Mr. Palmer: I'm going to add one more thing. (Laughs) If you draw a number line, you have a negative 1 and a positive 1, right?
> Ms. Becker: Exactly.
> Mr. Palmer: They're both odd. So what's in the middle?
> Ms. Becker: Zero.
> Mr. Palmer: An even. And every other number alternates. 1, 2, 3, 4 Odd, even. Even and odd. So if you have two odds, one on each side, you have to have zero to be even. Because if you look at a number line with whole numbers or integers, the pattern is even-odd-even-odd-even-odd. So by looking at the pattern, zero is even.

All group members began to further increase their understanding that the zero indicates no units, tens or any other place values when written in a number (Example: 209, where the zero indicates no tens). This was dramatically different from representing zero as nothing or just a "placeholder." As Ms. Becker indicated in the TPD session, the need to include in her teaching further clarification of zero to her students became important.

> I think when we're using decimals, to show the kids that anything to the right of the decimal number has no value. That's how the kids finally understand that. Decimal 80 [.80] is the same as decimal 8 [.8]. That a zero to the right of it doesn't change its value. So the only way the kids can really grasp that is when I put the zeros in front of a whole numbers and explain that that also doesn't change the value of 8. But now that she [facilitator] said it that way, I'm going to add that into it also. It has no tens, it has no hundreds. (Ms. Becker, TPD 1)

The second MKT problem (Figure 4.2) required the teachers to determine the best method for ascertaining if the number 371 was a prime number. The question had four possible answers with only one correct answer. The group quickly began to examine the number 371 and how they might determine if it was a prime number. During the discussion the group defined a prime number. In a short discussion amongst the teachers, they determined that any composite number can be prime factored and that any composite number must have prime numbers as factors. After a very long discussion that included me questioning and probing for their understanding, they realized that if a number is a prime number, they needed to test all prime numbers less than the approximate square root of the number; in this case for 371 it was twenty.

Figure 4.2 Elementary Content Knowledge Item: Learning Mathematics for Teaching Released Items, University of Michigan, Ann Arbor, 2008

2. Ms. Chambreaux's student are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution mehtod is correct? (Mark ONE answer.)

a) Check to see whether 271 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9
b) Break 371 into 3 and 71; they are both prime, so 371 must be prime.
c) Check to see whether 371 is divisible by any prime number less than 20.
d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Initially, all of the teachers had some difficulty articulating how they would determine if a number was a prime. As the discussion continued the group was able to articulate the reasoning behind the process of determining if a number could be a prime number. Mr. Juarez said he normally told his students to use the prime factorization to determine if a number was a prime number; however, he never knew why he used that process. He was finally able to articulate a working definition of a prime number:

“Basically it’s only divisible by one and itself if it is prime and then if it is divisible by another prime number it has to be [a] composite [number].” Mr. Palmer told me that his understanding of composite and prime numbers was weak until the discussion during TPD Session 1.

The first TPD extended into the first workgroup session due to the inability of the group to fully understand the concept lesson during the first TPD. This would not have occurred had we not begun the work in the TPD, since the teacher content specific workgroup sessions were designed to collaboratively develop a specific lesson by the group. The need to continue the discussion was evident when the first TPD session came to a close and the teachers did not have sufficient time to fully grasp the concept lesson and the multiple access paths for the solving the *Linking Fractions, Decimals, and Percents* task (Figure 4.3).

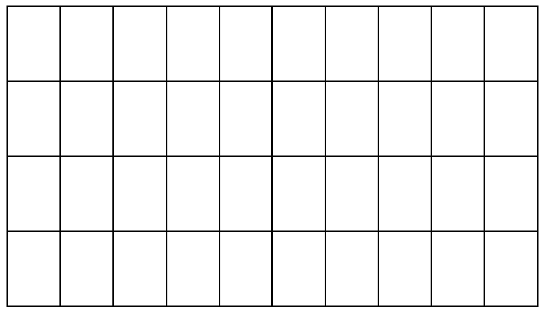
This task, characterized as *Procedures with Connections* (Smith & Stein, 1998), required the teachers to pictorially represent and explain the decimal, fractional and percent form of six shaded squares of the given 40 square diagram. The problem was relevant to the curriculum the teachers would be teaching during the fall semester. The task also provided an opportunity to expand the teachers’ knowledge about fractions. The task would be modified later in the semester by the teachers for possible use in a fifth or sixth grade classroom.

As the group moved through the workgroup session one they began to realize that they could represent the decimal pictorially by increasing the number of boxes to a 10 by 10 grid or 100 boxes. They discussed that as one doubled the number of squares, the number of shaded squares doubled leading to 12 shaded squares per 80 squares.

Figure 4.3 Linking Fractions, Decimals and Percents

Linking Fractions, Decimals, and Percents

Your task: Shade 6 of the small squares in the rectangle shown below. Then determine the percent, the decimal, and the fraction represented by the shaded squares.



Using the diagram, explain how to determine each of the following:

a. The percent of area that is shaded.

b. The decimal part of the area that is shaded.

c. The fractional part of the area shaded

Stein, M.K., Smith, M.S., Henningsen, M.A., and Silver, E.A. (2000). *Implementing Standards-Based Mathematics instruction: A Casebook for Professional Development, p. 4.*

Difficulty arose when they added 20 squares to the 80 squares to arrive at the 100 squares. Some of the teachers had difficulty understanding why they needed to shade 3 additional squares but after I asked some probing questions, the teachers were able to comprehend the process.

Mr. Palmer: My question is how did we get the 3? What…how do we – I mean, I understand how we got the 3. But how, let me rephrase my question. I can see what you did the first time. You basically, you wanted to take something. When you’re talking about a decimal or percent—you’re talking about 100.

Me: Yes.

Mr. Palmer: You're talking about 100. Now when you doubled that [referring to the forty squares], that all made sense. And when you add another 20 to get a full 100, that made sense. But how would you use the top two pictures to derive (sic) at the number 3….

Me: Now if I have 6 squares for every 40 squares, this is where that proportionality comes in. And that whole idea of proportionality. Then you say to yourself, "Well, I can't color 6 again because I don't have—The full 40.

Mr. Palmer: Right.

Me: What is the relationship between the 20 and the 40?

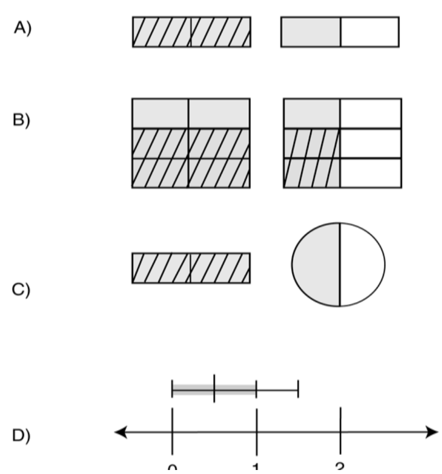
Mr. Palmer: One-half.

Me: So what is the relationship?

Mr. Palmer: One-half. And that's 3. Okay.

Figure 4.3 Elementary Content Knowledge Item: Learning Mathematics for Teaching Released Items, University of Michigan, Ann Arbor, 2008.

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately. Which model below cannot be used to show that 1 ½ x 2/3/= 1? (Mark ONE answer.)



The next TPD session, TPD 2, continued the group expanding their understanding of fractions. The group began with problem number 6 from the Mathematical Knowledge for Teaching (MKT) measures, Mathematics Released Items, 2008 (Figure 4.4).

The difficulty all of the teachers encountered resulted from a lack of knowledge of how a pictorial representation could demonstrate multiplication of fractions. In addition to choosing a model, the teachers were asked to explain their thinking about why the models worked and why the model that was not correct could not represent multiplication of fractions. Although some were able to determine the incorrect model, they were not sure if they could explain their reasoning. There was some concern that the students would not understand the representation. Mr. Palmer wondered aloud: "You think they will understand this?" He then commented to the group: "Well, you know, I've got to tell you I'm having trouble understanding this" (TPD 2). However, after there was more group discussion, the group realized that not all problems were to be used with their students but were to provide additional mathematical knowledge for the teacher. After further discussion all of the participants were able to explain the representations and were able to tie the incorrect model (C) to the concept of area. They realized that the square and the circle could not represent the same whole since they were not necessarily the same area. This idea led them to the concept of the whole and the importance of defining the whole correctly.

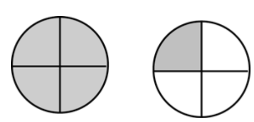
The teachers were given another MKT problem that led to a discussion of how to represent a whole (Figure 4.4). The discussion evolved into an in-depth exploration of the meaning of what the "whole" represents and how this idea related to the parts of a fraction; namely, the denominator and the numerator.

The teachers read the problem silently and then answered the question. Some determined the answer quickly. One teacher had more difficulty with the problem. During this problem, the teachers began to take ownership of the collaborative process and began to provide support for each other. The following excerpt demonstrates the increased capacity of the group and the increased understanding as they went through the problems that came out of the TPD 2.

> Ms. Talika: The two represents the whole of the two the part that is shaded of equal parts, there is one part, two parts, three parts, four parts, five parts out of the total eight which makes the two pizzas. So it's 5 out of 8.
>
> Me: And again, why is that powerful to students?
>
> Ms. Talika: Wow. …Because a whole can never, it can be different…
>
> Mr. Palmer: You define the whole. You can say a, you can say there's a piece of pizza left sitting here and if a quarter of the first whole, somebody can say does somebody want this whole piece of pizza meaning the whole fourth, the whole had to be carefully defined. And actually thinking about it, it's almost as though you have to define the concept of whole to define parts.
>
> Ms. Becker: And you know what, what she did is extremely, extremely common with kids in that they go right past that, the whole is of the two pizzas because I passed that, the whole is of the two pizzas. Because I had to kick myself and I'm here going no, and then I remember no, no, no, he's right because that is the whole pizza, because you skim right past that part and still see two wholes. Even though that sentence says the whole is all of that.
>
> Ms. Talika: The whole as a pizza [is] two pizzas. (TPD 2)

Figure 4.4 Elementary Content Knowledge Item: Learning Mathematics for Teaching Released Items, University of Michigan, Ann Arbor, 20085.

Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4
b) 5/3
c) 5/8
d) ¼

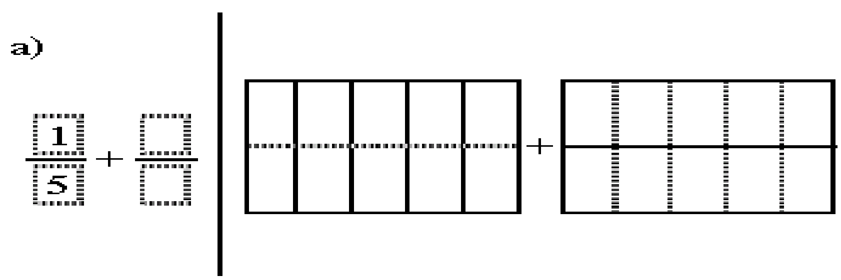
When the teachers were asked to think about the implications of this type of problem, they discussed how they consistently used single objects to represent a “whole,” and then dividing that single object into equal parts. They felt that this narrow description of a whole could lead students to a misconception about the definition of “the whole.” The application of the knowledge to the classroom during TPD Session 2 was an indication how the teachers were now beginning to transfer their TPD knowledge to the classroom.

Our next task was to revisit the first type of problem discussed earlier (Figure 4.3). I asked each teacher to make up a problem to see if they could represent the multiplication of fractions pictorially. This task proved to be challenging for all members of the group. They asked if we could do their individual problem that each created with the group since they were still uncertain how they draw figures to represent a multiplication of fractions. Their enthusiasm increased during the task and when we finally finished a few more problems, their understanding increased dramatically, as evidenced by their sharing of ideas and problems. This type of process allowed them to grow in their knowledge of mathematics specific to teaching, allowed them to experience the mathematics through their development of additional problems and provided them an opportunity to explain their thinking when prompted Sowder (2007).

Using grids, the teachers were further challenged to demonstrate the addition and subtraction of fractions as illustrated in Figure 4.5. Ms. Guzman used a similar problem in her class during my second classroom observation. Ms. Guzman was trying to change her instructional practices in her classroom by venturing beyond her textbook for her lessons and pushing her students to gain more understanding of mathematics.

The group struggled with portraying the addition of the fractional parts using the diagrams until they finally were able to see the process of superimposing the two representations upon each other. The teachers realized that the representation of addition of fractions could be used to understand subtraction of fractions as well. The multiple methods of representing fractions had increased their understanding. As Ms. Talika said, “My eyes have been opened. I love fractions; initially I wasn’t getting how you draw the pictures, and how you subtract from it….I understand now” (TPD Session 2).

Figure 4.5 College Preparatory Mathematics (CPM) 2002



In the third TPD session, TPD 3, we revisited the concept lesson *Linking Fractions, Decimals and Percents* introduced in TPD 1 and the group made some modifications to the lesson to accommodate a fifth or sixth grade implementation. After examining the lesson in depth and determining how they would implement the lesson, the group was guided through another lesson using fraction strips based on a *Connected Mathematics* lesson. The lesson provided an opportunity for the teachers to understand the relationships between factors used to find the Least Common Denominator. They were asked to fold 10 strips of paper into equal partitions: halves, thirds, fourths, fifths, sixths, ninths, tenths, and twelfths. After making the strips, the teachers were asked to determine the relationships between all of the partitions.

The teachers' interest in the problem led them into a discussion about how they could implement the fraction strips lesson in their classroom. Throughout this exercise I encouraged the teachers to explain their understanding of the relationships using the fraction strips and using mathematical terminology correctly. Only one teacher wanted to set the problem up by having the dotted lines on the strips so that the students would fold along the dotted lines. As Ms. Talika explained to the group:

> You know what? I don't know if this is the case with you and other teachers but my kids are really not that good. They're kind of clumsy with [using manipulatives] so I would probably copy one half dotted lines and just have them fold it along the dotted lines. (Ms. Talika, TPD 3)

I aksed the group if providing the lines on the paper for folding the strips would benefit the students. There was some hesitation in answering this question. After I prompted the group for possible consequences of providing the lines for folding, followed by a group discussion, they agreed that the lines would negate the effectiveness of the activity. The group decided that the students needed to fold the strips and see the relationship within the folding process: folding fourths into thirds to obtain the twelfths. The group continued folding all of the strips and discussed all of the connections between the different partitions. At the end of the meeting some of the group members decided to implement this particular task in their classes prior to our last workgroup meeting.

Throughout the TPD sessions findings suggest teachers were building their mathematical knowledge and increasing MKT. There were moments when they struggled; however, they consistently came back to work even harder to understand the concepts.

Increase in Making Mathematical Connections

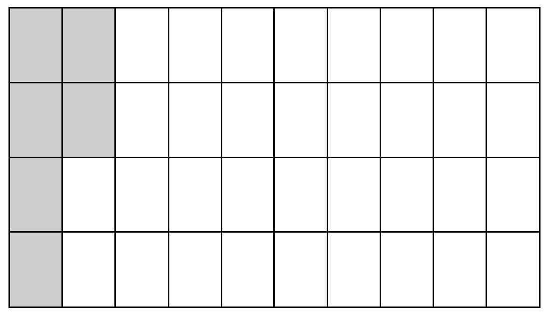
Although it is necessary to build mathematical content knowledge of the teachers when developing MKT, it is also necessary to build the pedagogical knowledge and the ability for the teachers to make connections between multiple representations (Ball, 2009; Smith & Stein, 2009). Findings suggest the TPD increased their ability to make connections. Evident in TPD sessions was my intentional use of multiple representations that would allow the participants to make mathematical connections. I continued to use questioning techniques based on Boaler's and Brodie's (2004) Categories of Questioning emphasizing questions that explored mathematical meaning and relationships and probing to encourage teachers to explain and extend their thinking. The findings suggest that all of the participants increased their ability to make connections between pictorial representations and procedural mathematics for fractions.

When the teachers re-examined the *Linking Fractions, Decimals and Percents* concept lesson (Figure 4.3) introduced in TPD 1, during workgroup session 1, the group turned their attention to the columns, each representing one tenth of the whole and the shading of the 6 squares as noted in Figure 4.6. The teachers exchanged ideas, pointed to the representation, made notes on their paper and sectioned off the columns. When prompted to look at the individual columns and what each column represented, understanding of this alternative method of looking at the problem became apparent. The teachers began to expand their knowledge and began to explore alternative pathways.

Figure 4.6 Linking Fractions, Decimals and Percents

Linking Fractions, Decimals, and Percents

Your task: Shade 6 of the small squares in the rectangle shown below. Then determine the percent, the decimal, and the fraction represented by the shaded squares.



Using the diagram, explain how to determine each of the following:

a. The percent of area that is shaded.

b. The decimal part of the area that is shaded.

c. The fractional part of the area shaded

Stein, M.K., Smith, M.S., Henningsen, M.A., and Silver, E.A. (2000). *Implementing Standards-Based Mathematics instruction: A Casebook for Professional Development, p. 4.*

Me: But what does each of these columns represent?
The Group: 10. 10%.
Mr. Palmer: I see. That's a good way to …
Ms. Becker: Ten and then half of ten, 15%. Oh man! That's cool. (Laughs)
Mr. Palmer: Yeah.
The Group: That was too easy. (Laughs)
The Group: Yeah. Yeah.
Mr. Palmer: So this one's 10%. [pointing to the first column]

The discussions did not conclude but continued on as another participant wanted to know what one square represented. The teachers led the discussion and stated their interest in connecting the pictorial representations with the procedural representations by extending the discussion to another representation of what each individual square represented.

Ms. Talika: I'm trying to see each square.

Mr. Palmer: It's 1/40th.
Ms. Talika: 1/40th. Why is it 1/40th?
Mr. Palmer: Because there's 40 squares total.
Ms. Talika: Ohhh…

Ms. Talika continued to ponder the 1/40th and began to think about the connections and then made even more connections.

Ms. Talika: So if you were to add all these, that would be 4 over 40, which is 1/10th….Okay, I got that a little bit. So 1/40th, 4 [of them] equals 4/40th, and that's 1/10th.
Me: So this was a good point that you brought up. 1/40, 1/40, 1/40, 1/40. What does that give you?
Ms. Talika: 10%.

This extension demonstrated the readiness of the teachers to expand their knowledge and to make even more connections. The final discussion was initiated by one of the teachers when she asked how you would use each row as representing 1/4 of the entire figure or 25% of the whole. This was more complex; however, the teacher wanted to figure out how this could be done as well. Evident throughout the TPD was the growth in making the connections while increasing mathematical knowledge.

Mr. Palmer noted that the mathematics that was being discussed may not necessarily be used specifically with his students but would provide more depth to his understanding. This was important since the goal was to build the mathematical content knowledge of the teachers. Mr. Palmer noted during the first workgroup session that perhaps only one method would be possible with his students.

> I wanted to make a comment here. I think that the first one that you did, would probably be the most direct and easiest for 6th graders. Because a lot of them [students], in order to do it this way [that last method], they have to [know] how to find ¼ and 25%. So it requires a deeper knowledge of math than that. I'm not saying that there's anything wrong with this. I'm just saying that the way you did it, the very first one? Is as close as purely pictorial as you can, and it seems to make more direct sense. That's just my own [opinion].

