Giant quantum freezing of nanojunctions mediated by the environment

A. Glatz, N. M. Chtchelkatchev, I. S. Beloborodov, and V. Vinokur

1Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
2Institute for High Pressure Physics, Russian Academy of Science, Troitsk 142190, Russia
3Department of Theoretical Physics, Moscow Institute of Physics and Technology, 141700 Moscow, Russia
4Department of Physics and Astronomy, California State University Northridge, Northridge, California 91330, USA

(Received 16 November 2011; published 1 December 2011)

We investigate the quantum heat exchange between a nanojunction and a many-body or electromagnetic environment far from equilibrium. It is shown that the two-temperature energy emission-absorption mechanism gives rise to a giant heat flow between the junction and the environment. We obtain analytical results for the heat flow in an idealized high-impedance environment, perform numerical calculations for the general case of interacting electrons, and discuss giant freezing and heating effects in the junction under typical experimental conditions.

DOI: 10.1103/PhysRevB.84.235101 PACS number(s): 05.70.Ln, 72.15.Jf, 73.63.−b, 85.80.Fi

I. INTRODUCTION

Electron transport in the presence of an electric field is always accompanied by heating of the charge carriers. This effect is especially pronounced in electronic devices, where overheating leads to instabilities in the current-voltage characteristics. \(^1,2\) This defines the urgent task of studying far-from-equilibrium heating of charge carriers.

In this paper we study the heat flow between tunneling electrons and an environment in nanojunctions using a non-perturbative technique based on a quantum kinetic equation. We show that a regime exists in which the interaction with the environment leads to an effective ("giant") environment cooling of the junction.

At high electron temperatures phonons play the role of the cooling agent. At low temperatures the direct energy transfer to the phonon bath becomes inefficient and the relaxation is dominated by energy exchange between tunneling electrons and an electromagnetic environment and/or environment of many-body excitations in the electrodes. \(^3\) In both cases, cooling follows the two-temperature emission-absorption mechanism: \(^1,3\) the emission of environment modes has a temperature equal to that of the tunneling charge carriers, \(T_e\), and the absorption of environment excitations having the temperature of the environment, \(T_{env}\). Moreover, not only temperatures, but also the distributions of emitted and absorbed environment modes, may be different in the far from equilibrium regime.

The coupling between the tunneling electrons and the environment has a dispersion characterized by the "cutoff" frequency \(\omega_{\text{max}}\). For example, \(\omega_{\text{max}} = E_c\) for a high-impedance environment, with \(E_c\) being the charging energy of the tunnel junction; \(\omega_{\text{max}} = 1/\sqrt{LC}\) for an environment represented by an \(L-C\) circuit; and \(\omega_{\text{max}} = 1/R TC\) for an ohmic environment, with \(R_T\) and \(C\) being the ohmic resistance and capacitance of the tunnel junction, respectively. \(^5\) We are interested in the regime where \(T_e, T_{env} > \omega_{\text{max}}\). In this case the large number of environment modes, \(N \sim \ln(\omega_{\text{max}} \tau_e(T_e)) \max(T_e, T_{env})/\omega_{\text{max}} \gg 1\) (\(\tau_e\) is the energy relaxation rate), participate in the heat exchange between the environment and tunneling electrons in the nanojunction. At low electron temperatures, when the environment has an electromagnetic or many-body origin, this regime is easy to reach. \(^6,7\) We show that in general the heat flux acquires the large factor \(N \gg 1\) in all orders in electron-environment interactions increasing the efficiency of the heat exchange. Using the Landauer scattering theory \(^8\) we express the density distribution function in the leads, resulting in

\[ \tanh(\beta E) \]

