THE INFLUENCE OF MANIPULATION AND COLLABORATION
ON THE EMERGENCE OF NUMBER CONSERVATION
WITH FOUR- AND FIVE- YEAR OLDS

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By

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ABSTRACT

THE INFLUENCE OF MANIPULATION AND COLLABORATION ON THE EMERGENCE OF NUMBER CONSERVATION WITH FOUR- AND FIVE- YEAR OLDS

By

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Children’s concepts of number conservation was investigated by Piaget some 45 years ago; he discovered that children tend to perceive an increase in the quantity of objects in a collection when these objects are spread out. The problem posed in this study is, how might children learn conservation of number best: by watching a teacher demonstrate it, by working with the materials in a “hands-on” way, or through conversation and manipulation? Sixteen pre-kindergarteners ranging from age four to five years old were split into three learning conditions: observing the teacher manipulate objects, manipulating the objects themselves, and in diads where the children both manipulated and discussed the problem collaboratively. It was hypothesized that children would learn the concept of ‘conservation of number’ best in pairs and least well (as measured by the number of trials in the ‘game’ needed to understand the concept) when they merely watch the teacher manipulate the pennies. Although the number of students or dyads in each group (4) did not allow concluding on any general statement, it appeared, in this experiment, that the children operating in pairs and who were allowed to collaborate and to manipulate understood the concept of number conservation faster than
in the other two conditions. Implications for math in the preschool setting are discussed in light of the Preschool Early Learning Foundations and other math-related initiatives for young children.
CHAPTER ONE

INTRODUCTION

According to the Glenn Commission Report (2000), the future well being of our nation and people depends on how well we educate our people, specifically on how well we educate them in science and in math. Mathematic achievement in the US has been identified as not meeting job requirements according to American educators and business leaders (Kilpatrick, Swafford, & Findell, 2001). Additionally, international testing has shown that American students are not doing as well as students in other countries in the sciences. Reports show that the achievement gap starts extremely early, as early as ages four to five (Klein, 2004).

Because there is a growing body of research reporting that early numeracy skills are significant predictors of later academic achievement (Duncan, 2011; Fuson, 2004), an interest in early childhood math skills for children in preschool is drawn into focus. The National Council of Teachers of Mathematics (2000) adds some precision by stating, “Historically, number has been a cornerstone of the entire mathematics curriculum internationally as well as in the United States and Canada.” (p. 32). Solid education in mathematics therefore means solid number sense development for the child from the beginning, that is during preschool. This focus on early childhood education is rendered even more important as more and more children attend daycare and pre-primary education, especially children from lower socio-economical status (U.S. Departement of Education, 2006).
Piaget’s conservation theory (1964) which emerged some 50 years ago remains foundational with regard to understanding how young children perceive number sense. Piaget states that there are cognitive developmental stages through which children grow. Between age four and age seven, children are supposed to be in what Piaget calls the pre-operational stage which, he writes, ought to be pre-logical and pre-numerical (Piaget, 1964). One characteristic of this stage is – among others – that children would perceive an increase in the quantity of objects in a collection when these objects are spread out. Perception of a larger quantity versus number sense is the object of this thesis.

**Statement of the Problem**

Are there conditions under which children develop math skills better than others? Before entering school, children already show spontaneous interest in mathematics and come to school with some arithmetic skills (Ginsburg, Inoue, & Seo, 1999), but how can math be best learned in preschool? It is the goal in this thesis to determine how much do preschool age children know about math, and what conditions allow them to learn better. Although various studies already explore the effect preschool curricula have on school readiness (National Center for Education Research, Institute of Education Sciences, U.S. Department of Education, 2008; Fram, 2012; Dang & al., 2011), this study will focus on three learning conditions, and may be useful to program coordinators working in the field of early childhood education.
Purpose of the Present Study

The problem posed in this study is how might children learn conservation of number best: by watching a teacher demonstrate it, by working with the materials in a ‘hands-on’ way, or through conversation and manipulation? The purpose of the research is to study the impact of manipulation and/or collaboration on the emergence and ideally mastery on conservation of numbers. The concept targeted in this research, number conservation, relates to how children realize that if nothing is added or taken away from a collection of objects, no matter how these objects are moved around and changed in appearance, the quantity remains the same.

The hypothesis is that if children are allowed to manipulate the objects, then the concept of number constancy will emerge more quickly than when all manipulations are done by the teacher or when the child is rendered a mere spectator of the rearrangements. Moreover, when children work in groups and exchange ideas, as they observe each other manipulating and they discuss concepts, it is predicted their skills will emerge faster than those who do not manipulate the objects or do not work with a peer. In short, are collaboration and manipulation catalysts for learning mathematical concepts?

Significance of the Study

The significance of the study is to observe first hand how children might engage the conservation of number task under varying conditions. The first assumption, supported by Piaget (1964) is that number conservancy is typically reached at age four. If the study shows that indeed, most of the students have not yet acquired the concept of
number conservation through this experience, but are capable of understanding it through some kind of experience, it could confirm to program coordinators that this is a concept to be considered as new in pre-kindergarten and that it should be tackled as new and not taken for granted in daily activities, which joins the concept of ‘intentional’ teaching suggested by the National Association for the Education of Young Children (2009).

When tackling new concepts, manipulation may be a key component of the learning experience. If it is proven that four years old children learn better when they are allowed to manipulate objects themselves, then teachers may be more likely to provide hands-on lessons along with current methods. Moreover, this test may be prolonged through the pre-operatory stage – until age seven – to measure how important is the impact of manipulation in understanding various mathematical concepts. If results show that children understand better abstract concepts when they manipulate, from preschool through second grade, and maybe above, then this would imply a necessity to include *ad hoc* manipulatives in each preschool and elementary classrooms, and that introductory activities should include manipulation in some ways.

Collaboration may enhance the understanding of new concepts (Kagan, 1990), adding in the component of scaffolding by peers. If the study demonstrates that four year old children learn better when they can communicate with each other while working on the same project, it will provide data regarding the benefits of working in groups and communicating among learners. This understanding would imply that cooperative learning is more efficient than individual performance. If this is the case, the classroom
organization and structure may be changed in order to always allow children to work
together and achieve goals as a team rather than on an individual basis.

**Terminology**

Certain terms are important to define because understanding the precise language of math underpins the clarity of understanding it in the first place. Math is all in the terminology!

*Cardinality:* The concept that the number name applied to the last object counted represents the total number of objects in the group (the quantity of objects counted). (California Department of Education, 2008)

*One-to-one correspondence:* One and only one number word is used for each object in the array of objects being counted. (California Department of Education, 2008) This concept is also used in the present work as the positioning of two collections of objects visually matching each object of one collection to one and only one object of the second collection.

*Ordinal:* Expressing order in a series. (Webster’s New World Dictionary and Thesaurus, 2002)

Ordination: term used by Piaget to refer to operations concerning ordinal numbers. (Flavell, 1963)

*Quincunx:* Arrangement of five objects in a square or rectangle, one at each corner and one in the middle. (Merriam-Webster’s Dictionary, 2005).
Subitize: The ability to quickly and accurately determine the quantity of objects in a small group (of up to five objects) without actually counting the objects (California Department of Education, 2008).

Preview of the Thesis

Chapter Two, the Literature Review will present the related works of Piaget that are very close to the subject of this thesis, and will be contrasted with contemporary research which, after delineating and describing various stages of number sense development, investigates the purpose, the use, and the potential danger of using standards in preschool. The review of other literature will reveal the role manipulation and the role collaboration may have on children’s learning process. Chapter Three, Methods, describes the process of selecting a pre-kindergarten class, the tools used to collect the data of the population divided into three groups and being filmed while operating under three different conditions, as well as the script used to interview and experiment with the students. Chapter Four, Results, is a written transcription of all the data collected, including quotes and references to precise moments of the bank of data on video support. The discussion of the results in presented in Chapter Five, Discussion, and confronts the findings to the important themes previewed in Chapter Four, but also presents unexpected findings that should lead to further studies and to maybe to future policy!
CHAPTER TWO

LITERATURE REVIEW

Introduction

This chapter will provide a review of the literature that is foundational to the current study. Piaget and Szeminska led experiments with young children that are very close to the ones considered in this thesis: the process of putting children in situations where they have to compare quantities of two collections of objects placed in one-to-one correspondence, to then compare them once again when the objects of one collection have been spread out was central to their research. The California’s Preschool Learning Foundations have also been influential because they include what is considered core content for mathematics for children 4-5 years of age. To understand the impact of manipulation on the emergence of number skills, reports from the National Association for the Education of Young Children (NAEYC), edited by Copple and Bredekamp (2009), will be reviewed as well. A final section explores cooperation between students when achieving a task presenting incentives to access higher thinking skills such as abstraction to numbers has been reviewed through the works of Kagan, Piaget, and Bandura.

The Groundbreaking Work of Piaget

Piaget (1966) found that the absence of quantity conservation is a clear indicator of the preoperational stage. He illustrated this absence using the experiment in which he empties a glass of water into a narrower glass or into a wider glass. Remarkably, he noted
that four to six year old children would conclude to an increase or a decrease in the quantity of water by solely considering the height of water, without considering the fact that the water is the same, just poured from one container to the other. There seems to be a focus on the configuration - the appearance rather than on the transformation. Although the transformation is considered, Piaget and Inhelder (1966) found that it is not conceived as a reversible change of shape leaving quantity unchanged. It is only at around seven or eight years of age that the child will argue that “c’est la même eau”, “on n’a fait que verser”, “on n’a rien enlevé ni ajouté” – “this is the same water”, “we just poured”, “no water has been added, nor taken away” (Piaget & Inhelder, 1966, p 94).

With regard to discrete quantities and numbers, Piaget and Inhelder (1966) state that numerical evaluation is, until the concrete operational stage, tied to spatial arrangement, in narrow analogy with figural collections. It only takes a spread from one another the objects of one of the two rows placed in optical correspondence for the child to stop accepting numerical equivalencies. However, the child steps toward the concept of number by establishing set equivalencies while putting in one-to-one correspondence objects of one collection with objects of another collection. Associating elements that are obviously corresponding, such as flowers and vases, or cups and plates, may be helpful, according to Piaget and Inhelder, to establish the one-to-one correspondence between two sets, and to lead to the concept of number units, which emerges from an abstraction of differential qualities.

Once the concept of number units has emerged, individual elements are classifiable according to the inclusion relation between subsets. In a related way, the concept of seriation implies that the children do not count the same element more than
Once. Thus, the concept of number is a combination of seriation and inclusion. Piaget and Inhelder (1966) lifted the veil on questions regarding the genetic perspective. What in the human nature allows the original synthesis of the steps of seriation and inclusion?

Piaget (1964) identifies what he refers to as the preoperational stage between age four and age seven. The stage, assumed to be pre-logical, ought to correspond, according to Piaget, to a pre-numerical stage in which children do not comprehend the conservation of numbers. Through his experimentations, he showed that children between age four and age five assume that the numbers of items in a collection varies – even when no objets are added or removed – according to the configuration of the collection. Specifically, children rely on their perception of the surface covered by the collection or its density instead of rationally considering the items in the collection. When children consider comparing conjointly the length of two rows of objects and their density, managing thereof a one-to-one correspondance between the two collections, “la correspondance terme à terme, même lorsqu’elle a lieu entre objets qualitativement complémentaires, ne suffit point à entraîner l’équivalence nécessaire et durable des collections correspondantes” [one-to-one correspondence, even when it takes place between objects that are qualitatively complementary, does not suffice to imply the necessary and constant numeral equivalency between the corresponding collections] (Piaget, 1964, p. 98).

Despite the fact that a one-to-one correspondence between the objects of two collections is rendered obvious by contact in an initial phase, it only takes a slight modification of the shape for pre-operatory children to conclude to a non-conservation of numbers. Piaget (1966) stated that when five to six year old children see a
correspondence between twelve red coins with twelve blue coins to verify that each row contains the same number of coins, it only takes the examiner to spread out the red coins or the blue coins for the children to conclude that the longest row contains more coins. The question becomes whether non-conservation is linked to a difficulty to conceive the reversibility of short displacements. So Piaget set the coins in two horizontal rows, and placed the red and blue coins in aisles following the shape of a fan. The row of blue coins placed on top had coins closer to each other, whereas the row of red coins placed on the bottom had coins more spread out from one another. Despite the correspondence red coin – blue coin made obvious by the fan structured set of aisles – allowing a motion of the coins in the aisles that made them meet – children maintain that there were more red coins than blue coins. Piaget explained that with the help of some colleagues, he elaborated a device allowing moving the spread-out red coins all at once upward along the aisles to make them meet with the tighter row of blue coins. The device did not change the children’s idea that there were more coins in the longer row, although they could see the red coins come tighter to meet the blue coins, to spread out again. In these experiments, Piaget did not specify if the children were allowed to move the coins themselves or if they had control on the device, nor does he specify whether the children were in groups and encouraged to collaborate and discuss the findings.

Piaget (1966) concluded that mental representations merely form a system of symbols during the transition from the preoperational stage to the operational stage. The representation children have of objects does not induce an operatory structure. In the operational stage, when children reach the age of seven or eight, representations are used as a basis for operations which are induced by actions, and not representation. Piaget’s
theory that operational structure is induced by action itself, and not by object representations, constitutes a strong incentive to allow children manipulation and to invite them to be the actors, or even the authors of the actions that will induce the operatory comprehension.

Contemporary Math Research

The Early Development of Number Sense

At an early age, cognitive development has much to do with physiological maturation. The development of the perceptions of objects or of collections of objects is largely dependent on the development of the brain and of the central nervous system (Copple & Bredekamp, 2009). Children around age three improve in their ability to perceive concepts related to number, and these abilities should improve with age.