The TPD was intended to intertwine the mathematical content and pedagogical knowledge while explicitly requiring the teachers to articulate the connections. The increase in mathematical difficulty was intentional to increase the capacity of the teachers. All of the teachers struggled at times with the content. Even as they moved into TPD 3, the anxiety of experiencing a new understanding of mathematics was clear. Ms Becker said, "We were getting frustrated trying to figure it ourselves, I mean it wasn't until we started doing the drawing that we started to [figure it out], look how long we've probably done this [referring to fractions, decimals and percents] in our entire lives."

This acknowledgement of their initial lack of making connections to alternative representations was evident as well as their increased capacity over time to make the connections once provided the opportunity in the TPD. This process of making connections was important for the teachers; however, transferring this to the classroom required that the teacher connect the idea of learning the "why" behind the mathematics to the need for understanding the "why" by the students. Findings suggest that three of the five teachers realized that the students needed to also understand the why behind the mathematics they were learning. As they progressed through the curriculum, findings also suggest that the same three teachers began to realize that the alternative access strategies to the material could provide that understanding within their teaching for their students as indicated by the increase in the cognitive demand of the lessons and the increase in cognitive demand of the questioning in the second observation (Table 4.2).

Connecting the "Why" to Student Understanding

Three of the five teachers began to realize that knowing the conceptual underpinnings for the mathematics was not only beneficial for them, but that it would also benefit their students. In addition, they felt that it was necessary to provide the alternative access strategies to deepen their students' understanding. Ms. Becker expressed her feeling that perhaps there needed to be more understanding of the concepts by the students during TPD 1.

> Well, based on what she just said [another group member was confused], that would make me stop and think, are my kids really understanding this? Maybe I need to ask more questions. Because maybe they're doing like her. Maybe they're just changing the answer because they saw somebody else had the right answer and they know that person's right. So maybe I need to rethink. Does that child really understand? What can I ask that child, that student, to see if they really are understanding? Because I have kids that do that.

Ms. Guzman considered how she would change her instructional practices when she reflected on the implementation of the collaboratively developed lesson. She had taught the procedural method for changing fractions to decimals prior to the group-developed lesson. During Workgroup Session 4, she related to the group that her students were not making the connections and that she would need to change the way she did the lesson. She concluded that although she pre-taught the procedures, perhaps she should have implemented the group developed lesson first so that the students would have a better understanding through the visual representations of the connections between fractions and decimals.

Ms. Guzman:	I would use, I would actually try to incorporate this [the visual representation] into it if I give a decimal, or a fraction they also use…individually, but yeah I would change it.
Me:	And that means you would probably put the procedural and the visual more together as opposed to separate?
Ms. Guzman:	Yeah, I need to work on that.

Ms. Talika began to understand that students should understand the mathematics and should be able to make connections with the mathematics. This understanding came after Ms. Talika discussed with the group her implementation of the group lesson during a Workgroup Session 4. Ms. Talika used the lesson that the group had developed after pre-teaching two lessons that covered the procedural aspects of the lesson that the group developed. Ms. Talika thought that her students would understand the connections between decimals and fractions; however, she discovered that the students did not understand the connections in workgroup lesson that she implemented after teaching the procedural aspects of the lesson. She had assumed that the lesson developed by the workgroup would be easy for her students since she had pre-taught the procedural methods; however, after she listened to her students during the lesson, she discovered that the students did not make the connections in the pre-taught lesson from the textbook and that the connections only became apparent to them after completing the workgroup lesson.

Ms. Guzman during her final interview, like two of her colleagues, Mr. Palmer and Ms. Becker, reiterated her need to have the students explain their reasoning. She felt that she needed to go beyond the procedural operations and ensure that her students understood the mathematics.

> …Well, again, it goes back to that…having that discussion with the kids in the sense of like asking them questions of why, for example, like when Delores (sic) wrote that fraction, is having them elaborate more on their process. It's something that I really didn't do. It's like, okay, let's move on as opposed to try and to see if they fully understand the concept. I think it goes beyond student achievement; it's just mechanical. Everyon can eventually achieve that, but do they truly understand what they are doing? That's I guess, the goal that I as a teacher want to accomplish, I mean, I don't think I will accomplish it 100% this year, but I know that I'm gonna work more towards accomplishing that goal.

During Workgroup Session 4, and after the group had a long discussion on how the collaboratively developed lesson should be implemented, Mr. Juarez summed up how they would implement the lesson that was more cognitively demanding. Although Ms. Talika wanted to "show" the students how to do the problems, Mr. Juarez said, "Like let them [the students] explore. Don't show them anything, just give them a preview and let them go." This ended the discussions and the group decided to let the students explore the concepts.

Although two teachers, after all TPD and workgroup sessions were concluded, did not appear to be ready to make instructional changes in the classroom to provide the opportunities for the students to understand the conceptual reasoning behind the mathematics, they were still interested in understanding the "why" for themselves. That was a beginning and cannot be overstated.

Classroom Instructional Impact

The intent of all of the TPD sessions was to not only increase the MKT of the teachers but to also impact instructional practices in the classroom. The observations were arranged according to the teacher's schedule and did not need to coordinate with the implementation of the lessons that were developed by the workgroup. This was intentional since I wanted to observe the teachers during a routine lesson. It was the teachers' decision if they opted to do a lesson that was developed in the workgroup for the observation.

The TPD effect on classroom instruction was mixed. Noting the changes from the first to the second observation (Table 4.1), four of the five participants included group discussion in their second lesson as opposed to only one teacher in the first observation. The groupwork provided a greater opportunity for students to interact and become part of the learning. For the Student Activity category in Table 4.1, three of the teachers included group work in the final lesson observed and encouraged students to collaborate when doing their work, which was also verified through an increase in the student-to-student interaction. Analysis of the teachers' questioning indicated that two of the participants used terminology appropriately during both observations and three consistently inserted and used appropriate terminologies only during the second observations. As Mr. Palmer said,

> I learned that actually anything you write in mathematics has a meaning. There's a reason why they put a zero somewhere. The reason why they put a number in a particular place value and why there's a decimal. It's very precise. (TPD Session 1)

The analysis of the questioning also indicated an increase in Exploring Mathematical Meaning and /or Relationships with three of the five teachers exhibiting more questions that directed students toward making connections in the mathematics and looking for relationships (Table 4.2).

The willingness of the teachers to change their practices, to use alternative methods of accessing the mathematics and then to reflect on their practices may be a factor in changing classroom instructional practices. Three of the five teachers tended to be more receptive to the multiple methods of accessing the mathematics than the other two teachers and they attempted to implement alternative methods of accessing the mathematics in their classroom. These teachers also consistently reflected on their teaching and how they might improve their practice in the classroom. Their perception of the success of their lessons in the classroom were more closely aligned to my observations and included realistic assessment of what worked in the classroom and what did not seem to work. The analysis and participation of the teachers within the TPD changed them from teaching that mainly modeled procedures to a model that included asking their students higher cognitively demainding questions and included students developing conceptual understadning. The change was indicated by the teachers' interest in determining how they might question their students to push them for more understanding. They began to embrace more student collaboration and to increase their student participation leading to the students taking a more active role in their learning. Mr. Palmer reflected in his last interview that he was doing less talking in the class and allowing the students to do more of the talking. He was also using more visuals in his classroom instruction.

> So, I'm trying to cut out the blah, blah, blah from me and have them actually be part of it [the lesson] so they'll, they won't, their minds won't be diverted into other things, but it's [using visuals] not infantile, but even if it was, there's an advantage there to keep 'em engaged.

Individual Differences

Ms. Guzman still had some concern about her knowledge base and how she would handle atypical questions from her students; however, she was open to moving forward with learning more about mathematics and learning more about teaching and questioning. Ms. Becker also wanted to learn more about why the mathematics was working and wanted to increase her knowledge about teaching her students in a manner that addressed the "why" in the work. She consistently reflected about her instructional practices and realized that although she understood how to do the procedural mathematics, she did not necessarily understand the "why" of the mathematical procedures.

Mr. Palmer was also interested in changing his instructional practices in the classroom; however, I needed to support him more through additional meetings to explain implementation of the collaboratively developed lesson. As he became more confident in his ability to make changes and grew more confident in the MKT he learned, his instructional practices in the classroom began to change. He was able to reflect on his work and to note realistically what worked and what did not work. He was willing to increase his knowledge throughout the study and he indicated that he wanted to continue building his knowledge through more professional development. Mr. Palmer, Ms. Becker and Ms. Guzman embraced the new knowledge and all decided to increase student participation in their classrooms by using group work and by asking students more in-depth questions that would probe for understanding.

Ms. Talika participated in the learning portions of the professional development; however, she often resisted allowing students to interact and work toward conceptual understanding in her classroom. Her intense anxiety about covering all of the California Content Standards tested on the CST and making sure that the work in the textbook was covered in every class was evident even when the workgroup was developing the lesson. She consistently referred to the textbook and the necessity to use the procedural methods provided in the text. When she implemented the lesson, although the lesson was designed to replace the section in the book, she still taught what was in the book first and then followed it with the workgroup lesson. Although the group discussed her method of implementation and she indicated that until her students did the collaborative lesson they may not have understood the lesson, she was still hesitant to try other methods of implementation. She attempted some of the ideas that we tried in TPD, but she did not think out the implementataion of the new methods carefully or she did not implement the methods with fidelity.

Mr. Juarez tended to be very quiet in the TPD and the workgroup sessions. Although he finally implemented the collaboratively developed lesson, it seemed to be implemented due to compliance from me asking about the lesson and also peer pressure to implement the lesson versus his interest in changing his teaching method in the classroom. His reflections about the effectiveness of the lessons that I observed did not align with my observations of the lessons. There appeared to be little change between the first and second observation. Although he asked his students questions in the classroom, many times he either did not wait for an answer or reworded a student answer to his satisfaction. The one area in which he seemed to increase his effectiveness was the insertion of correct mathematical terminology into the lesson.

Change does take time and the changes that occurred in this relatively short time period were real, observable and a beginning. Chapter Five will provide an overview of the results of this study, literature review and will conclude with recommendations for policy and practice and further research.

Chapter V

Over the past two decades professional development for teachers has shifted towards teacher generated reflective cycles in the form of lesson study, inquiry based learning or teacher initiated action research (Fernandez, 2005; Gellert, 2008; Alvine, 2007). Professional development may be teacher led and may provide an opportunity for content specific teacher workgroups to study their classroom practice through a co-developed lesson study format. This is a departure from traditional professional development where experts provide professional development to teachers with the expectation that the methods and lessons delivered to the teachers will be implemented in the classroom. The problem that exists is that although teacher workgroup collaboration is important, the mathematical knowledge for teaching (MKT) is an important element if teachers are to be successful in their practice. As a result of the need for teachers to increase their MKT, this research study focused on providing targeted professional development (TPD) that would build the MKT of the workgroup members.

Summary of the Study

The research study provided a setting for teachers to discuss their instructional practice and a separate setting to provide the participants an opportunity to develop and increase their content and pedagogical knowledge, which is collectively referred to as mathematical knowledge for teaching (MKT). The professional development used in this study was a Targeted Professional Development (TPD) that enhanced the mathematical and pedagogical knowledge of the teachers based on the needs of the teachers as determined by the teachers. The TPD was specific to the curriculum the teachers were teaching during the study. I developed the three TPD sessions based upon the teachers' need to learn how to teach fractions more effectively while providing greater conceptual understanding for their students.

In this study, the TPD designed to build MKT was delivered to a group of fifth and sixth grade teachers. The TPD was intended to build the MKT of the participants in the workgroup and to support the teachers as they developed common lessons that would be implemented during the semester. I collected and organized the data into five individual teacher case studies that examined the journey of each teacher and one case study of the collective workgroup and TPD sessions. The workgroup case study included seven workgroup sessions and three targeted professional development (TPD) sessions, observations and interview data. The research study addressed the following question:

> What is the impact of TPD sessions on teachers' understanding of and fluency with mathematical concepts and on the teachers' instructional practices in the classroom?

The individual case studies were used to analyze each participating teacher over the course of this study and the final workgroup case study was designed to study collectively the teacher workgroup through the intervention process of TPD and the impact on the content specific workgroup sessions. Each teacher case study allowed for in-depth analysis of the impact of the TPD on the teacher and provided an opportunity to note any similarities and/or differences of the impact of the TPD between the teachers. The workgroup case study allowed for analysis of the changes that occurred as the group experienced the TPD and the collaborative workgroup sessions. Tables 4.1 and 4.2 were used to collect and organize the initial data from the observations. These tables informed the baseline analysis and the changes that occurred between the initial and final observation.

Discussion

The baseline for analysis indicated that the teachers in this study had a procedural understanding of mathematics, lacked multiple access strategies to the curriculum and lacked an understanding of the "why" behind the mathematics. The TPD sessions addressed these areas by providing the teachers an opportunity to learn multiple strategies to access the curriculum and provided them opportunities to make connections between the various access strategies used. Ball and Bass (2009) indicate that professional development should include Mathematical Knowledge for Teaching and Rockoff (2003) further contends that the effectiveness of the teacher in the classroom is related to the teachers' mathematical knowledge for teaching. The TPD in this study specifically addressed increasing the MKT necessary to impact teacher effectiveness for fifth and sixth grade teachers.

In this study there appeared to be an impact of the Targeted Professional Development in the following areas: all teachers had an increase in the understanding of and fluency with mathematical concepts, all teachers had an increase in the ability to make connections to alternative representations of the mathematics, most of the teachers had an increased awareness of connecting the "why" to student learning, and three of the five teachers demonstrated changes in classroom instructional practices. Since the study was over a very short period of time, these changes may not endure if professional development, consisting of TPD and workgroup sessions, is not continued.

Although the observation data indicated change in practices of three teachers, lack of impact for two of the teachers may be attributed to the degree of the teachers' readiness or eagerness to change their teaching. Although the teachers were given the opportunity to decline participation, there may have been some peer pressure to be a part of the group from the teachers in the group. The principal came in during one of the sessions to thank the group for participating. This also may have increased the pressure on the teachers to participate in this study and may have affected both positively and negatively the impact of the TPD and workgroup sessions. It is difficult to determine if the changes in the instructional practices are permanent. The changes may have only resulted from my convincing the participants to expand their knowledge and to try some of the new methods of teaching in the classroom. After finishing the TPD some of the teachers may have gone back to their procedural methods of teaching due to the lack of support provided by me during the study. If growth in MKT is to continue for the teachers then the school needs to continue the support. In addition, the teachers must also have the opportunity to use the newly acquired knowledge through development of classroom lessons. Onetime only professional development models do not necessary ensure change in instructional practices.

The compensation provided by the school may have enticed some of the teachers to participate while others found it to be a nice bonus. The question arises if providing compensation would further enhance the implementation of professional development or if it would be better to include professional development within the school day, making it part of the schools' culture. Although the compensation was provided in this study, one teacher did not attend during his off-track time, so the impact of compensation may not have been as important to that teacher. The teacher who did not attend during his off-track time also did not have significant changes in his classroom practices (Table 4.1).

Implications

This study was limited in the number of teachers participating; however, the ability to connect the findings to a broader population may be possible. Since there is consensus that teachers must have specialized knowledge for teaching (Shulman, 1989; Ball, 2002; Ball & Hill, 2009), it is evident that if content specific workgroup members do not have sufficient knowledge the effectiveness of the collaborative workgroup is questionable. Merging the two professional development models of collaborative content specific workgroups and TPD must be considered. Providing the TPD at the school site based on the needs of the teachers may be a viable method of increasing teacher effectiveness in the collaborative workgroup. In addition, by providing workgroup sessions, there are additional opportunities to support teacher learning outside of the TPD sessions. In essence, collaborative mathematics workgroups must also have the support needed to build the MKT of the participants. It was clear that the workgroup in this study, with their procedural knowledge of mathematics, would not have developed higher cognitively demanding lessons if they were not provided the TPD sessions. The TPD sessions allowed the teachers to experience more complex problems and build their confidence in mathematics, a necessary goal for mathematics professional development (Sowder, 2007). This increased confidence in mathematics led to some of the teachers implementing more complex lessons.

Schools must not assume that providing either professional development to build MKT or providing collaborative content-specific workgroups will increase classroom instruction. The study showed that both kinds of professional development supported the work of the teachers and that each kind was enhanced by the other. TPD informed teachers' knowledge that was used in the workgroup sessions and the workgroup sessions informed the TPD curriculum.

The implications of this study are quite clear at the elementary level: elementary school teachers need additional professional development to build MKT. Research indicated that elementary school teachers have inadequate knowledge in mathematics for teaching (Flores, Patterson & Shippen, 2010; Tsao 2005; Welder & Simonsen, 2011). This study also indicated that the knowledge of the teachers was weak in MKT, as described in the Baseline Analysis in Chapter IV. As the teachers experienced the TPD there was an increase in mathematical knowledge. As they increase their knowledge, teachers are able to implement a more cognitively demanding lesson, as found in this study.

In addition, teachers need meet to discuss and use the newly acquired knowledge. This requires that schools provide the appropriate setting for teachers to meet and discuss their new knowledge and to put into practice the new knowledge through collaborative content-specific teacher workgroups that develop lessons that are implemented in the classroom. The impact in the classroom was noted with some of the teachers. The study suggests that change could occur in the classroom provided there is sufficient time to implement the TPD. The implication is that change will continue provided there is ongoing setting at the school for the teachers to develop lessons for the classroom that incorporate the learning from the TPD. Change cannot be expected to continue unless there is sufficient time for teachers to meet for both TPD and workgroup sessions.

The uniqueness of the TPD working in conjunction with the workgroup sessions implies that schools should not use a one-size-fits-all professional development model but instead should design professional development specific to the needs of their teachers. The TPD was developed specifically to meet the needs of the teachers and the curriculum that the teachers were to teach during the study. The TPD addressed the specific knowledge and pedagogical gaps of the teachers while allowing the teachers the opportunity to decide on the areas of greatest concern in their teaching. It was not a pre-designed program but was developed specifically for the teachers in the workgroup. In this study the TPD informed the workgroup sessions by providing the necessary content support to the workgroup members. Likewise, the workgroup sessions informed the TPD sessions by highlighting the areas of weakness in the teachers' mathematics knowledge and pedagogy. Schools that decide to implement a professional development model for mathematics that is similar to this study should consider designing their program specific to their teachers' needs, providing a setting for teachers to collaboratively develop lessons based on their new learning and should ensure that professional development is continued over time with the TPD changing according to the needs of the teachers. Many times schools will purchase a set professional development based on what experts think the areas of weakness are in the teachers' knowledge for teaching, thereby disengaging the teachers. By asking the teachers their areas of concern in teaching there is an intentional effort to include the teachers into the plan for professional development, an automatic buy-in.

In order to implement a program that is individually designed for a school, an expert that is well versed in MKT and the curriculum for the particular grade level is necessary. Districts that implement a program for increasing the effectiveness of mathematics instruction at the elementary level will need to ensure that the experts are well trained in both MKT and in facilitating collaborative workgroups. The type of training and experience needed for the experts to provide the support should be investigated. The experts will need to determine the needs of the teachers to develop appropriate professional development for the teachers, support the facilitation of both the workgroup meetings and the TPD and build the capacity of the teachers, it will be necessary to have experts with an extended knowledge of mathematics.