where \(\beta = 1/k_B T\). At low applied voltages, \(V \ll T_e\), and \(T_e \approx T\). In the opposite case, \(V \gg T_e\), we obtain \(T_e \approx V/2\). The function \(p(\varepsilon)\) in Eq. (1) is the weight function for a junction between two normal metals (Fig. 1) and can be calculated for any choice of the electron distribution function in the leads, resulting in \(p(\varepsilon) \approx 4\varepsilon/R_T\).
The energy relaxation time $\tau_e$ in the tunnel junction, resulting in the distribution function $f_{\text{eq}}(\varepsilon)$ of the emitted (absorbed) environment mode $\alpha$ enter the density matrix

$$\varrho = e^{-\sum_{\nu}^{(\text{ini})} i\varepsilon_\nu |\psi_{\nu}^{(\text{ini})}\rangle \langle \psi_{\nu}^{(\text{ini})}|} \times e^{-\sum_{\nu}^{(\text{fin})} i\varepsilon_\nu |\psi_{\nu}^{(\text{fin})}\rangle \langle \psi_{\nu}^{(\text{fin})}|}/T_{\text{env}},$$

where $T_{\text{env}}$ and $T_{\text{out}} = T_e$. Thus $N^{(\text{in})}_\nu = \langle \psi_{\nu}^{(\text{ini})}|c_{\alpha}^{(\text{ini})}\rangle = N_{\nu}(\omega,T_{\text{env}})$ and $N^{(\text{out})}_\nu = N_{\nu}(\omega,T_e)$. The quasiequilibrium approximation mentioned above corresponds to the Gibbs distribution of the environment modes: $\varrho_0 = \exp\{-\sum_{\nu} c_{\alpha}^{(\text{ini})}|c_{\alpha}^{(\text{ini})}\rangle \}$.

To estimate the magnitude of the heat flow $\dot{Q}$, we first expand the distribution function $P(\varepsilon)$ in Eq. (1), in the first order in $\rho(\varepsilon)$:

$$\dot{Q}^{(1)} \approx \frac{8}{R_T} \int_{\tau_e^{-1}}^{\infty} d\varepsilon \rho(\varepsilon)[n_\varepsilon(1 + N^{(\text{out})}_\varepsilon) - (1 + n_\varepsilon)N^{(\text{in})}_\varepsilon].$$

This expression becomes 0 if $n_\varepsilon = N^{(\text{in})}_\varepsilon = N^{(\text{out})}_\varepsilon$. If the distribution functions are not equal to each other, we can expand $\dot{Q}^{(1)}$ with respect to their difference. We consider the case where the voltage bias at the nanojunction is 0 but the temperatures of electrons at the leads and those that comprise the environment are slightly different, $T_e = T + \delta T/2$ and $T_{\text{env}} = T - \delta T/2$. Thus, $n_\varepsilon = n_\varepsilon(T + \delta T/2)$, $N^{(\text{in})}_\varepsilon = n_\varepsilon(T - \delta T/2)$, and $N^{(\text{out})}_\varepsilon = n_\varepsilon(T + \delta T/2)$, where $n_\varepsilon$ is the Bose distribution function. Expanding $\dot{Q}^{(1)}$ in the first order in a small parameter $\delta T/T < 1$, we find

$$\dot{Q}^{(1)} \approx 8 \int_{\tau_e^{-1}}^{\infty} d\varepsilon \rho(\varepsilon)[n_\varepsilon(T + \delta T/2)(1 + n_\varepsilon(T)],$$

where $n_\varepsilon(T) = d n_\varepsilon(T)/dT$. The index $\theta$ is 0 for the quasiequilibrium situation when the temperatures of emitted and absorbed environment excitations are equal and 1 for the nonequilibrium case (the index 1 is skipped throughout this paper). Since $n_\varepsilon(T)$ in Eq. (5) is always positive, the following inequality is valid: $|\dot{Q}^{(1)}_0| < |\dot{Q}^{(1)}|$, where $\dot{Q}^{(1)}_0$ and $\dot{Q}^{(1)}$ refer to the heat flux in the quasiequilibrium and in nonequilibrium cases, respectively. The interaction function $\rho(\varepsilon)$ in Eq. (5) quickly decays at energies higher than some characteristic frequency $\omega_{\text{max}}$. For temperatures $T > \omega_{\text{max}}$ we can approximate $n_\varepsilon(T) \approx T/\varepsilon \gg 1$ and find

