The Study of the Converse Magnetoelectric Coefficient

of a Composite Multiferroic Ring

A thesis submitted in partial fulfillment of the requirements

For the degree of Master of Science in Mechanical Engineering

By

Mario Lopez

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The thesis of Mario Lopez is approved:

__________________________________________                               __________________
Dr. George Youssef  Date

__________________________________________                                __________________
Dr. Peter L. Bishay  Date

__________________________________________                                __________________
Dr. Michael Kabo, Chair  Date

California State University, Northridge
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Abstract

The Study of Converse Magnetolectric Coefficient of a Composite Multiferroic Ring

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Master of Science in Mechanical Engineering

Composite multiferroic materials that consist of piezoelectric and magnetostrictive phases have been investigated in the past decade as an alternative to control magnetism. This is based on strain transduction from a piezoelectric phase to a magnetostrictive phase. An extensive review is introduced discussing first the fundamentals of each individual phase as well as the behavior of their composite. The focus of this research is to understand the dependency of converse magnetolectric coefficient on the bonding interface, frequency, applied electric field, and bias magnetic field of a composite ring of an outer axially and radially polarized piezoelectric (PZT) and an inner magnetostrictive rings (Terfenol-D, TD). The rings were epoxy-bonded in one arrangement and shrink-fitted in another. This resulted in three different sample arrangements: axially-poled PZT/epoxy/TD, axially-poled PZT/shrink/TD, and radially-poled PZT/epoxy/TD. An experimental design and theoretical modeling were used to characterize the composite rings and provide insight into the underlying strain transfer mechanisms. The composite rings were characterized by changing the electric field from 20-80kV/m, varying the frequency between 4-50kHz, and the bias magnetic field between 0-2300Oe. To accomplish the objectives of this research,
three hypotheses were investigated. It was found that the piezoelectric coefficients in the PZT ring are two orders of magnitude greater than those published. The piezoelectric coefficients were taken from the hoop strain, which has been theoretically shown to be responsible for the radial stress on the Terfenol-D. The radial stress on the Terfenol-D therefore, manipulates ferromagnetic domain walls and generates a magnetic flux within the ring. However, without a magnetic field these domain walls are strongly pinned to inclusions, resulting in a need for a magnetic field to de-pin the walls. Finally, because of the bonding layer and larger hoop strain generation, the axially-poled composite ring resulted in higher CME values than the other two rings.
Chapter 1   Introduction

1.1 Fundamentals

The evolution of electromagnetic energy has advanced steadily over the centuries while engineered composite materials to control magnetism are an active research area. Since the first electromagnetic motor, engineered in 1834 by Moritz Jacobi, electricity and magnetism have become ubiquitous in our daily lives, as they are essential in current technology. Nonetheless, energy efficiency is still a subject of concern, which requires a continuous improvement in the synthesis and fabrication of electronic devices. As a result, increased research interest has recently been placed on multiferroic materials as they promise higher device efficiency and size reduction. Multiferroic composite materials harness strain to control magnetism by means of an electric field, rather than current. Thus, an additional advantage of using multiferroic is the significance reduction in power consumption. Repeated investigations of composite magnetoelastic heterostructures demonstrated that the coupling coefficients are orders of magnitude larger than their single-phase counterparts [1]–[8]. Single-phase magnetoelastic materials have been studied since 1960 [9], [10]. For example, single-phase multiferroic Chromium Oxide has a magnetoelastic coupling coefficient of 0.001mV/cm-Oe, whereas a composite laminate consisting of Terfenol-D and PZT has a coupling coefficient of 4,700mV/cm-Oe [11]. The focus of this research is to experimentally investigate the effect of the bonding interface on the strain transduction of composite magnetoelastic composite material. Specifically, the coupling between the strain-generation, based on the application of electric field, and the strain-induced magnetic field is characterized in concentric multiferroic composite ring under varying electric fields, bias magnetic fields and frequency. The emphasis of the first
chapter is to motivate the significance and importance of magnetoelectric composites while providing in-depth scientific background.

1.1.1 Piezoelectric Materials

In 1880, Jacques and Pierre Curie discovered that a mechanical force could polarize certain crystalline structures. This coupling of the force and polarization is known as the piezoelectric effect. Additionally, a piezoelectric material subjected to an electric field will mechanically deform. Piezoelectric materials exhibit a hysteresis under a varying electric field. As the electric field increases, the corresponding polarization increases until it reaches saturation. Subsequently, a decrease in the electric field causes the polarization to linearly decrease. After reaching saturation polarization, the removal of the electric field will result in a remnant polarization and remnant deformation. In this hysteretic behavior, the loading and unloading path is different due to energy loss in rearranging the molecular structure, to be discussed later in the section.

Piezoelectric properties are hinged on its perovskite crystallographic structure. For example, the phase diagram of lead zirconate titanate piezoelectric is shown in Fig. 1.1, which represents the dependence of the crystal structure on temperature as well as on weight percentage of lead-titanate (PbTiO₃). Notably, the ratio between lead-titanate (PbTiO₃) and lead-zirconate (PbZrO₃) is tuned to optimize the electromechanical coupling (i.e., piezoelectric coupling coefficients) and dielectric constants, which are found to be maximized around the Morphotropic Phase Boundary (MPB). The MPB describes the transition between rhombohedral and tetragonal regions [12]. Above the Curie temperature, piezoelectric materials have a cubic centered structure (Fig. 1.2a). However,
below the Curie temperature and at PbTiO$_3$ concentrations slightly above 50%, spontaneous polarization (electric dipole) occurs causing a physical shift of the central atom (Fig. 1.2b). This physical shift changes the crystal structure from cubic to tetragonal. As a result, an applied electric field on the crystal can increase the electric dipole. Correspondingly, it causes an increase in the mechanical strain, where the greatest effective strain is reached in a tetragonal phase [13].

![Fig. 1.1: PZT (Lead zirconate titanate) Phase Diagram (modified from [14])](image1)

![Fig. 1.2: Dashed circle (Lead), Open circle (Oxygen), Solid circle (Titanate, Zirconate) (a) Cubic Structure of PZT b) Tetragonal Structure of PZT [15]](image2)
In the poly-crystalline state, domains have polarization in different directions, i.e. randomly polarized, as shown in Fig. 1.3a. An applied electric field will align the domains along its direction (Fig. 1.3b). Upon saturation and removal of the applied electric field, the polarization of the domains remain locked in the direction of the previously applied field (Fig. 1.3c). The domains in this remanent polarization state will not completely align as some domains will attempt to relax to their initial position, but will be resisted by adjacent domains. The relationship between the applied electric field (E) and polarization (P) is usually captured in a P-E hysteresis loop.

![Diagram showing random polar domain orientation, polarization under a DC Electric field, and remnant polarization after electric field removed.](image)

**Fig. 1.3:** a) Random polar domain orientation b) Polarization under a DC Electric field c) Remnant polarization after electric field removed ([16])

When studying the P-E hysteresis loop, the focus is on four characteristics, which are initial polarization, saturation polarization, remanent polarization, and coercive field. A typical P-E loop and the corresponding characteristics are shown in Fig. 1.4. Region 1 corresponds to the initial polarization process, where the domains are randomly oriented then become aligned to the direction of applied electric field. The 2nd point corresponds to the polarization saturation of the piezoelectric, where any further increase in the electric field will not yield any increase in the polarization of the material. The 3rd point corresponds to the remanent polarization of the material as the external electric field is completely removed. Finally, the 4th point corresponds to the coercive field, at which the material has
zero net polarization and it reverses the polarization. From a practical perspective in designing a load cell, for example, the linear region between remanent and saturation polarization is used to relate application of an external force to the change in polarization. This creates a standalone force sensor without the need of a power source [16].

![General PZT hysteresis curve i.e. Polarization vs Electric Field](image)

*Fig. 1.4: General PZT hysteresis curve i.e. Polarization vs Electric Field*

Similarly, the interaction between the strain ($\varepsilon$) and electric field (E) is captured in a $\varepsilon$-E hysteresis loop; an example of which is shown in Fig. 1.5. The $\varepsilon$-E hysteresis loop has one region and three points of interest analogous to those of the P-E hysteresis loop. Region 1 presents the initial strain as the material is initially polarized. The 2nd point corresponds to the saturation strain, which is the mechanical correspondent to the saturation polarization. When the electric field is removed, (3rd point) the piezoelectric crystal remains deformed but strain decreases. At this point when an opposing field is applied, the piezoelectric approaches an unstrained state (4th point). This unstrained state occurs at the coercive field.
The hysteresis loops (i.e. P-E and ε-E) are experimentally obtained to provide the piezoelectric properties of the material. To determine the P-E and ε-E loops, the polarization and strain are experimentally measured by a Sawyer-Tower circuit and strain gauges, respectively. Fig. 1.6 shows the experimental setup as depicted by Lynch [15]. The electric charges resulting from the electric dipole accumulate on the surface capacitor plate with surface area (A), and in turn cause a voltage (V) drop across the capacitor (C_f), where the polarization (P) is \( P = \frac{Q}{A} = \frac{C_f V}{A} \). On the other hand, the change in shape is measured by a strain gauge, where the signal is conditioned using a Wheatstone bridge such that the change in the resistance is related to the strain (\( \varepsilon = \frac{\Delta R}{R GF} \), where GF is the gauge factor).
Lynch initially subjected a piezoelectric material to a varying electric field while measuring the voltage drop across the reference capacitor ($C_f$) and later reproduced the experiment by varying applied stress. The resulting ferroelectric responses are shown in Fig. 1.7, in which the first four subplots (Fig. 1.7A-D) report the material response as a function of applied electric field while the remaining four subplots (Fig. 1.7E-H) are measured with respect to applied stress. Fig. 1.7A is the response of the polarization ($D_3$) along the direction of the applied electric field ($E_3$). The slope of the $D$-$E$ loop at zero applied field defines the permittivity ($\beta_{33}$) of the piezoelectric material. Fig. 1.7B is the strain response ($\varepsilon_{33}$) along the direction of the applied electric field ($E_3$), where the slope of the $\varepsilon_{33}$-$E_3$ loop at zero applied field defines the piezoelectric constant ($d_{33}$). On the other hand, Fig. 1.7C shows the strain response ($\varepsilon_{11}$) transverse to the applied electric field with the resulting slope defines the piezoelectric constant ($d_{311}$). Fig. 1.7D shows the ratio of $\varepsilon_{11}$ and $\varepsilon_{33}$, analogous to Poisson’s ratio, plotted with respect to the applied electric field. The ratio is approximately 0.5 indicating a conservation of volume [15]. However, Lynch interestingly noted that when the electric field reaches the coercive field, the volume appears to slightly
increase as shown by the increase in the $\frac{\varepsilon_{11}}{\varepsilon_{33}}$ ratio above 0.5. The deformation as function of the stress-cycle, Fig. 1.7E, is the strain response ($\varepsilon_{33}$) along the direction of the applied stress ($\sigma_{33}$). The resulting slope at the initial application of stress is the compliance ($S_{3333}$). Similarly, Fig. 1.7F is the transverse strain response ($\varepsilon_{11}$), where the resulting slope is the compliance ($S_{1133}$). Fig. 1.7G is the polarization ($D_3$) as a function of applied stress, where the resulting slope is the piezoelectric constant ($d_{333}$) under stress rather than electric field. Finally, Fig. 1.7H shows the Poisson’s ratio plotted against longitudinal strain ($\varepsilon_{33}$), which allows a direct comparison with the stress-strain curve in Fig. 1.7E. As the applied compressive stress increases on the polarized PZT the Poisson’s ratio increase then decrease.
Fig. 1.7: Experimental Results of a piezoelectric material [15]

Generally, the aforementioned material piezoelectric constants presented are used in the linear constitutive relationship (Eqn. 1 and 2).

\[ \varepsilon_{ij} = S_{ijkl}^{p} \sigma_{kl} + d_{nij} E_{n} \]  
\[ D_{m} = d_{mkl} \sigma_{kl} + \beta_{mn} E_{n} \]

Here, the generated strain tensor \( (\varepsilon_{ij}) \) is a function of the fourth-order compliance tensor \( (S_{ijkl}^{p}) \), stress tensor \( (\sigma_{kl}) \), piezoelectric coefficients tensor \( (d_{nij}) \), and electric field vector \( (E_{n}) \). Correspondingly, the resulting electric displacement vector \( (D_{m}) \) is a function of the
piezoelectric coefficients tensor \((d_{mk})\) and permittivity tensor \((\beta_{mn})\). This linear model is used to predict the piezoelectric response and is used in the design and analysis of stack actuators and sensors.