The questioning used in professional development must model the type of questioning used in the classroom to enhance the cognitive demand of lessons. As noted in the study, by my modeling the questioning techniques in the TPD, teachers understood the significance of the questioning and then began to write down the questions they needed to use in the classroom. The TPD included modeling appropriate questioning that would increase the cognitive demand of each session. As indicated by the study, appropriate questioning that increases the cognitive demand of the work is necessary if teachers are to use cognitively demanding lessons in the classroom. As the provider of the TPD, I consistently focused the group on how I used questioning to deepen their understanding and how this questioning could be used to increase the understanding of their students. Over the course of this study some of the teachers began to ask me what questions they could ask their students to push for deeper understanding of the concepts. Ms. Guzman and Ms. Becker actually scripted the questions and then used the questions when they implemented the lesson. As active learners, the teachers shared their knowledge and provided support to others in the group. Although some professional development may include the facilitator questioning the participants, there must be an explicit reference to the use of this type of questioning in the classroom.

In order for professional development to be effective there must be a trust developed amongst the teachers and the expert. If a provider of the TPD does not respect the knowledge that the participants bring to the discussions, the group will not be able to become part of the learning and the affective filter will be raised. As the Coordinator of Mathematics, I observed the affective filters of the participants when mathematics coaches presented themselves as the expert imparting knowledge to teachers and not as the facilitator of the teachers' learning. As Sowder (2007) indicates, professional development must be relevant and must allow the teachers to be active participants in their learning.

At the conclusion of this study, the assistant principal asked if I could continue the program next year when the school changed over to a single track school. The AP informed me that the teachers in this study had indicated to the faculty, in a panel discussion, that they wanted more professional development that they could use immediately as they were able to do in my study. In particular they referred to professional development that they received my study. They also stated that professional development should increase their knowledge in the content area as specifically referenced in the TPD in the study. I attribute the positive reaction of the teachers to key elements of the model: the active participation of the teachers in their learning and the safe environment of the TPD and workgroup sessions.

By combining appropriate grade levels for studying the mathematics it was possible to build mathematical knowledge for teaching at both grade levels while vertically articulating across grade levels. Since the standards for both fifth and sixth grade are incremental, the ability of the teachers to gain greater insight into the expectations for each grade level was evident. This cross over from one grade level to another allowed the teachers in this study to ask each other what the expectations were for each curriculum topic. This was an unintended result of this study. The study also suggests that there must be support from an expert in mathematics who can build the MKT of the elementary school teachers. The TPD cannot be a "one size fits all" approach for professional development.

Recommendations for Future Research

Although there appeared to be some growth in the MKT of the teachers in the study, it would be difficult to definitively say that the increase in MKT was directly related to the TDP. The opportunity for teachers to discuss their work and to discuss mathematics may have been the reason for growth and not the actual TPD sessions. In addition, although there may have been changes in the one classroom observation, I do not know if these changes continued once the study ended. I would like to further study the effects of TPD and the workgroup sessions over time and determine if the effect upon the classroom instruction is ongoing. Again, I am wondering if the changes were only due to my being in the classroom for that one observation so the teachers felt compelled to use the strategies we discussed or if the TPD knowledge was internalized by some of the teachers.

The intent of any professional development is to improve teacher instruction to improve student learning, but I am not sure if this study impacted the student learning. I would like to study whether an extended professional development that included TPD and workgroup session would affect student learning as measured by student achievement on standardized tests. The effect of professional development needs to be measured by the effect on student achievement. This study only examined the effect of professional development on the teachers over a relatively small period of time. Further research needs to be done on the effectiveness of sustained change in instructional practices over time when teachers are provided TPD that supports collaborative workgroup sessions.

Further research should also include questions about how the experts should be trained to facilitate the growth of the school's professional development model. In addition, there is a need to understand the characteristics of the experts needed to implement the TPD that is specific to teacher needs and how will they be trained to provide these services to the school. Although my district used mathematics coaches for school sites, Newton's (2005) study revealed that there were only a small number of schools within the district where the coaching model was successful. This again indicates a need to study how the expert or coach should be trained in order to provide the appropriate targeted professional development needed at the school site as well as support the collaborative workgroup. The use of a school site teacher expert, who is still in the classroom, should be researched and studied. This may provide an economically viable means for implementation, especially during times of budget constraints.

Professional development should also include opportunities for teachers to work collaboratively to discuss and use the learning received from the TPD sessions and to integrate the work from these sessions into their work. More research should be undertaken to study the infrastructure needed to provide professional development consisting of TPD and collaborative content-specific workgroups to a large school district and how to measure the success of this type of professional development.

Conclusion

As Shulman (1989) has indicated in his work with teachers, teaching is a difficult and challenging profession.

> After 30 years of doing such work, I have concluded that classroom teaching … is perhaps the most complex, most challenging, and most demanding, subtle, nuanced, and frightening activity that our species has ever invented. The only time a physician could possibly encounter a situation of comparable complexity would be in the emergency room of a hospital during or after a natural disaster.

If teachers are to increase their ability to address the needs of all children in mathematics they must have professional development to enhance the mathematical knowledge for teaching. If schools are to provide an environment that is conducive to collaboration, then teachers must have the appropriate settings at the school to discuss and develop their instructional practices. Merging of the two professional models could provide the necessary structure to build both teacher knowledge and collaboration, which would then impact instructional practices in the classroom.

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Appendix A

Informed Consent Form

PART 1: Research Description

Principal I: Patricia Pernin
Research Title: Providing Targeted Professional Development in mathematics to support content specific collaborative teacher workgroups.

You are invited to participate in a research study on effectiveness of professional development at your school site. You will be participating in an inquiry-based collaborative mathematics workgroup and in separate targeted professional development focused on mathematics. I will ask for your insights on the mathematics professional development prior to the workgroup meetings and the TPD, and after the workgroup sessions and TPD. **The study does not seek to assess the students, staff, or faculty associated with the school.** The goal is to better understand the effect of professional development upon classroom instruction and subsequent student achievement.

If you participate in the study, you will be asked to take part in collaborative workgroup meetings as determined by the principal, three (3) 2-hour mathematics professional development modules, and two (2) 45-minute interview sessions with the Principal Investigator. In addition, the Principal Researcher will observe you teaching a lesson twice during the study. The professional development will be interactive and will focus on fifth and sixth grade mathematics curriculum and classroom appropriate pedagogy. The workgroup meetings and professional development modules and the interview will be videotaped and subsequently transcribed. **Participation in the study is voluntary** and, if you elect to participate, you will not be obligated whatsoever to answer or respond to any question or to discuss anything that you are not inclined to answer or discuss.

If you consent to participate in the study, you will not be identified by name in the study's findings. All direct quotes from participants that are used will be identified by use of a pseudonym in order to protect the identity of each participant. To preserve accuracy all interviews will be audio taped and all professional development meetings will be videotaped. After a reasonable period of time for analysis and follow-up study, all secured materials will be destroyed.

Although you will not be paid for your participation in the study, you will benefit from participation, due to the TPD focusing on mathematics. Also, by being in the study, you will contribute to the findings of this study that may contribute to our knowledge of best practices and subsequent improved student achievement. The findings may also inform the district's policy and practices.

PART 2: Participant's Rights

- I have read and discussed the research description with the principal researcher. I have had the opportunity to ask questions about the purposes and procedures regarding this study.

- My participation in the research is voluntary. I may refuse to participate or withdraw from participation at any time without jeopardy to future employment or other entitlements.

- I may be withdrawn from the research at the Principal Researcher's professional discretion.

- If during the course of the study, significant new information that has been developed becomes available that may relate to my willingness to continue to participate, the investigator will provide information to me.

- Any information derived from the research that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.

- If at any time I have any questions regarding the research or my participation, I can contact, Patricia Pernin, who will answer my questions. Her phone number is (310) 612-1892. I may also contact the faculty advisor, Dr. Julie Gainsburg, at 818-677-6155.

- If at any time I have comments or concerns regarding the conduct of the research, or questions about my rights as a research subject, I should contact California State University, Northridge, Institutional Review Board. The phone number for IRB is ____________. I can write to IRB at -----------------------------------.

- Videotaping is part of the research. Only the Principal Researcher will have access to the written and taped materials. Please check:

( ) I consent to be videotaped and audio-taped

My signature means that I agree to participate in this study.

Participant's signature: ___________________________ Date: ___/___/___

Name: (Please Print) ____________________________

Investigator's Verification of Explanation

I, Patricia Pernin, certify that I have carefully explained the purpose of the research to ________________________________ (Participant's name). He/she has had the opportunity to discuss it with me in detail. I have answered all his/her questions and he/she provided the affirmative agreement to participate in the research.

Investigator's signature: ____________________________ Date: ___/___/__

Appendix B

Observation Protocol

Mathematics

Observer: ______________________________ School: ______________________________

Teacher: ______________________________ Grade(s): ______________ Local District ________

Date: ______________________________ Room: ______________

Time start: ______________________________ Time end: ______________________________

Students Present: ______________

Role	Gender	Ethnicity
Teacher		

PURPOSE OF THE LESSON

1. What was the purpose(s) or goal(s) of the lesson? What were the central ideas, skills, or concepts covered in the lesson?

Source: (ask teacher if possible) ☐ Teacher ☐ Observer

DESCRIPTION OF THE LESSON

3. a. What content was addressed in this lesson?

Content ☐ 4th ☐ 5th ☐ 6th ☐ 7th ☐ Alg. 1 ☐ Geom. ☐ Alg. 2

b. Which mathematical strands were covered?

☐ Number and operations ☐ Measurement ☐ Geometry
☐ Data Analysis ☐ Algebra

c. Which lesson activities were covered in this lesson?

☐ Engage ☐ Explore ☐ Explain ☐ Elaborate ☐ Evaluate

4. Activities that occurred: (Briefly describe the nature and range of activities you observed, including any relevant information about lessons surrounding the one observed to provide a larger context for the observation)

LESSON NARRATIVE

Include map of classroom and any narrative notes, noting time & engagement level every 10 minutes, keeping in mind what we are looking for in this observation protocol. The goal of the narrative is to recreate the experience of the students in the classroom.

Circle and shade in the dominant code PER CATEGORY, circle any other relevant codes. Try to limit to 3 codes or less.

Minutes	0-10	11-20	21-30	31 - 40	41-50
Teacher Activity	M GD MGW IP	M GD MGW IP	M GD MGW IP	M GD MGW IP	M GD MGW IP
Student Activity	IP GW NT	IP GW NT	IP GW NT	IP GW NT	IP GW NT
Direction of Discourse	T-only T-S S-S NA	T-only T-S S-S NA	T-only T-S S-S NA	T-only T-S S-S NA	T-only T-S S-S NA
Talk/ Questioning*	GI IT EM P GD LA ET OR EC	GI IT EM P GD LA ET OR EC	GI IT EM P GD LA ET OR EC	GI IT EM P GD LA ET OR EC	GI IT EM P GD LA ET OR EC
Questions					

Teacher Activity

M Modeling

GD Group Discussion

MGW Monitor Group Work

IP independent Practice

Student Activity

IP Independent Practice

GW Group Work

NT Note Taking

Direction of Discourse

T-Only Teacher does all the talking/presenting of ideas with little to no participation of students

T-S Conversations are primarily teacher to student

S-S Conversations are primarily student to student

NA Instruction does not involve oral discourse

Talk/Questioning:

GI Gathering Information, leading students through a method

IT Inserting Terminology

EM Exploring Mathematical Meanings and/or relationships

P Probing, getting kids to explain their thinking

GD Generating Discussion

LA Linking and Applying

ET Extending Thinking

OR Orienting and focusing

EC Establishing Context

Appendix C

Initial Interview Protocol

Thank you (NAME) for agreeing to participate in the interview today. In order for me to make sure that I have recorded all of your responses correctly I will be recording your responses. Is that ok?

The questions and your responses will be used to provide insights into how implementation of the mathematics collaborative workgroup sessions. Should you wish to expand any of the ideas please feel free to do so. All responses will be held confidential.

Before we begin please indicate the subjects you are teaching and the number of years that you have been teaching.

Thank you.

Now to our first question:

1. Thank you for allowing me to observe your classroom. As you reflect on your lesson today:
    a. What do you think went well today?
    b. Why do you think that it went well?
    c. Are there any areas that you think needed improvement?
    d. What might those areas be?

2. Having had collaborative workgroups at your schools, what effect, if any, has collaboration had on your practice?
    a. Can you give me an example of how this has impacted you in the classroom?
    b. What changes have occurred in your teaching practices? What changes do you envision will in the future occur due to this process?
    c. How have these changes that have occurred in teaching affected student achievement?
        i. Classroom performance, student behavior, etc.

3. What effect, if any, has the collaborative workgroup on developing your understanding of the teaching?
    a. Specifically, what are the insights that you may have gained about your teaching? *(Strategies, making connections in mathematics)*
    b. What insights have you gained about mathematics? *(Connections in mathematical ideas, deeper understanding of concepts, etc.)*

4. As you have worked in your collaborative workgroup, are there any areas of professional development that you feel you would benefit from that would help you

to increase your knowledge for developing effective lessons, such as professional development on differentiation or classroom group work?

5. What do you think are the most significant features of a professional development, the features that bring about the results you're looking for?
    a. If not mentioned, probe for:
        i. Do you think it's the teacher collaboration?
        ii. The focus on identifying or developing a student need?
        iii. The opportunity to learn from others?

6. What types of professional development have you experienced other than the collaborative workgroups?
    a. Did these professional development sessions have an effect on your classroom instruction? Why?
    b. Why were some of professional developments sessions more useful than others?

7. Other than stand alone professional development sessions and the collaborative workgroup setting, are there other resources that you use to support your learning as a teacher?

Appendix D

Final Interview Protocol

Thank you (NAME) for agreeing to participate in our second interview today. In order for me to make sure that I have recorded all of your responses correctly I will be recording your responses. Is that ok?

The questions and your responses will be used to provide insights into how implementation of the mathematics collaborative workgroup sessions and the targeted professional development modules is progressing. Should you wish to expand any of the ideas please feel free to do so. All responses will be held confidential.

Now to our first question:

1. Thank you for allowing me to observe your classroom. As you reflect on your lesson today:
    a. What do you think went well today?
    b. Why do you think that it went well?
    c. Are there any areas that you think needed improvement?
    d. What might those areas be?

2. As you reflect upon your class, has the collaborative workgroup and the Targeted Professional Development this year affected your classroom instruction?
    a. If practices have changed:
        i. In particular, how have you changed your practices?
    b. Why do you think these practices have changed?
    c. How have these changes that have occurred in teaching affected student achievement?
    d. If practices have not changed:
        i. Why do you think that professional development had no effect on your classroom?
3. What effect, if any, has the collaborative workgroup sessions had on developing your understanding of the teaching?
    a. Specifically, what are the insights that you may have gained about your teaching? *(Strategies, making connections in mathematics)*
    b. What insights have you gained about mathematics? *(Connections in mathematical ideas, deeper understanding of concepts, etc.)*

4. What effect, if any, have the Targeted Professional Development modules on developing your understanding of teaching?
    a. Specifically, what are some of the insights that you may have gained about their teaching? *(Strategies, making connections in mathematics)*
    b. What insights have you gained about mathematics? *(Connections in mathematical ideas, deeper understanding of concepts, etc.)*

5. Has the collaborative workgroup sessions and the TPD modules shaped the way you plan instruction? Please describe.

6. As you have worked in your collaborative workgroup, is there any other area of professional development outside of the workgroup and TPD that may be of use to enhance your workgroup meetings and instruction?
    a. Instructional strategies?
    b. Mathematical professional development?
    c. Support from others?

7. What do you think are the most significant features of professional development, the features that bring about the results you're looking for?
    a. If not mentioned, probe for:
        i. Do you think it's the teacher collaboration?
        ii. The opportunity to learn from others?
        iii. The TPD?

8. Would you like to share any other thoughts on professional development for teachers of mathematics?

Appendix E

The Journey of Ms. Guzman

Ms. Guzman is a young Hispanic teacher who is in her fifth year of teaching. This year the school opted to move her to the fifth grade so she could continue to instruct her fourth grade students from last year when they moved into fifth grade. Her students are English learners with levels ranging from Level 1 through Level 3 with Level 1 being those students with little if any English language capacity. This being Ms. Guzman's first year with the fifth grade curriculum has proven to be challenging, especially in mathematics. During her first interview she indicated her concern about how she needs to teach mathematics conceptually and not just procedurally.

> When we're doing our [math] concepts I don't know 'why', you just do it ….They [students] are asking a lot of questions but I feel bad because … I don't know 'why'…. What I am doing right now is I'm teaching them but not explaining why.

Not only has she changed grade level but she is also using a newly adopted mathematics textbook. As she indicated in her first interview, the training for the textbook was only one day and was provided almost a year prior to the implementation of the curriculum. She indicated to me that the training only showed the supplemental materials and the other components of the textbook. She was disappointed that she did not have an opportunity to use any of the materials prior to implementation.

Ms. Guzman was concerned about learning how she might teach the mathematics, how she might use alternative methods of teaching the mathematics and how she might learn how to make connections. She felt that her knowledge was not sufficient to provide alternative methods of teaching the mathematics: "I tend to have just one way of teaching…" In addition Ms. Guzman indicated that she does not use manipulatives or does she have the students work collaboratively since there may be a loss of control in the classroom: ”I need to have control.” Ms. Guzman was open to increasing her capacity to teach using various approaches and to deepen her understanding of mathematics.

I was unable to visit her classroom prior to beginning professional development so the first observation occurred after the first Targeted Professional Development (TPD) on fractions. The first TPD centered on finding equivalent fractions through multiplication and division by fractions equivalent to “one” with the teacher and students encasing the fraction equivalent to one in a large 1 figure. When I observed her classroom, Ms. Guzman used this strategy to emphasize the multiplication by a fraction equivalent to one.

Ms. Guzman’s classroom instruction included teacher modeling of the process, followed by student independent practice using a problem similar to problem that was modeled. When students were working, Ms. Guzman walked around the room and monitored their progress and provided support as needed. The problems that were modeled by Ms. Guzman and were practiced by the students comprised of tasks that could be classified as Procedures without Connections (Smith & Stein, 1998). Periodically Ms. Guzman would ask her students why they obtained the answers; however, answers from the students went through the procedures and never addressed why the procedures were used or why they were valid methods.

Her use of appropriate terminology was evident in the class as she repeatedly referred to “equivalent fractions.” The terminology that was evident in Ms. Guzman’s classroom was also consistent with the terminology used in the first TPD preceding the first classroom observation: “now using 42/56, find another equivalent fraction…give me a fraction equivalent to one." However, as Ms. Guzman continued through the problems, she began to refer to the process of multiplying by a fraction equivalent to one as multiplying by "whole number."