$$\frac{|\dot{Q}^{(1)}|}{|\dot{Q}^{(1)}_0|} \approx \int_{\tau_e^{-1}}^{\infty} \frac{T_{\text{env}} d\varepsilon}{\varepsilon} \rho(\varepsilon) d\varepsilon \approx \frac{T}{\omega_{\text{max}}} \ln(\omega_{\text{max}} \tau_e) \equiv N \gg 1.$$

Remarkably, at higher orders with respect to $\rho(\varepsilon)$, the nonequilibrium heat flow $\dot{Q}$ differs from the equilibrium flow $\dot{Q}_0$ by the same factor. This result holds even for a finite electric current flowing through the junction. Thus, the heat flow between the junction and the environment appears much larger than what the quasiequilibrium estimates predict.

**IV. OHMIC APPROXIMATION**

We now turn to the simplest case, an environment with a very high impedance compared to the quantum resistance, $R_Q$.

FIG. 1. (Color online) Illustration of the nonequilibrium heating effects in a nanojunction. Electrons traversing the junction absorb external photons (incident wavy lines) and emit them, leading to heating of the contact. Plots show the giant heating effect, $\dot{Q}$, as a function of the difference of electron and environment temperatures ($V = 0$) compared to the “quasiequilibrium” approximation, where the radiation density matrix is equilibrium, $\dot{Q}_0$. The full nonequilibrium analysis gives a heating effect that is at least 1 order of magnitude more pronounced than for the latter case: max$(\dot{Q}/\dot{Q}_0) \sim N > 10$.

FIG. 2. (Color online) (a) Illustration of electron-hole pair generation in the tunnel junction, resulting in the distribution function $n_\varepsilon$ [Eq. (1)] of these pairs (environment). (b) Comparison of the distribution functions for $T = 0$ in the leads and $T = V/2$. 
In this limit, tunneling electrons easily excite the environment modes. The spectral density \( \rho(\omega) \) of these modes is sharply peaked at the zero frequency, \( \omega = 0 \). For the correlation function \( J(t) \) the concentration of the environment modes at low frequencies implies that the expansion of \( J(t) \) over \( t \) up to second order yields \( J(t) \approx -i\omega t - (b/2)t^2 \), where coefficients \( a \) and \( b \) are defined as \( a = f_c^{-1}(1 + N^{\text{env}}_a - N^{\text{env}}_b)\rho(\omega)d\omega \) and \( b = f_c^{-1}\rho(\omega)B_d d\omega \). Using this expansion for \( J(t) \), we obtain the following result for the density function \( P(\omega) \):

\[
P(\varepsilon) = (1/\sqrt{2\pi b}) \exp[-(\varepsilon - a)^2/2b].
\]

Here the expansion parameter \( a \) can be estimated as \( a = a_0(1 + (T - T_0)/\Delta T), \) where \( a_0 = 2\int \rho d\omega \approx 2\rho(0)\omega_{\text{max}} \approx 2E_c, \) where \( E_c \) is the charging energy of the tunnel junction, \( T_e \) is the electron temperature in the junction, \( T_{\text{env}} \) is the temperature of the environment, and \( \omega_{\text{max}} \approx 1/(R_T C). \) Similarly to coefficient \( b \) in Eq. (7), we obtain \( b \approx a_0(T_e + T_{\text{env}}). \)

Substituting the density \( P(\omega), \) Eq. (7), into the heat flux \( \hat{Q} \), Eq. (1), we obtain our first main result for the typical heat exchange of the ohmic environment with the tunnel junction between two normal leads. The full temperature and voltage dependence is shown in Fig. 3.