Permanence of a collection of objects. Although permanence of objects – the notion that objects still exist when they are hidden from view – is different from permanence of collection of objects, which is a numerical concept, Steffe (2004) points out that the former is necessary to the latter. Once permanence of objects has been established, children may categorize items together (like the pennies on the table, or “your row of pennies”, and “my row of pennies”, or the dark pennies, and the shiny pennies …) and form a collection of perceptual items. However, once the permanence of the objects is established, it does not necessarily imply that “the collection per se has been established as a permanent object” (Steffe, 2004, p. 225). The distinction is crucial in the development of the concept of number.
Continuous versus discreet. Within the concept of quantity, children may develop a perception of continuous quantities without necessarily developing a perception of discreet quantities. There may be confusion while comparing two sets of objects between the number of objects in each set and the contour length of each set as children base their judgment on only one perceptual cue (Baroody, 2004) just as Piaget was stating in 1966. Piaget furthermore observed that the density of the collection was an additional influential perceptual clue. The children’s conclusion on the quantity of coins would be influenced by either one perceptual clue but could hardly be in accordance to all the features of the collection. It is therefore understandable that children tend to conclude that the longest row contains more coins than the shortest one does.

A one-dimensional way of thinking. In spite of their many advances, preschoolers can be one-dimensional in their thinking, and may hence miss principles such as the notion of conservation because of other varying factors (Copple & Bredekamp, 2009). At three or four year of age the world is experienced not in the single, integrated entity known to adults but rather is a multitude of separate, independent, and unrelated worlds (Griffin, 2004). Children will broaden their knowledge and categorize according to different attributes such as length, weight, and number, or color, texture and so forth.

Subitizing. To subitize is to see a quantity at a glance: to be able to tell what number of objects in a collection or to instantly recognize and name how many items of a small configuration (Clements, 2004). Children’s ability to subitize, like general mathematical knowledge, grows qualitatively. Children’s ability to see small collections of objects grows from perceptual to imagined, and to numerical patterns (Steffe, 1992;
Subitizing is a qualitative understanding of numbers – visually knowing the quantity of small groups of objects. It basically means that a child can look at four leaves and say “hey we have four leaves!” without needing to count them. It translates as giving the ability to conclude that two groups are the same in terms of quantity in a quick look (California Department of Education, 2008).

Richardson (2004) writes about her students whom she thought could understand dice patterns and relate them to numbers but then realized that when asked to reproduce the pattern, the number of dots did not always match the number represented. The children could not accurately imagine the pattern, and drew a conglomerate of dots representing an X shape to make a five, but the number of dots was not equal to five. Subitizing is an ability limited by the number of objects and by the child’s maturity.

Aspects of counting develop in a way that depends largely on individual children and their experiences (California Department of Education, 2008). The understanding of numbers is initially qualitative as children gain an understanding of “number-ness” with small quantities – subitizing. However, understanding of quantities and numbers is largely related to counting. Young children initially understand a quantity as an aggregate of single units and then relate counting and operations combining small sets of objects (California Department of Education, 2008).

A number can be used to represent so many things (Kilpatrick, Swafford & Findell, 2001), how can we bring different perceptual cues to the concept of number?
The Counting Process

Four categories in the enumeration activities have been identified according to levels of complexity (Seo & Ginsburg, 1999; Clements, 2004; Copple & Bredekamp, 2009; California Department of Education, 2008). These are reciting number words, ordinality, cardinality, and a dual method system of counting. Each of these will be described briefly. STEPHANE – a paragraph needs three sentences and though I left the one-sentence paragraph in the last section, two in a row is a bit much so I elaborated what was coming up.

**Reciting number words.** Number sense involves saying numbers, such as: I am five years old or in pointing numbers on the calendar and reading them out loud. It also consists in part in reciting number words in order starting from one. Counting consists of learning the list of number words.

**Ordinality.** After having learned the list of number words, there is a point in associating a number word to each one of the objects in a collection. Children are in the phase of connecting objects to spoken word to numerals (Clements, 2004). It may be viewed as a one-to-one assignment. It aims at assigning the numbers from the numeral list to each objects of a collection without counting the same object twice – tagging each object and associating it with a number word (Seo & Ginsburg, 1999). Ultimately the purpose is to enumerate objects.

**Cardinality.** Children may not always know how many items are in a collection after counting them. An additional concept needs to be acquired: cardinality. It is the concept of knowing that the last number named when assigning the numbers from the
numeral list to each of the objects of a group, without counting twice a same object, is the quantity of objects counted. It is a connection to make between the process of counting objects in a collection and the number of objects in the collection. The last number word enumerated during the process of counting is the number of objects in the collection. It allows concluding on the cardinality. The last ordinal number is the cardinal, the number of objects in the collection: One, two, three. There are three olives. The last number word said refers to how many items have been counted.

Two methods are used to find out if a collection of objects has more than another. Children may count the number of objects in each collection and find that the collection which has more is the collection which has the number word that comes the latest (Clements, 2004). They may also place the objects of the two collections in one-to-one correspondence from each set to see if there are more items in one collection then in the other. These processes contribute to learning about the concept of cardinality explained earlier. These four aspects are learned initially by different kinds of experience and need to become connected. These aspects are introduced with small numbers and then applied to increasingly larger numbers (Clements, 2004).

Relevancy to Everyday Life

The National Association for the Education of Young Children (Copple & Bredekamp, 2009) brings contemporary relevance to what Piaget stated when referring to the preoperational stages of development, emphasizing that children from two to seven were less capable in their thinking than older children. Copple and Bredekamp (2009) describe that when preschoolers were in a familiar situation, they were capable of greater
cognitive abilities, especially when the task was clearly explained to them, and that they could focus on one thing at a time. The ability to reason logically is enhanced when tasks are simple, in line with what they already know, and made relevant to their everyday lives (Copple & Bredekamp, 2009). The narrow focus on a limited amount of information at any given time is actually useful while children are learning so many things so rapidly. Overall, the most recent research (Clements, 2004) indicates that children will do better on cognitive tasks than once assumed.

Seo and Ginsburg (1999) found that children engage in all categories of the counting process – described in the following part – during their free play and that preschool and kindergarten children’s mathematics is advanced and powerful. This may be a basis to stand on if there were standards to be elaborated, and teachers may use this natural impulse that children have toward mathematics with artful guidance and challenging activities, understanding the mental lives and learning potential of young children.

**Making Math Matter in Early Childhood: The Use and Utility of Standards**

Copple and Bredekamp (2009) point out a necessity of guidelines in the domain of mathematics justified by the lack of mathematics proficiency by the end of second grade (Fuson, 2004). The goal is the implementation of a robust program in early childhood education. The curriculum is a guide presented as a progression of topics which has been elaborated by researchers (Copple & Bredekamp, 2009). Instructions need to be engaging to children, consistent with their developmental level, and focused on the important concepts and processes on which subsequent math learning will build.
Bredekamp (2004) mentions that the field of early childhood education in mathematics would tremendously benefit from some more specified content as it is done in reading and writing. A concerted effort to come to a consensus among early childhood educators and mathematics educators about what the standards should be for preschool and kindergarten, would lead to higher quality programs, proven to result in learning benefits into elementary school, including in mathematics (Bretekamp, 2004). Clements (2004) emphasizes that knowledge of what young children can do and learn, as well as specific learning goals, are necessary for teachers to have a vision of high-quality early childhood education.

In the next section, a description of math foundations will be offered along with various ways of measuring young preschoolers in their mathematical knowledge and skill. The section begins with a discussion of the California Preschool Learning Foundations (California Department of Education, 2008) regarding math. This is followed by the assessment component linked to the Foundations – the Desired Results Developmental Profile – Revised (DRDP-R, California Department of Education, 2006) along with guidelines established in a concerted effort of early childhood educators (Clements, 2004) and an integrated consideration of developmentally appropriate practice related to young children playing with a variety of math concepts (NAEYC, 2009). After describing the three resources for early childhood educators with regard to California’s Early Learning System, some caveats about working with standards and young children will be highlighted.
An Overview of the California Preschool Math Foundations

The California Department of Education (2008) in the *Preschool Learning Foundations* identifies sets of behaviors in mathematics typically expected to be observed and that can be used as guidelines rather than as aspirational expectations. These can be explored, as explained in the publication (see Table 2.1), under the condition of appropriate social-emotional, cognitive and language development, and take place in an environment encountered every day, through interactions, relationships, activities, and play.

**Reciting numbers in order.** The *Preschool Learning Foundation* (California Department of Education, 2008) indicates that developing number sense consists partly in reciting numbers in order with increasing accuracy: Up to ten at around 48 months of age and up to twenty at around 60 months of age. Reciting these numbers may be done while playing, walking, or singing. However, errors can be made; if some children do not recite by saying or by singing, the publication suggests using other means such as touching number cards or saying yes or no when an adult counts. Numeral association with the counting numbers can also be expected, although this skill will likely not be used by the students during the counting game with pennies.
Table 2.1

*The Preschool Learning Foundations: Number Sense*

<table>
<thead>
<tr>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.0 Children begin to understand numbers and quantities in their everyday environment.</strong></td>
<td><strong>1.0 Children expand their understanding of numbers and quantities in their everyday environment.</strong></td>
</tr>
<tr>
<td><strong>1.1 Recite numbers in order to ten with increasing accuracy.</strong></td>
<td><strong>1.1 Recite numbers in order to twenty with increasing accuracy.</strong></td>
</tr>
<tr>
<td><strong>1.2 Begin to recognize and name a few written numerals.</strong></td>
<td><strong>1.2 Recognize and know the name of some written numerals.</strong></td>
</tr>
<tr>
<td><strong>1.3 Identify, without counting, the number of objects in a collection of up to three objects (i.e., subitize).</strong></td>
<td><strong>1.3 Identify, without counting, the number of objects in a collection of up to four objects (i.e., subitize).</strong></td>
</tr>
<tr>
<td><strong>1.4 Count up to five objects, using one-to-one correspondence (one object for each number word) with increasing accuracy.</strong></td>
<td><strong>1.4 Count up to ten objects, using one-to-one correspondence (one object of each number word) with increasing accuracy.</strong></td>
</tr>
<tr>
<td><strong>1.5 Use the number name of the last object counted to answer the question, “How many …?”</strong></td>
<td><strong>1.5 Understand, when counting, that the number name of the last object counted represents the total number of objects in the group (i.e., cardinality).</strong></td>
</tr>
<tr>
<td><strong>2.0 Children begin to understand number relationships and operations in their everyday environment.</strong></td>
<td><strong>2.0 Children expand their understanding of number relationships and operations in their everyday environment.</strong></td>
</tr>
<tr>
<td><strong>2.1 Compare visually (with or without counting) two groups of objects that are obviously equal or nonequal and communicate, “more” or “same.”</strong></td>
<td><strong>2.1 Compare, by counting or matching, two groups of up to five objects and communicate, “more,” “same as,” or “fewer” (or “less”).</strong></td>
</tr>
<tr>
<td><strong>2.2 Understand that adding that adding to (or taking away) one or more objects from a group will increase (or decrease) the number of objects in the group.</strong></td>
<td><strong>2.2 Understand that adding one or taking away one changes the number in a small group of objects by exactly one.</strong></td>
</tr>
</tbody>
</table>

Subitizing. At around 48 months of age, according to the California Preschool Foundations (California Department of Education, 2008) children may – without counting – identify the number of objects in a collection of up to three objects. Subitizing – visually knowing the quantity of small groups of objects – may be expected to be performed as well for collections of up to four objects at around 60 months of age. The publication mentions that the support used to present a collection of items may use different senses, such as auditory (children perceive a certain number of beats without counting), tactiley (perceive with fingers bumps on a surface), and visually (seeing a number of objects). Move the sections that elaborate the Foundations (and remember to always italicize them) into the section I have identified for it above.

One-to-one correspondence. Establishing one-to-one correspondence between a group of objects and the list of number words may be expected for up to five objects at around 48 months of age and up to ten objects at around 60 months of age. These activities may be performed using blocks: as children build up a tower, they are encouraged to assign one and one number word only to each of the blocks used to make the tower using the correct order without repeating the same number word; during circle time: it is possible to assign a number to each of the students present or divided into small groups, such as boys, or students sitting on the bench. This skill may be used by students during the activity as they will be presented five pennies. The point however is to know how many objects there are in a collection.

Cardinality. Counting is completed by learning about cardinality, which is the concept of knowing that the last number named when assigning the numbers from the numeral list to each of the objects of a group, without counting twice a same object, is the
quantity of objects counted. This concept may be used at around 48 months of age and understood at around 60 months of age.

**Comparing.** Children may be expected to visually compare two groups of objects that are obviously equal or non-equal and communicate “more” or “same” at around 48 months of age. They may be expected to compare by counting or matching two groups of up to five objects and communicate “more”, “Same as”, or “less” at around 60 months of age. For example, at age five, children may be expected to count the number of rocks each of two children have and communicate: “five and five”, they have the same. (California Department of Education, 2008)

**What Are the Desired Results Developmental Profile for Four and Five Year Olds?**

The *Desired Results Developmental Profile – Preschool* (California Department of Education, 2010) is an assessment instrument designed to improve the quality of programs and services provided to children. It is designed to be used as a tool “by which educators can document the progress made by children and families in achieving desired results” (CDE, 2012).