Ms. Guzman then proceeded to the operation of reducing fractions. She modeled reducing the fraction 3/6 by dividing the numerator and denominator by the greatest common factor. She did not connect the process to the previous concept of multiplication by a number equivalent to one to obtain an equivalent fraction. Instead, she demonstrated that in order to find the greatest common factor it would be necessary to factor the numerator and denominator. Although there was an intentional effort by Ms. Guzman to push the students for deeper understanding by asking students why they performed the particular operation, the discussion evolved into students explaining the procedures that were used to obtain the answer. Again, the discussion spiraled into a Procedures Without Connections Task (Smith & Stein, 1998). As the period drew to an end, Ms. Guzman was unable to finish the process for reducing of fractions.

During the debriefing of the lesson that occurred after this class, Ms. Guzman indicated that she opted to use the strategy of multiplying by a number equivalent to one by drawing a large one around the fraction, used in the first TPD, when her students were having difficulty finding equivalent fractions. Ms. Guzman said that she "was kind of hesitant to use it [visual of the one] only because I thought it would confuse them a bit but I think that actually helped a little bit more." As the debriefing continued, Ms. Guzman noted that she knew that some of her explanations may not have been correct. In particular, indicating multiplication could be by "a whole number" could have led to a student miscomputation.

> I felt that I started off that way [stating a fraction equivalent to one] … then I was just getting tongue tied when it came to saying the whole phrase, so I just left it as whole numbers but I knew in my head that I was… incorrect. I just didn't know how to [say it].

Ms. Guzman stated she would be going through division the next day and would review the idea of multiplying and dividing by a number equivalent to one. Her openness and honesty revealed her desire to improve her teaching and her ability to reflect on her teaching.

As Ms. Guzman continued to attend the workgroup meetings and the remaining TPDs, her participation increased. She began to ask for more clarification about the work being done in the workgroup and even offered explanations of the work to fellow members of the group that were struggling. After the group agreed upon the common lesson and then implemented the lesson, each participant brought back the student work to a workgroup meeting. They discussed the implementation of the lesson the implications from the student work. The intent of collaboratively developing a lesson within the workgroup was to ensure that all of the participants would teach the lesson similarly. This turned out not to be the case; however, this lack of similarity provided the group an opportunity to compare and contrast some of the outcomes of the lesson based on the differences in implementation.

During the workgroup session, Ms. Guzman indicated that she taught the class the skills of the lesson and then taught the lesson as written. The lesson was intended to be a Task With Connections (Smith & Stein, 1998) and required that students use a ten by ten grid to represent fractions and decimals. Since the procedural process had been taught, the students were not able to make the connections. They immediately used the procedural process taught by Ms. Guzman and were not able to make the connections to the visual representation.

Ms. Guzman:	I would use, I would actually try to incorporate this [the visual representation] into it if I give a decimal, or a fraction they also use…individually, but yeah I would change it.
Researcher:	And that means you would probably put the procedural and the visual more together as opposed to separate?
Ms. Guzman:	Yeah, I need to work on that.

After the discussion about the student work and the knowledge gained by allowing students to make connections, Ms. Guzman realized that the next time she would allow her students to explore and would not lead them through the procedural process first.

The second observation found Ms. Guzman venturing out of her comfort zone and allowing her students to collaborate and explore. The lesson included an increased amount of time on student collaboration and the use of a task that could be classified as a Procedures with Connections (Smith & Stein, 1998). Prior to the observation I went to see her to arrange when I might meet with her. She told me that the students in the classroom did not understand how to change an improper fraction to a mixed expression although she taught them a procedural method with explicit steps. She also indicated that her students appeared to be confused and overwhelmed by the procedures, and when they had to use these procedures after they had added fractions if the answer was an improper fraction, they were unable to add these procedures when necessary. I suggested she go back to asking the students to represent, pictorially, the improper fraction and then to build on their knowledge base of what constitutes a "whole" for a given improper fraction. This targeted professional development specific to Ms. Guzman's request was intended to build her capacity to understand how to connect the visual representation with the algorithmic representation.

Ms. Guzman went home and developed a lesson that I observed the next day. She required the students to first draw a representation of the improper fraction using circles or any other shape that the student chose, change the improper fraction to a mixed expression and then explain the connection between the visual representation and the mixed expression. The first example was changing 3/2 to a mixed expression. A student came to the board and drew 2 circles, divided each circle into two parts to indicate the idea of halves, and then shaded in 3 halves indicating the fraction 3/2. Under the first circle the student wrote 2/2 and under the second circle 1/2, and wrote positive sign (+) between the two fractions, wrote and equal sign (=), followed by 3/2 followed by the number 1 ½. When asked by the teacher "Explain how you got that answer" the student explicitly explained the process using the pictures.

The students continued their explanations of the relationship between the drawing and the numeric representation. Although this proved to be challenging for some of the students, Ms. Guzman persevered. Her questioning included asking: "how can you show a whole," "explain what you did using the pictures" and "where did you get the one whole from in the picture?" As the class progressed the students were permitted to discuss what they were doing with other students, although Ms. Guzman did not explicitly indicate that they should work in pairs. This interaction was not allowed during the first lesson. All of the work that was done in the TPD and workgroups emphasized the need to have students collaborate when doing their work and the need to talk about the mathematics could increase their understanding. The relaxing of the "quiet mode" in Ms. Guzman's class was evident.

Each time that Ms. Guzman asked a student to demonstrate a given problem on the whiteboard at the front of the classroom; the student was to clearly articulate the process and to connect the process to the visualization of the problem. Although initially the students had some difficulty explaining their reasoning and making connections to the visualization, there was an increase in understanding as the class progressed. Ms. Guzman still had some reservations and her anxiety about providing a lesson that was not as operational was evident. When one student came to the board and wrote: 2/3 + 4/3 = 6/3 = 2 0/3. Ms. Guzman initially indicated that it was not correct since it was not written properly due to the 0/3. During our final debriefing of the lesson, the student's response and how she may have handled the answer was discussed. This example illustrated Ms. Guzman's lack of confidence in explaining answers that may be atypical.

This uneasiness with the changing practices in the classroom was further verified in the final interview. Ms. Guzman wanted to make sure that her students understand the concepts behind their work; however, the insecurity of the math knowledge was evident when she said,

> …on Friday we were doing a math problem and …I tried to change it and ended up not only confusing them, but then confusing myself….I had to redo the problem myself in order to be able to explain it. So, when they were explaining their different variations of how to solve their problem, I felt like a student where I'm like. Oh, show me how you did that, 'cause …I didn't understand how they arrived to that answer.

Her insecurity about how to teach the concepts in the classroom and to allow for atypical solutions continued to plague her. She indicated that she knew the answers were correct but then was torn between how to explain the answer, since it would not be seen in a test and she did not want to confuse the students. Again Ms. Guzman reiterated that she "didn't feel too confident in the subject matter". She continued to say that she "can teach the kids the process of it, but [I] struggle with teaching them the why portion of it." When pressed on the effect of the TPD and the effect on her practice Ms. Guzman said:

> It did [affect her practice]. Well, again, it goes back to that - having that discussion with the kids in the sense of like asking them questions of why, for example, like when Delores (sic) wrote that fraction, is having them elaborate more on their process, It's something that I really didn't do. It's like, okay, let's move on as opposed to try and to see if they fully understand the concept. I think it goes beyond student achievement; it's just mechanical everyone can eventually achieve that, but do they truly understand what they are doing. That's I guess, the goal that I as a teacher want to accomplish, I mean, I don't think I will accomplish it 100% this year, but I know that I'm gonna work more towards accomplishing that goal.

As indicated in Table 4.1, Ms. Guzman's questioning in the area of Exploring Mathematical Meanings and/or Relationships and Generating Discussion increased and Inserting Terminology, Gathering Information and Probing continued to be evident in each lesson (Boaler and Brodie, 2004). It should be noted that Ms. Guzman indicated that the TPD and the workgroups "definitely changed my way I see math to a certain degree." When asked, Ms. Guzman indicated that she would like to continue to build her capacity in math so that she would be able to build her capacity to teach mathematics.

As also noted in Table 4.2, the Task Analysis Guide, (Smith & Stein, 1998), her second lesson incorporated a task that was a higher level of demand and could be considered Procedures with Connections. Students were required to connect the visual representation to the procedural method and to explicitly verbalize their thinking. The implementation of asking the students to relate their thinking was difficult and the teacher did experience some frustration with the process as she reiterated after the lesson the difficulty of doing the lesson as well as her insecurity in the mathematics; however, the interest remained and she wanted to continue to try various methods of presenting the material.

Appendix F

The Journey of Ms. Becker

Ms. Becker is a Caucasian woman who is in her fifth year of teaching fifth grade. Teaching is a second career for Ms. Becker. Ms. Becker is very enthusiastic with a great deal of energy and this energy is transferred into her classroom. During both observations, students had numerous opportunities to interact and share their thoughts, although the student sharing in the first observation was not as mathematically driven as it was in the second observation. During both observations she promoted a classroom environment that was interactive and, unlike Anna's class, had a higher level of noise generated by student interaction. Ms. Becker monitored the students understanding during both observations.

Ms. Becker grew up in the South and when she came to California to teach she felt she needed to adjust her method of teaching to accommodate the English Learner. Ms. Becker felt that mathematics was easy for her; however, she felt that it was difficult for her to teach as indicated in her first interview:

> I'm really good in math, not as good at teaching it. And that's odd because I thought because I know math so well that I thought that would be my strong subject to teach, and when I started doing this group stuff [teacher workgroup] and we started talking, oh how am I going to teach that? How am I going to teach that? Oh, wait. Okay, maybe being good in math doesn't necessarily mean you're good in teaching math.

During the first classroom observation that occurred after our first TPD session, Ms. Becker was teaching a lesson on place value. In particular, she was teaching students how to "move the decimal point" to the left, adding zeros, when multiplying by a 0, 10, 100, 1000, etc. At first the students were asked to clap to indicate the number of zeros added when multiplying. When she asked "how many zeros are added if 70 were multiplied by 1" they were not to clap. The confusion occurred due to the existing zero in 70 so some of the students clapped once. During this time the students were standing. As the lesson progressed, the students were required to move the appropriate number of steps signifying the number of places the decimal point would move. This created even more confusion since some students were having difficulty with moving left and right, possibly due to a confusion of left and right. Ms. Becker felt that the movement would be helpful since she noted that many of her students were EL students. She indicated that the EL student had "language issues" and she decided to "put motion in, movement in to help kind of cement words that I want them to remember." In addition she "had them repeat the process of moving over and over again." This method of teaching was very procedural. The tasks that she asked her students to complete were more memorization where students were asked to recite the procedure, then move either left or right, following the teacher's directions. The tasks that Ms. Becker asked her students to complete were Memorization Tasks (Smith & Stein, 1999). Ms. Becker never asked students for any reasoning for the memorized tasks. As Ms. Becker proceeded with the lesson the students continued to move left and right depending upon the problem that Ms. Becker recited: "Seventy times 100." Most of the students followed the directions.

Ms. Becker never explained why the students moved the decimal point. During the debriefing of the first lesson, Ms. Becker was asked to reflect how she might improve her lesson. She thought about the question and then answered:

> How to explain why we move the decimal. If that's what they…I didn't want to go there today 'cause I didn't want to go into explanation, I really wanted then to see the [pause], but I wonder if understanding why would help them do that movement?

Her hesitation to include an explanation of the “why’ proved to be a dilemma for Ms. Becker and the realization that perhaps explaining the reason why they did the movement, may have provided more understanding for the students.

Ms. Becker decided to use movement in her lesson after she collaborated with a group of teachers the previous year and found that they used more kinesthetic activities in their teaching. Her openness to learning was evident in all of the sessions that she attended. When asked during her first interview about her effectiveness as a teacher Ms. Becker explained:

> Probably… I think about cheese. I think of Swiss cheese. I’ve got a lot of holes that … if you were to skim over the surface it just, it looks like it’s a whole piece and it’s fine, then … you, get a little more detail, when you go from concept to concept to concept then you realize, oh well maybe I don’t really have that as well as I do this. Like today with this lesson, I teach this, this is my fifth year and this is the first year I had kids struggle with the understanding of the decimal. Even though I’m doing more with them, so now I’m finding there a pocket there, a hole I need to fix.

Ms. Becker tended to see professional development as a lesson that could be brought back to the classroom immediately. She felt that the lesson should emphasize a set of procedures that she could demonstrate, step by step. Ms. Becker was always open to understanding more about teaching students’ mathematics:

> Even for as much as I know in math, I think that if I knew more, I think it would give me … that the direction I want to push them [students] to. I want to understand their learning this because this is where it’s going to take them to the next step.

As Ms. Becker progressed through the TPD and the workgroup meetings she actively participated to increase her understanding of mathematics. When she was challenged to learn new and different ways of teaching the mathematics that included more kinesthetic approaches, she was anxious to see how this would transfer into the classroom.

After the workgroup agreed upon the common lesson and then implemented the lesson, each participant brought back the student work to a workgroup meeting. They discussed the implementation of the lesson and the data they gathered from their student work. As noted before the intent of collaboratively developing a lesson within the workgroup was to ensure that all of the participants would teach the lesson similarly; however, the lack of similarity provided the group an opportunity to compare and contrast some of the outcomes of the lesson based on the differences due to the implementation. When Ms. Becker reported on the findings for her students she did not pre-teach the lesson as Ms. Guzman had done. Instead she allowed the students to do the problems one at a time. She noted that when they shaded the representation of the decimal on the 10 by 10 grid they were able to shade correctly. For the 1/4 shading many of her students counted three squares and then shaded the fourth.

As an extension of the lesson, Ms. Becker's students turned over their paper and divided the fractions. Ms. Becker explained that through this extension the students were able to make the connection between a fraction and the fraction's decimal representation. As Ms. Becker stated:

> We just did them one piece at a time. 2/100's, they knew that. 2 squares. .02 (point zero2) is 2/100's. So again they are working out the wording. They did it. We went through all four of the set then I had them turn it [their paper] over and [turn] the fractions into division problem and when they did that, then they're like oh it looks the same here and it equates the same here. Oh, we can use division to find the decimal because I had not taught them that yet, we hadn't gone over it, so they don't know, we'd not gone over how they can change a fraction to a decimal. So doing this and following up with this saved me having to teach them a lesson, well, I'm still going to teach them, but I mean it just gave me a heads up to teach them the decimal equivalents.

Ms. Becker's curiosity was further increased when she began to consider why the students counted three squares (1, 2, and 3) as noted by the marks in each box and then shaded the fourth when they represented the fraction one-fourth. When she was asked if she had questioned the students about their reasoning she responded: "I should have went back on that one since I saw them do that. That was my mistake….I'm curious now.'

The second observation included a lesson that had been demonstrated during the third TPD session. The emphasis was still on fractions. Each teacher was given ten (10) equal strips of paper, 8½ inches long by 1 inch. Each teacher folded one strip into each of the following partitions: halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, and twelfths and was asked to note how they folded the paper. A discussion followed that emphasized the connections between the partitions leading to a discussion about the fractions. The lesson was an adaptation of a Connected Mathematics lesson.

Ms. Becker was so interested in the lesson she decided to try the lesson with her students. The lesson was implemented two days before the students and Ms. Becker went off-track. Ms. Becker placed the students into groups of three requiring the students to work collaboratively. Each group was given a set of strips that they folded into halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, and twelfths. Since the class was shorter than normal, students worked together, with every person folding at least one strip. By allowing the students to work in groups, Ms. Becker was able to move into a discussion of the relationships between the partitions formed in the folding. Throughout the folding process, Ms. Becker continually monitored their progress. She also asked students to provide insight into how they folded the strips when she noticed that some folds were more difficult than others; in particular, thirds and fifths.

After all groups had finished the folding Ms. Becker began to ask questions about the partitions and the relationships between the partitions. She continually asked the students to relate the fraction strips to their explanations and to the concept that the denominators 2, 3, 4 and 6 were factors of 12 and that these strips could be folded to obtain the twelfths. Once the students folded the strips, Ms. Becker began with the twelfth strip and asked questions about the relationship to the other strips: "…How can we show how we can look at this [referring to the half strip} and get to six [equal partitions]?" Students told her how to fold the paper and she followed their directions. Then she asked "What fraction strip could I partition to make the one twelfth?" After the students answered the question she then asked what their reasoning was. This continued throughout the lesson until the math lesson was over after 45 minutes.

As also noted in Table 4.2, the Task Analysis Guide, (Smith & Stein, 1998), Ms. Becker's second lesson incorporated a task that was a higher level of demand and could be considered Procedures with Connections. Students were required to connect the visual representation to the procedural method and to explicitly verbalize their thinking. During the debriefing of the lesson in the second interview, Ms. Becker indicated that the implementation of asking the students to relate their thinking was difficult. She said that she experienced some frustration with the process, noting the difficulty of doing the lesson as well as her insecurity in the mathematics; however, she wanted to continue to try various methods of presenting the material even if there was some insecurity when doing the lessons.

During the debriefing Ms. Becker felt that the students really seemed to understand the concept. When asked why she thought they understood the problems and that they were gaining additional knowledge that she did not expect.

> From hearing what some of the things they were saying, you know, one-fifth, half of one-fifth is one tenth you know, using the language. So that was telling me, okay, they're getting the idea that … I think the one think amazing was that they were breaking a one-fifth in half and knowin' that they were gettin' a smaller fraction even though the number was bigger the denominator was bigger.

Ms. Becker continued to focus on the findings that the students were making in the lesson and was disappointed that she was not able to finish the lesson prior to the class going off-track. She then decided that she would revisit the concepts when they came back in six weeks and determined that this lesson would be a good review.

After the debriefing of the lesson Ms. Becker reflected on the TPD and the workgroup sessions during the second interview. In particular she reflected on the task that required her to shade in 6 blocks out of forty blocks and to represent the number of shaded blocks to the total number of blocks as a percent without dividing. She was still puzzled by the fact that she knew math well but had difficulty with the problem.

> I think I know math so well and then we were doin a shading activity and I couldn't wrap my head - I think it was when the 40 blocks and I couldn't take it to that 100 blocks and I understand it logically why you would do that , but I couldn't see it.

Then Ms. Becker explained that the TPD increase her knowledge and that new knowledge affected her teaching:

> Because I find even with the fractions, doing this fraction activity, I find that I was teaching it better to my students, so for me … what I'm trying to say is that the more training I have on a particular concept and going deep into that concept, you know, thinking about how to teach it, what kinda questions are gonna come up.

> Makin it rigorous enough that we challenge the students without being so difficult, havin the results, bringin it back, discussin the results and takin it back and goin to another – the next step. All those pieces help me to be a better teacher with that concept. Now to say that, that means that I would probably need to have training in each of the different classes I teach, you know, to be a very effective teacher, but I just realized that.

Ms. Becker indicated that she would like to continue with the TPD and she would like to continue with the workgroup sessions and especially with the analyzing the student work. She felt that the missing piece in all of her previous professional development was reflecting on the effectiveness of the strategies and lessons brought back to the classroom. Many times she felt that not meeting to discuss how effective the lesson was based on student work was in effect the missing piece. By looking at the student work Ms. Becker felt that

> It helped me to understand when I didn't under – bringing it back [student work to the group] and talking with other teachers helped me to understand parts when I didn't understand why my kids were doin this, why - why …I could say, you know, guys I've for this goin on, I don't understand why. I was able to get feedback from the other teachers and then it was like oh, okay.