**V. DYNAMIC COULOMB INTERACTION**

Next we discuss the more realistic situation where the tunneling junction is connected to two disordered conductors (leads). Following Ref. 9, one can find the spectral probability function \( \rho(\omega) \) corresponding to the electron-environment interaction,

\[
\rho_{ij}(\omega) = \frac{\omega}{2\pi} Im \sum_q \frac{(2\pi)^2(2\delta_{ii} - 1)\tilde{U}_{ij}(q,\omega)}{(D(q^2 - i\omega))^{1/2}(q^2 - i\omega)},
\]

where \( i,j = 1,2 \) are the lead indices, \( D_{1(2)} \) are diffusion coefficients within the respective electrodes, and \( \tilde{U}_{ij}(q,\omega) \) are the dynamically screened Coulomb interactions within (across) the electrodes. The form of spectral probability \( \rho(\omega) [\rho(\omega) = 2\rho_{12} + \rho_{11} + \rho_{22}] \) depends on the structure of the environmental excitations spectrum and, thus, on the external bias.

The system under consideration is shown in Fig. 4(a): two contacts are separated by distance \( d \) and their thickness is \( a \). The external bias is \( V \) and the contacts are kept at temperature \( T \) and the environment temperature \( T_{\text{env}} \).

The screened Coulomb interaction in Eq. (8) in Fourier space has the form \( \tilde{U}(q,\omega) = ([\tilde{U}^{(0)}(q,\omega) - 1 + \chi(q,\omega)])^{-1} \), where \( \tilde{U}^{(0)}(q,\omega) = u(q)L + v(q)\chi(q) \) is the bare Coulomb interaction and \( \chi(q,\omega) \) is the polarization matrix, respectively, with \( \chi(q) = \chi_D(q^2 - i\omega)^{-1}\chi_{\text{env}} \), \( \chi_{\text{env}} \) is the electron density of states at the Fermi surface in lead \( i \).

Below we concentrate on quasi-two-dimensional (2D) infinite leads. For this geometry with \( a \ll L \), where \( L \) is the characteristic lead size in the \( x \) and \( y \) directions, the bare Coulomb interaction has the form

\[
U_{ij}^{(0)}(\mathbf{r}_i - \mathbf{r}_j) = e^2 \int dz_i dz_j \frac{\delta(z_i - z_i^{(0)})\delta(z_j - z_j^{(0)})}{|\mathbf{r}_i - \mathbf{r}_j|},
\]

with \( z_i^{(0)} = (1/2 - \delta_i)L \), leading to \( u(q) = 2\pi e^2/q \) and \( v(q) = 2\pi e e^{-qd}/q \).

In the following, we consider the case of identical leads with the same diffusion coefficients \( D_1 = D_2 \equiv D \) and densities of states, \( \chi_{1} = \chi_{2} = \chi \). The dimensionless matrix elements \( \tilde{U}_{ij} \) of the dynamically screened Coulomb interaction (in units of \( e^2d \)) are then given by

\[
\tilde{U}_{ii} = \frac{4\pi}{q} \frac{\chi(q)}{\sqrt{v(q^2)}} = \frac{\tilde{U}_{ij}}{\sqrt{\chi(q)\coth^2(q)}},
\]

\[
\tilde{U}_{ij} = \frac{\tilde{U}_{ii}}{\chi(q)},
\]

where \( \chi(q) \) is the density of states at the Fermi surface. For identical leads, we have \( \chi(q) = \chi_{\text{env}} \).
where $\tilde{q} = dq/dq$ and $\tilde{\omega} \equiv \omega(d^2/d\gamma)$, with the dimensionless function $\chi(\tilde{q}) \equiv 1 + \coth(\tilde{q}) + \frac{4\pi e^2}{\tilde{q}^2 - i\tilde{\omega}}$. Using these expressions, we can write Eq. (8) as

$$\rho(\tilde{\omega}) = \frac{2e^2 d}{D} \tilde{\omega} \text{Im} \int_0^{\infty} \tilde{q} d\tilde{q} \tilde{U}_{11} \left[ 1 - (\chi(\tilde{q}) \coth(\tilde{q}))^{-1} \right] \left( \tilde{q}^2 - i\tilde{\omega} \right)^2.$$