The DRDP-P is meant to identify measures demonstrating achievement across development areas – among which “number sense”, at the center of the thesis. This publication reflects how centers can contribute to child development and may be used to hold programs accountable to the achievement of these desired results. It may be used to measure the quality of a program because it helps gather data for evaluating individual child development programs. At last, it is an assisting tool to improve practice in all child development programs.
Table 2.2

**DRDP Measure 32: Number Sense of Quantity and Counting**

<table>
<thead>
<tr>
<th>Exploring</th>
<th>Recites some number names not necessarily in order, identifies, without counting, the number of objects in a collection of up to three objects</th>
<th>“I have only one cookie. I want two.” “I see three dogs.” Names some numerals in a counting book as teacher points to them. Points randomly to objects and says, “1,2,4.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing</td>
<td>Recognizes and knows the name of some numerals; correctly recites numbers in order one through ten</td>
<td>Recites the numbers 1 to 10 correctly. Chants one to ten in order while jumping. Points to the number “3” on the bus and communicates “Three.”</td>
</tr>
<tr>
<td>Building</td>
<td>Counts at least five objects correctly, without counting an object more than once</td>
<td>Counts five bears in a story book, “1,2,3,4,5—there are five bears.” Brings the correct number of plates when an adult asks for six more plates or the snack table.</td>
</tr>
<tr>
<td>Integrating</td>
<td>Counts at least ten objects correctly; recites numbers in correct order up to twenty; demonstrates understanding that the number name of last object counted is the total number of objects</td>
<td>Paints a picture of ten flowers, then counts the flowers, and correctly indicates how many there are. Counts objects up to 13 during small group time: “I have 13 bears.” During small group for math, wants to see how many children are in the group, and counts 11 children correctly. Counts five spaces while advancing her game piece in a board game with dice and rolling a five. Counts, “one, two, three, four, five” and communicates “five”, “when asked, “How many boats do you have?”</td>
</tr>
</tbody>
</table>

Note: From the *Desired Results Developmental Profile – Preschool. Measure 33: Number sense of mathematical operations*. By the California Department of Education. Child Development Division. (2010).

In this table from the *Desired Results Developmental Profile* (California Department of Education, 2010), we find some aspects of number sense development as
they appear in *The Preschool Learning Foundation*: Recite number words, recognize and know some numerals, assign a number word to each item of a collection without repeating a number word twice or counting an object twice (one-to-one correspondence, ordinality), subitize, and find the cardinal of a collection of objects.

Table 2.3

**DRDP Measure 33: Number Sense of Mathematical Operations**

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploring</td>
<td>Demonstrates that items can be grouped and compared by quantity; communicates that result is “more” when objects from two groups are put together</td>
<td>Communicates, “We have the same,” when referring to the number of toy animals each child has.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Communicates, “Now we have more more,” when the teacher combines markers on the table with markers from the shelf.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Points to the group with the fewest objects when asked to do so.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Takes farm animals, places horses together, and counts, though may not count correctly.</td>
</tr>
<tr>
<td>Developing</td>
<td>Correctly identifies the larger of two groups without counting; adds or takes away objects from a group and communicates that the result is more or fewer</td>
<td>When there is a group of six cups and two cups, can point to the group that is “larger.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Communicates, “There are more kids on that team!”</td>
</tr>
<tr>
<td>Building</td>
<td>Compares by matching or counting two small groups of objects and identifies which as more, fewer, or whether they are the same; identifies the number of objects in a small group after one object is added or taken away</td>
<td>Counts the number of shells she has and the number a friend has and communicates, “Five and five: you have the same as me.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When setting the table for snack, puts out three cups, then communicates, “Oh, there are only two kids,” and takes one cup away.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When asked to take away one car from a block structure, removes a car and communicates, “Hey, now”</td>
</tr>
<tr>
<td>Competence</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>Integrating</td>
<td>Solves simple addition and subtraction problems with a small number of objects</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Brings over two more cups to a group of two and communicates that there are four cups.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Has two blocks and gets three more.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Communicates, “I have five blocks.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Removes one block from a collection of ten blocks and communicates, “She has nine now.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Removes three (of ten) ducks from the flannelboard, saying, “Three left, and seven stay,” when acting out a story.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Adds two more cups to a group of eight and communicates that there are ten cups.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Has three beads then takes another, and holds up four fingers.</td>
<td></td>
</tr>
</tbody>
</table>


In this table, competences deal with grouped items. They concur with the ones enunciated in the Preschool Learning Foundation (California Department of Education, 2008): compare the number of items in two collections using the words “same” and “more”, know that adding items will result in having more or that taking away will result in having less.

**Developmental Guidelines for Number and Operations (Clements, 2004)**

The following section concerns the guidelines established with a concerted effort of early childhood educators interested in engaging young children in mathematics.

**Finding how many items are in a set.** These guidelines mention that counting can be used to find out how many in a collection and that a key element of object-counting readiness is to represent nonverbally and to gauge the equivalence of small collections of objects. It is stated that at age two or three, children make and imagine small collections of up to four items nonverbally, such as seeing which is covered, and then pulling out added objects. They can find and match equal to a collection of up to four items, such as matching :: or 4 drum beats to collections of 4 with different arrangements, dissimilar items, or mixed items.

**Learning sequences of number words.** Another key element of object-counting readiness is learning standard sequences of number words, which may be facilitated by discovering patterns. At four years of age, children count by ones from 1 to 30 (and more) verbally with emphasis on counting patterns; e.g., knowing that “twenty-one, twenty-two,…” is parallel to “one, two, …”

**One-to-one correspondence between objects and number words.** Object counting involves creating a one-to-one correspondence between a number word in a verbal counting sequence and each item of a collection, using some action indicating each action as you say a number word. Children at age four can count the items in a collection and know the last counting word tells “how many” for up to ten items. They can also produce collections of a specified size (lag a bit behind counting items in a collection) for up to ten items. Clements (2004) mentions that number patterns can facilitate determining the number of items in a collection or representing it. Four year old children can verbally
subitize (see below) collections of up to five objects, and represent collections with a finger pattern.

**Comparing and ordering.** Clements (2004) delineates standards for five year old children: Comparing and ordering build on nonverbal knowledge and experience with real collections. At the age of five, children compare first visually, then using the verbal counting sequence. Learning language for ordinal numbers can build on children’s concrete comparing and knowledge of counting words.

**Subitizing.** Students use different approaches when dealing with small numbers than when dealing with larger sets of numbers. They may look at a small group of objects and recognize “how many”, i.e. subitize, but they may need to count a larger group. In these early years, students develop an ability to deal with numbers mentally, and without a physical support. This stage of development will vary from child to child, and some students will enter school with the ability to visualize mentally quantities in order to solve addition and subtraction problems, others will need to manipulate physical objects.

These developmental guidelines are to consider with four major caveats in mind. According to Clements (2004), qualitative different ways of thinking and learning of young children, does not appear. For example, the aptitude of subitizing does not explain the process for patterns to grow from being perceptual, being imagined, and finally being numerical (Steffe, 1992). A second caveat raised by Clements (2004) is that these tables do not give trajectories to be followed by teachers and their students; they do not give the level of details for great concepts or each mathematical topic. Finally, these guidelines do not constitute a curriculum. The competencies in the tables are not directions for
curriculum. The following paragraphs, indicate more precautions to consider when working with standards.

Caveats about Working with Standards and Foundations

**Individual variation.** Standards must be flexible enough to respond to inevitable individual variation because individual differences in children of any age may be greater than the change in behavior and skills they will experience over the course of one year (Clements, 2004). Copple and Bredekamp (2009) warn about the idea of a curriculum that would just be a scaled down version of a curriculum designed for older children, and not consider developmental gaps and differences between individual preschoolers.

With that in consideration, the *Preschool Learning Foundations* (California Department of Education, 2008) were developed as features to describe what typical development is rather than as a set of goals to be reached at each age under the best conditions. The *Foundations* are not intended as an assessment tool, with items against which children should be evaluated, but rather as teaching resource for teachers.

A major concern brought up by Bredekamp (2004) is that young children develop at very different pace, and that standards would tend to bring all individuals at a same level when the reality is that it may not always be appropriate to measure children’s performances on a skill or competence without considering individual, cultural, and linguistic variations in young children.

**Standards set as goals to reach may defeat the purpose of early childhood education.** What happens if children are pressured with intensive mathematics and material they are not ready to learn, such as harsh forms of instructions consisting of
written drills and practice in mathematics? The preschool role of nurturing curiosity would be lost. Seo and Ginsburg (2004) underline the importance of not using methods of instruction that would be developmentally inappropriate. Bredekamp (2004) warns that children can learn numbers up to 100, but also asks whether or not all children should be expected to know numbers to 100 in pre-kindergarten. It is not because children can that they should.

Bredekamp warns teachers about the use of “all children should” in the National Council of Teachers in Mathematics publication: Principles and Standards for School Mathematics Standards (2000) from pre-kindergarten through second grade. The standards have been established by researchers in the field with the wisdom of the classroom and should be envisioned with developmentally appropriate practice. The standards do not allow the teacher to make judgment of the development of the child in the specified field, but rather provide guidance for the activities to be set.

The power of free play. Seo and Ginsburg’s study (1999) indicated that children do engage in mathematical activities during their free play. Clements (2004) recommends that teachers engage very young children in mathematics by offering them experience within their play and in their natural relationships with others in activities, interests, and questions. Working with standards must not reduce the place of free play in an early childhood program, but help maintain children’s mathematical interest, curiosity, motivation, and competences by observing how they engage in mathematical activities during their free play.
A mind structured for mathematics. Humans are born with a fundamental sense of quantity and young children possess an informal knowledge of mathematics that exceeds adults’ expectation in its breadth, complexity and level of sophistication (Clements, 2004). Early childhood educators have a special opportunity to unveil this knowledge – existing concepts – in situations that will entice children to demonstrate their ways of solving problems – making sense – and dealing with interesting and engaging math-based situations. Steffe (2004) does not assume that children come to school as blank slates but rather have already constructed mathematical concepts and operations before. They come to school with already having complex ways and means of operating mathematically. It seems hence important to start working on mathematics with young children by unveiling what children know rather than confront them with standards established standards.

Manipulatives and Collaborative Learning

Manipulatives

Children often manipulate a wide variety of things, including materials in mass (sand, rice, water), but also discrete objects that may be sorted, classified, arranged in patterns, and counted (Copple, 2004). Copple and Bredekamp (2009) emphasize that teachers can help children “mathematize” their everyday encounters, practicing and learning math skills and concepts when playing with blocks or manipulatives, or doing movement activities, or even computer time where it is assumed that children are in control of the machine. Here again, it is not mentioned how really manipulating would allow for enhanced learning experience, but it demonstrates how children from age three
to age five are naturally inclined to move, to touch, and to manipulate. Hence the reasons to believe that learning mathematical concepts and skills ought to be enhanced through manipulation.

Clements (2004), at the conclusion of the symposium on standards in mathematics in prekindergarten and kindergarten, differentiates in the process of counting the knowledge of the sequence of number words, the ability to assign a number to each object of a collection, and the designation of the last number word as the total amount. For young children, Clements (2004) writes, the ideal situation for the counting process, after having learned – as we learn a song – the sequence of counting words, is to have a small collection of objects, aligned in a row, in a way that allows children to touch and move with their fingers each object as the assign them with numbers. Clements (2004) points that touching is essential to realize the connection between number words and objects – ordination – at an early age. To count objects requires considerable practice to coordinate which can be facilitated by having children touch the objects they are counting. The next step is to realize that the last number word uttered is the quantity of objects: the cardinal number. As children acquire skills and practice they will be able to cope with larger quantities without the need to touch and move objects.

According to Piaget (1969) when children have manipulated and explored with their senses before being invited to conceive their abstract properties, the concepts they acquire subsequently consist in a genuine realization of active schemes already visited. What Piaget (1969) seems to emphasize is that practical intelligence and sensori-motor skills need to be solicited and used to actively explore objects and props that relate directly to an abstract concept before even talking about it. If children are exposed
without prior experience involving manipulation to a verbal version of a concept, accompanied with formal practice exercises and no foundation of a preliminary experimental substructure, the activities will be uninteresting and not as meaningful as they should. Piaget refers to the Active Method as to the method featuring a preliminary manipulation of the quantities and surfaces. To illustrate a system in which teachers would guide students by making them be actors, rather than limiting the interaction to listening to dictating lessons.

Piaget (1972) refers to “traditional” teaching as lacking in giving opportunities to students to experiment and to allow them to discover the process of experimentation. Piaget states allowing children to touch things but under the obligation to follow a prescribed procedure, puts them also in an assisted position, where they are not real actors. In both cases, the approach will not teach them the general rules of all scientific experimentation, such as the variation of one factor (parameter) while neutralizing other factors. According to Piaget, an experiment that we do not do ourselves, with the entire liberty to initiate action, is not an experiment, but mere conditioning with no learning values.

**Collaborative Learning**

**Influences of social interactions and play.** Peer social skills emerge at school as complex, sophisticated, and multifaceted (California Department of Education, 2008). As preschoolers achieve insight into others’ feelings and thoughts, through their own elaboration of “theory of the mind,” they become capable of greater cooperation with other preschoolers and coordination in their shared activities. In turn, social-emotional
skills contribute to the development of confidence in initiatives in learning, to more participation in class activities and to greater achievement in a positive school experience, and nurtures the child’s approaches to learning (California Department of Education, 2008).

According to the California Department of Education (2008) approaches to learning are important predictors of school achievement. It is defined as children’s classroom engagement, motivation, and participation. In preschool, there are significant differences among children in terms of enthusiasm, motivation, and self-confidence. The California Preschool Foundations (2008) actually makes a distinction in children’s approaches to learning which is between the performance orientation, where children tend to perform for the appraisal of others and tend to fear difficult tasks that may lead to failure, and the learning for mastery, where children will bring efforts into their task in order to gain in ability and will not fear challenging or unknown tasks. It seems, according to Kagan (1990), that cooperative learning, where children are not competitive, but rather cooperative, promotes the latter trend of approach to learning: “Students in cooperative learning classrooms outperform those in individualistic and competitive classrooms.” (Kagan, 2009)

Interacting with a familiar adult, and other adults, as well as with peers, is part of the goals mentioned in the Learning Preschool Foundations (California Department of Education, 2008), with the precision that the interaction can be comfortable and competent as children naturally tend to seek security and warmth in their relationship with caregivers. The ability to engage in these interactions constitutes an important component of a successful transition to the school years. At 48 months, interactions with
peers in shared activities become cooperative efforts, and at around 60 months of age, cooperation with each other becomes more active and intentional.