Through analyzing the student work and through Ms. Becker's probing for deeper understanding from her students, she began to realize that she tended to "assume more [student learning occurring] than they actually [could] show me …Yeah, I found that maybe I was doin a lot of assumptions with my students."

As indicated in Table 4.1, Ms. Becker's questioning in the area of Exploring Mathematical Meanings and/or Relationships, Probing, and Generating Discussion increased and Inserting Terminology and Gathering Information continued to be evident in each lesson (Boaler and Brodie, 2004). As noted in Table 4.2, the Task Analysis Guide, (Smith & Stein, 1998), Ms. Becker's second lesson incorporated a task that was a higher level of demand and could be considered Procedures with Connections. Students were required to connect the visual representation to the procedural method and to explicitly verbalize their thinking. Ms. Becker wants to continue to work collaboratively and to learn more about mathematics and indicated that she would like to continue to attend more TPD and workgroup meetings in the spring.

Appendix G

The Journey of Mr. Palmer

Mr. Palmer is a Caucasian teacher who is in his ninth year of teaching and taught sixth grade mathematics for three years, previously taught science and a combination of English, ESL and history. Mr. Palmer previously taught in a middle school in the same local district. The change to the K-6 configuration at a year round school was a difficult transition for Mr. Palmer. He was an active participant in the meetings and did not worry about asking questions when he was confused. Mr. Palmer did finally implement the lesson; however, it was approximately one month after the intended implementation date due to his feeling of being overwhelmed with the new job requirements and his insecurity in implementing the lesson. At one point Mr. Palmer thought that he might need to drop out of the study, but decided to continue after we discussed his anxiety. After the three months, Mr. Palmer began to adjust to the elementary school setting and to embrace the changes that he went through during the first three months.

The first observation, occurred after our first TPD session, and one workgroup session, Mr. Palmer was reviewing material with his students. His intent was to review the work he had previously done with the students. He provided the students with five problems that could be characterized as Memorization Tasks that involved using procedures that were previously covered in class and did not require the students to make connections to the concepts or meaning that underlie the facts (Smith & Stein, 1998). The problems involved order of operation and finding the Greatest Common Divisor and Least Common Multiple. Throughout the lesson Mr. Palmer asked the students to focus and to quiet down. The entire math period of one hour twenty-four minutes was spent on the five problems: $3^3 + 5(2+3)$; 1/3 x 10; Simplify 45/100; Find the GCD: 6, 8, and 12; Find the LCM of ½, 2/5, 1/10.

During the lesson Mr. Palmer would ask questions such as: "Ok, let me take a step back, when we add, subtract or compare fractions, you usually do what first...you have to have what first? Is that also true with multiplication and division?" These low level cognitively demanding questions were indicative of all questioning during the period. Mr. Palmer asked why the operations were used; however, the responses that were accepted were procedural and did not provide the concepts behind the operation. Normally the instruction was led by Mr. Palmer and he also provided the steps that were to be used for each problem. Mr. Palmer tended to do all of the modeling and only asked questions that required a single word answer. There was no in-depth analysis of the process used. When students came to the board to write down the solution, Mr. Palmer usually explained the work.

When the students did not know how to change 10/3 to a mixed expression Mr. Palmer went through a very long story that was to help them remember how to set up the division so they could change the improper fraction to a mixed expression. The story was to help them change the 10/3 into the division problem. This story was about a cowboy (dividend) riding a horse (divisor) and the horse (divisor) stays outside and the Cowboy (dividend) goes inside the house (under the division symbol). He did not insert any mathematical terminology such as numerator, denominator, dividend, divisor or quotient but pointed and drew where the numbers would be placed. After this explanation, there were still students that did not really understand how this related to dividing 10 by 3.

During the first interview when we debriefed the lesson, Mr. Palmer indicated that he thought the lesson went well because “the fact that by and large I felt that I was able to keep my kids engaged.” When asked why it went well he indicated that it was the “Combination of them (students), their interest in wanting to learn this, the level of students and again, maybe this is wishful thinking, maybe a little bit was me.” This seemed to be different than observed as the review of 6 problems seemed to take entirely too long for material that had previously been taught. I observed some students off task and some students still able to solve the problems. When asked what he might improve he indicated that he would like his lesson to be more organized so that it would flow.

> Stay a bit more focused…They [the students] have a tendency sometimes to ask questions that are not related to the subjects, they kind of interject other outside things. I, from my point of view would like to see myself conduct a lesson like that more seamlessly, maybe plan it out a little bit better.

Mr. Palmer also said that he tended to teach the students by ‘basically following a sequence of instructions” and the instructions needed to be “embedded in their minds….so they know - do one thing before another.” The intent of the lesson observed was to review the work that they had previously covered; however, the results of the lesson indicated that the students did not understand all of the work.

Mr. Palmer indicated during the first interview that when he worked with other teachers he learned from them but was unable to articulate what he learned and how he used the information he learned in his classroom. He hoped that by participating in the study he would be able to learn even more about teaching mathematics. His said that his teaching practices had evolved over the years.

> I used to think teaching, and this is really important for me personally, was a linear process in which you start here and you do this and you build on this, But I’m learning that it’s not. It’s. I don’t know the right term to use but it’s more

> holographic. It's more experiential. I have seen teachers teach and their lesson are not completely sequenced. They go in and out of different directions depending on the need of the students at the time, it almost like organized chaos.

Mr. Palmer realized that he taught mathematics "basically through rote work"; however, he was "learning to see math in a more rational way." He indicated that he is now beginning to see that the 'whys' and that the procedures need to be included. He was anxious to learn how he could combine these two aspects of mathematics, concepts and procedures, in his teaching.

As Mr. Palmer continued to attend the TPD sessions and the workgroup sessions he began to understand the mathematical reasoning behind the material that he was teaching. He was very open to exploring new methods of teaching mathematics; however, there were times that he was unsure how he would implement the lessons in the classroom. In particular, Mr. Palmer did not implement the collaboratively developed lesson when the other workgroup members did. When asked if he would implement the lesson he said that he would but he was still unsure about how he would implement the lesson.

In the workgroup meeting, the other teachers then went through how they implemented the lesson and he began to feel a bit more confident. I also agreed to meet with him prior to the implementation of the lesson to go over each part with him and provide more one-on-one assistance. After he implemented the lesson he was quite happy about how the students performed. He indicated that he was surprised how engaged they were in the activities. He also told the workgroup that he thought they all "got it" except for the problem 25 divided by 50. He noted that the students were "actually interacting together" since they had to work in groups. He was very excited about the students working together and learning and doing all of the work. When asked what his next steps in the lesson he indicated that he will have the students explain what they did, choosing students who did the problems in different ways.

The lesson covered during Mr. Palmer's second observation was based on defining a ratio, writing the ratio as a fraction, simplifying the ratio and then moving into a discussion of equivalency of ratios and the unit rate. There was more group discussion and students were asked to explain their reasoning when they shared their answers. The teacher spent time at the beginning of the period gathering information and inserting terminology and modeling the process. Students were required to explain their thinking during this lesson using appropriate terminology and mathematical concepts. This process was different than what had occurred during the first observation when the lesson was clearly more procedural. Mr. Palmer used such words as confirm, compare, show you work and explain in his questions to the students. He continually probed for greater understanding. He made a concerted effort to tie all of the material in the lesson together.

Mr. Palmer said that he felt the lesson went well. He referred to the different strategies that he used and the impact of students working collaboratively as well as the student engagement. He indicated that

> The kids engaged. I think the examples I used and things that I said were not over-wordy, but specific. I did use – have them do interactive stuff on the white board and they would talk to each other as a class so I think – I like to think that helped them stay engaged.

When asked about what he might change in the lesson he reflected and said that he would organize and manage the time better when implementing the lesson,

> I think there are – even though they did a lotta talking, it wasn't all me, it was interactive, I think I should have stopped and had em 10 or 15 minutes of practice

> to get the concept gelled. In other words, I should have chunked it a little bit better. So I sort of ran on the direct lesson a little too much.

This reflection was thoughtful and was what I had observed in the lesson. Considering where he started, this lesson provided a dramatic change from his first lesson where he did all of the talking and inserted very little terminology. He decided in this lesson to change his approach and in his second interview he said that he was trying to have the students do more of the talking and discussing so they could be part of the lesson.

> So, I'm trying to cut out the blah, blah, blah from me and have them actually be part of it [the lesson] so they'll, they won't, their minds won't be diverted into other things, but it's [using visuals] not infantile, but even if it was, there's an advantage there to keep em engaged.

When asked about the effect of the TPD and the workgroup sessions, Mr. Palmer thought that the combination of the sessions helped him to see how he could change his teaching practices.

> Well the professional developments we did and the lessons that we did after we did them created a cohesion that I was able to put into my classroom, you know, a little bit at a time. In other words I think it did improve my teaching.

Mr. Palmer further explained that learning math in the TPD allowed him to examine different methods of accessing the mathematics. He understood that although he could do the mathematics, he now understood that there are other ways to do mathematics related to the many different ways a camera in a movie can readjust to look at the scene in different ways but it is still the same scene.

> You know, I learned mathematics in a…way, I mean, very mechanical and when you stop and think about it you realize there's a little bit different perspectives you can look at it. It's kinda like if you shoot a movie scene and you're watching

> it, you never realize that they're shooting it from a perspective like you can have a camera in the scene readjusted to look at the same thing in different ways and I think that math is more of that than you realize. It's not just do this, do that, do that. There are many – it's more holistic, holographic sort of, that I began to think of math as….Look I am not a mathematician, but I'm beginning to see math differently.

As indicated in Table 4.1, Mr. Palmer's questioning in the area of Exploring Mathematical Meanings and/or Relationships, Probing and Inserting Terminology were evident in the second lesson and Gathering Information continued to be evident in each lesson (Boaler and Brodie, 2004). As also noted in Table 4.2, the Task Analysis Guide, (Smith & Stein, 1998), his second lesson incorporated a task that was a higher level of demand and could be considered Procedures with Connections. Students were required explicitly verbalize their thinking and reasoning while his first lesson incorporated tasks that were Procedures without Connections (Smith & Stein, 1998).

Mr. Palmer has continued to attend professional development I am providing at the school and is continuing to hone his skills as a mathematics teacher.

Appendix H

The Journey of Ms. Talika

Ms.Talika is an African American teacher in her eleventh year of teaching. She has taught fifth grade for the past four years and previously taught first and second grade. Her first classroom observation occurred after the first TPD. During this observation Ms.Talika tended to ask low cognitively demanding questions that required single work answers. She spoke very quickly and many times left very little time wait time for students to respond. The classroom was somewhat chaotic and noisy and many of the students were not on task as I observed them doing other work or simply talking to their friends on subjects unrelated to mathematics. Answers to Ms.Talika's questions were yelled out by a few of the students.

The first observed lesson was focused on multiplication of two digit numbers (1.5 X 1.7). Ms.Talika used the smart board to model the multiplication process. The modeling lacked interaction and the Smart Board was used as a tool similar to using a PowerPoint. All of the steps were written out prior to the lesson and displayed on the Smart board. Mathematical terminology was limited and tended to be informal. Since the problems were displayed on the whiteboard, Ms. Talika could just point to each level of multiplication and lead the students through the process.

As she showed the multiplication she instructed them to multiply as if the number were whole numbers. After she finished the multiplication she then asked the class, "Are we finished?" and the students responded "no." Then she proceeded to ask the students to count the "points" and later would ask the students to count the "factors after the decimal point," an indication of her misconceptions of the terminology in mathematics.

Even when she asked students to read the decimal 1.812 a student said "1 point eight hundred twelve thousandths" indicating that the word "point" was used in lieu of the "and" normally used for decimals.

There was no collaboration between the students focused on the mathematics. The lesson was whole group and teacher directed. The direction of the discourse was Teacher-Student. The task could be categorized as *Memorization* (Smith & Stein, 1998) as evidenced by procedural steps that were reinforced by the teacher consistently throughout the lesson. When students were required to do their problems they were guided through the steps that were listed on the Smart Board. When Ms.Talika asked "What does one point five hours mean?" the students responded "one hour and five minutes" and "one hour and thirty minutes." Ms. Talika did not explain why the first answer was incorrect, and merely accepted the correct answer. This was a missed opportunity to explain the student's misconception.

Ms. Talika walked the students through the same problem again noting how to count the number of places to the right of the decimal point and how to move the decimal point once the multiplication was completed. Ms. Talika then decided to use a problem related to going to the market: "Imagine I am going to Sprouts to get some peanuts." Ms.Talika continued to explain that she was going to buy 3.02 pounds of peanuts and would then multiply the 3.02 times the cost per pound. When she wrote the problem she used .6 times 3.02. She then asked "What is the value of the decimal (referring to the 0.6) and number of students responded "6 tenths cents" to which she responded "No, 60 cents" and then moved on with the lesson. Again, there was no clarification of the misconception. Again, she dismissed the answer that she considered incorrect and cited the correct answer.

After the students tried to work out a problem she asked for volunteers to come to the board to write up their problems. After they began to write the problem on the board, Ms.Talika continued to explain what the students had written "So we start at the back and count." After this explanation the students went back to do some independent work at their seats. The students that needed help came up to Ms.Talika's area in the front of the room and waited in line to receive some help. When the line grew a bit too long, Ms.Talika decided to do more work at the board and explain the problems to the students.

When the students were required to work independently, Ms.Talika did not monitor the students' progress by roaming around the room. The math terminology used during the lesson included decimal point and multiplication. When referring to the zeros that were placed in the multiplication process of multi-digit problems, Ms. Talika referred to the zeros as placeholders. When students needed to determine the placement of the decimal point they were told to move with no mathematical reason for the "move."

When we debriefed the lesson, during the first interview, Ms.Talika indicated that she was building on the "skill" of multiplication by using the same procedure she taught for whole numbers. She also felt that she "explained it clearly" but then said "I tried to explain it as clearly as I could." When asked what she might change in the lesson she has a concern that she did not understand the mathematics behind the multiplication algorithm and in particular the decimal representation.

> Maybe for me knowing really why, because I still really don't know why, when you multiply decimals why you don't align [the decimals points as in addition] – I couldn't really explain that to them….about the fact that it was commutative.

When asked about any collaborative work that she had done at her school, Ms.Talika said that she had collaborated with other teachers. The collaboration appeared to be centered on developing what would be taught out of the book and what the pacing would be for the work they would be doing. When asked how this impacted her classroom instruction, she said that she would need to think about that since she had "never even reflected on that." The only other mathematics training that Ms.Talika experienced was the training that Ms. Guzman had with the new textbook adoption. Ms.Talika also noted that the textbook training was intended to show you "how to use the program." Throughout the first interview, Ms.Talika talked about the lack of time to cover the material and to integrate activities into the lessons. In particular, there was a resistance to using groups in the classroom since she felt that it was too time consuming. Her idea of teaching was modeling how the problems were solved.

Ms.Talika said that she felt her math skills were limited and she wanted to be able to explain the math to the students. She decided to participate in the group because she wanted "to learn different ways of doing different types of problems…so [I] can be more apt at what I'm doing in math." During the past four years of teaching math at the fifth grade level, Ms.Talika felt that she began to make more connections in the mathematics. She wanted to continue to make the connections and to become more knowledgeable in mathematics. As Ms.Talika continued to attend the TPD and workgroup meetings she continued to learn more about mathematics. She was an active participant and would ask questions whenever she was unsure about the work we were doing.

After the group agreed upon the common lesson and then implemented the lesson, each participant brought back the student work to a workgroup meeting. The intent of collaboratively developing a lesson within the workgroup was to ensure that all of the participants would teach the lesson similarly. Ms.Talika had actually covered the lesson from the book during the previous week prior to implementation of the collaboratively developed lesson indicating that she thought that she needed to do the sections from the book prior to the lesson.

Ms. Talika said that when she introduced the grid and she asked the students to note what they saw about the grid. She said that the students said there were 100 boxes per grid. She was surprised when some of her students indicated that the grid also represented tenths, with each column representing one-tenth. She had not taught that relationship prior to implementation. After a short introduction Ms.Talika allowed her students to work on the problems and she walked around the room and monitored their progress. Ms.Talika reported that they seemed to understand the work although one student asked a question about 25/50 and she was unsure on how she might address that question.

> Overall I think mine, it was, the issue was it was like Ms. Morris, 25 out of 50. And I'm like okay, yes. So we talk about parts of a whole and I said what's the whole and they said well 50, I said well why don't you just do 25 and she said yeah, and another girl she said well I'll just do half and I said why did you do half and she said well because 25 over 50 equals ½ so I figured since we talked about equivalent fractions she said, isn't that the same thing. I said it is the same thing, I said if you did 25 out of 50 what would that represent out of the whole 100, so some kids got that because that's really ¼ but ¼ between 25 over 50 they're not equal. So I thought that was kind of, I didn't know how to really explain that. I, but I told her when she did her, when she shaded half, it was true because 25 over 50 equals 1/2.

When Ms.Talika examined the effect of teaching the lesson from the textbook prior to this lesson she indicated that she thought that it may have helped but then decided that the students really did not understand the connection between equivalent decimals and fractions until the students experienced the grid representation.

> I think it did [helped the lesson] but I think at the same time getting them to understand really that a decimal is the same as it's fraction form they weren't that clear on it until today because they started to notice when I say 2/10's and, I'm sorry 2/100's [decimal form] and 2/100's [fractional form] that they were the same and a little girl and a couple kids oh, they're the same. Look they're the same. They're just going different ways and I was like, I was trying to ignore so I don't think they got that last week that they're synonymous with one another.

The idea of parts and whole was covered in great depth during the TPD and what the "whole" represented, especially in a fraction. Ms.Talika said that she used questioning more frequently with her students as she recounted what she did in the lesson. She indicated that if her students were able to use multiple representations and able to make the connections, this could provide more understanding of the mathematics.

> …I feel like if they are able to manipulate and count and do all that different, that means they really know it because they realize that, we were talking about this being a ¼ they know how to do the fraction parts but to just understand that they can design and make different things it doesn't matter how it looks it all equals the same. Like it is all out of a whole so no matter how the picture looks, … 25 parts out of 100, they can do it any kind of way and it pretty much becomes their own because they really know how to do it.

During the second observation, Ms.Talika was getting ready to go off-track for six weeks. Prior to the observation, Ms. Talika asked if she needed to do the second lesson that we had developed. I indicated that I would be observing her and the lesson was her choice as was the case with all of the observations. Ms.Talika's lesson focused on adding and subtracting fractions with like denominators. She illustrated the process by drawing fraction bars. The examples illustrating the process were directly from the book. Ms.Talika did all of the talking during the first 20 minutes, modeling all of the work. She asked more probing questions of the students after the initial modeling and demonstration. There was a continual reference of part to whole and why all of the pieces were equal. Insertion of terminology was explicit and included denominator, factor, divisibility, prime and improper fractions. The lesson included drawings of the fractions and not just procedural methods of adding like fractions. When she asked the students why the fraction of 8/9 could not be simplified any further, the students eventually arrived at the answer that 8 and 9 do not have any shared factors other than 1.