Using this expression we can calculate the heat flux $\dot{Q}$ in Eq. (1) between environment and nanojunction with dynamic Coulomb interaction. The typical energy scale is given by the Thouless energy for a junction of distance $d$, $E_{th} = D/d^2$, which we use to rewrite all expressions in dimensionless units. For a typical temperature $E_{th} \approx 100\, K$, the temperature and voltage dependence is numerically calculated and shown in Fig. 4(b). Again, the nonequilibrium heat flow $\dot{Q}$ is up to an order of magnitude larger than the quasiequilibrium approximation $\dot{Q}_0$. We remark that, in this case, the function $\rho(\omega)$ introduces a natural cutoff for $J(t)$ which behaves as $\sim - |t|$ for large $t$.

VI. DISCUSSION

Above, we have assumed that the density of hot electrons is high enough so that the electron-electron scattering time is smaller than the time of energy relaxation (this time is large because of the quasielastic nature of interaction between the electrons and the environment). In this case the electron distribution function is close to an equilibrium one with an effective temperature well exceeds the high-frequency cutoff $\hbar\omega_{\text{max}}$. From the experimental point of view the temperature regime in which the effect is present is readily accessible. However, one needs to measure the time dependence of the junction temperature in order to extract $\dot{Q}$, which could be technically challenging for a nanojunction, and the presence of a substrate might need consideration.

One can expect that our results, in particular, the giant freezing effect, will be important for electronic transport in junction arrays, which will be the subject of a forthcoming work.

ACKNOWLEDGMENTS

We are grateful to J. Pekola and F. Heikkila for useful discussions. This work was supported by the US Department of Energy Office of Science under Contract No. DE-AC02-06CH11357. I.B. was supported by an award from the Research Corporation for Science Advancement and the Materials Theory Institute at ANL.

APPENDIX A: HEAT FLOW RATE

In this appendix we present a derivation of Eq. (1) for the rate of the heat flow. A general formula for the heat current going from the left electrode (1) toward the right electrode reads

$$I^{1\rightarrow}_{q} = -(\Gamma^{1,\text{in}}_{q} - \Gamma^{1,\text{out}}_{q}).$$

where $\Gamma^{1\rightarrow}_{q} (\Gamma^{1\rightarrow}_{q})$ is the heat transfer tunneling rate calculated in the left electrode:

$$\Gamma^{1,\text{out}}_{q} = \frac{1}{R_i} \int_{\epsilon_{\text{ee}}} (\epsilon - \phi_i) f^{(\text{2})}_{e} \left( 1 - f^{(\text{2})}_{e} \right) P(\epsilon - \epsilon'),$$

$$\Gamma^{1,\text{in}}_{q} = \frac{1}{R_i} \int_{\epsilon_{\text{ee}}} (\epsilon' - \phi_i) \left( 1 - f^{(\text{1})}_{e} \right) f^{(\text{2})}_{e} P(\epsilon - \epsilon').$$

Using the gauge transformation $\phi \rightarrow \phi - \partial \delta k$, the distribution functions transform like $f^{(i')}(\epsilon) \rightarrow f^{(i)}(\epsilon + \partial \delta k)$. Therefore the rates and the heat current defined above are gauge invariant.