According to Copple and Bredekamp (2009), the importance of peers appears during the years from three to five, and children interact much more than during their toddler years. Their interactions appear also to be more complex than just parallel play, as they engage in social dramatic play where they agree on which role they will be playing. Between three and five years of age, children have better understanding of others’ thoughts and feelings (Copple & Bredekamp, 2009). Emerging social skills such as cooperating, helping, negotiating, and talking with other people to solve problems will develop in real collaborative skills.

Kagan (1990), in his rationale for cooperative learning, mentions cooperative work and collaborative work as means for improving classroom climate, lowering anxiety, using of peer support for language development, increasing ratio student/teacher talk, increasing self-esteem among students, internal locus of control and intrinsic motivation, role taking abilities (situations in which bilateral and multilateral communication are necessary probably increases the general sense of interdependence among students, and a induces a higher level of morality), increasing time on task, attendance, and liking for school and learning. All these improved aspects of learning conditions are a result, according to Kagan, of the implementation of a collaborative setting. Although Kagan does not mention the age range target of his analysis, we can imagine these impacts on children entering the operatory stages (age seven). Kagan points out that collaborative settings tend to improve social relations amongst students, which in turns, affects the cognitive development of the child. The first aspects he finds
improved in collaborative settings are a diminution of the achievement gap between various social categories, such as races and socio-economic status, improved race-relations among students, and improved social and affective development among all students. In turns, global academic achievement is improved in schools where collaborative learning is implemented, compared to control groups of schools favoring individual achievement and competition among students.

In trying to answer the question “why does cooperative learning work?” Kagan (1990) acknowledges the complexity that faces the researcher as there are multiple factors and myriads of situations arising in cooperative settings. It is very difficult for the research to isolate explanations, causes, as there are many possible ways cooperative techniques produces gains in learning experience. The topic is so complex, says Kagan, that it is a real nightmare for the researcher who wants to analyze in what ways cooperative work affects positively the students learning experience, however, what appears to be a nightmare for the researcher, turns out to be a dream for the educator because of the favorable impact on number of educational outcomes.

However some key factors have been identified, such as the practice spent by students on items they need to learn most. Cooperative learning promotes social and affective development and provides motivation and reward. Peers become supportive and rewarding of cooperativeness among their teammates. Although cooperative learning methods do not feature explicit rewarding systems, students are subject to frequent and immediately following learning successes peer supported rewards such as praise, which are for students of any age as important, and sometimes more important than delayed and less frequent parental or teacher praise. Moreover, Kagan (1990) points out that students
show, in cooperative settings, more positive pro-academic norms, involving more facilitative and encouraging interactions among students. Peer acceptance and support is important, especially for groups that tend to refer to parental norm and teacher norm that may be very different for minority students.

Piaget and Inhelder (1966) nuance two very different emotional aspects of child-adult interactions and child-child interactions, when addressing socio-emotional interactions, they consider them as not being possibly dissociated from cognitive development, as the former drives behavior and the latter structures behavior. The first one is of child-adult interactions as one-way interactions, where the adult is the source of linguistic and cultural values, as well as moral feelings for the child. It is possibly the nature of this relationship that may bias an adult-child interview. The second one which may be of adult-child interactions, but mostly of children interactions, is a continued and constructivist socialization process, and not of one way communication. Piaget and Inhelder (1966) point out that it is during the early childhood years (before age seven) that children present maximum social interactions and interdependence, but that the socialization effect is lessened by a lack of structure. This point of view may be supported by an experiment where interaction between children is encourages, but the activity is structured by the adult who will use as a guideline early childhood cognitive development research results.

Socio-emotional development is so interrelated with cognitive development that it seems nonsense to Piaget and Inhelder (1966) to wonder if cognitive cooperation – in the sense of performing reversible actions together – will generate individual operations – in the sense of reversible action – or if it is the opposite. It does appear to them that in the
preoperational stage, it is maybe more appropriate to refer to pre-cooperation rather than
to cooperation, as children at this stage will have a perspective centered on their own, and
will not be capable of the type of cooperation involving the adaptation to someone else’s
perspective. The egocentric view will in a way impair the nature of cooperation itself,
because the maturity of preoperational children will not allow them to center their point
of view outside of their own.

However, although the intention to adopt someone else’s perspective and to
communicate an intention may be absent in the socio-cognitive developmental stage of
pre-operational children, these children will find themselves in communion and in
synergetic mode when working with peers, without paying attention to the details of what
their neighbors are doing. Although, all advantages described by Kagan (1990) may not
really occur during the cooperation of young children, some of them, such as the self-
esteem, the time on task, peer reward, and lowered anxiety may allow these children to
improve their performance. Moreover, the fact that these children are allowed to
manipulate, one child will be able to observe the actions of the other child and be
influenced by these in a constructivist way, and it has been demonstrated that in an
enticing activity children will show less of the egocentric attitude and collective
monologues than in free play, where they would tend to listen to themselves rather than
listening to others.

One might expect that children will learn from each other’s actions. Bandura
supports that imitation is a behavior which contributes greatly to children’s cognitive
development and that children actually learn even when the reinforcement is only
vicarious (Bandura, 1969). Learning occurs when children observe what other children or
adults do, eventually seeking to reproduce what they observe. The experimenter’s signs of satisfaction would reinforce children’s attempts, but, according to Bandura, learning will occur even if reinforcement is offered either overtly or vicariously. For instance, when the children build on each other’s actions, children working together will observe each other’s actions and reproduce them, and if successful in sustaining the interaction (if deemed pleasurable) they will be more likely to repeat them (Bandura, 1986).

**Synthesis of the Review of the Research**

The works of Piaget and Szeminska help establish the design of the research, and are complemented by the incentives manipulation and cooperation may provide to the experience of the children going through the procedure. Understanding what mental processes involves counting and comparing the numbers of objects in two collections will help identify the various steps taken by the children to access number conservation. The guidelines established by various entities, including the California Department of Education, frame our expectations and give precious indications for our script and dialogue with the students.

In the next chapter, the methodology for the current study is presented. The process of informing the parents to ask their consent for letting their child participate in the study will be explained. The instruments used to interview students and to collect data will be presented with the design of the study and of the procedures necessary to the study. At last, the script, intended to be used with the students is provided in detail.
CHAPTER THREE

METHODS

Introduction

In the current experiment, a popular concept, “conservation of number” (that objects can be moved around spatially, but the number of objects remains the same) was observed in preschool age children under three different conditions: 1) with the teacher manipulating the objects, 2) with the child manipulating the objects, and 3) with two children working together and manipulating the objects.

Often math is directly taught by the teacher, where he or she demonstrates something and the children are expected to learn it by watching. This is represented by the first condition in the study where the teacher manipulates the objects, in this case 10 pennies. However, emphasis on hands-on-learning can be highly effective, as is proposed in the second condition when the child manipulates the pennies. The third condition, children working in pairs, is derived from a social-constructivist approach that posits learning with another can be beneficial for both and in this condition, two children work together manipulating the coins to solve the problem. It was hypothesized that children would learn the concept of ‘conservation of number’ best in pairs and least well (as measured by the number of trials in the ‘game’ needed to understand the concept) when they merely watch the teacher manipulate the pennies.

This chapter will describe how the students were grouped to play the game, how the children were videotaped while playing the game, and why the design used for this
qualitative study was a flexible experimental design. In other words, the details of the procedures used in this project are detailed in this chapter.

**Participants**

**Sampling**

**Number of students.** Eighteen four-year old children from the same preschool class were randomly grouped into the three conditions C1, C2, and C3. There were four students in C1, four students in C2, and the remaining 10 students randomly paired to result in four dyads in C3.

Table 3.1

*Operating Conditions*

<table>
<thead>
<tr>
<th>Condition 1 (N=4 children)</th>
<th>Condition 2 (N=4 children)</th>
<th>Condition 3 (N=4 pairs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult manipulates objects</td>
<td>Child manipulates objects</td>
<td>Children manipulate objects and discuss in pairs</td>
</tr>
</tbody>
</table>

**Random grouping of students.** In order to achieve the reandom grouping of students into the three conditions the student’s first names were placed in alphabetical order and were numbered from 01 through 18. Then the table of random numbers (Gay, 2009) was used to place them into groups as follows. The list of random single digits were taken by pairs to form two digits numbers. As numbers from the group of 01 through 16 appeared, students were placed in order into group C1 until group reached
four students, then in C2 with four more students, then C3 for the rest and for which they placed by pairs as their numbers appeared.

Description of Students

**Age distribution.** In this pre-K classroom the ages of the children ranges over one year. The youngest child was four years and four months old (4.4) when the series of games ended; the oldest was five years and four months old (5.4). The series of experiments lasted seven weeks. The mean which happened to be also the median age of the class was five years and ten months 5.10.

**Language used with the students.** The school in which this study took place is a bilingual program in a French/American school in Los Angeles. However, English is the primary language of each of these students, as reported by the parents at the time of enrollment. Therefore, even though French is used at school, the language that will be used during the study is English, as it is understood by all. All children were born in the USA, except for one child who was born in the Philippines. All children were declared, at the time of enrollment, citizens of America, except for the child born in the Philippines, who was declared French citizen, though her primary language at home is reported to be English.

**Instruments**

A video camera set on a tripod was used to film the students while they were manipulating or shown the 10 pennies. The math game took place during the morning daycare, before circle time, during recess, and after class, during the afternoon daycare at
a table in an area behind a fence juxting the playground area. Thus there was some random timing of the experiments as a result of teachers’ planning for the day and the children’s interest. After a pilot test, it was determined to use a picnic table outdoors for the greatest consistency in the filming and focus of the children.

Before starting to gather data, the testing procedure was pilot tested with Kindergarten students. The use of the camera set on a tripod, the quality of the sound and the precision of the image as well as the appropriate frame needed to be tested before hand with some other students. Attention was brought to the restitution of the sound and the image, but also to the script in how to clarify the task.

Research Design

A Model Described by Piaget

The research is an experimental study. The model followed was as flexible as possible. The experimenter tried to follow the spirit Jean Piaget used with the children he interviewed and played with, according to the description he makes in his book “Génèse du nombre chez l’enfant” (Piaget, 1964). Jean Piaget emphasized on using free conversation with the child and allows the conversation itself to be guided by the issue investigated. However, he writes, the interviewer ought to follow the thoughts of the interviewee as the child wanders in a spontaneous construction of the mind. The conversation with the subjects, says Piaget, ought to be more thriving when children are allowed with adequate props, or tools, to act first and then talk about their actions. Although the experimenter has a goal, and his questions were oriented toward that goal, it was his desire that children felt free to ask questions, to go in other directions, and participate in the activity.
A Design Addressing Each Hypothesis

The four students operating under the C1 condition were sitting, in turns, at a table, in front of the experimenter who was manipulating the coins. They were observing the coins as the experimenter was moving them around. The experimenter explained the layout of the experience, and assigned one row of coins to the student, and one row of coins to himself. The questions that followed were: “Do you have more coins? Do I have more coins? Or do we have the same number of coins?”

Under the C2 condition, the four students operated in turns at a table in front of the experimenter, and started with a stack of pennies. They were asked to make two rows, one row for them, and one row for the experimenter, with the same number of pennies in each row. When they were down, they were asked the question: “Do you have more? Do I have more? Or do we have the same number of pennies?” If the answer was that there weren’t the same number of pennies, then the experimenter asked if the student could make the rows in a way that there would be the same number of pennies in each row. Then, they were asked to spread out the pennies of one of the rows, and asked again the same question: “And now, do you have more? Do I have more? Or do we have the same number of coins?” Whatever the answer, the following question would be “How can you tell?”

Under the C3 condition, eight students operated in pairs. They remained with the same partner during the whole process. They were subject to the same conditions as in C2, with the only difference that they were allowed to interact with each other to solve the problem of more or the same number of coins.
Note change in table of contents, if need be – it was just hanging by itself Procedure

The Lycée International de Los Angeles is a school which consists of four campuses spread across Los Angeles. The campuses are located in Pasadena, Los Feliz, Orange County, and Tarzana. Because the experimenter is the director of the Tarzana campus, where he conducted the project, he first needed an authorization from the Head of School. He then asked authorizations to the pre-K teacher and how there could be a way to manage the conduct the study without disturbing in anyways the class activities.

All parents were informed by a letter explaining the main lines of the study. The eighteen families of the pre-K classroom from the Lycée International de Los Angeles, Tarzana Campus, received an email sent by the campus director to each parents before the winter break in order to give the families some time to read the documents presenting the study, and to discuss it. Attached to the email, were the Bill of Rights, the Letter of Informed Consent, and the Video Permission for Stéphane Plancke.

When families got back from the winter break they either brought the forms printed from home or found the paper copy of the forms in their personal folder at school. Either one of the parents, or both, signed the forms and returned them to the director’s office or the campus’ office. The experimenter personally chatted with the parents and reassured them that is was not different from the activities teachers would be leading with the children for the sake of playing and learning. However, the experimenter needed their special approval because the study was intended for a Master of Arts study and was subject to the informed consent requirements for the Protection of Human Subjects. The parents were told that the director would respect their choice if they decided that their child would not participate. He explained he did not need the participation of all the
children from the class, so it would not be a hurdle in any ways if some children did not participate. Parents were told that their children would not feel ostracized if they did not participate formally in the study because the director could play another similar game if they wanted to play anyways, like their friends.

Some parents were very curious about the study and asked the experimenter to give them a report of their child’s number sense. He also mentioned that the report would not compare children, or even evaluate children. The point of the study was to evaluate the impact of manipulation and collaboration on the emergence of a mathematical concept. Nonetheless, some parents wanted to see the videos of their child, and also know if their child was progressing well with regards to others in the class.