Although Ms.Talika did ask more questions of her students than in her first observation, she still tended to rush through the procedures and tended to answer the questions for the students. Ms. Talika did tend to monitor the progress of the students as they worked on problems at their seat by walking around the room. There was some insertion of terminology and some probing; however most of the class talk was directed toward gathering information and leading students through a process. The tasks used in the class were Procedures Without Connections The tasks completed by the students required no explanations and the focus was solely on describing the procedures that were used (Smith and Stein, 1998).

During the second interview and the debriefing of the lesson, Ms.Talika felt that she was strategic in her lesson by using example problems with the students that were relevant and that used some type of visualization. She felt that approximately 85 % of the students understood the lesson. The lesson was to be a review of previously learned material and included revisiting the numerator and denominators and the procedure for adding fractions with like denominators. Since this was a review, the indication that all students did understand all of the work was not clear as I observed the work.

Ms.Talika felt that the TPD did have an impact on her teaching since she now was able to use various strategies and "It [TPD] has allowed me to use various strategies…It has allowed me to show them [students] different ways of doing things…it made it more interesting for me." Ms.Talika said that the TPD and the workgroup meetings were productive; however, she did not think that she would have time to continue in the spring since she wanted to work on her National Board Certification.

As indicated in Table 4.1 Ms. Talika's questioning in the area Inserting Terminology was utilized in the second lesson and Gathering Information continued to be evident in both lessons (Boaler and Brodie, 2004). Ms. Talika did not seem to consciously reflect on her practices as consistently as Ms. Guzman, Ms. Becker or Mr. Palmer. Her interest was geared toward "getting through the material and not toward making sure that the material was understood and internalized by the students.

As noted in Table 4.2, the Task Analysis Guide, (Stein, et, al), her second lesson incorporated a task that was a higher level of demand, Procedures with Connections (Smith & Stein, 1998); however, on the continuum of cognitively demanding tasks it was still relatively low. Students were not required to connect the visual representation to the procedural method and were not explicitly required to verbalize their thinking.

Appendix I

The Journey of Mr. Juarez

Mr. Juarez is a Hispanic teacher who taught fifth grade for the past seven years and previously taught third and fourth grade. His room is very organized and his students remain quiet when he is teaching the class. His first observation did not occur until the beginning of November due to scheduling conflicts. Mr. Juarez did not attend the first TPD nor the first work group session; although, he was notified and offered compensation.

During the first classroom observation, Mr. Juarez began with a review of the answers for the homework assignments. The students did not work in groups nor discussed any of the work collaboratively. They were not directed to take notes. Although the room was very quiet, it appeared that not all students were engaged and only a small number of students raised their hands or offered answers to the questions. The classroom was tense and only a few key students responded when asked. When students answered incorrectly, Mr. Juarez normally said “no” and then went on to another student. Very few of the students wrote down notes about the work being covered and later I observed that he would often tell them to put down their pencils since they could take notes later.

Mr. Juarez was the center of the instruction and lectured almost the entire period. The interaction was primarily T-T as indicated on the chart of predominant interaction recorded during the first observation. He periodically asked questions and probed for understanding; however, he would recap the student answer sometimes using his own interpretation and explanation, sometimes correcting students that may not have been completely correct. The corrections were not noted to the students as corrections. There were many missed opportunities to delve deeply into the student misconceptions. If a student answered incorrectly he immediately said “no’ to the answer and then moved on to a student that answered correctly.

The homework was on representing fractions pictorially and applications of fractions. Mr. Juarez read the answers aloud and students corrected their answers. During this time he asked if there were any questions and periodically he asked the students for an answer, congratulated them on the correct answer and then proceed to explain their answer: “let me tell you how Henry (sic) got the answer,” The problems that he covered were not generated by student questions about the problems but were chosen by Mr. Juarez. He continued to ask students for their reasoning, and he continued to restate never using the student words although many times the student explanation was very clear. When students answered it was almost impossible to hear them in most parts of the classroom.

After about 25 minutes, Mr. Juarez began the day’s new lesson that focused on representing fractions on a number line. He only used questions that were from the textbook. Again he asked students to give answers to the problems, while he sat at his desk, indicating if they were correct or incorrect. He then assigned them some problems to do, with a time limit of four minutes. He never monitored their progress by walking around the room. He was so anxious to do the problems that in 1.5 minutes he read through the answers very quickly and then asked the students if they had any questions. Although I noticed that some of the students had not written down anything on their paper, those same students never asked question. Many of the students had not finished the problems and did not have an opportunity to write the answers down on their paper. The tasks that the students were assigned were Procedures Without Connections since the students were required to describe the procedures that were used with very little emphasis on "the connections to the concepts or meaning that underlie the procedure being used" (Smith & Stein, 1998). Although Mr. Juarez asked the students to explain what they were doing, he many times dismissed their answers and stated them in his own words: "Let me restate in my own words."

During the first interview I asked Mr. Juarez to reflect on lesson and the parts that he felt went well. He indicated that he thought the students understood the lesson and that the students were engaged.

> Um, the kids were interested for the most part. They understood the lesson well. It is like, it was like the most basic concept of fractions, so I sort of anticipated a good understanding for the lesson. Like I said before, there was complete understanding on the lesson. I didn't see anybody that was completely lost or unclear about the concept. They were able to do the work independently.

My observations did not align with his observations. Most of his attention was given to those students that were answering his questions. Those students were primarily in the front of the room to his rights. When he was asked what he might do differently in the lesson he thought that he should have given the students more opportunity to practice what they had learned.

> Would have probably given more opportunities to all the students to do more exercises…I would have had, instead of just using what the book provided maybe providing additional sheets, hand them out and have the kids complete them independently. Collect them at the end, look over them after to see who fell through the cracks.

This response verified that there was little opportunity for all students to demonstrate their understanding. When he asked them to do the problems, he gave them very little time to do the problems and only questioned the small groups that usually understand the work in the class.

When asked during the first interview his experience with collaboration, Mr. Juarez said that he did not have any experience with collaboration at the school site. Although he indicated that "The impact [of collaboration] is normally positive and I would appreciate more opportunity to do that kind of activity as a teacher" I was not sure if he truly believed in collaboration or if he was answering because he felt it was what was required of him to do. When asked about other professional development, Mr. Juarez did have some professional development delivered by a mathematics coach and also had professional development related to the new textbook. Mr. Juarez indicated that he wanted to have professional development that was related to his teaching that targeted the areas that he was having difficulty teaching.

> Excellent staff developments, because those are things that are related to our teaching at the moment. When things have to do with you, then you use them, you know. Especially when you are struggling how to do, knowing how to do something and then they teach you that particular skill, then you feel empowered. Like whoa, this is exactly what I need to do for that particular thing I'm struggling with.

It appeared that he wanted a professional development that he could immediately take back to the classroom, premade and ready to implement. Mr. Juarez also wanted to have tools to use in the classroom and felt that the district should tell teachers what areas students are having trouble understanding and how to teach that area; "Why not just identify, give us the materials and the training and say, 'Look, guys. You guys as a group don't know how to teach A, so here's this. And this is how you teach it'." Mr. Juarez also believed that the district should teach the teachers how to teach the skills. He did not consider that the lack of student understanding of the skills may be due to the concepts behind the skills:

> I think that every kid can get just about any skill. It's just that time and effort has to go into teaching those particular skills. Over and over and over and over. It's the drill and the repetition that makes them stick in there. Because sometimes you may teach something, and they'll forget it. So you have to re-teach it. There are plenty of teaching opportunities.

When I mentioned that he did ask his students to explain their thinking, the related his experience learning mathematics and the effect it had on his teaching:

> Well, you know, I grew up, going through my years in school, not really understanding. I mean, I added and divided the fractions and multiplied them. But what did it really mean? ….I went through all of my college experience not knowing, you know. It wasn't until I started teaching that I really understood what they mean one-half divided by one-third.

He also noted that the work we did in our sessions also had an impact on his thinking about teaching and in particular understanding what it was to multiply fractions and use a pictorial representation.

Mr. Juarez continued to attend the workgroup sessions and attended the next two sessions of the TPD. He did not attend the first TPD or the first workgroup meeting since he was off-track and did not want to come in for the meetings although the school would have been compensated. Mr. Palmer was also off track but he decided to attend the meetings.

He participated in the sessions; however, he was reluctant to implement the lesson but finally did after some coaxing. When he shared the student work he noted that he didn't see any reason to do multiple representations. This verified his idea that the lessons should be taught using one method and that for divergent thinking was not necessary; however, if it happened that would be acceptable.

> Encourage them [his students] to do that [different representations], because I didn't see and particular use for this….But some of them, you know, they kind of deviated from the norm and they went their own way. It was real interesting….They did it without hesitation. You know, they went right at it and got it done. Some had the wrong answers. So after we were done with the project I pulled out those kids and you know we did it as a group first then I spoke to those kids and said, 'What would you do now that you understand how to do it?' just to make sure that they accurately got the concept.

In addition, the method that he used when working with students that did not understand required pulling them out and working with them as a group. This indicated that he "taught' them the material but they did not understand, so he would keep them in for extra tutoring. Mr. Juarez equated understanding the concept with understanding the procedure. Again this seems to underscore his procedural method of presenting the materials.

When asked if the students asked him questions, he reported to the group that "…oh ton…you know what , it's like they wanted to take it like beyond what I was giving them you know like, "can we do the other ones and we come up with our own decimals and fractions?" They got really motivated." These student questions did not display a high level of cognitively demand and illustrated Mr. Juarez's lack of understanding that the questions the students asked were merely replicating a procedure that they were taught.

During the second observation, Mr. Juarez was teaching a lesson on geometry. This lesson occurred after the winter break since he did not go off-track as some of the members of his workgroup. The lesson revolved around the two areas of study: geometry and defining the types of triangles and the second area was a lesson about adding fractions with like denominators. During the first part of the lesson, Mr. Juarez tied the shape of the triangle to the polygon family. He first defined the equilateral triangle by separating the word into ‘equi’ prompting for equal and ‘lateral’ prompting for side and then he placed the two meanings together to obtain the idea of equal sides. Most of the questions posed were of low level cognitive demand that normally required single word answers. When students were asked to explain their answer, the students referred to the definition of the selected figure.

The lesson was repetitious and Mr. Juarez continued to follow his original format of using only work from the textbook. Since our TPD did not cover geometry, there was no specific work related to geometry that could have been fused into this lesson easily; however, high cognitively demanding questions were modeled and marked consistently. Mr. Juarez continued with the lesson going through the problems from the textbook. He tended to do all of the talking throughout the lesson and did not encourage students to make connections between the figures. Since the lesson on geometry and types of triangles took all but the last ten minutes of the class, he quickly changed over to adding fractions. The lesson on the geometry was entirely too long and did not utilize any kinesthetic representations.

When he finally moved into the fraction part of the lesson, Mr. Juarez asked a student to read the introduction out of the book. Mr. Juarez then lectured about the reading. There were no opportunities for the students to share their understanding of the reading. Again the lesson was teacher centered with some teacher-student interaction when he asked question. During this lesson he did have the students do some group work as opposed to the first lesson when students did not work in groups. He did insert terminology consistently in the geometry lesson that included: angles, right triangle, isosceles triangle, equilateral triangle, polygon, angles and equal sides. The tasks could be identified as Memorization Tasks that could be categorized at a lower level than the first lesson (Smith & Stein, 1998).

During the second interview, Mr. Juarez again thought that the students learned the materials and that the lesson went well.

> What I think went well was the students learned the material. They were able to do it successfully. Some struggled, but then eventually I think I had 100% understanding. They did the assignment. During tutoring, I had a chance to help five students and they did it independently.

Although he talked about using visualization in the second interview and how it was important to allow students to access the information in multiple ways, this did not appear to transfer to the classroom lesson that I observed. Mr. Juarez also began to explore the use of the grid and expanding the ideas used.

> The grids and how the kids were able to visualize things in different ways. It was really interesting and to see how different brains operate in different manners and even though – even the students learned during that grid activity that not all of the answers had to be identical to be right. They saw different approaches from attacking the problem and at the end they realized that no matter what end product is, the answer was still acceptable and math is pretty much set and there is no – you can't deviate from the standard, but when it comes to creating materials such as decimals and grids and all of that, then there is some flexibility.

Mr. Juarez also noted that working with his peers he was able to feel a great deal of support and appeared to support finding alternative ways of teaching the curriculum. This philosophy was not evident in the first or second observation.

> It helped me realize that a lot of my colleagues were having the same kind of issues that I was encountering and like when we decided to focus on one particular mathematics area, how we were all in agreement that fractions was one of the areas where we, as instructors, needed more assistance and when you create your own activities to do in the classroom, then you make it yours….You take ownership of it and then you can mold it the way you want it to go and then you teach math in a different way than just following the textbook and doing whatever they tell you to do, you know.

When he was asked if there was any effect on his teaching he indicated that he feels more confident in his instruction and that he felt he increased the number of different approaches to the classroom. He also felt that he has been planning more for his lessons.

> Well, for one thing, I have more confidence. I have more confidence. I prepared ahead of time more than before. I was able to bring in more different approaches than I would of had without having done it. I actually spend more time doing math now than before and I actually planned ahead more. I sort of like planned out my year, based on this, on this plan that we got going. I started looking at prior assessments and using data more than I had before.

The evidence during the two observations did not seem to align with his comments. Although his first observation was on November 4th, after attending workshops and a TPD, Mr. Juarez did not exhibit any of the strategies used in the sessions while Ms. Guzman, who was observed for the first time on November 5th did implement strategies used in the workgroup sessions as well as the TPD.

As indicated in Table 4.1, Mr. Juarez's questioning in the area of Inserting Terminology and Generating Discussion increased during the second lesson and Gathering Information continued to be evident in each lesson (Boaler and Brodie, 2004). The growth in the implementation of lessons that required higher order thinking during the observations was not apparent. As also noted in Table 4.2, the Task Analysis Guide, (Smith & Stein, 1998), the second lesson incorporated a task that was a lower level of demand and could be considered Memorization. Students were required identify geometric figures based on a definition. When students did not identify the figure correctly the teacher reinforced the information by reciting the definition.

Mr. Juarez continued with professional development offered.

Appendix J

The Journey of the Workgroup

The following case study follows the journey of the workgroup through a total of ten sessions: seven collaborative workgroup sessions with three Targeted Professional Development (TPD) sessions interspersed between the workgroup sessions. The collective nature of the workgroup and the specific occurrences in each session provided more insight into the learning that may have occurred during the study. The original group of teachers consisted of four fifth grade teachers and two sixth grade teachers. After the first session one of the sixth grade teachers dropped out of the study due to a change in their teaching assignment.

Targeted Professional Development 1: September 23, 2010

The first Targeted Professional Development, TPD, was designed to cover concepts the teachers would be teaching during the fall semester – fractions. Mr. Juarez did not attend this session since he was off-track and decided not to attend the session, although he was offered compensation. The TPD began with a discussion of release items from the Mathematical Knowledge for Teaching (MKT) Measures, University of Michigan at Ann Arbor, 2008. The problems were specifically tied to the topics the teachers taught during the duration of the study. The first problem that the group discussed was a problem that asked the teachers to either mark "yes, no or I'm not sure" for each of the following statements: a). 0 is an even number, b). 0 is not really a number. It is a placeholder in writing big numbers and c). The number 8 can be written as 008. The discussion that ensued highlighted some of the misconceptions that the workgroup members had.

As the discussions began, answers from the teachers included such phrases as zero was “not a number,” that zero means “nothing,” that “it’s a placeholder,” “it’s a neutral number,” that it “can be in the middle of a positive and negative number,” “it’s on its own” and that zero is a number. This wide range of answers allowed the group the opportunity to have a very rich discussion. When asked if zero was an even number, one of the teachers did say it was even. The teacher, Mr. Palmer, went further to explain the idea of an even zero by referring to a number line and finally to the pattern of even and odd whole numbers:

Mr. Palmer:	I’m going to add one more thing. (Laughs) If you draw a number line, you have a negative 1 and a positive 1, right?
Ms. Becker:	Exactly.
Mr. Palmer:	They’re both odd. So what’s in the middle?
Ms. Becker:	Zero.
Mr. Palmer:	An even. And every other number alternates. 1, 2, 3, 4. Odd, even. Even and odd. So if you have 2 odds, one on each side, you have to have zero to be even. Because if you look at a number line with whole numbers or integers, the pattern is even-odd-even-odd-even-odd. So by looking at the pattern, zero is even.

The teachers’ discussion of zero as an even number allowed them an opportunity to engage in an in-depth discussion of the characteristics of odd and even numbers. As the discussion continued one of the teachers had to work through the idea that zero was an even number. It was only through her understanding of distance on the number line and the distance between the numbers 2, 4, 6 and 8 did she agree that when going backwards 8, 6, 4, 2,and then to zero, that zero was also an even number.

Once there was agreement that zero was an even number, the teachers tackled the idea of zero as a place holder in multi-digit multiplication. Evident in the discussion was the increase in the number of questions from the teachers that evolved during the session. As the session moved forward the teachers continued to challenge each other and to delve into more depth with the mathematics. After given the number 20, the teachers were asked what zero represented in the number. The group determined that the zero indicated zero units, zero ones or absence of ones and the 2 represented the number of tens. When given the number 008 and asked about the two zeros preceding the 8, the group determined that the zeros had a meaning and that they meant no 100's and no 10's. What the group began to realize was that the zero actually had a meaning when written in a number. As Ms. Becker said;

> I think when we're using decimals, to show the kids that anything to the right of the decimal number has no value. That's how the kids finally understand that. Decimal 80 [.80] is the same as decimal 8 [.8]. That a zero to the right of it doesn't change its value. So the only way the kids can really grasp that is when I put the zeros in front of a whole numbers and explain that that also doesn't change the value of 8. But now that she said it that way, I'm going to add that into it also. It has no tens, it has no hundreds.

I asked what they learned from this discussion and Mr. Palmer began to realize that what you write mathematically has meaning and that one needs to be precise.

> "I learned that actually anything you write in mathematics has a meaning. There's a reason why they put a zero somewhere. The reason why they put a number in a particular place value and why there's a decimal. It's very precise."

Ms. Becker thought about how she might use this information and began to question whether her students really understand the mathematics she was teaching and that perhaps she need to ask more questions to ascertain their understanding.

> Well, based on what she just said [another group member was confused], that would make me stop and think, are my kids really understanding this. Maybe I need to ask more questions. Because maybe they're doing like her. Maybe they're just changing the answer because they saw somebody else had the right answer and they know that person's right. So maybe I need to rethink. Does that child really understand? What can I ask that child, that student, to see if they really are understanding? Because I have kids that do that.

Even though Mr. Palmer knew that zero was an even number, he was still unsure about his mathematical knowledge and he doubted his ability to explain the reasoning behind the answers.

> To me, I knew that it was an even, I just didn't know how to back it up or support it. It was something, I felt like it was just taught and I had to know it. But I never really knew why. And then with B, I kept going back and forth, just because of the wording. I think the way I tried to rationalize it. I kept going back and forth. Am I agreeing with it or am I not agreeing it. So that was my issue, where I was stuck.

The discussion then led to the zero in the multiplication of multi-digit numbers. This discussion grew out of the observation of Ms. Talika's multi-digit multiplication lesson that I observed. Ms. Talika had a multi-digit problem on her board so the group was able to examine the meaning of the zero in the problem. Ms. Becker shared that the meaning of zero and its significance came to her as well when she was teaching.