Similarly, we can find

$$I^{2\rightarrow}_{q} = -(\Gamma^{2,\text{out}}_{q} - \Gamma^{2,\text{in}}_{q}).$$

where $\Gamma^{2\rightarrow}_{q} (\Gamma^{2\rightarrow}_{q})$ is the heat transfer tunneling rate calculated in the right electrode:

$$\Gamma^{2,\text{out}}_{q} = \frac{1}{R_i} \int_{\epsilon_{\text{ee}}} (\epsilon - \phi_i) f^{(\text{2})}_{e} \left( 1 - f^{(\text{2})}_{e} \right) P(\epsilon - \epsilon'),$$

$$\Gamma^{2,\text{in}}_{q} = \frac{1}{R_i} \int_{\epsilon_{\text{ee}}} (\epsilon' - \phi_i) \left( 1 - f^{(\text{2})}_{e} \right) f^{(\text{2})}_{e} P(\epsilon - \epsilon').$$

The gradient of the heat current, $\nabla I Q$, at the contact is

$$\nabla I Q = I^{2\rightarrow}_{q} - I^{1\rightarrow}_{q} = \{ \Gamma^{1,\text{out}}_{q} - \Gamma^{2,\text{out}}_{q} \} + \{ \Gamma^{2,\text{in}}_{q} - \Gamma^{1,\text{in}}_{q} \}.$$

Finally, we find

$$\nabla I Q = (\phi_2 - \phi_1) I + \frac{1}{R_i} \int_{\epsilon_{\text{ee}}} (\epsilon - \epsilon') P(\epsilon - \epsilon') \times \left\{ f^{(\text{1})}_{e} \left( 1 - f^{(\text{2})}_{e} \right) + f^{(\text{2})}_{e} \left( 1 - f^{(\text{1})}_{e} \right) \right\}.$$

On the other hand, the conservation law demands

$$\dot{Q} + \nabla I Q = \mathcal{E} I,$$

where the right-hand side is the Joule heat, which is related to the work of the electric field. The Joule heat is dissipated in the bulk of the electrodes at distance $l_E$ from the junction, where $l_E$ is the energy relaxation length. The heat $\dot{Q}$ is the heat dissipated into the environment:

$$\dot{Q} = \frac{1}{R_i} \int_{\epsilon_{\text{ee}}} (\epsilon - \epsilon') \sum_{i,j} f^{(\text{1})}_{e} \sigma_i \sigma_i \left( 1 - f^{(\text{2})}_{e} \right) P(\epsilon - \epsilon').$$

where $\sigma_i$ is the Pauli matrix. Equation (A4) can be rewritten in terms of "Bose" distribution functions as follows:

$$\dot{Q} = \int_0^{\infty} d\epsilon \epsilon \rho(\epsilon) [n_T \rho^{\text{C}}(\epsilon) - \left( 1 + n_T \right) \rho^{\text{T}}(-\epsilon)],$$

with $\rho(\epsilon) = 4e^2/R_i$ and $n_T = [\epsilon + (\epsilon - V) N_B (\epsilon - V, T) + (\epsilon + V) N_B (\epsilon + V, T)]/2e$. Equation (A5) coincides with Eq. (1) in
the text. In the zero-voltage limit, Eq. (A5) agrees with the corresponding expression in Ref. 5.

APPENDIX B: HEAT FLOW AT SECOND ORDER IN $\rho(\varepsilon)$

Below Eq. (3), we derived the heat flow $\dot{Q}$ in the leading (first) order in spectral function $\rho(\varepsilon)$. In this appendix we show that the heat flow $\dot{Q}$ at second order in the electron-environment interaction and at first (leading) order in the temperature difference $\delta T = T_e - T_{env}$ leads to the same enhancement as the first order term.

The heat flow can be written as a sum of two terms, $\dot{Q} = (W_1 + W_2)\tau$, where

$$W_1 = \frac{1}{2} \int_{-\infty}^{\infty} \! \! d\varepsilon \varepsilon p(\varepsilon) [\hat{\rho}_n(n_e)] P(\varepsilon), \quad (B1)$$

$$W_2 = \int_{0}^{\infty} \! \! d\varepsilon \varepsilon p(\varepsilon) n_e \tau \{ P(\varepsilon) - P(-\varepsilon) \}. \quad (B2)$$

Here $\hat{T} = (T_e + T_{env})/2$. Typically, $W_1 \lesssim W_2$; therefore we concentrate on contribution $W_2$ below.