### 1.1.2 Piezomagnetic Materials

In 1847, Joule discovered that iron, a ferromagnetic material, deforms when subjected to a magnetic field. This coupling of magnetism and deformation is termed magnetostriction. Conversely, a ferromagnetic material is magnetized in response to a mechanical force in the presence of bias magnetic field. When the magnetic field is increased, the resulting magnetization will increase until it reaches saturation. After reaching saturation, the removal of the magnetic field results in a remanent magnetization (below the Curie temperature) and a corresponding residual strain. This response can be either addressed at the atomic level using quantum mechanics or using the effective response of magnetic domains in the continuum sense. Since this research study is focused on the macroscopic response, magnetostriction discussion herein is based on continuum mechanics rather than quantum mechanics.

Macroscopically, the properties are measured by magnetization experiments using a magnetometer. A magnetic field is applied by controlling the current passing through a coil, also known as the primary coil, while the induced magnetization is sensed using another coil (i.e., secondary coil) that is wrapped around the sample. The induced magnetic field \((H)\), described using Eqn. 3, is directly proportional to the number of turns in the coil \((N)\) and the current \((I)\) passing through, while it is inversely proportional to the length of the coil \((L)\). A magnetic field acting on the secondary coil will create an electromotive
force ($V_{emf}$), which is also related to the number of turns in the coil and the time rate of the magnetic flux ($\phi$) as shown in Eqn.4.

$$H = \frac{NI}{L}$$ \hfill (3)

$$V_{emf} = -N \frac{\Delta \phi}{\Delta t}$$ \hfill (4)

In essence, magnetic flux density ($B$), magnetization ($M$), and magnetic field ($H$) are interrelated through the material. A sample with an area cross section ($A$) and an applied magnetic field created by the primary coil will have a magnetic flux $\phi_{applied} = \mu_o HA$, where $\phi$ are magnetic lines of force (Fig. 1.8). In response, the secondary coil will sense the induced magnetic flux $\phi_{induced} = BA$. Under an applied magnetic field, the sample will form north and south poles, where the strength of the poles is referred to as magnetization. The magnetization will add lines of magnetic force to the $\phi_{applied}$ depending on the magnetic permeability of the material. It is noted that $\phi_{induced} \gg \phi_{applied}$ in ferromagnetic materials due to the magnetization [17]. The combination of the sets of magnetic lines of force is, therefore, the magnetic flux or lines of induction ($B$). The relationship between $H$ and $M$ is

$$B = \mu_o (H + M)$$ \hfill (5)

where, $\mu_o$ is the permeability of free space ($4\pi x 10^{-7}$ N/A$^2$).
Similar to their ferroelectric counterparts, the properties of a ferromagnetic material are magnetic field dependent and are usually depicted in a B-H hysteresis loop (Fig. 1.9). Here, the B-H hysteresis loop is also divided into one region and three points. The first Region corresponds to the initial magnetization of the material, which increases with increase in the applied magnetic field. Although the underlying mechanism is different, the induction of ferromagnetic materials and polarization of ferroelectrics show the same behavior on the hysteresis loop. The slope of a line extending from the origin to a point on the initial B-H curve, Region 1, defines the material’s Permeability ($\mu$). 2nd point corresponds to the saturation of the material. Nonetheless, the saturation is better observed in the M-H line (dashed line on Fig. 1.9) since the curve levels off rather than continuously increasing as on the B-H curve (solid line). The latter phenomenon is elucidated in Fig. 1.8, where the magnetic flux density from the sensing coil accounts for the material (i.e. permeability) and the external magnetic field. The 3rd point corresponds to the spontaneous or remnant magnetization when the external magnetic field is removed. Finally, the 4th point corresponds to the coercive field, at which a positive magnetization is switched to a negative magnetization as the applied field decreases or vice versa.

Fig. 1.8: Longitudinal cross-section of an Iron sample (Modified from [17])
The interaction of magnetic domains and external magnetic field is shown in Fig. 1.10, which is the underlying mechanism responsible for the hysteretic behavior discussed previously. Ferromagnetic materials intrinsically are broken into domains and must be forced into a magnetized state by an applied external field. Schematically, Fig. 1.10a shows two magnetically saturated and opposing domains separated by a domain wall. This arrangement results in a zero net magnetization. When subjected to an external magnetic field, Fig. 1.10b, the domain in the general direction of the field grows as the domain wall moves to minimize the opposing domain. This process continues as the magnetic field increases until the domain wall is annihilated as shown in Fig. 1.10c. This results in a net magnetization relative to the original \( M_s \) and orientated respective to the direction of the applied field. Finally, once the magnetization is completely aligned with the applied magnetic field, the material is referred to as magnetically saturated.

Fig. 1.9: Magnetization curves and hysteresis loops [17]
Similar to magnetization, the magnetostriction ($\lambda$) (Fig. 1.11) can be related to magnetic domains as illustrated in Fig. 1.12, where the arrows indicate the magnetization direction. Magnetostriction is the strain generated under a magnetic field. Each domain is at saturation magnetization and saturation magnetostriction [17]. Above the Curie temperature, the materials do not exhibit any ferromagnetic or magnetostriction behaviors (Fig. 1.12a). Below the Curie temperature, the material spontaneously deforms (Fig. 1.12b). The application of an external magnetic field will result in a reorientation of the domains along the field direction, which in turn results in deformation (Fig. 1.12c).
The effect of mechanical stress is similar to that of an applied external magnetic field (e.g., Fig. 1.10). Specifically, Fig. 1.13a shows zero net magnetization of a material due to the random orientation of the magnetic domains. The application of stress results in the movement of magnetic domain walls (Fig. 1.13b). As the stress increases, magnetic domains will merge and align in the direction of applied stress (Fig. 1.13c). Subsequently, the application of external magnetic field in presence of the mechanical stress causes the annihilation of the remaining domain walls (Fig. 1.13d). Basically, the mechanical stress has two effects on the magnetic state of the material, which are: 1) change in the
magnetization due to the domain wall movement; and 2) create an easy axis of magnetization that requires lower applied magnetic field to reach saturation.

Finally, the magnetostrictive response is used to calculate the piezomagnetic coefficients ($\gamma$). These material constants are used in the linear piezomagnetic constitutive relationship (Eqn. 6 and 7) in design and analysis of sensors and actuators. The generated strain tensor ($\varepsilon_{ij}$) is a function of the fourth order compliance tensor ($S^m_{ijkl}$), stress tensor ($\sigma_{kl}$), piezomagnetic coefficient tensor ($\gamma_{nij}$), and magnetic field vector ($H_n$). Correspondingly, the resulted magnetic flux density vector ($B_m$) is a function of the piezomagnetic coefficients tensor ($\gamma_{mkl}$) and permeability ($\mu_{mn}$).

$$\varepsilon_{ij} = S^m_{ijkl}\sigma_{kl} + \gamma_{nij}H_n$$  \hspace{1cm} (6) \\
$$B_m = \gamma_{mkl}\sigma_{kl} + \mu_{mn}H_n$$  \hspace{1cm} (7)
1.1.3 Magnetoelectric Materials

In 1820, Oersted discovered that an electric current generates a magnetic field as it passes through a conductor. Thereafter, Ampere quantifiably showed (Fig. 14) the mathematical relationship between current passing through a wire and a magnetic field (Eqn. 8), where the magnetic flux density ($B$) is a function of current ($I$), permittivity of free space ($\mu_0$), and the radius ($r$).

$$B(r) = \frac{\mu_0 I}{2\pi r}$$ \hspace{1cm} (8)

This simple concept is the underlying physical principle on which most current technological advancements are hinged. That is, electric current (or moving charges) has been the only method to control magnetism.

Nonetheless, a major setback is the increase in Ohmic losses as the size of the wire shrinks down to the nanoscale due to the increase in the resistance (i.e. $R = \frac{\rho L}{A}$, where $\rho$ = resistivity, $L$ = length of wire, $A$ = cross-sectional area of wire). This in turn presents a scientific barrier for future miniaturization, reduction in the power consumption (i.e. $P = I^2 R$, where $I$ = current, $R$ = resistance), and improve energy efficiency of electronic devices.
To better illustrate this scientific barrier, consider a 1m long copper wire with a radius of 0.1mm and 1mA electric current passing through it. Assuming all the electric energy is lost as heat, this scenario will generate 53W of heat. Now, consider the same wire but with a reduction in the radius to 0.01mm. This will increase the power dissipation to 5347W. Thus, a decrease by a one order of magnitude in the radius corresponds to two orders of magnitude increase in the power dissipation. That is to say, the coupling of electricity and magnetism on the nanoscale requires a completely new physical perspective.

In response to this scientific challenge, multiferroic materials have been investigated and interest has increased over the past few decades. Pierre Curie first hypothesized these materials, where magnetization and strain can be coupled in a manner analogous to his discovery of piezoelectric materials [18]. Thereafter, Debye coined the term magnetoelectric based on a theoretical hypothesis that such material could exist [19]. Landau was successful in analytically proving that single-phase magnetoelectric materials exist [20]. Shortly after, Astrov experimentally discovered that Chromium (III) Oxide (Cr₂O₃) exhibit multiferroic properties, i.e. the first magnetoelectric material [10], [21]. However, the majority of intrinsic homogenous multiferroic materials have weak magnetoelastic coupling at room temperature [10]. On the other hand, composite multiferroic have been shown to be effective at room temperature. For example, Van Suchtelen and van den Boomgaard created the first strain-mediated multiferroic composite that consisted of barium titanate (BaTiO₃) as the ferroelectric phase and cobalt ferrite (CoFe₂O₄) as the ferromagnetic phase [22]. Based on the scientific and practical importance of multiferroic materials, the focus of this research is on composite multiferroic.
Multiferroic composite structure for magnetoelectric (ME) coupling consists of piezoelectric and magnetostrictive constituents. Fig. 1.15a presents a typical composite laminate design, which can be used in two different ways. First, Direct Magnetoelectric Effect (DME) uses ME composites to generate an electrical field by applying a magnetic field (Fig. 1.15b). That is, the magnetostrictive strain is transferred to the piezoelectric material; in turn, the strain manipulates the electrical domains generating electrical charges. Thus, the DME coupling coefficient is the ratio of the generated electric field to applied magnetic field. On the other hand, Converse Magnetoelectric effect (CME) is the result of magnetic flux density in response to the applied electric field (Fig. 1.15c). Here, the piezoelectric strain is transferred to the magnetostrictive layer resulting in a magnetic flux density.

Fig. 1.15: Magnetoelectric Composites a) general laminate representation b) DME c) CME
1.2 State-of-the-art in Multiferroic

An exhaustive literature review revealed a pattern in research of multiferroic materials, which is illustrated in Fig. 1.16. First, researchers focused on a single-phase material, and then research evolved to focus on a macroscale laminated composite that consisted of two or more different ferroic materials. Thereafter, the direct and converse magnetoelastic coupling of various material systems were experimentally measured and characterized. Based on the experimental observation, simple models were first developed then, more detailed and sophisticated models to account for demagnetization, shear lag, and anisotropy followed. Once understood at the macroscale, the next step was to focus on microscale modeling and fabrication. The present thesis continues the experimental investigation in Experimental Mechanics Laboratory (EML) [23] by following the same trajectory as a priori research reported in the literature. Specifically, the experimental and numerical simulation results of concentric composite ring geometry are the prime emphasis of this research study. It is the focus then of the following section to review the available literature with emphasis on discoveries of piezoelectric, piezomagnetic, and multiferroic materials.
An extensive review of piezoelectric materials was done with emphasis on the macroscale response to external applied electric field and applied stress. Of specific interest to this research is lead zirconate titanate, denoted as PZT hereafter, to which majority of the reviewed studies is attributed. Mitrovic et al. presented a detailed analysis on the change in elastic modulus and piezoelectric properties in terms of energy density [24]. The piezoelectric response has a dependence on preload and the number of cycles. Lynch and Fan recently presented a method to characterize piezoelectric properties to determine the linear coefficients and to report on the nonlinear behavior [15], [25]. They also reported a nonlinear representation of constitutive relationship that uses a mechanical model to describe polarization switching, which was found to agree with experimental data. Randall and Ricote observed the ferroelectric domain configurations for PZT ceramics by use of microscopy and were able to visually analyze the domain formation [26], [27]. Additionally, Masys, Zhang and others experimentally and analytically showed that the material properties of piezoelectric materials have a dependence on frequency, composition, and applied stress [28]–[33]. For example, Masys et al. presented the piezoelectric response of two different PZT compositions, PZT EC-65 and PZT EC-69, under various frequency as well as DC and AC electric fields (Fig. 1.17). Fig. 1.17a presents the piezoelectric response to frequencies up to 1kHz of a PZT EC-65, where a ~13% decrease of the coefficients is noted. On the other hand, the response of PZT EC-65 to applied AC electric field at various applied DC electric fields is shown in Fig. 1.17b. As the AC electric field increases, the piezoelectric coefficient increased by up to 140% while an increase in the DC electric field generally decreases the coefficients (Fig. 1.17b). Alternatively, Fig. 1.17c shows the frequency response of a PZT EC-69, where the
piezoelectric coefficients appear to be frequency independent in comparison to those reported for PZT EC-65 counterpart (Fig. 1.17a). The piezoelectric coefficients have a similar trend that increase with the AC field. Contrary to the PZT EC-65 composition, the piezoelectric coefficient increases as the DC field is raised (Fig. 1.17d).