> "The placeholder represents groups of numbers written in – you know how we do place values? You learn it like base 10. It's 1, 10, 100, 1000. So when I show them that, and I didn't realize, a light bulb went off in me like maybe 2 years ago. When I showed them that it was like, Oh my God! That zero is the next group, the ones you're talking about there [pointing to the ones]. You're talking about what you're doing, the ones, now you're doing the 10s."

Two of the other teachers came to the realization that they were told that the zero was a placeholder and that this knowledge made them think that the zero in multi-digit multiplication had no meaning. They realized that the discussions in the TPD could be of value in the classroom. Ms. Becker said that " just teaching like that, I learned you have to teach them why, and then they'll get it. It'll click and go 'oh that's why! Oh, okay, I get it now'."

The second MKT problem required the teachers to determine the best method for ascertaining if a particular number was a prime number. The question had four possible answers with only one correct answer. The group quickly began to examine the number 371 and how they might determine if it was a prime number. During the discussion the group defined a prime number, and then discovered that any composite number can be prime factored and that any composite number must have prime numbers as factors. After a very long discussion that included questioning and probing their understanding, they realized that if a number is prime, they needed to test all prime numbers less than the approximate square root of the number, in this case for 371 it was twenty.

Mr. Juarez said he told his students to use the prime factorization to determine if a number was prime; however, he never knew why he used that process. He was able to articulate a working definition of a prime number: “basically it’s only divisible by one and itself if it is prime and then if it is divisible by another prime number it has to be composite,” but he still felt that his understanding of composite and prime numbers was weak.

After the lengthy discussion of the MKT problems we moved on to the next area of study. I decided to use a concept task, Linking Fractions, Decimals, and Percents, from one of the District’s Instructional Units (Figure 1). The most challenging part of the task for the teachers was using a pictorial representation to determine the answer rather than using a procedural method, such as division, to determine a fraction’s decimal representation. Since the group now sensed that the normal procedural methods of obtaining the answer would not be acceptable in the TPD sessions, they were challenged to use other methods to find a solution to the task. Since our time came to an end, we decided that we would continue with Linking Fractions, Decimals, and Percents task at the next workgroup session. After working more than two hours, the teachers gathered up their materials to go home for the day.

Workgroup Session 1, September 27, 2010

The first workgroup session was a continuation of the TPD. The members of the workgroup were still having difficulty understanding the multiple access paths for the solving the Linking Fractions, Decimals, and Percents task (Figure 1). The task could be characterized as Procedures with Connections (Smith & Stein, 1998) that required them to pictorially represent and explain the decimal, fractional and per cent form of six shaded squares of the given 40 square diagram. This problem was relevant to the mathematics they would be teaching during the fall semester. The task also provided an opportunity to expand the teachers' knowledge about fractions. The task would later be modified by the teachers for possible use in a fifth or sixth grade classroom.

As the group moved through the process they began to realize that they could represent the decimal pictorially by increasing the number of boxes to a 10 by 10 grid or 100 boxes. They methodically noted that as one doubled the number of squares, the number of shaded squares doubled leading to 12 shaded squares per 80 squares. Difficulty arose when they added 20 squares to the 80 squares to arrive at the 100 squares. Some of the teachers had difficulty understanding why they needed to shade 3 additional squares.

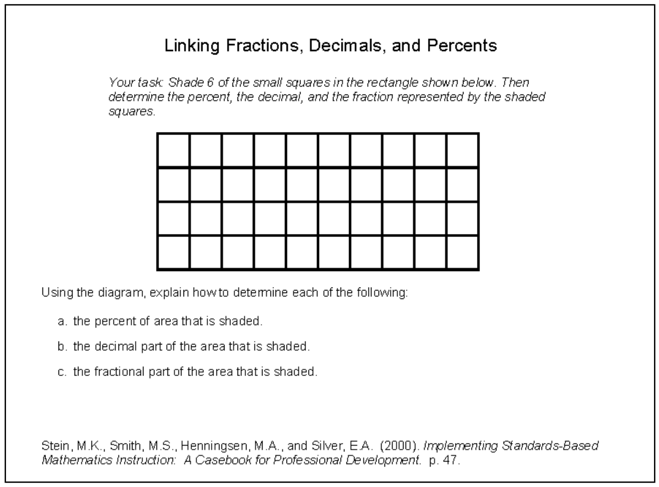


Figure 1

Mr. Palmer: My question is how did we get the 3. What, how do we – I mean I understand how we got the 3. But how, let me rephrase my question. I can see what you did the first time. You basically, you wanted to talk something. When you're talking about a decimal or percent—

….

Mr. Palmer: -- you're talking about 100.

Researcher: Yes.

Mr. Palmer: You're talking about 100. Now when you doubled that, that all made sense. And when you add another 20 to get a full 100, that made sense. But how would you use the top two pictures to derive (sic) at the number 3….

….

Researcher: Now if I have 6 squares for every 40 squares, this is where that proportionality comes in. And that whole idea of proportionality. Then you say to yourself, "Well, I can't color 6 again because I don't have—The full 40.

Mr. Palmer: Right.

Researcher: What is the relationship between the 20 and the 40?

Mr. Palmer: One-half.

Researcher: So what is the relationship?

Mr. Palmer: One-half. And that's 3. Okay.

Eventually the problem led them to the needed 15 shaded squares per 100 squares leading to 15 hundredths and then to 15 percent of the squares shaded; but this only evolved after may guiding questions that probed them for greater understanding.

The group then looked at each column representing one tenth of the whole and the shading of the 6 squares as noted in Figure 2. The teachers exchanged ideas, pointed to the representation, made notes on their paper and sectioned off the columns. When prompted to look at the individual columns and what each column represented, understanding of this alternative method of looking at the problem became apparent.

Researcher: But what do each of these columns represent?
The group: 10. 10%.
Mr. Juarez: I see. That's a good way to …
Ms. Becker: Ten and then half of ten, 15%. Oh man! That's cool. (Laughs)
Mr. Juarez: Yeah.
ALL: That was too easy. (Laughs)
ALL: Yeah. Yeah.
Mr. Palmer: So this one's 10%. [pointing to the first column]

The discussions did not conclude but continued on as another participant wanted to know what one square represented.

Ms. Guzman: I'm trying to see each square.
Mr. Palmer: It's $1/40^{th}$.
Ms. Guzman: $1/40^{th}$. Why is it $1/40^{th}$?
Mr. Palmer: Because there's 40 squares total.
Ms. Guzman: Ohhh…

….

The discussion continued and the group began to understand the even adding the first 4 squares in the first column could be written as one-fortieth plus one-fortieth plus one-fortieth plus one-fortieth and the result would be four-fortieths or one-tenth or ten percent. Through this discussion the concept was tied back to each column being equivalent to ten percent of the whole.

Ms. Talika: So if you were to add all these, that would be 4 over 40, which is $1/10^{th}$….Okay, I got that a little bit. So $1/40^{th}$, 4 [of them] equals $4/40^{th}$, and that's $1/10^{th}$.'

…

Researcher: So this was a good point that you brought up. 1/40, 1/40, 1/40, 1/40. What does that give you?

Ms. Talika: 10%.

At this point I then asked Ms. Talika to show the group how the ten percent relates to the one-fortieth and to the diagram.

As the session continued, the group explored that each row was equivalent to one-fourth of the whole or twenty-five percent of the whole. Although this concept was more difficult to grasp by some of the teachers, the group continued to grapple with all of these ideas and began to realize that not all of these methods would be evident in the classroom discussions. Mr. Palmer commented that:

> I wanted to make a comment here. I think that the first one that you did, would probably be the most direct and easiest for 6th graders. Because a lot of them [students], in order to do it this way, they have to [know] how to find ¼ and 25%. So it requires a deeper knowledge of math than that. I'm not saying that there's anything wrong with this. I'm just saying that the way you did it, the very first one? Is as close as purely pictorial as you can, and it seems to make more direct sense. That's just my own [opinion].

As the group continued to think about how they might use some of their learning with their students in their classroom, they began to discuss how they might ask their students questions that would push them for greater understanding. As the meeting drew to an end, I asked what they thought about the problem they experienced during the session.

Mr. Palmer: Abstract thinking. For me it was abstract because I didn't realize – well, I know that fractions, decimals and percentages all relate. But I honestly didn't realize, when you were talking about, that's not equivalent fractions. To me, I don't know, would you call it expanded fractions? I don't know what you would really call it. But it's basically parts of the whole. And that's what I got. And just taking, my students just trying to let them walk their way through it, you know, and think about it. So you can see that "aha" moment. Like you saw it.

Ms. Becker: Yeah, because I was – as you were talking, it was making me think that we usually teach in 100ths to make it easier for our kids to catch on. And so I don't think we're really stretching their thinking, because we want them to make – we want them to be successful in this, in the understanding of the equivalence between the percentages and fractions.

Ms. Talika: It would frustrate me, though.

Ms. Becker: Yeah, because we were, we were getting frustrated trying to figure it ourselves. I mean it wasn't till we started doing the drawing that we started to –look how long, we've probably done this in our entire lives. But looking at this, it gives us a tool to stretch their [Students'] way of thinking. So that we really know if they understand it. Not just because they memorized certain parts to equal that 100.

Ms. Guzman: Yeah.

The teachers began to grapple with the idea of using more questioning in their lessons that would lead to greater student understanding, using visual representations in their work to stretch their students understanding and the idea of providing opportunities for their students to build conceptual understanding.

Although the session was short and within the school day, the intensity of the work resulted in great deal of new learning for the teachers. As the session ended the teachers quickly left to pick up their students from psycho-motor.

Workgroup Session 2, October 11, 2010

During Workgroup Session 2 the group determined a mathematics topic that they felt was difficult to teach to their students. The goal of the meeting was to determine the topic that we might use for our lesson planning and the approximate date the teachers would implement the lesson. The teachers were given a notebook that included: Theory of Action for the study, Elementary Mathematics Cognitively Demand information and Instructional Unit Outline for fourth and fifth grade; Unit Concept Organizer for Sixth grade; and the Content Specific Workgroup Protocol used to develop the lesson. This was the first meeting when all teachers were present.

The group had difficulty determining what they might be teaching in November but finally decided that the lesson should be related to fractions, decimals and/or percents. They agreed that they would come to the next meeting and bring any materials related to this area of study. It was obvious at this time that the teachers had not done a great deal of planning collaboratively to develop a single lesson and had as a group not examined common student work. At first they thought they would independently develop a lesson and discuss what they did in the class. After further clarification, they understood that the lesson was a collaboratively developed lesson that was implemented by each person and then reflected upon using the student work as evidence of student learning. After determining that the lesson would focus on fractions, decimals and/or percents the teachers left to pick up their students from psycho-motor. Sessions that occurred during the school day were normally more difficult to manage due to the short time that was allocated.

Workgroup Session 2, October 18, 2010:

During this session, we looked at various problems that dealt with multiplication of fractions using some of the materials from *College Preparatory Mathematics* (CPM). Although the workgroup was asked to bring samples of lessons that they had done with fractions, none of the teachers brought any work. This verified their overwhelming dependence on using the textbook strategies, examples and lesson structure for their teaching.

The first strategy introduced was the area model of multiplication. Mr. Palmer said he remembered a professional development that used this type of multiplication; however, he was not told that the strategy was based on area. He was happy that he now knew why this multiplication did work. We continued through this area model of multiplication using two digit numbers, 59 times 46, by actually drawing a diagram. We drew a diagram that indicated a length of one side as 59 (50 + 9) units by 46(40 + 6) units (Figure 3). The difficulty was delineating linear versus area. Mr. Juarez also said that area versus perimeter is a difficult concept for students to understand. Ms. Guzman thought that this would be easier for her students who have difficulty multiplying since there is no borrowing that is needed except when adding the four areas.

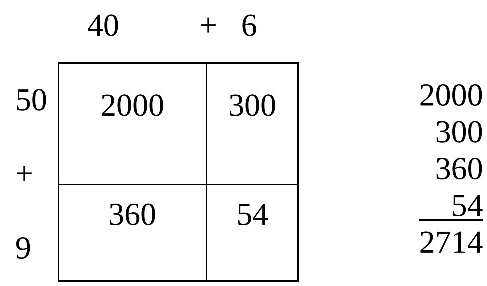


Figure 2

The teachers then discussed how students can multiply decimals using the same model and how students determine where the decimal point should be placed. The use of vocabulary was stressed throughout this workgroup session and included a discussion about digit versus factor. This provided an opportunity to clarify the misconceptions about this terminology used in Ms. Talika's class. The materials that were introduced were used by Ms. Guzman during the first observation and Mr. Palmer said that he would correct his use of the word "digits."

In addition, the group learned how to use and explain the "Giant One' problem as well as the underlying principle of multiplying by a number equivalent to one to find an equivalent factions. They then used the ratio table as another method to find the LCD. And finally we returned to the area ideas and worked with the rectangular fraction model

(Figure 4). The rectangular fraction model proved to be more challenging for the teachers and would be the one strategy that we would revisit in a later session.

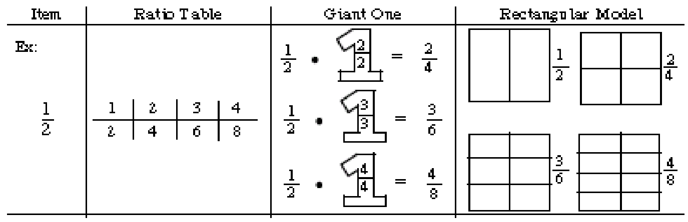


Figure 3

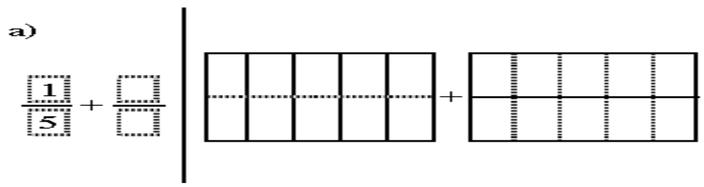


Figure 4

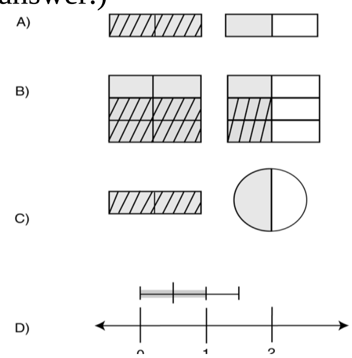
(CPM, Foundations for Algebra: Year 1, 2002)

The challenge again during this session was the time allotted for the workgroup session. Since this session was during the school day, the time that we had together was approximately forty-five minutes. After school workgroup sessions always ran longer than the designated one hour. When the group was able to become entrenched in the work, the ability to lengthen the time proved to be an asset to the process and also allowed the teachers the time to interact and challenge each other's thinking. Although the participants did not bring any of lessons or strategies to share it proved to be a productive session and provided a base from which the teachers could build a lesson.

Targeted Professional Development Session 2, October 20, 2010

The second TPD continued with the examination of the fractions. The group began with problem number 6 from the Mathematical Knowledge for Teaching (MKT) measures, Mathematics Released Items, 2008. The teachers were to determine the one model out of four possible models that could not be used to show that 1 ½ x 2/3 = 1.

> 6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately. Which model below cannot be used to show that 1 ½ x 2/3/= 1? (Mark ONE answer.)



The difficulty that each of the teachers encountered resulted from a lack of knowledge of how a pictorial representation could demonstrate multiplication of fractions. In addition to choosing a model, the teachers were asked to explain their thinking why the models worked and why the model that was not correct could not represent multiplication of fractions. Although some were able to determine the model they believed to be incorrect, they were not sure if they could explain their reasoning. There was also some concern that the students would not understand the representation. Mr. Palmer wondered "You think they will understand this?' and then commented to the group "Well, you know, I've got to tell you I'm having trouble understanding this."

Two of the teachers did state that "C" was not the correct model. After Ms. Talika said "I can't explain it" I asked her "Why? What do you think?"

> Ms. Talika: I think it's not C because they're not in equal parts, they're not equal parts…
> Researcher: Equal parts…What do you mean by equal parts?
> Ms. Talika: Equal parts in the same…
> Mr. Palmer: Area

Ms. Talika: Same Area. Yeah…

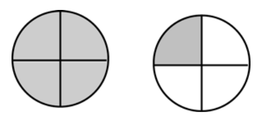
The conversation took on a little different route when the teachers looked at the 3 parts and moved away from the idea of equal areas until Ms. Becker said “This (pointing to the rectangle) and this (pointing to the circle) are not the same area, so if I say two thirds – I would say this (the rectangle), I would say this is 2 one –thirds and this won’t be the same area as this (pointing to the circle).” She indicated, as the group continued to work , that one cannot use C since “there’s no equality” in the two figures. The others then saw that since the areas were not necessarily the same for the circle and the rectangle, part C was unacceptable.

Even more challenging for the some of the teachers to understand were the models used for fraction multiplication. Each of the acceptable models was analyzed to ensure that all teachers understood the reasons why they modeled fraction multiplication. Time was given for all teachers to grapple with the models. The teachers began to increase their understanding and began to expand their understanding to other areas.

> Mr. Palmer: Well, what you’re actually doing is your increasing the 2/3 and you’re basically you’re making it 2/3 larger to one but it’s still smaller than the one and a half. Normally when you think of multiplying whole numbers where you have each factor which the product is going to be bigger than all factors that go in there, but in a fraction your product is going to be, it might be bigger than one of the factors but it can’t be bigger than the largest factor.

Once all understood the problem the teachers were asked to show how the problem could be represented pictorially by changing the order of the factors. This created even more challenges for the group members; however, they persevered and went on to another problem. The group had a discussion on what the “whole” represents and in particular, how the “whole” relates to the denominator of a fraction. The teachers were asked to do another problem from the MKT released items. The discussion evolved into an in-depth exploration of the meaning of what the "whole" represents and how this idea related to the parts of a fraction; namely, the denominator and the numerator.

> 5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)
>
> a) 5/4
> b) 5/3
> c) 5/8
> d) 1/4



The teachers read the problem silently and then answered the question. Some determined the answer quickly while one had more difficulty with the problem.

Ms. Talika	The two represents the whole of the two the part that is shaded of equal parts, there is one part, two parts, three parts, four parts, five parts out of the total eight which makes the two pizzas. So it's 5 out of 8.
Facilitator	And again, why is that powerful to students?
Ms. Talika	Wow…Because a whole can never, it can be different…
Mr. Palmer	You define the whole. You can say a, you can say there's a piece of pizza left sitting here and if a quarter of the first whole, somebody can say does somebody want this whole piece of pizza meaning the whole fourth, the whole had to be carefully defined. And actually thinking about it, it's almost as though you have to define the concept of whole to define parts.
Ms. Becker	And you know what, what she did is extremely, extremely common with kids in that they go right past that, the whole is of the two pizzas because I passed that, the whole is of the two pizzas. Because I had to kick myself and I'm here going, no, and then I remember no, no, no, he's right because that is the whole pizza, because you skim right past that part and still see two wholes. Even though that sentence says the whole is all of that.
Ms. Talika	The whole as a pizza, two pizzas.