The eighteen four-year old children were grouped randomly into three groups C1, C2, and C3 according to the sampling method describe in part 2 of this chapter. There were four students in C1, four students in C2, and ten students randomly paired to result in four groups in C3. The math game was originally planned to take place as a part of the normally occurring free-choice activities offered to the children for 40 minutes each morning.

However it was determined best to schedule the observations during their recess time (10:40 a.m. to 11:05 a.m.), and daycare hours (8:00 a.m. to 8:30 a.m., and 3:25 p.m. to 5:00 p.m.). The goal was to maintain participation in the games twice a week for each child. Hence, children were invited to participate during their recess time. It happened a couple of times children did not want to participate because she was in the middle of some important game with their friends, so other children were observed; ultimately everyone was happy to have a minute or two of individual attention.
Instrumentation

The flip-camera used in the study was very easy to set on a tripod, not voluminous and actually did not draw too much attention because of its very light and compact design. It was very easy to use because of its integrated USB key which allows after each session to plug it in the computer, download the videos, and recharge the batteries at the same time. The camera recorded the day and time of the recording (just adding one hour to the actual hour it was taken). At first the camera only focused on the hands of the subjects. I realized after visioning the first videos that filming the faces of the children as they performed the task, provided a lot of information essential in uncovering what and how children did to solve the conservation of number task.

Script

Once the child or children are comfortably installed at the table meant for the experiments, the experimenter brought out 10 pennies… saying:

“I have some pennies here. Can you (or in condition 1 the experimenter will do everything) put them into two rows so that there is the same number of pennies in each row?” Experimenter waits until the child(ren) agree that each row has the same number of pennies. “Now this (pointing to the row closest to child(ren) is your row and this (pointing to the other row) is mine. Do we have the same number of pennies? Do you have more? Or do I have more?” Wait for response.

“Now can you move your pennies apart, spread them out?” The child (or experimenter in condition 1) moves the pennies in one row apart. “Now do you have more pennies, do I have more pennies or do we have the same number of pennies?”
The response of the child(ren) was not important, but the next question was the same regardless of whether children said one row had more or not… “How do you know?” or “How did you figure that out?” or “What happened?” as examples of prompts to have the child(ren) tell about their problem solving strategy.

Once the child has demonstrated conservation of number, that is confirming that the number of coins remains the same in the two rows no matter the spacing in one row, he did not need to continue further. Children were invited to participate in the ‘number game’ for up to four weeks, twice/week. Each trial took about five minutes or less.
CHAPTER FOUR

RESULTS

In this study, the hypothesis is that when children manipulate objects they will understand emerging mathematical concepts, such as number conservation, faster than when they are a spectator merely sitting in front of an instructor. Moreover, if they are encouraged to collaborate with peers on the task and can exchange thoughts as they manipulate, they will master the targeted concept even faster.

This chapter first presents the results of the data collection in terms of number of trials before the acquisition of the concept, number conservation, acquired when the child responds that the quantities in each row of coins are the same whether the coins are spread out or not. However, it appears that acquiring the concept of number conservation involves different cognitive stages and moves when manipulating the coins. This is the reason why, after a list of answers given by the children in each condition along the trials, there is a description of what are thought to be turning points in terms of progressing toward the mastery of number conservation. The final part of this chapter consists of the notes taken during the games that are thought to be relevant to the development of the children in terms of manipulating quantities and working together.

Number of Trials before Mastery of Conservation

The sample used for the research is a group of 16 pre-K students with ages ranging from year 4.3 to year 5.4 (M=4.9). Four children were a part of the C1 condition – with no manipulation, nor collaboration, four others played individually in the C2
condition – with manipulation, but no collaboration, and eight children were grouped in pairs to collaborate, and were allowed to manipulate the coins as they wished in C3

**Results for Condition One: No Manipulation, Nor Collaboration**

Table 4.1 shows the conclusion each student made after spreading out the coins and being asked about whether or not there were more coins in the spread out row. As indicated, the word “more” is written to mean that the student answers that there are more coins in the spread out row. When “more” is assigned by a student to a row, it is always the spread out row. The word “same” is used to mean that the student said that there is the same amount of pennies in each row, no matter their spacing. The number of trials varies from six to eight, but stops when the stage of number conservation seems to have been reached. Their frequency varied from two to three times a week.

Table 4.1

*Child’s Response to Quantity of Pennies in the Spread Out Row in Condition C1*

<table>
<thead>
<tr>
<th>Children:</th>
<th>Jips (5.4 years)</th>
<th>Insu (5.1 years)</th>
<th>Inom (4.5 years)</th>
<th>Teso (4.11 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st}) trial</td>
<td>More</td>
<td>More</td>
<td>More</td>
<td>No answer</td>
</tr>
<tr>
<td>2(^{nd}) trial</td>
<td>More</td>
<td>More</td>
<td>More</td>
<td>Same</td>
</tr>
<tr>
<td>3(^{rd}) trial</td>
<td>More</td>
<td>More</td>
<td>More then same as prompted to count</td>
<td>Same</td>
</tr>
<tr>
<td>4(^{th}) trial</td>
<td>More</td>
<td>More</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>5(^{th}) trial</td>
<td>More</td>
<td>More</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>6(^{th}) trial</td>
<td>More</td>
<td>More</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>7(^{th}) trial</td>
<td>More</td>
<td>More Confusion as prompted to count, then no answer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8(^{th}) trial</td>
<td>More</td>
<td>More</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

48
In this condition there is almost no noticeable evolution of the concept of conservation of number. The only real changes are triggered by prompts to count, and even with that, not all children are able to come to the conclusion that the rows are the same. Even after eight trials Jips and Insu always conclude that the number of pennies increases when spread out. However, Teso after hesitation on the first trial, for which she does not answer, comes with the conclusion of conservation of number for all following trials. Inom, who concludes that the number increases when spreading out the coins on the first two trials, concludes conservation of number after being prompted to count on the third trial and continues to answer correctly for the remaining three trials. To summarize, two children do not learn to conserve number, and possibly two did learn conservation of number, but this is confounded by the potential that Teso possibly already knowing how to conserve number before the experience.

**Results for Condition Two: Manipulation, but no Collaboration**

Table 4.2 shows the conclusion each student made after moving the coins around and being asked whether or not there were more coins in the spread out row. The number of trials it took for the child to answer “same” varies from five to 10, but stops when there is no more time for further trials or when no evolution is noticed from one trial to the next one.
Table 4.2

*Child’s Response to Quantity of Pennies in the Spread Out Row in Condition C2*

<table>
<thead>
<tr>
<th>Children:</th>
<th>Djmu (4.9 years)</th>
<th>Seon (4.3 years)</th>
<th>Teom (4.3 years)</th>
<th>Tivj (4.6 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; trial</td>
<td>More</td>
<td>More</td>
<td>More</td>
<td>More, then Same (after counting)</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; trial</td>
<td>More</td>
<td>More</td>
<td>More</td>
<td>Same</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; trial</td>
<td>More</td>
<td>More</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More (prompt to count) then Same</td>
<td>More</td>
<td>More</td>
<td>Same</td>
</tr>
<tr>
<td>5&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More</td>
<td>Same</td>
<td>More</td>
<td>Same</td>
</tr>
<tr>
<td>6&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More</td>
<td>Same</td>
<td>More</td>
<td></td>
</tr>
<tr>
<td>7&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More</td>
<td>More</td>
<td>More</td>
<td></td>
</tr>
<tr>
<td>8&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More</td>
<td>More</td>
<td>More</td>
<td></td>
</tr>
<tr>
<td>9&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More</td>
<td>More</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10&lt;sup&gt;th&lt;/sup&gt; trial</td>
<td>More</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There is little evolution in terms of getting to the conclusion of conservation along the trials in C2 condition. However, various stages in the development of the concept of numbers are observed. The starting point varies according to the children and some children started by putting five coins in two rows. In spite of little evolution towards the conclusion of equivalency between the sets, there is a lot of evolution in terms of getting to make two equivalent rows. The experimenter uses the phrase “can we share the stack of pennies, and make two rows, one for you and one for me?” as a prompt to help the subjects make the two equivalent rows to start with. This phrase was used in all cases, although it was not originally in the script. It seems like the children in C1 are more successful than in C2 – with one child getting conservation in the manipulation context. This will be discussed in Chapter 5.
Results for Condition Three: Manipulation, and Collaboration

Table 4.3 shows the conclusion each student made after being asked whether or not there were more coins in the spread out row. Trials stopped when the experimentation did not bring anymore new elements and became repetitive.

Table 4.3
Children’s Response to Quantity of Pennies in the Spread Out Row in Condition C3

<table>
<thead>
<tr>
<th>Children:</th>
<th>Efen (5.0) &amp; Moen (5.0)</th>
<th>Eska (4.9) &amp; Neas (4.4)</th>
<th>Dusi (5.2) &amp; Umow (5.0)</th>
<th>Iwem (4.6) &amp; Tuqj (4.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st trial</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>No Conclusion</td>
</tr>
<tr>
<td>2nd trial</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>3rd trial</td>
<td>Same</td>
<td>No conclusion</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>4th trial</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>5th trial</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>6th trial</td>
<td>Same</td>
<td>Same</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th trial</td>
<td>Same</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8th trial</td>
<td>Same</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There is almost no evolution in these results as almost all the dyads conclude from the start that the number of pennies was the same in each row, before and after having spread out the pennies in one row. In some cases (Eska, Neas; 3rd trial) and (Iwem, Tuqj; 1st trial) there were no conclusion brought to the question. This was the result of an activity that took too long, or for which the elaboration of a configuration of two numerically equivalent rows with one row spread out and one row more compact was not successful.
Turning Points

There are some key moments during the various trials, which can be described as turning points. These are either times when the children shift from one conclusion to another, but also times when they tackle important counting process milestones which are described in Chapter Two as part of the developmental process in understanding conservation of number (Piaget, 1964). Below, the observed turning points are presented for each condition.

**Turning Points in Condition One**

The most obvious turning point is when Inom (4.5) is asked to count the number of objects in each collection. Although on the first two trials she answers that there are more coins in the spread out row, when she counts five coins in each row at the third trial she concludes that there are the same number of coins in each row; she comes with the same conclusion on all following trials. Another turning point is when Teso says it is the same number of coins “because you put the same” (Teso; 01/24; 1’10”).

**Turning Points in Condition Two**

Djmu (4.9) had some difficulties making two rows with the same number of coins in each row using all 10 pennies, but finds her way when she makes smaller groups of pennies, such as two pennies or three pennies in each group, leaving the remaining pennies on the side. In some situations she even starts by giving only one coin to each row, and then increases by one the number of pennies in each part - as prompted - to try to use all the pennies.
Seon’s manipulations revealed a lot of learning going on. For instance, he had difficulties making two rows with the same number of pennies, but upon advice to put both rows right next to each other (Seon; 01/27; 2’55”) he managed to establish one-to-one correspondence and made two equivalent rows. At another trial, he seemed to find that the two rows he has prepared are not the same; he decided to put away some coins to reduce the quantities to a two and two configuration, stating “we have the same” (Seon; 02/06; 2’15”). However, when prompted to add more coins to each row, he managed to make two separate rows of five coins (Seon; 02/06; 2’50”), then put them next to each other to establish one-to-one correspondence (Seon; 02/06; 3’05”). After spreading out his coins, Seon stated “we have the same, because they are separated, and before they were like this” [he puts them back together], then spreads one row and the other to establish one-to-one correspondence (Seon; 02/06; 3’50”). Although Seon fails to see the spread out pennies as being equivalent to the compact set on the following trials, he manages to proceed methodically to build two rows of five coins from the ten given pennies (Seon; 02/15; 1’00”).

Teom concluded that the number of coins is the same in the spread out row and in the compact row on the third trial (Teom; 01/27; 4’30”) as she is accompanied by her two year old friend “baby Nepf” (a younger child who is observing her), but concluded that there are more coins in the spread out row in each other trial, when no one was watching. On the sixth trial, Teom (4.3) made two quincunxes and says “now we have the same” (Teom; 02/08; 1’55”), without explaining why. Teom manipulated a lot the coins before establishing two equivalent rows; she tended to group all ten of them on one same line, then voluntarily took out two coins of the whole set to establish two equivalent rows.
However, she succeeded in making two rows with the same number of coins after the 4th trial, and spread out the coins perfectly. On seventh trial, Teom tried to make two row of equal quantity by adjusting the length.

Tivj said there are more when they are spread on the first trial but immediately had doubt and decided to count them to conclude then that there is conservation. “1, 2, 3, 4, 5: we have the same!” Tivj emphasized the last number word enumerated (Tivj; 01/11; 2’15”).

**Turning Points in Condition Three – Pairs Collaborating**

In each pair of children the dynamics of how they figured manipulated the coins, determined whether the rows contained the same number of pennies and the children discussed their decision-making strategies. Each groups’ process is described to follow.

**Efen and Moen.** On the first trial, Efen (5.0) and Moen (5.0) spent some time trying to adjust the number of coins they have in their respective rows. Many exchanges of coins occurred before succeeding in making the number in each row equal (Efen, Moen; 01/17; 2’10”). However, the students who did not place the two collections in visual correspondence needed to count to figure whether they have the same or not. They proceed by trial and error to adjust, but ultimately moved more than one coin at a time from one set to the other. Eventually, Moen withdrew two coins and put them away, to conclude “we have the same.” Efen counted his pennies successfully to four, Moen counted and concluded “four” like Efen. The experimenter asked the students to put their respective rows next to each other when Efen noticed that two coins have been left on the
side. Efen reached for them and placed one in each row. He counted out loud by pointing to each penny with his finger.