Zhang investigated the piezoelectric coefficient response of two types of PZT, namely PZT-5A and PZT-5H under the applied stress (Fig. 1.18). As the stress is increased, the piezoelectric coefficient ($d_{33}$) decreased by ~38% and 30% for both PZT-5A and PZT-5H, respectively. In short, the composition of the piezoelectric materials contributes to the overall response.
Similarly, an extensive review of ferromagnetic domain interactions was done in order to comprehend the macroscopic mechanical response. Brown discussed in detail the macroscopic effect of ferromagnetic domains and magnetization, where the manipulation of the domains by magnetic field or mechanical stress leads to spontaneous magnetization \[34\]. Additionally, Ferro experimentally demonstrated that magnetic domains can align parallel (negative magnetostriction) or perpendicular (positive magnetostriction) to compressive stress \[35\]. Further insight of domain interactions have also been shown along with importance of crystal orientation on the performance \[36\]–\[40\]. For example, Al Jiboory presented evidence using Lorentz microscopy of the interaction of domains under a magnetic field \[41\]. However, experiments and models have been done in order to understand the domain interactions in ferromagnetic domains; this is still a subject of study.

Of specific interest to this research is Terfenol-D. Zhao and Lord reported the response of Terfenol-D along different crystal directions as well as the demagnetization effecting \[42\]. Specifically, they showed that as demagnetization increases, so does the number of “easy” magnetization directions. Therefore, there will be a fraction of domains
magnetization in different directions resulting in a decrease of magnetization in certain directions. Moffett and others characterized Terfenol-D and found the dependence of its properties on frequency, applied magnetic field, and applied stress [43]–[53]. For example, the piezomagnetic coefficients become independent of stress at high magnetic fields (~1.5kOe) [43], where the piezomagnetic response is dominated by the magnetic fields rather than applied stresses. Davino reports that Terfenol-D becomes magnetically harder as stress is increased, resulting in a decrease of piezomagnetic coefficients [46]. At low pre-stresses (3.72MPa) and magnetic field that range between 0 and 314Oe, the piezomagnetic coefficients are ~50% greater than at large pre-stresses (28MPa) when tested within the same range of magnetic field. Additionally, the magnetic field was shown to also change the magnetic permeability, where an increase in external magnetic field corresponds to a decrease in the magnetic permeability, thus a decrease in magnetization.

Throughout the aforementioned experimental investigations, the overall behavior of Terfenol-D was found to be nonlinear and highly dependent on applied magnetic field and stress resulting in difficulty in predicting their response. Therefore, models of ferromagnetic materials were formulated based on their magnetostriction and magnetization hysteresis curves [54]–[66]. These models discuss physical obstacles that affect Terfenol-D and generally all ferromagnetic materials, i.e., crystalline anisotropy, demagnetization, and change in the elastic modulus (‘Delta E’). For example, Kellogg presented detailed data on the effect of the apparent Young’s modulus (‘Delta E’) within a range of applied field [0 – 200kA/m] and at different levels of stress (Fig. 1.19) [45]. As the applied field increases, the modulus decreases to minima then increases again. Kellogg extracted the modulus minima of the elastic modulus and plotted it against the
corresponding magnetic field as shown in Fig. 1.20. They noted the minimum elastic modulus monotonically increases in response to increase in externally applied stress. Additionally, the magnetic field corresponding to the minimum modulus was found to also increase as function of increasing applied stress [45].

Fig. 1.19: Young’s Modulus w.r.t. Magnetic field at Different Stress Levels [45]
Kellogg also found an interesting correlation between the modulus minima and the induction rate extreme labeled in Fig. 1.21, where the induction rate extreme is largest induction rate relative to each experiment. This correlation dictates that there exists combination of stress and magnetic field, which results in an apparent modulus minimum and therefore an induction rate extreme. In summary, the stress and applied field conditions affect the overall stiffness of the materials, which subsequently affect the magnetic induction.
Fig. 1.21: Induction rate w.r.t. Applied Stress at different Applied H Fields [45]

The Mangetoelectric effects of laminate, disk-ring, and layered ring-ring composite structures have been studied analytically and experimentally [67]–[83]. Li et al. [84] have reported DME coefficients of 1900mV/cm-Oe for disk-ring composites. Jia et al. [85] are one of the first to study the CME effect on a PMN-PT/Terfenol-D/PMN-PT laminate and reported a maximum CME of 450mG/V. Wu et al. studied the CME effect on a piezofiber/Metglas laminate and reported a maximum of 66.2mG/V [79]. Hockel et al. studied the CME effect on a PZT/Terfenol-D/PZT laminate and reported a maximum CME of about 4700mG/V [86]. Chang and Carman have proposed a quasi-static theoretical model investigation of shear lag and demagnetization on the DME of laminate composites indicating a DME dependence on material choice, excitation frequency, magnetic bias field, and electric bias fields [87], [88]. The resonance frequencies have been seen to have the largest magnetoelectric effects [85], [89]–[107]. These resonances have been termed
magnetomechanical resonance (MMR) and electromechanical resonance (EMR), respectively. For example, Hockel et al. reported an EMR frequency near 35kHz as shown in Fig. 1.22.

![Graph: Frequency response of the CME coefficient](image)

**Fig. 1.22: Frequency response of the CME coefficient** [86]

Particularly, Hockel’s investigation of the CME response in a PZT/TD/PZT laminate is shown in Fig. 1.23, where the maximum was shown at 10V (rms) AC electric field, 700V DC electric field, and 250Oe bias magnetic field. The resulting CME coefficients increase with the increase of DC voltage on the laminate (Fig. 1.23). Piezomagnetic coefficients of the laminate under a bias DC voltage are shown in Fig. 1.24. The added pressure on Terfenol-D from the piezoelectric phase based on the DC voltage result in increase in the piezomagnetic coefficients, which can be correlated to the increase in CME with respect to the DC voltage (Fig. 1.23).
Despite the extensive research done thus far on multiferroic heterostructures, ring structures have been the least investigated with even less attention given to composite cylinder structures [22], [108]–[113]. Zhang, Leung, and Li have notably experimentally investigated the ME effects of a magnetostrictive/piezoelectric ring structures[81], [114], [115]. Hou reported a method for solving the transient response of the plane strain problem of magneto-electric-elastic hollow cylinder under dynamic loads [116], [117]. Also, Wang
reported on a method to solve the axisymmetric plane strain dynamic problem of a special
non-homogeneous magneto-electro-elastic hollow cylinder under dynamic loading [118].
Although several analytical studies of cylindrical multiferroic composites have been
conducted solutions for the CME coefficient, experimental efforts to determine the CME
of composite multiferroic cylinder are lacking.

1.3 Research Hypothesis

The present study will present the research on concentric multiferroic composite
rings (Fig. 1.25) to understand the effect of shape and bonding interface on the
performance. In response, three hypotheses are investigated.

![Concentrically Bonded Multiferroic Composite Ring](image)

**Fig. 1.25: Concentrically Bonded Multiferroic Composite Ring**

The research hypotheses are:

1) The converse magnetoelectric coefficients of the composite ring are greater than
in existing laminate structures due to the change in geometry;

2) Solution of Lamé’s equation for concentric composite multiferroic structure
reports reasonable agreement with experimental results; and

3) Since an applied stress re-orient the magnetic domains in Terfenol-D, the bias
magnetic field can be replaced by applying a pre-strain on the composite.
The remainder of the thesis is divided into four chapters. In the following chapter, the formulation of theoretical models as well as the experimental setup are discussed. Chapter 3 includes the results of all theoretical models and experimentations. Discussion of results are presented in Chapter 4. Finally, future work based on the outcomes of this research are included in Chapter 5.
Chapter 2    Modeling and Experiments

The process of answering the hypotheses requires the synthesis of new theoretical models and experimental design (which follow the Design of Experiment (DoE) process) for characterizing the composite rings. On the one hand, the objective of the experiment is to determine the CME coefficients of three differently constructed composite rings. To accomplish this, four physical variables were controlled, namely frequency, applied electric field, bias magnetic field, and bonding interface. The experimental setup previously developed by Chavez [119] at the Experimental Mechanics Laboratory was adopted herein. On the other hand, the models were formulated and validated using the experimental results. The objective of these models is to help understand the underlying mechanics of strain transduction in strain-mediated multiferroic composite rings. In this Chapter two theoretical models will be first introduced, which will then help explain the experimental results from the individual and composite rings.

2.1    Theoretical Modeling

Two theoretical models were developed to help explain the experimental results. In the first attempt, the geometry and compliance of the bonding layer were neglected, which resulted in the introduction of the interface quality factor in the general piezoelectric and piezomagnetic constitutive relationships. In essence, the ability of the interface to allow strain transduction between the piezoelectric to the piezomagnetic phases was determined based on the losses during the strain transfer process. This approach, in general, yielded good agreement with the experimental results but it provided no insight into the strain distribution within the composite structure. Thus, a second modeling attempt was pursued, in which the geometry and mechanical properties of the bonding layer was included in the
initial formulation. In turn, the strain distribution in the piezoelectric, bonding, and the piezomagnetic layers can be easily calculated. In the next two subsections, the details of the model development are discussed.

2.1.1 Quality Factor Model

The foundation of the quality factor model is based on the boundary conditions of the strain continuity and force equilibrium at the interface for a laminate composite as shown in Fig. 2.1 [78], [85]. We start by recalling the generalized constitutive relationships that were discussed in Chapter 1. Here, we reduce the equations to focus on the interaction between two orthogonal directions, where the electric field ($E_3$) and magnetic field ($H_1$) are applied and the resulting stress ($\sigma$) and strains ($\varepsilon$) are in the 1-direction.

\begin{align*}
\varepsilon_{11}^p &= S_{11}^p \sigma_{11}^p + d_{31} E_3 \\
D_3 &= d_{31} \sigma_{11}^p + \beta_{33} E_3 \\
\varepsilon_{11}^m &= S_{11}^m \sigma_{11}^m + \gamma_{11} H_1 \\
B_1 &= \gamma_{11} \sigma_{11}^m + \mu_{11} H_1
\end{align*}

As mentioned, the first boundary condition is force equilibrium at the interface can be expressed as

$$\sigma_{11}^p t_p = -\sigma_{11}^m t_m$$
where, $t_p$ and $t_m$ are the PZT and Terfenol-D thicknesses, respectively. On the other hand, the second boundary condition is strain transduction at the interface defined as

$$\varepsilon_{11p} = \xi \varepsilon_{11m} \tag{14}$$

where, $\xi$ is the newly introduced quality factor and takes values greater than 1. When $\xi=1$, there is a perfect strain transduction and as the value of $\xi>1$, the transduction decreases. The steps for solving the constitutive equations (Eqn 9-12) by imposing the previously stated boundary conditions are as follows. Substituting Equations 9 and 11 into Equation 14 and 12, neglecting $H_1$, based on assumption of no direct interaction between electric and magnetic field in this configuration, i.e. dielectric materials.

$$\sigma_{11}^p S_{11}^p + d_{31} E_3 = \sigma_{11}^m \xi S_{11}^m \tag{15}$$

$$-\sigma_{11}^m \frac{t_m}{t_p} S_{11}^p + d_{31} E_3 = \sigma_{11}^m \xi S_{11}^m \tag{16}$$

$$d_{31} E_3 = \sigma_{11}^m (\xi S_{11}^m + \frac{t_m}{t_p} S_{11}^p) \tag{17}$$

$$\sigma_{11}^m = \frac{d_{31} E_3}{\xi t_p S_{11}^m + t_m S_{11}^p t_p} \tag{18}$$

$$B_1 = \frac{\gamma_{11} d_{31} E_3}{\xi t_p S_{11}^m + t_m S_{11}^p t_p} \tag{19}$$

Now, by rearranging Eqn. 19 in accordance to the definition of the converse magnetoelectric coefficient such that CME is the ratio between induced magnetic flux and applied electric field, it yields
\[ CME = \frac{y_{11} d_{31}}{\xi t_p s_{11}^m + t_m s_{11}^p} \]  

The thickness and material property interaction shown in the Eqn. 20 provides an insight to the CME response, as shown in Fig. 2.2. Three important interrelation between the CME, geometry, and material properties were noted. First, as the thickness increases for either phase, denoted by the downward pointing arrow in Fig. 2.2a, the CME values decrease. This states the performance of thinner composite structure is superior in comparison to that of a thick composite, which is justified since the underlying mechanism is strain generation and transfer. That is, the strain of a slender beam is higher than of a prismatic beam in response to the same loading conditions. Second, degradation in the quality of the bonding interface (i.e., value of \( \xi \)) between the two phases results in a decrease of CME values. As previously stated, the greater the value of the quality factor, the worse the strain transduction quality is (Fig. 2.2b). Finally, the effect of the material properties is shown in Fig. 2.2c, where steps of 10% decrease of the magnitude in piezoelectric coefficients results in a large difference of CME values. However, this model is sufficient to quantify the quality of the interface for a laminate; it does not provide insight into the strain distribution in the piezoelectric, bonding, or magnetostrictive layers. Hence, the motivation to seek a more descriptive model.