When the teachers were asked to think about the implications of this type of problem, they discussed how they inadvertently used single objects to represent a “whole”, and then divided that single object into equal parts. They felt that this narrow description of a whole could lead students to a misconception about the definition of “the whole”.

Our next task was to revisit the first type of problem discussed and to have each person make up another problem to see if they could represent the multiplication pictorially. This proved to be more challenging so the group decided that they needed to cover a few more problems together. Their enthusiasm increased during the task and when we finally finished a few more problems their understanding increased dramatically evidenced by their sharing of ideas and problems.

Our next area of study was to share other strategies of multiplication of fractions. In particular we practiced multiplication of fractions using a grid to display the multiplication which was introduced in the previous session. The interest was still high and the teachers tried all of the problems with eagerness. The use of addition and subtraction of fractions using grids (College Preparatory Mathematics, 2002) would be later seen in one of the classroom visitations (Figure 5).

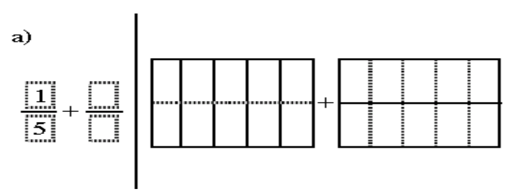


Figure 5

(CPM, Foundations for Algebra: Year 1, 2002)

The group struggled with portraying the addition of the fractional parts using the diagrams until they finally were able to see the process of superimposing the two representations upon each other. The teachers realized that the representation of addition of fractions could be used to understand subtraction of fractions as well. The multiple methods of understanding fractions had increased their understanding. As Ms. Talika said, "My eyes have been opened. I love fractions; initially I wasn't getting how you draw the pictures, and how you subtract from it….I understand now."

The group looked through some of the other materials distributed and then as the session drew to an end, they gathered their materials and were reminded that during their next workgroup meeting we would develop the lesson that would be implemented the following week. Enthusiasm was still high and a few of the members stayed behind to discuss more mathematics that included using the fraction bars that were distributed with the student textbook

Workgroup Session 3, October 25, 2010

The workgroup session focused on the lesson that was to be developed collaboratively. There was only one person missing from the meeting, Ms. Becker. Ms. Talika wanted to know the format of the lesson and if it needed to follow the seven-step format. They decided to use their original idea of writing a lesson placing fractions and decimals on a number line, and then Ms. Guzman suggested that the students "be able to find or differentiate between fractions and decimals [that are] equivalent or not." As they continued the discussion it went from fractions to decimals and then to percents. The content that they wanted to cover became massive. Ms. Talika immediately went to her textbook and began to align each section with a specific day that the section would be taught. The section that they were to teach according to her counting off of the sections was on shapes. It was at this time that the group decided to change their focus and move back to fractions.

When asked about what they do for fractions, Ms. Talika went back to her book to determine what should be done. During this session, we did have a discussion that included connecting fractions, decimals and per cents and specifically connecting these ideas to a 10x10 grid, an idea that we had examined before. The group decided that with the midweek holiday and with all of the other school activities, they would not be able to teach the lesson until the week of November 8th.

The idea of writing a detailed lesson plan appeared to be foreign to some of the members of the group. One group member said that "it won't take us that long" not realizing that there was a need for detail and that it was important for all to implement the lesson similarly. Explaining how to write the lesson did take time. The adjustment to the protocol was needed since the group members had never worked in a lesson study type of collaboration. The group finally determined that they would focus on a lesson that would be a precursor to a number line lesson. The use of pictorially representing fractions and decimals using a 10 by 10 grid introduced during a TPD seemed to be the best possibility for the lesson.

Ms. Talika then referred back to the book and began to read an example from the section that they were going to cover in the lesson. She tended to take over the discussion in the group. Ms. Talika's method of delivery for a lesson always moved toward showing the students what to do, while never allowing students to explore problems to develop their understanding. As the facilitator, I began to ask how they might use the grids to represent various fractions and decimals. Again, Ms. Talika dominated the conversation; however, the other group members nodded in agreement with the new direction and indicated through their gestures that they were okay with the lesson. Mr. Palmer thought that this might be a good review for his sixth grade students. Ms. Talika consistently wanted to "show it to the kids" and to do all of the work for them on the board. Mr. Juarez finally summed it up for the group: "Like let them explore. Don't show them anything. Just giving them a preview, then let them [go]."

The group finally decided upon a lesson that let the students explore, drawing representations of fractions and decimals using the 10 by 10 grids. I agreed to type up all of their notes, including the questions that the teachers could ask their students. They agreed to follow the lesson using the agreed upon problems and format. Ms. Talika still had difficulty understanding that the lesson we wrote would take the place of the lesson in the book but seemed to finally grasp the concept. The group left with the understanding that I would email the grids and the questions and we would discuss via email any changes that needed to be made.

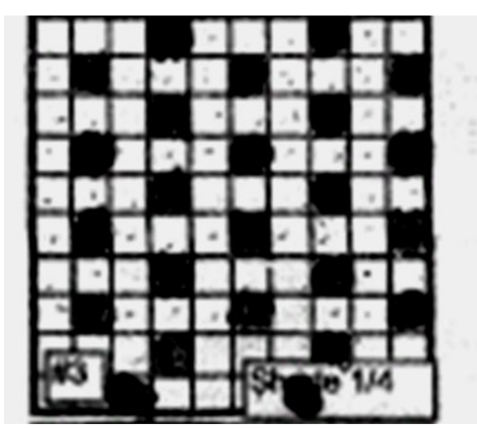
Workgroup Session 4, November 15, 2010

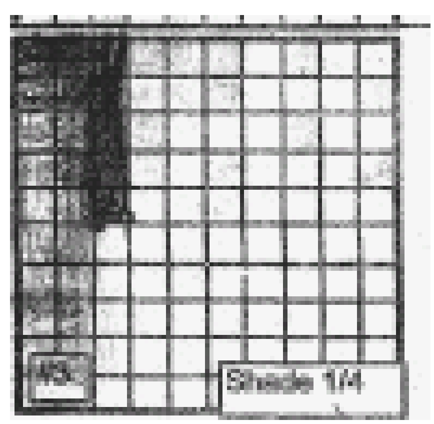
During this session some of the teachers discussed the student work and the implementation of the lesson. Two of the teachers, Mr. Palmer and Mr. Juarez did not implement the lesson. They would implement at a later. Overall the group thought that the implementation went well and found their students were engaged. The implementation of the lesson was not the same by all teachers; however, the differences in implementation provided some interesting talking points during the meeting. Ms. Guzman did not treat the task as a lesson but thought of the task as an assessment. Ms. Talika and Ms. Becker used the task as the lesson; however, Ms. Talika decided to teach the material prior to the implementation of the lesson. This apparently was an issue from the previous session since Ms. Talika was reluctant to forego the textbook lesson for a lesson developed by the group that covered the same material.

When the group learned that Ms. Talika had implemented lessons that would address this task prior to the agreed upon lesson, they wanted to know if there was an impact on the lesson. She felt that the in some instances the work did not present problems to the students; however, there appeared to be some challenges to Ms. Talika when students asked her questions:

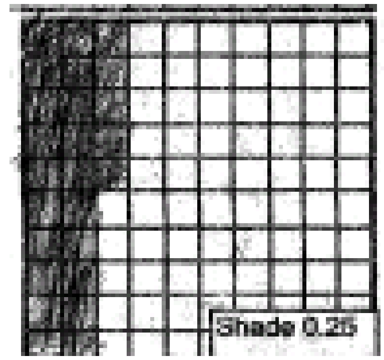
> Overall I think mine, it was, the issue was it was like ‘Ms. Morris, 25 out of 50.’ And I’m like okay, yes. So we talk about parts of a whole and I said what’s the whole and they said well 50, I said well why don’t you just do 25 and I said yeah, and another girl she said well I’ll just do half and I said why did you do half and she said well because 25 over 50 equals ½ so I figured since we talked about equivalent fractions she said, isn’t that the same thing. I said it is the same thing, I said if you did 25 out of 50 what would that represent out of the whole 100, so some kids got that because that’s really ¼ but ¼ between 25 over 50 they’re not equal. So I thought that was kind of, I didn’t know how to really explain that. I, but I told her when she did her, when she shaded half, it was true because 25 over 50 equals 1/2.

The issues of how to explain the equivalency and how she might deal with the questions should they arise again were discussed in the debriefing. A continuing theme of how to define the whole and how that might be represented was discussed and how many of their students may have addressed these ideas as they looked at the student work.





Example 1: Shade 1/4 Example 2: Shade 1/4



Example 3: Shade .25 Example 4: Shade .25

Example 1 indicated that the students may have taken the 1/4 as one part out of 4 parts should be shaded. The students may have determined that the “whole’ consists of 4 partitions and the one refers to 1 partition. In Example 2 the student may have known that each column is 1/10 of the whole and then went to 2 and 1/2 columns of the large grid to equate to 25 parts of the 100. With Example 3, although the student was asked to shade a decimal representation .25 the student continued to count as they did for Example 1 and may have known that 1/4 is the same as .25. In Example 4 the student actually may have shaded 25 immediately since the number is 25 hundredths. The group discussed how this looking at the student work could impact their teaching and also what they would change in the lesson based on the work that they examined. As Ms. Becker indicated, she needed to work them through the idea of equivalency but had difficulty helping them to see the connection.

> They were only seeing the 50, they couldn’t, with mine, it’s almost like they couldn’t make the connection that 25/50’s is the same as 50/100’s. They, they had to stop it at 50 shade out 25, x out the other 25. They counted to 50 and then they did it again. And I said well then what is that? How many did you shade in total? 50. Okay, well how many boxes were there total? 100. I said so 25/50 is equivalent to what? I don’t know. They couldn’t make that connection. Even though they had it right there in front of them.

As the group moved forward, they began to look at representing 25/50 and 50/100 on the grid, noticing that they would look differently. The question then arose during this meeting – how would one might rectify this discrepancy: “(looking at the two different drawings) Wait a minute, 25/50 is not equivalent to 50/100.” As the discussions continued we were brought back to a previous idea discussed in the TPD, area equivalency. So the group then understood that as a representation, 50/100 and 25/50, in terms of the grid were not equal, due to the interpretation of the whole and that the “whole was represented as two different “whole” values, 50 versus 100.

The teachers were surprised that the students actually demonstrated different access strategies to answering the questions. Some of the teachers asked their students how they arrived at the answer and some while monitoring their progress noted what the students were saying as they were doing the problems.

> Ms. Becker: Yes. I was curious while they’re still doing that – I noticed that one of my kids did that. I understood what he did on the fourth (1/4) – but I wonder, do you think that maybe she knew that ¼ is the same as .25, so for her to make sense of it she had to…Well, she did exactly the same thing. And I knew she counted, ‘cause her … Denise and Guadalupe were all doing the same thing and I was like what are you all doing, ‘we’re counting. 1, 2, 3, 4.’ And then I didn’t go back over there to see this, so I didn’t know they had done it.

Ms. Becker was particularly interested in what they were thinking; however, she did not go back to check with the group to really see if how they knew that 1/4 was the same as .25. As she said later in this session “I should have went back on that one since I saw them do that. That was my mistake.” As an extension of the problems, Ms. Becker had her students on the back of their paper change the fractions into decimals. The students then were able to see that not only were the diagrams the same, the decimal and fractional representations were the same.

Ms. Guzman taught the lesson after she introduced a more procedural method of linking fractions and decimals. After reflecting on the lesson, she concluded that she would use the conceptual task first before going into the procedural because she felt they could have been more successful had they understood the concept prior to her modeling the procedural. This appeared to come out of Ms. Becker's discussion of how her students were tying the pictorial representations to the actual changing of fractions to decimals. The session ended quickly as all the teachers needed to pick up their students from psycho-motor.

Workgroup Session, November 29, 2010

When we came back I asked Mr. Juarez if he had implemented the lesson. He indicated that he "Couldn't find the time to do it"; however, he did say he would implement the lesson "this week" and said it would be fine if I visited the class during implementation. I asked if anyone would like to discuss any post reflections on the lesson. Ms. Becker said that she decided to review their learning from the lesson since the students had been off for a week due to the Thanksgiving holiday. She said that some of her students thought some of the problems were more difficult than others. This was a nice segue into the discussion that followed about the new lesson we would develop. Ms. Becker discovered that one of her students found decimals were easier to represent on the grid than fractions, in particular the .2 which he saw as 20 hundredths. When I asked her what would happen if the grids were no longer 10 by 10 but were similar to the problem we did earlier in the TPD, the 40 box problem, she asked reflected and asked herself a key question : “I wonder if they [the students] could really understand this fraction, if they would really understand fractions or decimals if it was out of something other than 100?”

Mr. Palmer then chimed in with a wondering “40 squares, 6 out of 40 squares.” As the group continued to discuss the problem using 40 squares they began to talk about possible ways that the lesson could be implemented. The biggest concern was how their students would transfer their knowledge from the 100 square implemented task to the 40 square task. The teachers discussed questions that they would ask the students if the students were not able to figure the problem out. Ms. Guzman was concerned about the type of questions that would be asked “What is the question that you’re going to lead to [having the students] changing it [the diagram] to the hundredths?” The workgroup continued to discuss how they would implement the lesson focusing on questions that they would pose to students to enhance and deepen understanding. Their focus changed from the teacher doing the modeling for the class to having the students work in either pairs or groups and then acting as a facilitator through appropriate questioning. As we came to the end of the session their interest was increased as we discussed what our next TPD session would entail. I decided that I would draft a lesson based on the discussion and notes. The energy was high as they left the meeting to once again pick up their students from psycho-motor.

TPD Session, December 1, 2010

Our meeting began with a lesson that incorporated the concept task we had done at the first TPD and first workgroup session with adjustments based on the previous workgroup meeting discussion on November 29th. Since the original problem was a seventh grade concept lesson, our discussion focused on modifications for fifth and sixth grade implementation. During the first half of the TPD, the group went through the student task as a review. This turned out to take longer than anticipated. Most of the discussions were centered on how to question the students if they were not able to do the task. The teachers went through the task and analyzed and changed some of the questions. As we went through the task, the questions about implementation increased. Throughout the TPD I modeled the type of questions that might be asked, stressing questions that required explanation of how the students arrived at their answers. Ms. Talika still wanted to go to a procedural process since she was having difficulty determining how to explain her reasoning and relating that to a visual representation. Ms. Guzman continually jotted down notes and questions she would ask her students including "what is the meaning of the number in the numerator, where is the answer represented in the picture and how did you arrive at the answer." Throughout the entire lesson, the teachers were asked to respond based on anticipated student responses. The definition of the "whole" was considered again.

Ms. Becker:	The next question is confusing. What is the fractional part of the sum of the shaded and the non-shaded squares to all the squares? Wouldn't that be forty-fortieths?
Researcher:	Which is?
Ms. Becker:	One whole. Okay, so that was deliberate?
Researcher:	Why did you think I asked that deliberately?
Ms. Becker:	To help them understand that that is a whole, even though it's not a hundred, it's still a whole. ...Because that's where the kids get lazy because they think a whole is only used as a hundred.
	…
Ms. Becker:	So on the next part where you're asking if you were to add the fractional part of shaded, which is four-fortieth to the non-shaded would be thirty-six over fortieth and that's going to show the kids that the denominator stays the same, you just add a numerator, you will get forty over forty which is one whole.

After we had discussed the new lesson, Mr. Juarez, who finally implemented the first collaboratively developed lesson, discussed how his students did with the lesson. He said that “they all did it without hesitation. You know, they went at it and got it done. Some of them had the wrong answer.” He then said that he “pulled out those kids and we did it as a group first then I spoke individually to those kids and said “What would you do now, now that you understand how to do it?” just to make sure that they accurately got the concept.” He was happy that the students asked him questions and then wanted to take it beyond the exercise and he said that the student asked if they could “do others and we [the students} come up with our own decimals and fractions?” Mr. Juarez indicated to the group that they were really motivated. The group continued to examine Mr. Juarez’s student work and began to think of questions that they might ask the students to check for conceptual understanding. When we examined his student work there appeared to be some understanding; however, the degree of understanding did not seem as great as Mr. Juarez had indicated.

As another activity that tied to fractions, the group was guided through a lesson using fraction strips based on a *Connected Mathematics* lesson. This lesson provided an opportunity for the teachers to understand the relationships between factors used to find the Least Common Denominator. They were asked to fold 10 strips of paper into equal partitions: halves, thirds, fourths, fifths, sixths, ninths, tenths, and twelfths. After making the strips, the teachers were asked to determine the relationships between all of the partitions. They marked the partitions and began to make all of the connections. This activity excited a number of the teachers as they discovered how this could be used for their class. Only Ms. Talika wanted to set the problem up by having the dotted lines on the strips so that the students would fold along the dotted lines. As she told the group: "You know what? I don't know if this is the case with you and other teachers but my kids are really not that good. They're kind of clumsy with [using manipulatives] so I would probably copy one half dotted lines and just have them fold it along the dotted lines." We discussed why it would not benefit the students if the lines were indicated and would negate part of the activity that included discussing how they folded the strips and the relationship within the folding process: folding fourths into thirds to obtain the twelfths. The group continued though all of the strips and the connections between the different partitions. At the end of the meeting some of the group members decided to try this particular problem prior to our last workgroup meeting.

Workgroup Session, December 13, 2010

The last session allowed the group to wrap up and reflect upon their work during the sessions and to arrange observations and interviews for those teachers going off-track. In addition, Mr. Palmer reported to the group about the implementation of his lesson. I had met with him prior to implementation since he felt he needed extra time to go over the implementation. When asked about the effectiveness of implementation of the lesson he told the group that:

> They learned it pretty good. I think they've got it except for one of them that was a little bit challenging for them. 25/50. Because they have to break up the square into 2 50-units and they were having a little trouble with that. But that's what I wanted to see. Then when I look at it, then I can figure out how to go back and talk to them.

As Mr. Palmer continued to reflect on the lesson he was surprised with the fact that the students were working together so well in the groups.

> They were working together. They were actually interacting with each other. Because I had them do it in groups. Some of them actually said, "No, no, no.

> This is the way you do it." "But this is what it says." "No." And I did not, I gave them very little coaching. I had them figure it out. I wanted to see if they could get it."

Although this was the last meeting, the group was still planning how they would implement the lesson covered in the last TPD meeting. Ms. Becker opted to do the fraction folding exercise on Thursday before she went on winter break and Ms. Juarez indicated that after the winter break she would go back and have the students share more of their work. During this meeting the teachers discussed with ease the implementation of lessons and connections that could be made between the lessons. They also began to share the effect of group work for their students. Ms. Becker said that she likes her students to work in groups "because I find that in math, my kids do better when they're working with someone else….solving problems. They seem to work better when there's more than [one].'

They began to expand their thinking to mixed numbers and improper fractions and representing these numbers using alternative representations that included pictorial representations. They began to readily think about using alternative methods of teaching concepts. They even began to talk about the food that they were buying for their holiday party and how they included the students in the discussion. Ms. Becker had a nice story about her students talking about "fractioning' the pizza and chips.

When some of the members had to leave, I met with the remaining members and we continued our discussion on prime factorization and a method of determining least common denominator and the greatest common factor. After we finished going through these methods, the very next day, Ms. Talika used the strategy with her students.