At second order in electron-environment interaction [function $\rho(\varepsilon)$], we obtain the following result for the heat flow:

$$\dot{Q}^{(2)} \propto \int_{0}^{\infty} \! \! d\varepsilon d\varepsilon_1 d\varepsilon_2 \varepsilon p(\varepsilon) \rho(\varepsilon_1) \rho(\varepsilon_2) n_{e_1}^{(12)} (1 + N_{e_1}^{(out)}) \times N_{e_2}^{(in)} \delta_{\varepsilon_2 - \varepsilon_1 - \varepsilon_2} + (1 + n_{e_2}^{(12)}) N_{e_2}^{(in)} \delta_{\varepsilon_2 + \varepsilon_1 - \varepsilon_2}$$

$$+ (1 + n_{e_2}^{(12)}) N_{e_2}^{(in)} \delta_{\varepsilon_2 - \varepsilon_1 + \varepsilon_2} - (1 + n_{e_2}^{(12)}) \}$$

$\times N_{e_1}^{(in)} N_{e_2}^{(in)} \delta_{\varepsilon_1 - \varepsilon_1 - \varepsilon_2} - (1 + n_{e_1}^{(12)}) (1 + N_{e_1}^{(out)})$

$$\times N_{e_2}^{(in)} \delta_{\varepsilon_2 - \varepsilon_1 - \varepsilon_2} - (1 + n_{e_2}^{(12)}) N_{e_2}^{(in)} (1 + N_{e_1}^{(out)}) \delta_{\varepsilon_1 + \varepsilon_1 - \varepsilon_2} \}. \quad (B3)$$

At low frequencies ($T \gg \varepsilon$) we find the $W_2$ contribution to $\dot{Q}^{(2)}$ as follows:

$$n_{e_2}^{(12)} (1 + N_{e_1}^{(out)}) (1 + N_{e_2}^{(out)}) - (1 + n_{e_2}^{(12)}) N_{e_2}^{(in)} N_{e_1}^{(in)} \approx n_{e_2}^{(12)} (T_e) - (1 + n_{e_2}^{(12)}) \frac{(T_{env})}{\varepsilon}$$

$$\approx \cdots + (T_e - T_{env}) (1 + 2n_{e_2}^{(12)}) T_e + \frac{T_{env}}{2 \varepsilon} + \cdots. \quad (B4)$$

Here dots represent the terms that finally cancel in Eq. (B3).

We mention the presence of a large enhancement factor ($1 + 2n_{e_2}^{(12)} \approx (T_e + T_{env})/\varepsilon \gg 1$ in Eq. (B4).

In the quasiequilibrium case we do not have this large factor. Indeed, in this case we have

$$n_{e_2}^{(12)} (1 + N_{e_1}^{(out)}) (1 + N_{e_2}^{(out)}) - (1 + n_{e_2}^{(12)}) N_{e_2}^{(in)} N_{e_1}^{(in)} \approx n_{e_2}^{(12)} (T_e)^2 - (1 + n_{e_2}^{(12)}) \frac{(T_e)^2}{\varepsilon}$$

$$\approx \cdots + (T_e - T_{env}) \frac{T_e + T_{env}}{2 \varepsilon} + \cdots. \quad (B5)$$

To conclude, at second order in function $\rho(\varepsilon)$, the heat flow $\dot{Q}$ is enhanced by the same factor ($T_e + T_{env})/\varepsilon \gg 1$ as at first order. Similar results can be proved at higher orders in $\rho(\varepsilon)$.

---

4. Since the current-voltage characteristic $I(V)$ becomes linear only at high voltages, $R_e$ is defined at large $V$.