![Fig. 2.2: Insight on Quality Factor Model in terms of varying a) thickness b) quality factor c) piezoelectric coefficients](image-url)
2.1.2 *Lame’s Approach*

The solution to Lame’s equations in the cylindrical coordinate system is based on the pressure due to interference between concentric rings, as shown in Fig. 2.3. This is done by solving system of equations for the unknown Lame’s coefficients (A and B). The resulting solution is then used to describe the stresses within each ring in the composite structure. The hoop, radial, and axial stresses are defined using Equations 21-23.

\[
\sigma_H = A + \frac{B}{r^2} \quad (21)
\]

\[
\sigma_r = A - \frac{B}{r^2} \quad (22)
\]

\[
\sigma_a = 0 \text{ "open – ended cylinder"} \quad (23)
\]

Where, constants A and B are determined by radial pressure boundary conditions on the outside and inside surfaces of the respective layer.

![Concentric Rings Schematic](image)

**Fig. 2.3: Concentric Rings Schematic**

The aforementioned equations are applied three times to find the A and B constants for the piezoelectric, bonding, and magnetostrictive layers, respectively, based on corresponding boundary conditions. The boundary conditions are stated as
\[
\begin{align*}
\sigma_{ro} &= 0 \; \text{for Outer Layer} \\
\sigma_{ri} &= -P_p \\
\sigma_{ro} &= -P_p \\
\sigma_{ri} &= -P_m \; \text{for Middle Layer} \\
\sigma_{ro} &= -P_m \\
\sigma_{ri} &= 0 \; \text{for Inner Layer}
\end{align*}
\]

where, \( P_p, P_m, r_o, \) and \( r_i \) are the pressure on the PZT, pressure on the Terfenol-D, outer radius, and inner radius, respectively.

After the definition of the boundary conditions, Eqn. 21 and 22 were solved as simultaneous algebraic equations for each of the three layers. The Lame’s constants for the piezoelectric layer were obtained as:

\[
\begin{bmatrix}
1 & -1 \\
-1 & r_o^2 \\
1 & -1 \\
r_i^2 & r_i
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
0 \\
-P_p
\end{bmatrix}
\]

(25)

\[B^e = P_p \left[ \frac{r_o^2 r_i^2}{r_o^2 - r_i^2} \right]\]

(26)

\[A^e = P_p \left[ \frac{r_i^2}{r_o^2 - r_i^2} \right]\]

(27)

It is worth noting that for reminder of this Chapter, sub- or superscript “\(p\)” indicates association with the piezoelectric layer, “\(b\)” for bonding layer, “\(m\)” for magnetostrictive layer, “\(o\)” for outer radius, and “\(i\)” for inner radius. Similarly, for the bonding layer the equations are as follows.

\[
\begin{bmatrix}
1 & -1 \\
-1 & r_o^2 \\
1 & -1 \\
r_i^2 & r_i
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
-P_m \\
-P_p
\end{bmatrix}
\]

(28)
\[ B^b = (P_p - P_m)(r_i^2r_o^2)/(r_i^2 - r_o^2) \]  

(29)

\[ A^b = (P_pr_o^2 - P_mr_i^2)/(r_i^2 - r_o^2) \]  

(30)

Lastly, the boundary conditions imposed on Lamé’s equations for the magnetostrictive layer yield the A and B constant for Terfenol-D ring.

\[
\begin{bmatrix} 1 & -\frac{1}{r_o^2} \\ -\frac{1}{r_i^2} & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -P_m \\ 0 \end{bmatrix}
\]  

(31)

\[ B^b = P_m(r_i^2r_o^2)/(r_i^2 - r_o^2) \]  

(32)

\[ A^b = P_m(r_o^2)/(r_i^2 - r_o^2) \]  

(33)

Now that the stresses for each layer are described, the associated hoop strains can be calculated using 3-D Hooke’s law

\[ \varepsilon_H = \frac{\sigma_H}{E} - \nu \frac{\sigma_r}{E} - \nu \frac{\sigma_\alpha}{E} \]  

(34)

where, \( \varepsilon_H, \nu, \) and \( E \) are the hoop strain, Poisson’s ratio, and Young’s modulus, respectively.

Combining Eqn. 21, 22, and 23, the hoop strain for the outside and inside faces of the piezoelectric layer are shown in Eqn. 35 and 36, respectively.

\[ \varepsilon_{H_o}^p = \frac{2P_p}{E_p} \left[ \frac{r_i^2}{E_p \left( r_o^2 - r_i^2 \right)} \right] \]  

(35)

\[ \varepsilon_{H_i}^p = \frac{P_p}{E_p} \left[ \frac{r_i^2 + r_o^2}{E_p \left( r_o^2 - r_i^2 \right) + \nu_p} \right] \]  

(36)

While, the bonding layer hoop strains are
The interference, a result of the piezoelectric and magnetostrictive displacement, is the difference between the inner and outer diameter of their respective rings. The latter is calculated by

\[ u = r \varepsilon_H \]  

(41)

where, \( r \) defines the strain at a radial point of interest and \( u \) is the displacement. Thus, the interference (\( \delta \)) is

\[ \delta = r [ \varepsilon_{Ho} - \varepsilon_{Hi} ] \]  

(42)

Using this definition, the interference between each subsequent layers was calculated. For instance, the interference between the PZT ring and the bonding layer is defined as

\[ rE_3d_{31} = r [ \varepsilon_{Ho} - \varepsilon_{Hi} ] \]  

(43)

\[ E_3d_{31} = \frac{P_p}{E_p} \left[ \frac{r_i^2 + r_o^2}{r_o^2 - r_i^2} \right] + \nu_p - \frac{1}{E_b} \left[ \frac{P_p r_i^2 - P_m r_i b^2}{(r_i^2 - r_o^2)} + \frac{(P_p - P_m) r_i b^2}{(r_i^2 - r_o^2)} \right] + \nu_b \frac{P_p r_o^2 - P_m r_i b^2}{(r_i^2 - r_o^2)} - \frac{(P_p - P_m) r_i b^2}{(r_i^2 - r_o^2)} \]  

(44)
Where, \( rE_3d_{31} \) (for the axially-poled, \( rE_3d_{33} \) for the radially-poled) is the displacement due to the piezoelectric response. Here, Eqn. 44 shows the interaction between the applied electric field and resulting strain based on the geometry and constitutive properties. Thus, overcoming a major shortcoming of the model introduced in Section 2.1.1. The same concept applies for the interference between the bonding layer and the Terfenol-D, although magnetostriction is used instead of a piezoelectric strain.

\[
rH_1\gamma_{11} = r\left[\varepsilon_H^b - \varepsilon_H^m\right]
\]

\[
H_1\gamma_{11} = \frac{1}{E_b}\left[\frac{P_p\rho_2^2 - P_m\rho_2^2}{(\rho_2^2 - \rho_2^2)} + \frac{(P_p - P_m)m\rho_2^2}{(\rho_2^2 - \rho_2^2)}\right] - \frac{\nu_b}{E_b}\left[\frac{P_p\rho_2^2 - P_m\rho_2^2}{(\rho_2^2 - \rho_2^2)} - \frac{(P_p - P_m)m\rho_2^2}{(\rho_2^2 - \rho_2^2)}\right] - \frac{P_m}{E_m}\left[\frac{\rho_2^2 + \rho_2^2}{(\rho_2^2 - \rho_2^2)}\right] + \nu_m
\]

Where, \( rH_1\gamma_{11} \) is the displacement due to the piezomagnetic response. The above-mentioned equations are mainly function of the pressures in the piezoelectric and magnetostrictive layers as well as the applied electric and magnetic field, respectively.

These equations can then be rearranged in a matrix form to facilitate the solution for \( P_p \) and \( P_m \) as shown in Eqn. 47 below.

\[
\begin{bmatrix}
\sum_{i=1}^{5} \alpha_i \\
\sum_{i=1}^{2} \beta_i \\
\sum_{i=1}^{5} \eta_i \\
\sum_{i=1}^{5} \delta_i
\end{bmatrix}
\begin{bmatrix}
P_p \\
P_m
\end{bmatrix} = 
\begin{bmatrix}
d_{31}E_3 \\
q_{11}H_1
\end{bmatrix}
\]

Where \( \alpha_i, \beta_i, \text{and} \eta_i \) are defined as

\[
\begin{align*}
\alpha_1 &= \frac{1}{E_p} \left[ \frac{\rho_2^2}{(\rho_2^2 - \rho_2^2)} + \nu_p \right], \\
\alpha_2 &= \frac{1}{E_b} \left[ \frac{-\rho_2^2}{(\rho_2^2 - \rho_2^2)} \right], \\
\alpha_3 &= \frac{1}{E_b} \left[ \frac{-\rho_2^2}{(\rho_2^2 - \rho_2^2)} \right], \\
\alpha_4 &= \frac{\nu_b}{E_b} \left[ \frac{-\rho_2^2}{(\rho_2^2 - \rho_2^2)} \right], \text{and} \\
\alpha_5 &= \frac{\nu_b}{E_b} \left[ \frac{-\rho_2^2}{(\rho_2^2 - \rho_2^2)} \right]. \\
\beta_1 &= \frac{\rho_2^2}{E_b(\rho_2^2 - \rho_2^2)}, \\
\beta_2 &= \frac{\rho_2^2}{E_b(\rho_2^2 - \rho_2^2)}
\end{align*}
\]
\[
\eta_1 = \frac{1}{E_b} \left[ \frac{r_i^2 b}{(r_i^2 b - r_o^2 b)} \right], \quad \eta_2 = \frac{1}{E_b} \left[ \frac{r_o^2 b}{(r_i^2 b - r_o^2 b)} \right], \quad \delta_1 = \frac{-r_i b^2}{E_b (r_i^2 b - r_o^2 b)}, \quad \delta_2 = \frac{-r_o b^2}{E_b (r_i^2 b - r_o^2 b)}.
\]

\[
\delta_3 = \frac{v_b}{E_b} \frac{r_i b^2}{(r_i^2 b - r_o^2 b)}, \quad \delta_4 = \frac{v_b}{E_b} \frac{-r_o b^2}{(r_i^2 b - r_o^2 b)} \quad \text{and} \quad \delta_5 = \frac{1}{E_m} \left[ \frac{-(r_i^2 m + r_o^2 m)}{(r_i^2 m - r_o^2 m)} - \nu_m \right].
\]

The corresponding pressures are,

\[
P_p = \frac{\delta}{\alpha \delta - \beta \eta} d_{33} E_3 - \frac{\beta}{\alpha \delta - \beta \eta} q_{11} H_1
\]

\[
P_m = \frac{\alpha}{\alpha \delta - \beta \eta} q_{11} H_1 - \frac{\eta}{\alpha \delta - \beta \eta} d_{31} E_3
\]

Where \( \delta = \sum_{i=1}^{5} \delta_i \), \( \beta = \sum_{i=1}^{2} \beta_i \), \( \alpha = \sum_{i=1}^{5} \alpha_i \), and \( \eta = \sum_{i=1}^{2} \eta_i \).