On the first trial, Efen struggled to spread out his coins. The concept of spreading out seemed hard to understand. The word “separate” was used and led the children to divide the coins into two groups. The experimenter then pointed at the smaller groups and asked them to separate those coins as well. Once done with this, both Efen and Moen concluded that they have the same number of pennies. At one point (Efen, Moen; 01/17; 5’30”) Efen wanted to take Moen’s pennies to spread them out in one-to-one correspondence to his spread out row. Since the experimenter asked Efen not to spread out that row, Efen counted them out and concluded that the number had not changed; they both had the same number of pennies.

On the second trial, Efen and Moen declare having the same number of coins in each row, and when asked why, they said “because I just separated them.” When the experimenter points out that one row is longer than the other row Efen explained that separating is the same, while Moen puts the separated pennies back together. When the experimenter asked them to separate the other row, and to compare, the conclusion was still the same, the rows are the same.

On the fourth trial, while making the initial row of coins, Efen and Moen noticed that they did not have the same number of coins, without the need to establish a one-to-one correspondence. They actually put their hands around their row and when asked whether or not they have the same number, without counting out loud, nor pointing with their finger, they concluded, after taking the time to look at each set, that they did not
have the same number of coins. When asked to make it the same, Efen seemed to know that he needed to take two from Moen who did seem ready to give away his coins. The experimenter asked them to put their rows next to one another. Efen then repeated this request in his own words to Moen, as if he was asking kindly for his cooperation.

On this same fourth trial, Efen and Moen, responded to the experimenter together “we have the same!” (Efen, Moen; 01/27; 2’00”). But when asked why, Moen stuck to the method he used to answer the question on the first trial: he showed equivalency by modifying the configuration. While spreading the pennies of his row out to reestablish visual one-to-one correspondence, he said “we have the same, see, I can separate mine” (Efen, Moen; 02/08; 2’50”). Efen also wanted to show equivalency with one-to-one correspondence when asked to put their rows next to one another he made the coins touch, and when the experimenter asked him not to make them touch, but to put them just close enough, Efen explained that he was doing this to see if it is the same (Efen, Moen; 01/25; 2’10”).

**Neas and Eska.** Neas and Eska failed to separate the coins while leaving them in a row. Instead, Neas separated them in different directions (Eska, Neas; 01/11; 4’40”), which still complies with Piaget’s hypothesis that the influencing perception is of the surface occupied by the coins. Moreover, when Neas seemed to understand the fact that the coins needed to remain on the same line, he exaggerated the displacement and brought the coins all the way to the end of the table, and at one point to the next table. Although this may seem as goofing around, it does appear that he is playing with the perception of the spaces between the coins which are typically overlooked by children.
who conclude non-conservation of the number. Neas and Eska then counted the coins in each of two displaced row, they both make mistakes but corrected one another. One coin was missing, and they concluded correctly “no, we don’t have the same” (Neas, Eska; 01/20; 5’14”). Then, Neas said “because we are missing a coin.”

At that point, the experimenter decided to continue despite the missing coin, and showed his row containing four coins but on a longer row than theirs which had five coins, saying “my row is longer, do we have the same?” The response of the children was to make their line even bigger. After failing to count correctly (attributing two consecutive numbers to the same object), and spreading out the coins, Neas had an idea: “Hey, what if we copy each one out of one that people have, and that’s how we separate them.” He mapped the two sets of coins with one-to-one correspondence. He made two equivalent sets with visual one-to-one correspondence (Eska, Neas; 01/20; 6’15”). Neas counted the pennies, he made mistakes while counting (again attributing two consecutive number words to the same object, or repeating the same number word for two different pennies), but started again and ended up by counting correctly, and concluded “5 and 5, we have the same” (Eska, Neas; 01/20; 6’45”).

When prompted to move the spread pennies back together in only their row and asked the question again, both children counted and concluded that they still had the same number of coins (Eska, Neas; 01/20; 7’30”). The experimenter asked why. He pointed out that one row was shorter than the other one. The question seems to have intrigued the children, but at the 8th minute of this activity the experimenter and the
children decided this was enough! The point was made: the number is the same so we have the same.

However, on the 5th trial, on 01/24, after sharing the coins, Neas burst out “we have the same,” and Eska said, “no, one, two, three, four, and one, two, three, four, five, six,” counted again, and said “we don’t have the same, you have more” (after 1 minute). Although this note should have been a turning point in the process, Neas did not seem to understand Eska, and decided to make the constellations corresponding to five objects, and made the point by counting the number of objects in each collection and concluding that the two constellations were equivalent. However, when prompted to put the coins in a row, Neas made the same reasoning as Eska, and even gave one of his coins to Eska to conclude on the equivalency by counting to five after several tries (01/24;2:50).

On trial number six, after having spread out the coins, Neas counted all the coins up to 10. His coins were spread out and he concluded that he has more. However, Eska, who just looked at him do the counting, said “no, these are mine”; he counted his coins again and then Neas coins to conclude they had the same. Neas repeated the process and concluded on the equivalency. It is interesting to note that on this sixth trial, the children had taken possession of a row each, whereas at the previous trials, they had a row for both of them and the instructor had a row. At one point Neas said “but mine is bigger than his”, and Eska to reply, “but mine is going to be bigger than yours tomorrow, like now!”, and he spreads out his coins (02/01; 2’30”) to make a bigger line than Neas’. They count again each collection and concluded and agreed that they have the same, but Eska’s was bigger, and Neas said that his could be bigger too and spreads out his coins even
more (02/01; 3’00”). While Eska is playing with the camera, the experimenter talked one-to-one with Neas and had him tighten his row, and asked who has more. The process is of counting each set was repeated and conclusions on equivalency were made!

On the seventh trial, Neas said that since Eska’s row is longer he has more. Eska’s counted the objects in front of him. Neas repeated the process and concluded on the number equivalency.

**Dusi and Umow.** Dusi and Umow observed each other, repeated the process the other was doing, and ended up synchronizing their count (Dusi, Umow; 01/24; 1’25”) and arrived at the same conclusion. The fact that they each had their own row and got to get coins to make their own set, and agreed to share to reach number equivalency shows that they understood the concept of number. When they failed to count together in harmony they waited until one of them was done counting to start counting their own. They seemed to understand that they could count their own row, and just needed to compare the number they reached to conclude on the numeral equivalency.

**Iwem and Tuqi.** After spreading out the pennies of one row on the second trial (Iwem, Tuqi; 01/17; 2’45”), Tuqi concluded, “we have the same more,” Iwem agreed, “we have the same,” when asked why, he counted each set to five after correcting his own mistake of counting the whole collection of ten pennies.

When Iwem and Tuqi were asked, on the third trial, whether one row had more or whether they had the same number of pennies, Iwem took the initiative to count (Iwem, Tuqi; 01/18; 2’11”) each row and Tuqi concluded “we have the same!” The conclusion
on the fourth trial was very clear (Iwem, Tuqi; 01/24; 3’05”). Tuqi concluded, without counting that “we both have the morest,” while Iwem counted systematically, making the mistake to count the quantity contained in both sets: the whole ten coins, but corrected himself, and counted again to the cardinal of each subset to say “we have the same.” And when notified by the experimenter that one row was longer than the other, Iwem replied: “Yeah, it’s because they are spread out!”

**Transcription of the Videos**

The following is a more systematic transcription of the children’s words and actions as they are operating in their own conditions along the trials. This data gives an idea of how the whole series of experiments went and provides dates and time of various moments to be used to illustrate various points in Chapter Five

**Transcription of Experiment in Condition One**

Children in Condition One (C1) accepted that they needed to keep their hands from touching the coins. None of the children tried to manipulate the coins as the experimenter was placing them on the table. The experimenter, by mistake, had C1 student Jips manipulate on the fifth trial. Jips tried to manipulate the coins and had to be reminded not to at all the following three trials.

**Jips.** After two trials Jips came to the conclusion that the wider row had a greater number of coins so Jips was invited to count the pennies (Jips; 01/20; 1’15”). He counted to five in each row of pennies and got confused. He reversed his statement that the compact row had more. On the next trial, Jips still concluded that the longer row had
more and when asked to explain why, he answered “because I say so” (Jips; 01/24; 4’45”). On the fifth trial, the experimenter mistakenly allowed Jips to manipulate the coins. However, the conclusion was the same: the longer row had more coins. On the sixth trial, Jips explained: “you have more because you spread them out” (Jips; 02/01; 0’50”), and always concluded on more for the spread out row. The experimenter mentions that there has been no coin added, nor taken away (identity), but time is up! Same results on seventh trial when Jips answered the question why with: “Because I say so” (Jips; 02/08; 1’00”).

On the eighth trial, the experimenter insisted on the word “number,” in “we do not have the same number of coins?” (Jips; 02/10; 1’35”), which triggered Jips to count. However, Jips counted the coins but did not use the cardinality of each set. Jips counted the coins associating each coin with a number word of the sequence up to ten, but did not count each set separately. Jips’ conclusion upon that counting process was that, even though there are ten whether the coins of one of the sets are spread out or not, one of the set still had more coins. At the same trial, the experimenter asked the subject to count each set. Jips found five for each set. “There is five and five.” After being asked “so do you have more, do I have more, or do we have the same?” the answer was “we both have five but you have more because you spread them out” (Jips; 02/10; 2’50”). On the same trial, Jips stated: “I can say that we don’t have the same because I am looking at the coins” (Jips; 02/10; 3’40”). The conclusion remained that the most spread out row had more.
**Insu.** Insu had no hesitation to say that there were more coins in the spread out row on the two first trials. On the third trial, as she could not answer the question “how do you know?” the experimenter brought back the coins to visual correspondence and asked the same question, and Insu answered that there were the same quantity of coins. However, on the fourth trial, Insu’s answer to the question “why?” was “you spread them out, so you have more” (Insu; 01/27; 0’55”). During this period of time, Insu refused three times to come and play the penny game. She accepted to come back when the experiment suggested that friend came with her. Her friend sat on the bench behind her and did not interfere with the activity. On the 5th trial the test was short and no prompts were given other than the usual, the answers remained the same.

However, on the 6th trial, the experimenter reversed the process several times (spreading out and bringing back together) and asked the same question. Insu responded, “because we do like that [shows the action of spreading out] there is more.” (Insu; 02/08; 0’50”) On the 7th trial, the experimenter prompted Insu to count the coins in each row when one of the rows had been spread out; prompt she executed and came with the answer “5.” The experimenter asked again if there were more in one of the rows or if there were the same number of coins in both rows. Insu seemed confused (Insu; 02/10; 1’15”), for a while. There was no conclusion as the experimenter told her that we would have time to think about that later…

**Teso.** Teso hesitated but answered on the first trial that the two rows placed in visual correspondence had the same. She did not bring any answer, nor comment, and stayed quiet when the coins in one row were spread out (Teso; 01/11; 2’00”). On the
second and third trials she found that the two rows were the same whether they were spread out or not, but did not give any explanation. On the fourth trial Teso said there was the same number of coins “because you put the same” (Teso; 01/24; 1’10”). No more explanation than “we have the same” came out of the 5\textsuperscript{th} and 6\textsuperscript{th} trial.

**Inom.** On the two first trials Inom answered that there were more coins in the spread out row. She was asked to count the number of objects in each collection. Although, when she counted five coins in each row, at the third trial, she realized that there was the same number of coins in each row. She started to conserve, conclusion she carried on the 4\textsuperscript{th} and 5\textsuperscript{th} trial.

**Transcription of Experiment in Condition Two**

**Djmu.** Djmu made two rows of only four pennies each and left at the center two pennies. She was asked to use all the pennies, and added the two remaining pennies to her row. When asked if we had the same number of pennies she answered in the negative and when asked to make the rows in a way that we have the same number of pennies, she took the two pennies she added to her row and put them back at the center, returning to the four-four configuration (Djmu; 01/11; 1’00”). She then spread out her pennies as requested by the experimenter and concluded: “We don’t have the same” (Djmu; 01/11; 1’50”); and that she had more pennies. On the second trial Djmu made two rows of only two pennies each. However, she used all the pennies and made two rows of five each when prompted to use all the pennies (Djmu; 01/17; 0’45”). Djmu spread out her pennies and concluded she had more. On the third trial Djmu made a row of six pennies for
herself and four pennies for the experimenter, but when asked “do we have the same number of pennies?” she answered “no” (Djmu; 01/18; 0’55”).

When she is asked to change to make it such that we have the same number of pennies, she spread out her six pennies, and when asked if we had the same she said that she had more because she spread them out (Djmu; 01/18; 1’20”). When asked to make it such that we had the same number of pennies, she spread out the experimenter’s four pennies to match the length of her spread out six pennies, and concluded: “We have the same, because I spread them out” (Djmu; 01/18; 1’55”). However, on the fourth trial, Djmu managed to make two equivalent rows of five pennies each. Still, after spreading out her pennies she concluded that she had more pennies. The experimenter asked her to count the pennies in each row, and Djmu counted five pennies in her spread out row and five pennies in the experimenter’s row (Djmu; 01/20; 2’40”). To the question about who had more asked once again, Djmu answered “we have the same number of pennies” (Djmu; 01/20; 3’10”).

The same scenario with the same answers occurred on the fifth trial. On the 6th trial, Djmu made her row with four pennies and the experimenter’s row with six pennies. She answered by the negative when asked if both rows had the same number of pennies, and spread her row out to match the length of the experimenter’s row when asked to make it such that each row had the same number of pennies (Djmu; 01/27; 1’20”). On the seventh trial, although Djmu managed to put five pennies in each row, she said that she had more pennies (her row was longer). When asked to rearrange the pennies to make two rows with the same number of pennies she spread out her row to match the
experimenter’s row, and established a visual correspondence between the pennies (Djmu; 02/03; 1’00”). After spreading out her pennies, she concluded that she had more pennies.