The concentric ring geometry and material property interaction in Lame’s approach provides an insight on the mechanics, as shown in Fig. 2.4. The hoop stress of the composite ring is depicted in Fig. 2.4a as a function of the radial distance at various applied magnetic fields (\(H\)). The hoop stress resulting from the strain expand the PZT while compressing the Terfenol-D. The inset of Fig. 2.4a shows the compression of the bonding layer as the magnetic field increases. The radial stress is depicted in Fig. 2.4b as a function of the radial distance. The continuity in the radial stress agrees with the boundary conditions stated previously. The hoop strain shown in Fig. 2.4c accounts for both radial and hoop stresses (Eqn. 34) and shows the dependence on the material properties. The effect of magnetostriction is greater than the piezoelectric strain resulting in a hoop strain on the PZT greater than in the Terfenol-D. The increase in modulus of the bonding layer (\(E_b\)), denoted by the arrows on Fig. 2.4d, results in an increase of strain transfer from one layer to another. In the end, the radial stress in the Terfenol-D is combined with constitutive Eqn. 12 to predict the magnetic flux then the CME coefficients. Such that, the CME
coefficient is defined by the ratio of the magnetic flux \((B)\) and the RMS value of the applied voltage.

![Graphs showing mechanical behavior of concentric rings](image)

**Fig. 2.4:** Insight on the mechanics of the Concentric Rings. a) Hoop Stress b) Radial Stress c) Hoop strain d) Hoop strain with various \(E_b\)

### 2.2 Response of Individual Ring

#### 2.2.1 Sample Preparation

The theoretical models has shown the importance of the material properties to predict the CME coefficients, therefore three rings used in the composite rings were studied. The first ring is an axially poled polycrystalline PZT (APC International, Type 841), while another ring of the same type is radially poled. The last ring is a polycrystalline Terfenol-D ring ([120]) with an axial [112] crystallographic axis as shown in Fig. 2.5 in a
rod form. Relevant manufacturer’s material properties are shown in Table 1 and Table 2, respectively.

Table 1: PZT Type-841 Material Properties

<table>
<thead>
<tr>
<th>E_{p,11}</th>
<th>E_{p,33}</th>
<th>\nu_e</th>
<th>d_{33}</th>
<th>d_{31}</th>
</tr>
</thead>
<tbody>
<tr>
<td>76GPa</td>
<td>63GPa</td>
<td>0.3</td>
<td>300pm/V</td>
<td>109pm/V</td>
</tr>
</tbody>
</table>

Table 2: Terfenol-D Material Properties

<table>
<thead>
<tr>
<th>E_m</th>
<th>\nu_m</th>
<th>\gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>18–55GPa</td>
<td>0.25</td>
<td>6-10nm/A</td>
</tr>
</tbody>
</table>

Fig. 2.5: Terfenol-D rod schematic showing the crystallographic axis [45]

2.2.2 Experimental Setup

The principal strains at specific points of interest of an axially poled PZT ring and Terfenol-D ring were separately measured in response to applied electrical field and bias magnetic field, respectively. Fig. 2.6 shows schematic of the experimental setup used in
determining the piezoelectric and piezomagnetic coefficients of the individual rings. In which, a bi-axial strain gauge (SGD-2/350-XY11) was adhered to the PZT inside diameter face to measure the hoop and axial strains. The Terfenol-D ring has three bi-axial strain gauges, one on the inside face, second on the top face, and a third on the outside face. The strain gauges measure the respective hoop, axial, and radial strain. These strain gauges were connected to a quarter Wheatstone bridge (MR1-350-127), where a shunt calibration test confirmed the calibration equation provided by Micro-Measurements ($\varepsilon = \frac{4\Delta V}{GF \times 10^{-3} - 2\Delta VGF \times 10^{-6}}$, where GF the gauge factor) [121]. After which, the strain gauges were measured under a magnetic bias and did not result in a change of voltage. The change in voltage, due to strain, was measured using a wide band strain gauge input module (SCM5B38). The analog signal from the Gaussmeter was measured using a narrow band voltage input module (SCM5B30). An ISO-RACK 16/P board connected to a PCI-DAS6014 DAQ was used to power the Wheatstone bridge and for acquisition of experimental data.

In the design consideration of this experiment, an emphasis was given to the interface at which strain transduction takes place from the piezoelectric ring, i.e. inner diameter. Additionally, of specific importance to the validation of previously described models is the difference between the hoop, radial and axial strains in the Terfenol-D ring. This importance is stemmed from the effect of shape anisotropy and strain distribution on the induced magnetization.
2.3 **Response of Composite Ring**

2.3.1 **Sample Preparation**

In this research, concentric composite ring structures consisted generally of an outer lead-zirconate-titanate (PZT, APC International: P/N-841) ring and an inner magnetostrictive Terfenol-D (ETREMA Products) ring. The nominal dimensions of the PZT rings were 30mm-OD and 20mm-ID, while the Terfenol-D has a nominal outer diameter of 25mm and 2.5mm wall thickness. All the rings were 5mm in height. Three different sample configurations were constructed and characterized. Fig. 2.7 shows the 3D and cross-sectional views of all three-sample configurations, where the electrodes are dramatized and only a fraction of a millimeter. First (Fig. 2.7a), an axially poled PZT and Terfenol-D rings were adhered together using a polymer epoxy (Pro-Set LAM 125/226). The two-part polymer epoxy was first thoroughly mixed, and the mixture was then applied to the inner surface of the PZT and the outer surface of the Terfenol-D rings. The latter was then inserted into the PZT ring, and the two rings were rotated with respect to one another to ensure all surfaces were properly wetted and to allow any entrapped air bubbles to
escape. The composite ring was placed in an oven at 100°C for one hour for the epoxy to fully harden.

In the second sample configuration (Fig. 2.7b), a Terfenol-D ring was cold shrink fitted into an axially poled PZT ring, which created a negative allowance (interference). In the shrink fit process, the PZT ring was kept at room temperature while the Terfenol-D ring was cooled down in a dry ice bath. The shrunken Terfenol-D ring was then carefully inserted into the PZT ring and the assembly was kept at room temperature to reach thermal equilibrium. Finally, in the third configuration (Fig. 2.7c), a radially poled PZT ring was adhered to Terfenol-D using conductive silver epoxy (MG Chemicals 8331-14G). The assembled composite annulus was then left to cure in ambient conditions for 24 hours. In the first and second configurations, the top and bottom surfaces of the PZT rings were coated with silver and were used as electrodes to apply the electric field. In the third sample arrangement, the outer surface of the PZT and the Terfenol-D ring acted as the outer and inner electrodes, respectively. A copper search coil was wrapped radially around the composite structure to measure the strain-induced magnetic flux density, where each search coil consisted of 16 turns of 134-AWP wire.

Fig. 2.7: Composite Ring Boundary Schematic
2.3.2 Experimental Setup

The objective of this research is to study the effect of electric field, bonding interface, bias magnetic field, and frequency on the converse magneto-electric coupling through strain transduction between the PZT ring and the Terfenol-D ring. Fig. 2.8 shows a schematic of the experimental setup used in the characterization of the converse magneto-electric coupling of concentric composite multiferroic rings. In which, a function generator (Agilent, 33210A) was used to generate a sinusoidal waveform with frequencies between 4kHz – 50kHz. The output waveforms were then amplified using a high voltage amplifier (TREK, PZD700A) with a set gain of 100. The amplified signal was applied across the silver electrodes (Fig. 2.6) while being monitored using high voltage probe on a digital oscilloscope (Tektronix, DPO 2012). Each sample configuration was tested at four electrical fields that ranged from 20kV/m to 80kV/m. Throughout the experimental investigation, the composite ring structure was placed between the poles of an electromagnet (GMW Associates, 3470) and were subjected to DC bias magnetic field that ranged from 0Oe to 2300Oe. The strength of the bias magnetic field was measured using a gaussmeter (FW Bell, 6010). The strain-induced magnetic flux density was measured through a Helmholtz coil and Lock-In amplifier (Stanford Research Systems, SR830DSP). Fig. 2.9 shows a schematic of the concentric composite multiferroic ring as well as the location and alignment of the Helmholtz search coils. The electromagnet, the sample holder, the sample, and the gaussmeter were enclosed in magnetically shielded chamber to eliminate electromagnetic interference. The experimental setup closely mimicked the one used by Chavez et al. [119].
During the experimental characterization, first, an electric field was applied across the height and the wall thickness of the PZT ring for axially-poled and radially-poled sample configurations, respectively. The bias magnetic field was then adjusted from 0Oe to 2300Oe with step size of 125Oe. The upper limit of the bias magnetic field was selected above the field required for saturation (i.e., 2000Oe) [47], [58]. At each bias field level, the root mean square value of $V_{emf}$ was measured and the magnetic flux density was calculated as a function of the frequency based on the number of turns in the search coil [119]. Lastly, the converse magneto-electric coefficient (CME) was calculated as the ratio of the induced magnetic flux density to the applied voltage. These steps were repeated at different applied electric fields; namely 20kV/m, 40 kV/m, 60 kV/m, and 80 kV/m.

Fig. 2.8: Schematic of experimental setup for the Composite Ring
Fig. 2.9: Composite Ring Schematic
Chapter 3  Results

This chapter includes results from the theoretical models and the experiments to characterize individual and composite rings. Overall, the composite ring with an epoxy-bonding layer reported higher CME coefficients than its shrink fit counterpart. The response shown from the search coils coincides with the strain response of a single Terfenol-D ring; i.e. shape anisotropy. Generally, the theoretical predictions agreed well with the experimental data. For example, Lame’s approach provided an insight on the internal mechanical response of each layer within the composite ring structure, in which the radial stress was used to predict the CME response using the average of the piezomagnetic coefficients over the thickness of the ring. In this chapter, the theoretical results are first compared to a low frequency composite ring experiment. Then the strain results of the individual PZT and Terfenol-D rings are presented. Finally, the CME response for both H-aligned and H-transverse coils are presented for the three composite ring structures.

3.1  Model Results

From discussion in Chapter 2 about the development and expected performance of the theoretical models, it is important to recall that the quality factor model is used to quantify the efficiency of the strain transduction and the assembly process while disregarding the properties and thickness of the bonding layer. On the other hand, the model developed based on Lamé’s equations is used to elucidate the strain distribution in the piezoelectric, bonding, and magnetostrictive layers, respectively, which in turn are used to calculate the CME coefficients. Herein, the piezoelectric coefficients are based on the measurements of the strain response in an individual PZT ring at a frequency of 4kHz and
electric field of 20kV/m. These conditions were selected because the results will later be compared to the 4kHz composite ring experiment. The piezomagnetic properties are based on averaging the coefficients calculated from the hoop strain at the inner, middle, and outer surface of the Terfenol-D ring at each respective bias magnetic field. As discussed in Chapter 1, the elastic modulus of Terfenol-D is dependent on the magnetic field and it was reported as a range by the manufacturer. In turn, a value of 10GPa was assumed for the elastic modulus of Terfenol-D, which is relatively softer than the values reported by the manufacturer that was tested at DC field. The elastic modulus of PZT was taken to be 76GPa (which is the value of $E_{11}$). The elastic modulus of the Pro-set epoxy bonding layer was taken to be 3.39GPa, while the silver epoxy had an elastic modulus of 760MPa [122], [123].

The resulting quality factors of the shrink-fitted and polymer-bonded axially-poled configuration was 60, while it was 45 for silver-epoxy bonded radially-poled configurations. Since higher quality factor values indicate reduction in the strain transfer, the silver-epoxy bonded radially-poled configuration has superior strain transduction as indicated by the lower quality factor. Fig. 3.1 shows the iterative process to determine the value of the quality factor. That is, the quality factor value was increased until the prediction best fit the low frequency experimental data. This was done for each composite ring configuration.
Fig. 3.1: Process of obtaining quality factor value

Fig. 3.2 shows the comparison between the experimentally measured data and Lamé-approach model predictions. Specifically, CME data collected at 4kHz for both the axially-poled and radially-poled composite rings and were compared with results of the model represented in Eqn. 12 and 22 listed in Chapter 2. That is, the calculation of the radial stress on Terfenol-D from Eqn. 22 led to the prediction of the magnetic flux and in turn the CME. Notably, the maximum CME from experiment and prediction has a 35% and 40% difference between the axially-poled and radially-poled composite rings, respectively. This model is highly dependent on the input material properties (notice the properties in Eqn. 48 and 49 in Chapter 2); thus results should be only compared to quasi-static experiments since none of the models account for frequency.
3.2 Individual Ring Results

In the previous section, the performance and limitations of each model was reported and compared with experimentally determined data. In what follows, the results from strain response of each individual ring are reported. The measurement to investigate the strain response of the PZT ring focused on the interrelation between the piezoelectric behavior, the frequency, and electric field. The measurement to investigate the strain response of the Terfenol-D ring focused on the dependence of the piezomagnetic behavior on the bias magnetic field.

3.2.1 PZT Results

The piezoelectric coefficients for axially-poled (solid line) and radially-poled (dashed line) PZT rings are shown in Fig. 3.3, which are reported as a function of frequency and applied electric field. The piezoelectric coefficients are defined as the ratio of the
largest strain to the applied electric field at each frequency. For each poling configuration, the hoop and axial piezoelectric coefficients at the inner diameter face are reported at applied electric fields that range between 20 and 80 kV/m, and frequencies that range between 1kHz and 10kHz (Fig. 3.3). The applied electric fields are in the linear region, well below the coercive field for the PZT (354kV/m). Fig. 3.3a and 3.3b show the piezoelectric coefficients for the axially-poled PZT ring are larger than those for radially-poled as the frequency of the applied electric field increases. Regardless of the poling direction, the piezoelectric coefficients show a dependence on frequency but an independence on the magnitude of the applied electric field. The hoop piezoelectric coefficient (Fig. 3.2a) is ~30% larger than those found in the axial direction and will be discussed later for their order of magnitude difference with manufacturer’s properties [16].
Fig. 3.3: Piezoelectric Coefficients vs frequency a) Hoop Coefficients b) Axial Coefficients

3.2.2 *Terfenol-D Results*

The Terfenol-D ring exhibits a shape anisotropy effect when under an applied magnetic field as shown in Fig. 3.4. The response was recorded in the up- and down-scale directions of change in the bias magnetic field, which resulted in the hysteretic effect shown in Fig. 3.4. In the figure, the 90° location is associated with the location of H-aligned coil, while the 0° is associated with the location of H-transverse coil and the 180° indicated the face opposite to 0°. Fig. 3.4a shows the hoop strain of the outside face at the 0°, 90°, and
180° locations. The strain at the 90° location increases rapidly when magnetic fields between 200-500Oe. At higher fields, the strain continues to increase but at a steady rate. While Fig. 3.4a shows a tensile strain at 90°, Fig. 3.4b shows the radial strain is under compression as the magnetic bias field is increased. Fig. 3.4c presents the latter response for the axial strain. Fig. 3.4d depicts the hoop strain over the entire ring. Since the strain was collected using strain gauges at discrete locations, sharp points on Fig. 3.4d represent the location of the strain gauge, while the remaining points were obtained by linearly interpolation. The hoop strain response over the entire ring under magnetic fields from 0 – 2300Oe are collected in Appendix A.

Fig. 3.4: Terfenol-D shape anisotropic response under magnetic bias fields a) Hoop strain b) Radial strain c) Axial strain d) Hoop strain on the entire ring
3.3 Composite Ring Results

Now that the results of the theoretical models and the response of individual rings were presented in the previous sections, the CME measurements of the composite multiferroic rings are discussed next. Based on the prior discussions in Chapter 2 and results presented thus far, change in the CME response is expected to vary with frequency and in turn identify the EMR frequency. The CME response is also expected to be significantly lower at the H-transverse location due to the effect of shape anisotropy on the magnetostriction of Terfenol-D experiment. First, the H-Aligned coil experimental results for each composite ring will be discussed, finally the H-Transverse coil.

3.3.1 H-Aligned Coil

The overall behaviors of the converse magneto-electric coefficient for axially-poled epoxy-bonded, axially-poled shrink-fitted, and radially-poled epoxy-bonded multiferroic composite rings are shown in Fig. 3.5, 3.6 and 3.7, respectively, as a function of the bias magnetic field and the frequency of the applied electric field. For each ring configuration, CME is reported at applied electric field of 20, 40, 60, and 80kV/m, frequencies that range between 4kHz and 50kHz, and bias magnetic field that varies from 0Oe to 2300Oe. Fig. 3.5, 3.6, and 3.7 show that CME\textsubscript{max} occurs at a single frequency regardless of the bias magnetic field or the applied electric field. That is, the maximum CME happens at the resonance frequency for the composite ring, which is dependent on the interface boundary condition. In comparison to existing CME studies, shown in Table 3, the multiferroic composite ring reports ~400mG/V more than most other laminate structures except for the case by Hockel et al. [86]. The latter discrepancy is elucidated by considering the difference in the geometrical dimensions, material properties, and type of bonding adhesive, all of
which affect the generated and transduced mechanical strain; i.e. the main parameter of interest. For example, the piezoelectric coefficient ($d_{31}$) is -109pm/V and -340pm/V for PZT materials in our study and Hockel et al., respectively. There was also a significant difference in the piezomagnetic coefficient ($\gamma_{11}$), where Terfenol-D in this study has an average maximum of .28ppm/Oe while in Hockel’s study has higher coefficient of .5ppm/Oe. Finally, the height of the PZT ring is 5mm, which is 500% thicker than the PZT plates used in layered laminates in the study by Hockel and colleagues. In light of this discussion, it is expected then that a ring structure with similar properties, dimensions, and adhesive layer will generate a similar or greater CME than a laminate made of similar materials.

![Fig. 3.5: CME Response of the axially-poled polymer bonded composite ring w.r.t. frequency and Magnetic Bias in H-aligned Coil location; a) 20kV/m b) 40kV/m c) 60kV/m d) 80kV/m [119]](image-url)
Fig. 3.6: CME Response of Axially-poled shrink fitted composite ring w.r.t. frequency and Magnetic Bias in H-aligned Coil location; a) 20kV/m b) 40kV/m c) 60kV/m d) 80kV/m [119]

Fig. 3.7: CME Response of Radially-poled composite ring w.r.t. frequency and Magnetic Bias in H-aligned Coil location [119]
Table 3: Comparison of maximum CME for different composite configurations [119]

<table>
<thead>
<tr>
<th>Configurations</th>
<th>Converse Magnetoelectric Effect_{MAX}</th>
<th>Schematic</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT/Terfenol-D Composite Ring</td>
<td>535 mG/V</td>
<td><img src="image1.png" alt="Schematic" /></td>
</tr>
<tr>
<td>Current Investigation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT/Terfenol-D/PZT Laminate</td>
<td>4800 mG/V</td>
<td><img src="image2.png" alt="Schematic" /></td>
</tr>
<tr>
<td>Ref. 86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMN-PT/Terfenol-D/PMN-PT Laminate</td>
<td>105 mG/V</td>
<td><img src="image3.png" alt="Schematic" /></td>
</tr>
<tr>
<td>Ref. 85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Piezofiber/Metglas Laminate</td>
<td>66.2 mG/V</td>
<td><img src="image4.png" alt="Schematic" /></td>
</tr>
<tr>
<td>Ref. 79</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On the one hand, Fig. 3.8 shows CME_{max} for the epoxy-bonded ring decreases by ~20% from 535mG/V (at 36kHz, 20kV/m, and 413Oe) to 443mG/V (at 34kHz, 80kV/m, and 784Oe) showing a dependency in the applied electrical field. While the maximum occurs at 413Oe for 20, 40, and 60kV/m, the magnetic bias field increases to 784Oe for 80kV/m and occurs at a different frequency. It is important to note, at resonance (36kHz, f_r), CME of the epoxy-bonded composite ring decreased from 535mG/V to 455mG/V at applied field of 20kV/m and 60kV/m (Fig. 3.8a), respectively, while the bias magnetic field remained constant at ~413Oe (Fig. 3.8b). Additionally, CME_{max} at 80kV/m, while decreases relative to other applied field amplitudes, occurs at a lower bias magnetic field of ~287Oe (Fig. 3.8b).
On the other hand, Fig. 3.9 shows that $\text{CME}_{\text{max}}$ for the shrink-fitted ring is 330mG/V, which is approximately 40% less than the maximum for the epoxy-bonded interface. Another important difference between ring configurations is the shift in the resonance frequency. While the resonance frequency for epoxy-bonded ring is 36kHz (at low fields), it was reported to be 34kHz for the shrink-fitted ring. The shift in the frequency is attributed to the change in the overall effective stiffness of the structure due to change in the interface. The CME for the shrink-fitted ring is shown to be insensitive to amplitude (less than 10% change) of the applied electric field and occurs at an average bias magnetic field of 1141Oe, i.e. 297% more than the bias magnetic field demonstrated for the epoxy-bonded ring.
Fig. 3.10 shows that $CME_{\text{max}}$ for the radially-poled silver epoxy ring is 498mG/V, which is approximately 7% less than and 50% greater than the maximum for the Pro-set epoxy-bonded and shrink-fitted interface, respectively. The radially-poled ring resonance was reported to be 34kHz just as the shrink-fitted ring but reached CME values only 7% less than the axially-poled epoxy ring. This response is attributed to the improved overall effective stiffness due to the silver epoxy but still under the axially-poled epoxy ring. As it has been shown in theoretical discussion section, the stiffness of the bonding layer can improve or degrade the CME response. CME for the radially-poled ring is shown to decrease from 498 to 468mG/V with an increase of the applied electric field from 20 to 80kV/m and occurs at an average bias magnetic field of 748Oe, i.e. 160% more than and
34% less than the bias magnetic field demonstrated for the axially-poled epoxy bonded and axially-poled shrink-fitted rings.

![Graph showing CME and H(0e) at CME max vs. Frequency (kHz)](image)

**Fig. 3.10: Radially-poled epoxy bonded composite ring, H-aligned Coil a) Maximum CME w.r.t. frequency at various Electric fields b) Magnetic Bias at Max [119]**

Here, we emphasize the importance of the bonding conditions where the shrink-fit structure underperformed in transducing the mechanical strain resulting in a lower CME than the axially-poled epoxy-bonded and radially-poled epoxy-bonded composite rings. This is not consistent with assumption in Section 3.1, where the quality factor for the axially-poled shrink-fit and axially-poled epoxy bonded ring ($\xi = 60$) were the same, thus should have obtained similar CME values but did not. While the radially-poled epoxy bonded ring (with $\xi = 45$) should have performed better than the other two rings. Using these values of the quality factor, the analytical predictions agreed well with experimental results only at low frequencies. On the other hand, the axially-poled epoxy bonded ring with larger CME
values than the radially-poled epoxy bonded ring validated Lamé’s-approach. Although, Lamé’s-approach did not account for frequency it does account for the geometry and bonding layer. Resulting in predicted CME values larger for the axially-poled bonded ring than the radially-poled bonded ring. Therefore, at higher frequency, nonlinear constitutive models are required to account for the dependencies of the piezoelectric and piezomagnetic coefficients on frequency [29], [43], [63]. Finally, the reason for the poor strain transfer in the shrink fit is the interference allowance due to the geometrical tolerances during the shrink-fit fabrication process. That is, the difference between the maximum outer diameter of the inner Terfenol-D ring and the minimum inner diameter of the outer PZT ring defines the degree of the interference: light fit, medium fit, or heavy fit. The latter represents the maximum negative allowance, while the light fit represents a light negative allowance. In other words, however the gap between the ferroelectric and ferromagnetic layers is not easily observable; such gap significantly affects the strain transduction. Since the generation of magnetic flux density is dependent on the strain transduction, a larger gap compared to no gap at all will not transfer the strain from the piezoelectric to the piezomagnetic, resulting in a loss of magnetic flux density generation. In short, further improvements in the shrink-fit process are warranted. Nonetheless, the results clearly demonstrate the effect of the bonding conditions on CME and validate Lamé’s-approach.

Finally, Fig. 3.8, 3.9 and 3.10 can be used to find the optimal conditions to operate the composite ring structure based on the results and discussion listed above. The optimal conditions are found by noting the values of maximum CME at frequencies other than the resonance frequency and the corresponding minimum required amplitude of the applied electric field and magnitude of magnetic bias field ($f_{opt}$). For the shrink-fitted multiferroic
composite ring, the optimal conditions are 20kV/m and 400Oe at 36kHz. While, the recommended conditions for the epoxy-bonded ring can be extracted from Fig. 3.8 at 38kHz. Regardless of the operating frequency, Fig. 3.8 shows that 80kV/m corresponds to the lowest bias magnetic field.

Fig. 3.11, 3.12 and 3.13 show the dependency of the converse magneto-electric coupling coefficient for all three-sample configurations on the bonding interface. These figures also elucidate the dependency of the coupling coefficient on applied electrical field and the frequency (for example, Fig. 3.11a-d). However, the measurements were done over a range of frequencies (4-50kHz) as shown in Table 4, the figures herein only include the results in the vicinity of the resonance frequency (highlighted yellow in Table 4) where the maximum CME (highlighted green in Table 4) is reported in all three-sample configurations. Generally, the resulting values of the CME coefficient are based on the contribution to the mechanical strain from: 1) the piezoelectric effect due to the applied electric field; 2) the magnetostriction effect due to the bias magnetic field; and 3) the interface between these phases. It is important to note that the generated piezoelectric and magnetostrictive strain depend on the piezoelectric and piezomagnetic properties of the specific type of PZT and Terfenol-D used in the composite structure, respectively, which have been shown previously to depend on the frequency [28], [29], [32], [38], [55], [100]. In all, the experimental results expose the contribution of each of the experimental variables (electrical field, bias magnetic field, frequency, and bonding interface) to the values and behavior of CME.
Here, we emphasize four important observations based on the experimental results. First, the epoxy-bonded axially-poled PZT/Terfenol-D composite ring (Fig. 3.11) outperformed the other two configurations (Fig. 3.12 and Fig. 3.13) in magneto-electric coupling, where the maximum CME of 535 mG/V is reported at the lowest applied electrical field of 20 kV/m and a bias magnetic field of 415 Oe. The striking differences between the CME behaviors when comparing all three-sample configurations can be explained by considering the interface. The quality of the bonding interface between the ferroelectric and ferromagnetic phases of the ring structure affects the strain transduction and in turn the indirect coupling between electrical and magnetic energies [119].