At the eighth trial, Djmu made two rows of the same length but one with six pennies and the other with four pennies and stated: “We have the same” (Djmu; 02/08; 1’00”). After spreading out the row of four pennies, she concluded that this was the row that contained the most (Djmu; 02/08; 1’25”). After bringing all the coins back together, she managed to establish visual correspondence by switching row to one penny and said: “We have the same, because I put them together” (Djmu; 02/08; 3’05”), and concluded that the experimenter’s row had more after spreading them out. On the ninth trial, seeing the same mistake occurring when Djmu placed six coins for her row and four coins for the experimenter’s row, the experimenter used the terms “can you share so that we have the same number of coins in each row.” Djmu started to spread out the row of four coins to match the length of the row with six coins, and then established visual correspondence and passed a coin from one row to the other to reach numeral equivalency (Djmu; 02/10; 0’45”). After spreading out a row, the conclusion was still that the longest had the most, despite the many actions of, in turns, bringing back the coins together and spreading them out.

On the tenth trial Djmu put four coins in her row and six coins in the experimenter’s row, and stated that the experimenters had more coins (Djmu; 02/15; 0’50”). When asked to make the rows in a way to have the same number of pennies in each row, she took one penny out of the experimenter’s row [the lengths of each row now match; and Djmu seems to try to establish visual correspondence], she stated that the
number of pennies was the same in each row. When the experimenter asked her to bring the rows close to one another (Djmu; 02/15; 2’20”), Djmu established one-to-one contact, making evident that one row contained one coin more than the other. She decided to take one coin out to make the two rows equal: one-to-one contact was made between two rows of four coins each (Djmu; 02/15; 2’55”). Djmu spread out one row and concluded that there were more coins in the spread out row. The experimenter asked if more coins had been added, or if coins had been taken away, and then asked Djmu to count the coins in each row. The answer was four, for each row. This process is repeated several times. After counting the pennies in each row and noticing that the number was the same, Djmu seemed unsure, but still maintained that the spread out row had more (Djmu; 02/15; 8’05”), because “it’s bigger”.

Seon. On his first trial, Seon made two rows of four pennies and stopped. When the experimenter asked him to use the two remaining pennies on the side he added them to his row (Seon; 01/11; 2’10”). When asked if the rows had the same number of pennies, Seon counted each penny of the two rows, up to ten, with no conclusion. When asked again if the two rows had the same number of pennies, he stated that he had more pennies (Seon; 01/11; 2’50”).

On the second trial Seon still hesitated before using all the pennies, and eventually made two rows with five pennies each. However, he stated that the longer row had more. When he was asked to put the two rows next to each other, he made a one-to-one correspondence and concluded on the equivalency (Seon; 01/25; 1’30”). However, after
spreading out his coins, Seon concluded that he had more, “because I made more” (Seon; 01/25; 2’35”).

On the third trial, Seon put all the coins in a line on his side and said “I have all the coins!” (Seon; 01/27; 1’35”), when asked to share he has difficulties adjusting five and five. He somewhat got stuck on the four and six, and six and four configurations because he moved two coins at a time from one row to the other (Seon; 01/27; 2’10”). Upon advice to put both rows right next to each other (Seon; 01/27; 2’55”), Seon managed to establish one-to-one correspondence and made two equivalent rows. Then, he did not know how to spread out the coins and – as if it were a necessary intermediate step – Seon divided his row into two groups of coins, and concluded that the two rows still had the same number of coins. But after spreading out his coins like expected, he concluded that he had more coins.

On the fourth trial, the difficulty in making two initial equivalent rows of five persisted, and Seon made one row of four, and three rows of two, explaining “this is my row, this is your row, this is my row, and this is your row” (Seon; 02/01; 2’05”). Eventually, Seon established a one-to-one correspondence (Seon; 02/01; 3’35”). Remarkably, after spreading his coins out and being asked if we still had the same, Seon spread out the coins in the experimenter’s row and concluded that there were the same (Seon; 02/01; 4’10”).

On the fifth trial Seon’s still stopped at small numbers of pennies in each row and struggled to reach five: he used a technique which was giving one to each party at a time, but got confused by the end and ended up by giving six pennies to one row and four to
the other (Seon; 02/06; 0’10”). He recognized that the quantities were not the same and got back to a two and two configuration, stating “we have the same” (Seon; 02/06; 2’15”). Prompted to add more coins and to maintain the same number of coins in each row, Seon managed to make separately two rows of five coins (Seon; 02/06; 2’50”). He then put the two rows next to each other to establish a one-to-one correspondence (Seon; 02/06; 3’05”).

After spreading out his coins, Seon stated “we have the same, because they are separated, and before they were like this” [he puts them back together], then spread one row and then the other and established one-to-one correspondence (Seon; 02/06; 3’50”).

On the sixth trial (Seon; 02/08), Seon started a one-to-one correspondence by contact, making two equivalent rows of five, and agreed that we had the same. However, the conclusion made on prior session did not hold this time: Seon stated that the spread out row had more coins (Seon; 02/08; 1’20”), but when asked to bring back his coins together and to spread out the other row, he said “we have the same” and explained “because we move them like this” and spread out the compact row to match with one-to-one correspondence the spread out row (Seon; 02/08; 2’00”).

Seon then voluntarily took off the coins, mixed them up and made the one-to-one correspondence one his own. He says that we have the same. When asked to spread his coins out one more time he concluded that he had more (but maybe the activity had been going on too long!)

On the seventh trial, Seon showed the same difficulty as before in making two rows with five coins each, he made two rows with four coins each, stopped and watched
for approbation (Seon; 02/10; 0’40”), then added the two spare coins to his row, to conclude he had more. He then repeated the sharing process starting with two coins, added coins one by one, established a one-to-one correspondence by contact (Seon; 02/10; 2’20”). Seon said he had more after spreading out the coins (Seon; 02/10; 3’45”). When repeating the process of bringing back together, Seon stated that we had the same, but as soon as one row was spread out he concluded that it had more coins (Seon; 02/10).

On the next trials, Seon proceeded methodically to build two rows of five coins each from the stack of ten pennies (Seon; 02/15; 1’00”). When the five pennies were placed in one-to-one correspondence he said there were the same number, but when one row was spread out he said there were more in that row (Seon; 02/13; 02/15).

**Teom.** Teom succeeded in making two rows of five coins on the first trial by picking the coins one by one from the center stack to each side of the table in a row (Teom; 01/11; 0’50”). Teom struggled to “spread out,” the concept of spreading seemed new and the experimenter explained by using the word “separate.” Teom concluded on the first trial that the spread out row had more pennies (Teom; 01/11; 2’50”).

On the second trial, after her 15 days break, out of school, Teom made one row with six coins and another with four, and concluded that there was the same number of coins in each row (Teom; 01/25; 0’50”). When prompted to put the two rows parallel to each other and each coin in correspondence, she realized that something was wrong, she had two coins left! The experimenter gave her a hint and told her to put one and one in correspondence (Teom; 01/25; 2’05”) which she did and managed to have her two
equivalent rows. However, after spreading her coins, she found that she had more (Teom; 01/25; 2’50”).

On the third trial, Teom comes with a two year old friend who did not interfere in the activity. She made the same mistake as in the second trial by putting two more coins in one of the rows. She was invited by the experimenter to place the coins in one-to-one correspondence, and it took four minutes to end up with two rows with the same number of coins (Teom; 01/27; 4’10”). However, when asked to spread out the coins, she found that the rows were equivalent: “we have the same” (Teom; 01/27; 4’30”).

Teom managed, on the fourth trial, to make two rows with the same number of pennies and this time using all ten pennies. She wanted to check by counting, but counted all ten pennies instead of counting the pennies in each row, which did not allow her to draw any conclusion on her hypothesis (Teom; 02/01; 2’02”). This time, after having spread out the coins of one of the rows, she found that one row had more.

On the fifth trial, Teom started making groups with the same numbers but without using all pennies, she was encouraged to do so until she came up with two rows of five pennies, and concluded that the spread out row contained more coins than the other one.

On the sixth trial, Teom wanted her two year old friend to come again, but the experimenter told her that her friend would come later. Teom started the activity by putting the coins in various configurations: in a pile (Teom; 02/08; 0’50”), in two rows – one of four and one of six (Teom; 02/08; 1’05”), and when asked if they had the same, she started making rows of two coins each, and then made two quincunxes with the
pennies and said “now we have the same” (Teom; 02/08; 1’55”), without any explanation.

On the seventh trial, Teom made one row of four and one row of six, and mentioned that we did not have the same. When asked if we could do it in a way that we had the same, she spread out the coins to have the two rows match in length (Teom; 02/10; 0’36”). Then she was encouraged to make the coins in each row to touch each other, which she did, and stated that we did not have the same. When prompted again to make it so that we had the same, she decided to put all the pennies together and stated “this is both of our rows. That is how we get the same” (Teom; 02/10; 2’05”). The prompt then became: “Can we share?” Again, Teom started by making two rows of two coins each and put the rest on the side. When encouraged to add more coins in each row she added more coins, saying “I think I can make it the same!” and managed to make two rows of five pennies each (Teom; 02/10; 2’45”). However, after spreading out her coins, she concluded that she had more coins (Teom; 02/10; 3’25”), and could not explain why, but agreed that her row was bigger.

On the eighth trial, Teom made one row of seven spread out, and one row of three compacted, and concluded that we did not have the same (Teom; 02/10; 1’50”). When asked if she could make it the same number of coins, she made two pentagon shaped groups of five, tried to see if they were the same, counted the pennies up to ten, seemingly without being convinced that this was a good way to be sure the groups were the same, and when asked if the groups were the same, she answered by the negative (Teom; 02/14; 3’00”). She was prompted to put both stacks in a line. She started making
two rows with one-to-one correspondence, matched up to four coins, and decided to take away the two remaining coins as if handling all the pennies were too much for her (Teom; 02/14; 3’35”). She however managed to make two spread out rows of five (Teom; 02/14; 4’00”). When asked to bring back together the coins, or/and to spread them out, she consistently concluded that the longest row contained more, and when they had the same length they were the same. Teom seemed to understand that something was wrong and appeared to be a little confused, demonstrating a likely state of disequilibrium (Teom; 02/14; 6’05”).

**Tivj.** On his first trial Tivj established the two rows of five pennies by immediately splitting the set of pennies in two groups of equal number (Tivj; 01/11; 0’15”). Although he did not seem to understand “row”, he quickly overcame this difficulty when referred to the term “line.” “Spread out” seemed to be a term that Tivj did not understand (Tivj; 01/11; 1’15”), but ultimately understood (Tivj; 01/11; 1’45”). Tivj without hesitation said, with his spread out row, “I have more pennies” (Tivj; 01/11; 2’00”). But soon after, said “I think ….”, and started to count the pennies in each row to five, and concluded “1, 2, 3, 4, 5: we have the same!” Tivj insisted on the last number word being enunciated (Tivj; 01/11; 2’15”).

**Transcription of Experiment in Condition Three – Dyads Manipulating the Objects**

**Efen and Moen.** On the first trial, Efen and Moen managed to make one row for each of them. Moen found that the two rows did not have the same number just by glancing at them, while Efen counted each row, up to four for one row, up to six for the other. However, it seemed difficult to find right away how many coins needed to be
transferred from one row to the other (Efen, Moen; 01/17; 2’25”). Efen seemed to have difficulties understanding what to do when asked to spread the coins out, but finally succeeded, following the experimenter’s instructions to “separate the coins” (Efen, Moen; 01/17; 4’30”). On the second trial, Efen told the experimenter that Moen did not know what “separate” means, and shows Moen how to separate by separating his own coins when prompted to help him out. I love that! This language thing is so central! Collaboration in this episode last almost one minute with both children’s full attention (Efen, Moen; 01/17; 5’50”). The whole process of putting two rows of coins next to one another and the coin separation in one of the rows takes some time, but eventually Efen and Moen seem to understand the distinction of two groups of five coins. Efen systematically counts the pennies in each row and catches his own mistake when counting all ten pennies (Efen, Moen; 01/27; 2’10”).

On the second trial, Efen and Moen made their own row of pennies, apparently paying attention to both what they were doing and what their peer was doing. At one point Efen counted out loud his pennies, an action to which Moen appeared very attentive. Because the answer to the question came from Efen and not Moen, the experimenter directed the question specifically to Moen, and asked whether or not there were the same number of coins in the spread out row and the non-spread out row. While Efen, counted the pennies one more time, Moen regrouped the separated pennies into their initial position, reversing the action of spreading out, as to show that “nothing has changed” in terms of quantity.
On third trial, Efen played with the pennies: he put them in a row, separated them, and put them back together again. Moen decided where to put his row of spread out coins, and separated them much further apart than what had been given to students in C1, while Efen was attentive to his actions (Efen, Moen; 01/25; 3’50”).

Efen explained to the experimenter that Moen “did not know what separate meant” (Efen, Moen; 01/25; 4’20”). When the experimenter asked Efen to explain Moen, Moen said: “it is when you go like this” [he separates his row of coins].

**Eska and Neas.** Eska and Neas hesitated a lot on the first trial, and some time elapsed by before they started their actions. Neas decided eventually to separate the dark pennies from the light pennies: “I like to put the dark ones with the dark ones. That’s how I separate them” (Eska, Neas; 01/11; 1’40”). Then he brought more attention to the engravers on the penny than on the need to separate them into two rows. A Canadian penny drew Neas’ attention. But after discovering the novelty of the pennies, Neas and Eska separated the collection into two unequal groups and Eska said: “He has more” (Eska, Neas; 01/11; 1’22”), and eventually after another separation (Eska, Neas; 01/11; 2’20”) there was a first attempt to comparing the cardinalities: using “more”.