### Table 4: Summary of the maximum CME at different frequency and different bonding interface [124]

<table>
<thead>
<tr>
<th>Freq. (kHz)</th>
<th>Axial Epoxy</th>
<th>Axial Shrink</th>
<th>Radial Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CME&lt;sub&gt;max&lt;/sub&gt; (mG/V)</td>
<td>H (Oe)</td>
<td>E (kV/m)</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>417</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>34</td>
<td>415</td>
<td>80</td>
</tr>
<tr>
<td>12</td>
<td>35</td>
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</tr>
<tr>
<td>34</td>
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<tr>
<td>36</td>
<td>536</td>
<td>415</td>
<td>20</td>
</tr>
<tr>
<td>38</td>
<td>285</td>
<td>914</td>
<td>20</td>
</tr>
<tr>
<td>45</td>
<td>87</td>
<td>1163</td>
<td>60</td>
</tr>
<tr>
<td>50</td>
<td>98</td>
<td>1290</td>
<td>60</td>
</tr>
</tbody>
</table>
Second, the shrink-fit based composite structure underperformed despite the expectation otherwise due to poor strain transfer through the interface. The maximum CME for axially-poled, polymer-epoxy sample occurred at 36kHz, was reported at 34kHz. The slight shift in the resonance frequency is mainly attributed to the change in the effective stiffness of the composite structure ($\omega = \sqrt{k/m}$), which can be realized by considering a mechanical spring analogy of three springs connected in series to represent PZT, Terfenol-D, and interface between, respectively. The middle spring, i.e. the interface, changes in stiffness from one sample configuration to another due to change in the epoxy type or the removal of epoxy (shrink-fit). For example, the elastic modulus of the polymer-epoxy is 3.39GPa as it compares to merely 0.76GPa for the silver-epoxy; such a difference will reduce the overall stiffness and in turn cause shift in the resonance frequency. The latter will also affect the overall generated piezoelectric and magnetostrictive strains since the properties are frequency-dependent.
Finally, the CME values in the samples with radially-poled PZT are 7% lower than those with axially-poled rings, the CME appeared to be clipped off at ~500mG/V between ~400Oe and ~1100Oe. When the electric field is applied across the wall-thickness, the generated piezoelectric strain that is transferred to the Terfenol-D ring is higher than in the axially poled configuration, in which the transferred strain is based on the Poisson’s effect because the electric field is applied across the height. As a result, the radial piezoelectric strain applies pressure on the Terfenol-D ring, thus lowers the magnetostrictive strain and limits the strain-induced magnetic flux. The pressure-imposed limitation in magnetostriction is the primary reason for the maximum CME to occur at 20kV/m.
The overall behaviors of the converse magneto-electric coefficient for axially-poled epoxy-bonded, axially-poled shrink-fitted, and radially-poled epoxy-bonded multiferroic composite rings at the H-Transverse coil are shown in Fig. 3.13, 3.14 and 3.15, respectively, as a function of the bias magnetic field and the frequency of the applied electric field. Over all the composite rings the H-transverse coil measured CME values an order of magnitude smaller than the H-aligned coil. The axially-poled epoxy bonded composite ring decreased by ~87% from 535 to 67mG/V at an electric field of 20kV/m and frequency of 36kHz. The axially-poled shrink-fitted composite ring decreased from 330 to 66mG/V, while the CME\textsubscript{max} occurred at the same 80kV/m electric field and at a frequency of 34kHz. Similarly, the radially-poled epoxy bonded composite ring also decreased by ~90% from 498 to 50mG/V at an electric field of 20kV/m and 36kHz.

The CME response at the H-tranverse location validates the expectation that it would be significantly lower at the H-transverse location. This is due to the effect of shape anisotropy on the magnetostriction of Terfenol-D experiment. The bias magnetic field
causes a large demagnetization effect on the Terfenol-D ring, which results the shape anisotropy.

On the other hand, the H-transverse coil has shown a second resonance frequency. The axially-poled epoxy bonded composite ring shows a second peak of 26mG/V at a frequency of 24kHz (Fig. 3.14). This second peak is predominately shown in the radially-poled epoxy bonded composite ring with a value of 28mG/V (Fig. 3.16). It is hypothesized that since this peak did not appear in the H-aligned coil, where the CME response is greatest on the composite rings, the peak is due to the resonance frequency of Terfenol-D. This is due to the decrease in magnitude of the CME values allowing the resonance in the Terfenol-D to be visible.

![Fig. 3.14: CME Response of axially-poled polymer bonded ring w.r.t frequency and Magnetic Bias at H-transverse Coil location; a) 20kV/m b) 40kV/m c) 60kV/m d) 80kV/m](image_url)
Fig. 3.15: CME Response of shrink-fitted composite ring w.r.t frequency and Magnetic Bias in H-transverse Coil location; 
a) 20kV/m b) 40kV/m c) 60kV/m d) 80kV/m

Fig. 3.16: CME Response of radially-poled epoxy bonded composite ring w.r.t frequency and Magnetic Bias in H-transverse Coil location; 
a) 20kV/m b) 40kV/m c) 60kV/m d) 80kV/m
Chapter 4  Discussion

The following chapter will discuss the results presented in Chapter 3 based on knowledge previously published on this topic and the newly introduced models. For example, the underperformance of radially polarized ring in comparison to axially polarized ones will be detailed. Maxwell’s equations on magnetism are used to explain the anisotropic response of the Terfenol-D. The discussion of these examples helps to explain the response of the multiferroic composite rings. Specific focus is given to the difference in the magnitude of the CME values for each composite ring, shift in the CME response with applied electric field, anisotropic CME response at different areas of investigation, and the frequency dependence on the CME values; to name a few.

4.1  Strain Experiment Discussion

4.1.1  PZT

The results from the strain experiment of a single piezoelectric ring show that axially polarized PZT and radially polarized PZT rings have higher piezoelectric coefficients than laminate piezoelectric coefficients. Compared to Masys, who studied plates, (Fig. 1.17) the piezoelectric coefficients in the PZT rings are two orders of magnitude greater and have a non-linear response to frequency. The piezoelectric coefficients in the PZT ring were also larger than the published properties from the manufacturer shown in Table 1. This shows that the ring geometry affected the piezoelectric coefficients.

Additionally, the piezoelectric coefficients in the hoop direction have been shown to be greater than those in the axial direction, which was attributed to the geometry of the
ring. Take the hoop strain for ring and the strain for a plate represented in Eqn. 50 and 51 respectively.

\[
\varepsilon_{\theta\theta} = \frac{1}{r} \frac{du_{\theta}}{d\theta} + \frac{u_r}{r} \quad (50)
\]

\[
\varepsilon = \frac{du}{dx} \quad (51)
\]

Where, \(u_{\theta}\) and \(u_r\) is the hoop and radial displacement, respectively. The resulting hoop strain from the application of the electric field on a PZT ring is a sum of the strain due to the hoop and radial displacements, as shown in Eqn. 50. The additional component of strain due to the hoop displacement is the reason the PZT ring generates piezoelectric coefficients greater than the plate, which only has one component (Eqn. 51). Therefore, the piezoelectric coefficients obtained in this research cannot be assumed for a laminate form of the PZT and merit further investigation.

The radially-poled PZT has underperformed compared to the axially-poled PZT, this can be explained in terms of stress on the material. The polarization direction dictates the largest normal displacement. From Hooke’s law, a larger strain dictates a larger stress. Therefore, from Eqn. 50, it can be said that the radial displacement is due to the piezoelectric response while the hoop strain is mechanical strain in the ring resulting in a stress induced in the ring. Zhang has shown in Fig. 1.18 that the stress on the material decreases the piezoelectric coefficients. Therefore, theoretically a radially poled PZT ring should outperform an axially-poled PZT. Instead, because of the stress is along the polarization direction, it will decrease its performance. The axially-poled PZT ring does not have the stress along the polarization direction thus not limiting the strain performance.
This response between the poling direction of the PZT rings merits further investigation for a definite answer.

A frequency dependence of the PZT ring is due to the elastodynamics. The elastodynamics is a function of the shape and mechanical properties. As shown in Fig. 3.3, the piezoelectric coefficients were found to be dependent on the frequency of the applied electric field. To demonstrate the dependence of the PZT response on the frequency, COMSOL Multiphysics Simulation software was used to simulate a PZT ring with same geometry as those used in the experiment. In which, the required piezoelectric mechanistic constitutive relationships were applied and the response was observed using frequency analysis. Fig. 4.2 shows the results of the frequency response of the PZT ring using the material properties previously mentioned. The resonant frequency found corresponded to the experimental resonant of 36kHz.

Fig. 4.1: COMSOL PZT frequency study

4.1.2 Terfenol-D

The Terfenol-D strain experiment showed an anisotropic response with the change of magnetic bias fields, where the greatest and least strains occurred at the H-aligned coil
and H-transverse coil, respectively (Fig. 3.4). Before the shape anisotropy effect is discussed, a few aspects should be acknowledged first. Terfenol-D is a poly-crystalline ring with an axial [112] grain oriented axis. In the literature, the magnetic field is applied along this axis but in this thesis it was applied in the plane perpendicular to the <112> axis. According to manufacturer the in-plane crystal directions include collective volume fraction of <111> and <110>, where the <111> crystal direction is the primary magnetic easy axis. This observation was also confirmed by Engdahl [125]. Thus, the difference between the results herein and those reported in the literature is attributed to the orientation of the easy axis with respect to the applied magnetic field.

The ring geometry causes a demagnetization that explains the shape anisotropy response of Terfenol-D presented in Chapter 3. Maxwell’s equations of magnetism state that the magnetic flux density normal to the face at the boundary of two different medias are equal and the perpendicular magnetic fields are equal (the two medias are air and the Terfenol-D material). This means that the greater the component of magnetic flux normal to the face of the ring will result in a decrease of magnetization because the magnetic flux density in air is less than within Terfenol-D. The decrease of magnetization results in a decrease of magnetostriction, shown at 0°, and 180° of the ring. Maxwell’s equations were solved for the Terfenol-D ring using COMSOL in order to show the demagnetization (Fig. 4.3). The background in Fig. 4.3 represents the air and the magnetic flux density is color-coded with its direction represented by arrows. COMSOL simulations corroborates with experimental results, where the 90° location of the ring holds the largest magnetic flux.

Maxwell’s equation on magnetism explains the shape anisotropy of the ring but fails to explain the difference in magnetostriction from the outside-diameter, top, and
inside-diameter faces. In addition to the demagnetization effect explained from Maxwell, there exists an internal demagnetization effect. The internal demagnetization effect is due to the polycrystalline configuration. Consider two adjacent crystals as shown in Fig. 4.4; the magnetization of one crystal generates a magnetic flux that circles around the crystal like a normal magnet. The magnetic flux extends out toward the neighboring crystal effectively acting as an applied magnetic field. The magnetic field opposes the magnetization, thus results in a demagnetization. This demagnetization effect explains the magnitude difference observed across the thickness.

![COMSOL analysis applying Maxwell’s equation to a Terfenol-D Ring](image)

**Fig. 4.2: COMSOL analysis applying Maxwell’s equation to a Terfenol-D Ring**
The two states that can exist in a magnetic ring is an onion state or a vortex state. Klauui and others have shown these states [22], [109], [110], [112]. A vortex state is defined by the closure of the magnetic flux within the ring, while the onion state is defined by the presence of two opposite head to head walls [109]. The head-to-head walls are the north and south poles, which the mangetic flux will flow from one to other. In the presence of a vortex state, there are no poles. Fig. 4.3 depicts a state of magnetism referred to as the onion state. The face perpendicular to the applied magnetic field will create a nucleation zone, in which the magnetization will bound to the edges of the geometry rather than simply passing though [109].

Additionally, the dependence of piezomagnetic coefficients on the magnetic field (Section 3.2.2 and Appendix A) can be explained by ferromagnetic domain theory. Before a magnetic field is applied, the ferromagnetic domains are oriented in a random low energy state [17]. An applied magnetic field moves the domain walls, reorienting the net
magnetization closer to the direction of the magnetic field (as discussed in Section 1.1.2). For example, at low magnetic fields of 0 – 500Oe the rapid increase of magnetostriction is a response of the domain wall motion overcoming pinning sites (Fig. 3.4) [34], [42], [43], [46], [56], [126], [127]. At magnetic fields greater than 500Oe, the domain rotation dominates causing a slow increase in magnetostriction as depicted in Fig. 1.10c-d. The pinning sites are imperfections in the sample, which the domain walls are “pin”-ed to and require magnetic energy to be “de-pin”-ed, depicted in Fig. 4.5. The vertical line represents the domain wall pinned to an inclusion, represented as a square. Fig. 4.5a-c depicts the domain wall still pinned to the inclusion as the domain wall moves, until a sufficient amount of energy is applied to de-pin the domain wall (Fig. 4.5d) [17]. In summary, the ferromagnetic theory explains that at low magnetic fields, the rapid magnetostriction is a result of de-pinning and at high magnetic fields the domain rotation is a slow response.

Fig. 4.4: Schematic of domain wall pinning sites [17]

4.2 Composite Ring Discussion

The summary of the experimental results discussed in Section 3.3 lead to a few interesting phenomena, i.e. there is, a large difference in CME values, a shift of resonance frequencies, and CME$_{\text{max}}$ occurs at different magnetic fields between each composite ring. First, the bonding layer effect will be addressed. Second, the CME response curves shifting
to the left, magnitude decrease, and the behavior of the response will be addressed. The latter is explained in terms of competing energies between stress and magnetic field. The former is due to the mechanics of the bonding layer itself.

4.2.1 Bonding Layer effects

Three different interface configurations were experimentally investigated; 1) shrink Fit, 2) Pro-set Epoxy, and 3) silver based Epoxy. CME responses at their respective resonant frequencies, shown in Fig. 4.6 a-c for each of the interface configuration, respectively. The shrink fit composite ring was found to have the lowest CME, which was attributed to the lack of a continuous contact between the two rings, based on imperfect circular shape of the rings. This resulted in an ineffective transfer of strain from one ring to another. Once the interface is replaced by polymer epoxy, the magnitude of CME was shown to be ~200mG/V higher than shrink-fitted configuration. There was also a 5% difference between the CME reported using polymer and silver epoxies, which was contributed to the piezoelectric coefficients and the material properties of the epoxies.

Fig. 4.5: Recall of the CME Response at resonance for each composite ring

Lamé’s approach provided insight into the effect of the elastic modulus of the bonding layer and piezoelectric coefficients for the axial and radial composite rings at low frequencies. The model predicted a 40% difference in CME values between the two
composite rings, which was validated by the 35% difference seen in experiment (Fig. 3.2). Theoretically, this occurs because the piezoelectric coefficient in the axial ring is approximately 1.3 times greater than that of the radial ring. The model predicts a minimal change (of .3%) in the CME due to the elastic modulus of the bonding layer. This minimal change leads to the conclusion that the CME response for the composite rings differs due more than the simple bonding layers elastic modulus. Moreover, the elastic modulus of pro-set epoxy is frequency dependent, which is consistent with general properties of polymers. This can be seen in Fig. 3.2 where at low frequency experiments the radial composite ring outperforms the axial composite ring and theoretically, it is vice versa. The variable to consider is the material properties of the bonding layer in response to frequency.

4.2.2 CME Response Discussion

The bonding layer effect has not only shown the difference in maximum CME values but the behavior as well. Atherton and others have discussed the effect of stress on ferromagnetic materials, based on which the following interpretation will be used to explain the CME response [35], [43], [45], [46], [58], [128]–[130]. Terfenol-D is a positive magnetostrictive material; therefore, a compressive stress moves the domain walls to align the magnetization perpendicular to the applied stress, similarly shown in Fig. 1.13. An applied magnetic field similarly moves domain walls to align the magnetization. Ultimately, magnetic and strain energies compete to manipulate the domain walls or magnetization. At each level of applied magnetic field, a corresponding stress level exists to induce the maximum magnetic flux. The greater the magnetic field the larger the stress is required to obtain a maximum magnetic flux. That is, the ferromagnetic material effectively becomes mechanically harder as the magnetic field increases because of the
domains are aligned with the applied magnetic field and become harder to move with application of stress, described in Fig. 1.19. Overall, the hardening results in a decrease of magnetic flux within the material.

Davino studied the same effects in terms of piezomagnetic coefficients for Terfenol-D [46]. Fig. 4.7 depicts the piezomagnetic coefficients as a function of bias magnetic field at different stress levels (the labels of the subplots represent the stress levels). As the stress is increased, the piezomagnetic coefficients decreases, while at high fields the piezomagnetic coefficients also decrease.

![Piezomagnetic coefficients vs H at different constant stress](image)

**Fig. 4.6: Piezomagnetic coefficients vs H at different constant stress [46]**

The competing magnetic and mechanical energies are essential in understanding the CME response shift as electric field is increased on the composite rings since stress is the mechanism that generates the magnetic flux. The shrink-fit composite ring response shown in Fig. 3.6 reaches maximum at a magnetic field higher than the other two rings. As the magnetic field increases the magnetostriction strain, this improves the contact between the
rings allowing an increase of pressure at the interface. The resulting radial stress then manipulates the domain walls to increase the magnetic flux. On the other hand, the axial composite ring has an improved interface but still require a magnetic energy in order to depin the domain walls, thereafter the stress will manipulate the domain walls. The increase in magnetic field theoretically should decrease the generation of magnetic flux but the magnetic flux increases. The increase of elastic modulus of the bonding layer would increase in the radial stress on the Terfenol-D as shown by Lamé’s approach. The response of radially-poled composite ring is based on the same competing energies mechanism. Although, due to the short distance the dipole travels, as previously described, results in a plateau type of response signifying a saturation of stress on the Terfenol-D.

Additionally, the effect of the stress in the composite ring was shown from the change in electric field in Fig. 3.8, 3.9, and 3.10. The increase of electric field results in the piezoelectric response, increases the pressure of the Terfenol-D. This increase of pressure decreases the piezomagnetic coefficients, as shown by Davino [46], resulting in a decrease of magnetic flux density. This is experimentally shown in Fig. 4.5, the CME response decreases as the electric field increases in the composite rings at resonance.

Notably, the CME response is significantly lower in the H-transverse coil than the H-aligned coil locations, due to the shape anisotropy, revealing the resonance frequency of Terfenol-D. The maximum CME occurs at 24kHz at high electric fields as shown in Fig. 3.20. This is interpreted that the resonant frequency of Terfenol-D is near 24kHz. The reason it shows in the H-transverse coil and not in the H-aligned is due to the fact the piezoelectric response dominates at this area because the magnetostriction response is lowest at this location. Fig. 1.19 provides detail data on the effective elastic modulus for
Terfenol-D at different stress levels. COMSOL was used to determine the resonant frequency range for the various elastic modulus, resulting in possible resonant frequencies between 20-40kHz. It is plausible the 24kHz can be the resonant frequency for Terfenol-D.
Chapter 5  Future Work / Conclusion

In conclusion, this thesis followed the DoE process to synthesis new theoretical models and experiment design to characterize three multiferroic composite rings and answer three hypotheses. The experiment successfully determined the CME coefficients for the three composite rings by controlling four physical parameters, namely frequency applied electric field, bias magnetic field, and bonding interface. The experiment results successfully validated the quality factor and Lamé’s models, providing insight to the mechanics of the strain-mediated control of magnetism. The characteristic responses from the rings were interpreted, with the use of literature review, in terms of the competition of strain energy and magnetic energy. The following chapter will answer the hypothesis stated in Chapter 1, then discuss future work, and finally conclude with some final thoughts.

5.1 Hypothesis #1

One of the underlying mechanism, which results in large CME values, is the material properties for both the piezoelectric and piezomagnetic phases. Therefore, it can be said that certain laminate combinations can outperform the composite ring structure and vice-versa. Although, it has been shown here that the PZT ring shape generates piezoelectric coefficients two orders of magnitude larger than the manufacturer’s specifications. This improvement of piezoelectric response results an increase of pressure on the piezomagnetic phase resulting in CME values greater than in a laminate structure composed of the same materials.

5.2 Hypothesis #2

Lamé’s approach discussed in Section 3.2.2 has reported a reasonable agreement to experimental results. In spite of the magnitude difference between experimental and
theoretical, the difference between the two rings in both cases only differs by 5%. This is significant due to the fact the PZT ring is the only changing variable between the two composite rings. The piezoelectric coefficients extracted from the strain experiment successfully shows the theoretical the same CME magnitude difference between the axial and radial composite rings.

The magnitude difference seen between theoretical and experimental is simply due to the piezomagnetic coefficients and limitations of the model. The piezomagnetic coefficients used were extracted from the individual strain experiment, which was not under an applied stress. Davino showed the change piezomagnetic coefficients under an applied stress. Thus, the piezomagnetic coefficients decrease and the model does not account for this material property change.

A second aspect that the model does not account for is the demagnetization effect within the ring. Section 4.2.2 discusses the demagnetization effect within a ferromagnetic material. The composite ring produces a magnetic flux density within the material resulting from the manipulation of magnetization. Simply, the existence of demagnetization fields is inevitable and Lamé’s model neglects this, resulting in theoretical CME values greater than experiment.

5.3 **Hypothesis #3**

A lot of discussion has been placed on the stress on Terfenol-D, which is the main contributor to the magnetic flux generation, but have always been accompanied with an applied magnetic field. The reason is, as discussed in Section 4.1.2, due to the polycrystalline composition. All polycrystalline ferromagnetic materials will have impurities within which pin domain walls. An application of stress has been shown to move
domain walls but without the applied magnetic field, the domain walls are strongly pinned to the inclusions. A pre-strain replacement would not be sufficient to replace the applied magnetic field.

5.4 Future work

This thesis has provided extensive results for the three multiferroic composite ring structures. Further insight can be obtained by:

Improving the sophistication of the theoretical model with further strain experiments on the individual rings and composite ring structures and account for losses previously neglected. Material properties extracted from the strain experiments of the composite ring properties using the model will be a function of the previously stated physically controlled variables. Further study on demagnization and other magnetic losses should also be imbedded in the model to provide an even further accurate model.

COMSOL will be used to combine Maxwell, piezoelectric, piezomagnetic, and frequency models in order to predict the composite ring’s performance. A COMSOL model was developed during this research but is still in the initial stage of development. After which, micro-magnetics can be applied to the model and be able to accurately design and predict the performance.

The next step, after a clear understanding of the physics at the macro-scale, is to scale down the composite rings to the micro-scale. This will require knowledge in fabrication at this scale such as Photolithography, Chemical Vapor Deposition, Physical Vapor Deposition, or E-beam Lithography. The polycrystalline losses will be essentially removed and plans for application can finally be done.
Bibliography


L. Ruiz De Angulo, J. S. Abell, and I. R. Harris, “Magnetostrictive properties of


The complete magnetostrictive response of the Terfenol-D ring is shown in Fig. 2, 3 and 4 as a function of the external magnetic bias field. The hoop response was measured at three different faces, OD face, ID face, and Top face, at 8 different locations of 45° increments measured from the center of the ring, where the 0° vector is aligned with the external magnetic bias field. The magnetostriction of each face (Fig. 2.a, 3.a, 4.a) shows an anisotropic response. The greatest magnetostriction occurs at the 90° and 270°, while the least occurs at 0° and 180°. Although, the magnitude of magnetostriction increases from ~150ppm at the OD face to ~325ppm at the Top face to ~710ppm at the ID face. The slope of a line from the origin to the point of interest can be taken as the piezomagnetic coefficients shown in Fig. 2.b, 3.b, 4.b, as a function of the magnetic bias. The ring shape has an advent effect on the piezomagnetic coefficients, which increases by over 200% from the OD to ID face when compared to Hockel’s experiment which required a DC bias electric field to increase the piezomagnetic coefficients.
Fig. 2: Terfenol-D OD hoop response w.r.t. Magnetic Bias measured at various locations; a) Magnetostriction b) Piezomagnetic Coefficients
Fig. 3: Terfenol-D Top hoop response w.r.t. Magnetic Bias measured at various locations; 

a) Magnetostriction 
b) Piezomagnetic Coefficients
Fig. 4: Terfenol-D ID hoop response w.r.t. Magnetic Bias measured at various locations; a) Magnetostriction b) Piezomagnetic Coefficients
Fig. 5: Terfenol-D hoop response (shape anisotropy) w.r.t. Magnetic Bias