Still on this first trial, Neas had his attention brought to qualitative aspects rather than quantitative aspects of the coins. When Neas understood that he had to make two rows, he “liked to make it zigzag”. Neas and Eska each take possession of a stack of pennies and exchange from one group to the other, back and forth. Until the experimenter realized himself that there was a penny missing, so the division into two equivalent groups was not possible.
However, in this first trial, while Neas proceeded by trial and error transferring a random number of pennies from one row to the other, but concluding that the number of pennies was not the same (gross comparison), Eska made a brief attempt, with his finger, to count the pennies. Eska did seem sure, and seemed to prefer to observe rather than to act. Ultimately, both agreed on the disposition when asked one more time “can you make it so that there is the same number of pennies in each row” (Eska, Neas; 01/11; 3’30”). To the question “who has more?” the answer was “we both have more” (Eska, Neas; 01/11; 3’50).

When asked to separate the coins from each other, another difficulty arose: the constraint to maintain the coins on the same line (to form a row). Neas spread the coins out in all directions, without leaving them on a line. When asked to put them on “a same line,” “on one row,” Neas chose to put them into a special configuration which is not a line (Eska, Neas; 01/11; 4’20”). The experimenter decided to move on, given that the time spent on the task reached already five minutes, and asked “and now, who has more? Do you have more? Do I have more? Or do we have the same?” The answer was “We have the same” from Neas (Eska, Neas; 01/11; 4’50”). Eska seemed to agree, but no explanation was given to explain that we still had the same number of pennies and the experiment ended (Eska, Neas; 5’00”).

On the second trial, though the pennies were in a pile, Neas and Eska moved them into two rows: one for the experimenter, and one for Neas. Eska seemed to remain a spectator at this point – because all the manipulation was made by Neas – but participated into the debate. The experimenter realized that it would have been better to ask “make
two rows, one row for Neas, and one row for Eska” instead of: “Make two rows, one row for you, and one row for me.” However, both engaged in the process of counting, pointing at each penny with their finger, and cooperated as Neas, who seemed to dominate the manipulating process, listened to Eska pointing at coins that needed to be moved from one row to the other. They counted up to four the coins in the experimenter’s row, and up to five the coins in their row, but still concluded “we have the same,” until they realized two coins were piled on each other and decided to move them apart, to start the counting process again. They counted up to five and concluded “we have the same.” The cooperation and agreement between the two subjects was shown with a smile of apparent satisfaction on their faces!

On this second trial, Neas and Eska managed rapidly to make two rows. While Neas manipulated the coins to make the two rows, Eska said that the experimenter “has all of these coins” as to show Neas that there was more in one row (Eska, Neas; 01/13; 0’50”). Neas moved one coin from one row to the other and counted “1, 2, 3, 4. 1, 2, 3, 4, 5” as there were two coins stacked on one another (Eska, Neas; 01/13; 1’00”). But still said “we have the same,” noticed the stacked coins and put them in line (Eska, Neas; 01/13; 1’15”). Eska’s answer was based on Neas’ count and stated “we have five”, and “you have four” (Eska, Neas; 01/13; 1’20”). Neas rectified: “No. 1, 2, 3, 4, 5. 1, 2, 3, 4, 5. We have the same.” (Eska, Neas; 01/13; 1’24”)

Then the experimenter guided the children a lot to help them achieve a row. The children decided to go on their way (Eska, Neas, 01/13; 1’40”). Until Neas put the pennies into a quincunx and said “I can make a squ…[he stops], I can make a five”
To end the experiment, in front of a row of coins and a quincunx, the experimenter asked who had more, and both children counted the pennies to five, and conclude “we have the same” (Eska, Neas; 01/13; 2’50”).

On the third trial, as Neas made three rows of coins, Eska asked “what is “two rows”? ” (Eska, Neas; 01/19; 0’45”) The experimenter explained that the coins needed to be on a line but Neas did not follow instructions and said “look I made a diamond!” (Neas, Eska; 01/19; 1’50”) Nonetheless, Eska count, “we have two coins,” and you have one, two, three, four, five, six, seven [omitting one]” (Eska, Neas; 01/19; 2’08”). It seemed that getting to making two rows is more difficult than actually counting the coins, or maybe just getting to the needed configuration required a lot of manipulation and counting!

Eventually, Eska and Neas managed to make two equivalent rows and agreed on the count of each row as they share the task (Eska, Neas; 01/19; 3’10”). Spreading out the coins, using the word “separate,” remained a challenge as the children made various types of configurations without leaving the coins aligned, such as making quincunxes (Eska, Neas; 01/19; 3’40”).

Eventually, in front of their spread out row, and the experimenter’s compact row (Eska, Neas; 01/19; 4’55”), the experimenter managed to ask the question of whether there were more coins in the spread out row or not. What happened reveals a good understanding of the counting process but not of subitizing collections of five, and the size of their spread out row seemed to influence their answer. Eska counted to coins in each row, but made a mistake when counting their row to six (Eska, Neas; 01/19; 5’10”).
It seemed that it was in reaction to that count, and maybe to the size of their row that Neas immediately took one coin from their spread out row to put in on the experimenter’s compact row (Eska, Neas; 01/19; 5’20”). Now Neas concluded that “he had the morest” (Eska, Neas; 01/19; 5’35”).

Neas seemed confused and decided to rearrange the coins into two rows in one-to-one correspondence (Eska, Neas; 01/19; 6’00”); they agreed that they had the same (Eska, Neas; 01/19; 6’30”). After spreading the coins out again in their row, they counted again when asked which one had more. Accidentally Eska counted to six in their spread out row, and concluded they had more, but then counted again and seemed to be intrigued that the number was five. It seemed that he was expecting more coins in the spread out row and miscounted the spread out row to six (Eska, Neas; 01/19; 7’30”). Although the children had exceeded their attention span and were going in all directions, there was a tentative to count in each row to compare the quantities, without success (Eska, Neas; 01/19; 7’40”); however, there was good comparison of groups (Eska, Neas; 01/19; 8’50”)

On the fourth trial Neas and Eska started by saying “we are missing some coins!” (Eska, Neas; 01/20; 0’5”). Eska attempted to count them all but counted too fast to 11, and concluded, “no (we are not missing any).” Then Neas, disagreed, “yes, we are missing some!” Although this was a good start for an interesting discussion, the experimenter tried to bring back the focus of the children to making two rows which was much harder than at the previous sessions.

Neas and Eska then counted the coins in each of two displaced row, they made mistakes but corrected one another. They concluded “no, we don’t have the same” (Neas,
Eska; 01/20; 5’14”). The experimenter showed his row containing four coins but on a longer row than theirs which had six coins, saying “my row is longer, do we have the same?” The response of the children was to make their row even bigger. After failing to count correctly (attributing two consecutive numbers to the same object), and spreading out the coins, Neas had an idea: “Hey, what if we copy each one out of one that people have, and that’s how we separate them.” He mapped the two sets of coins with one-to-one correspondence. He made two equivalent sets with visual one-to-one correspondence (Eska, Neas; 01/20; 6’15”). Neas counted the pennies, he made mistakes while counting (again attributing two consecutive number words to the same object, or repeating the same number word for two different pennies), but started again and ended up by counting correctly, and concluded “five and five, we have the same” (Eska, Neas; 01/20; 6’45”).

When prompted to move the spread pennies back together in only their row and asked the question again, both children counted and concluded they still had the same (Eska, Neas; 01/20; 7’30”). The experimenter asked why, and said that one row was shorter than the other one. The questions which seemed to intrigue them eventually seemed too much because by the eighth minute of this activity the experimenter and the children decided this was enough! The point was made: the number was the same so we have the same.

However, on the fifth trial, on 01/24, after sharing the coins, Neas bursted out “we have the same”, and Eska said, “no, one, two, three, four, and one, two, three, four, five, six”, counted again, and said “we don’t have the same, you have more” (after 1 minute). Although this note should be a turning point in the process, Neas did not seem to
understand Eska, and decided to make quincunxes, and made the point: counted the number of objects in each collection and conclude that the two constellations were equivalent. However, when prompted to put the coins in a row, Neas made the same reasoning as Eska, and even gave one of his coins to Eska to conclude on the equivalency by counting to five after several tries (01/24;2:50).

On trial number six, after having spread the coins out, Neas counted all the coins: up to 10. His coins were spread out and he concluded he had more. However, Eska, who just looked at him do the counting, said “no, these are mine,” and counted his coins again and then Neas’ coins, to conclude they had the same. Neas repeated the process and concluded on the equivalency. It is interesting to note that on this sixth trial, the children have taken possession of a row each, whereas at the previous trials, they had a row for both of them and the instructor had a row. At one point Neas said “but mine is bigger than his”, and Eska to reply, but mine is going to be bigger than yours tomorrow, like now!” and he spread out his coins (Eska, Neas; 02/01; 2’30”) to make a bigger line than Neas’. They counted again each collection and concluded and agreed that they had the same, but Eska’s is bigger, and Neas to say that his can be bigger too and spread out his coins even more (Eska, Neas; 02/01; 3’00”).

While Eska is plays with the camera, the experimenter talked with Neas and had him tighten his row, and asked who had more. The process of counting each set was repeated and conclusion on equivalency made!
On the seventh trial, Neas said that since Eska’s row was longer he had more. Eska counted the objects in front of him. Neas repeated the process and concluded on the number equivalency.

**Iwem and Tuqj.** On their first trial, Iwem and Tuqj (Iwem, Tuqj; 01/10; 1’00") separated naturally the pennies between the three parties involved and Iwem perceived the game as the game “whoever has more wins.” The experimenter guided them to put their coins together and compare the number of pennies they had with the number of pennies the experimenters had. The concept of row did not seem clear, thus the experimenter used the alternative prompt “line” (Iwem, Tuqj; 01/10; 2’20") and showed with his hand the desired shape. Iwem and Tuqj managed to establish the two rows: one for them, and one for the experimenter (Iwem, Tuqj; 01/10; 2’45”). Iwem said: “How much do we have? I am going to count them so we can see if we have the same.” (Iwem, Tuqj; 01/10; 2’45”).

Iwem and Tuqj started to count the pennies in each row which have been placed in visual correspondence (Iwem, Tuqj; 01/10; 2’50”). However, Iwem missed one coin when counting his row and concluded that they did not have the same. Tuqj pointed at the two rows and said: “this and this are the same” (Iwem, Tuqj; 01/10; 3’20”). Iwem stopped paying attention, as if this thing bothering him drove him away. Nonetheless, he brought his focus back to the task and said: “[you have more pennies] because you have six and we have four” (Iwem, Tuqj; 01/10; 3’40”). The experimenter decided to continue and asked the children to separate their pennies, to move them apart. The term separate had been used instead of “spread out,” maybe this was the reason why the children
formed two groups with their row: one for each of them. When asked to say who had more, they counted their parts. No conclusion could be brought at this point and the activity had to end because class was starting.

On the second trial (Iwem, Tuqj; 01/17), the children took two pennies each when asked to make two rows of pennies with the same number of pennies. The experimenter asked them to use all the pennies, which they did, managing to make two equivalent rows, without counting but just glancing at what the other had. However, when asked to place their rows next to each other, they understood (Iwem, Tuqj; 01/17; 1’05”) that “it is to see if it is the same size”, and that “it is to see if it is the same number”, and said “Oh! It is the same number!” Iwem decided to count them out, followed by Tuqj, and they both concluded “we have the same, because hers is five and mine is five” (Iwem; Tuqj; 01/17; 1’40”). After spreading out the pennies of one row, Tuqj concluded that they had the same more, Iwem agreed that they had the same. When asked why, he counted each set to five after catching his own mistake of counting the whole collection of ten pennies.

On the third trial, the concept of making two rows, one for the experimenter and one for the children seemed to be understood. The children started off by putting only two coins in each row, and Iwem commented: two and two is four (Iwem, Tuqj; 01/18; 0’20”). After trying different configurations with the coins, upon the insistence of the experimenter to make only two lines, Iwem and Tuqj managed to make two rows of five coins each (Iwem, Tuqj; 01/18; 0’55”). It seemed that Tuqj managed to subitize and said without counting “I have five,” and Iwem counted the coins in one row, concluded on five, and let Tuqj count the other row, concluded on five and Iwem stated “we both have
the same” (Iwem, Tuqj; 01/18; 1’00”). When asked to separate and spread out the coins, Tuqj understood and showed Iwem, who did not seem clear about the idea of spreading out the coins while leaving them on a line (which is quite an abstract concept!) However, when asked whether one row had more or whether they had the same, Iwem took the blunt initiative to count (Iwem, Tuqj; 01/18; 2’11”) each row and Tuqj to conclude “we have the same!”

On the fourth trial (Iwem, Tuqj; 01/24), however, the experimenter just asked the children to make two rows and each child took a row for themselves, unlike previous trials where they had to make a row for the experimenter and a row for both of them. They started by taking only three pennies and stated that they had the same, but the examiner asked them to use all the pennies. Again, it seemed that Tuqj subitized to judge that the two sets had the same number, whereas Iwem systematically counted the coins to do so.

Iwem mentioned that he talked to his mom about this game (Iwem, Tuqj; 01/24;2’20”) and that she said “you just started a game?” it is unclear what Iwem said after, something like “pfff, I never said that!”

The conclusion on this fourth trial was very clear (Iwem, Tuqj; 01/24; 3’05”). Tuqj concluded, without counting “we both have the morest,” while Iwem counted systematically, making the mistake to count the quantity contained in both sets: the whole ten coins, but corrected himself, and counted again to the cardinality of each set and said “we have the same”. And when notified by the experimenter that one row is longer than the other, Iwem replies: “Yeah, it’s because they are spread out!”
On the fifth trial, Iwem and Tuqj counted and concluded we have five, we have the same. After displacing the pennies to put the two rows next to one another, they counted again in French this time, after spreading out the coins, they counted again in French and concluded with the amusement of having counted in French, that they had the same (Iwem, Tuqj; 02/03; 2’10”), moreover, after the examiner said but this one is longer, how can we have the same, Iwem said “it is because you spread them out.”

**Dusi and Umow.** Dusi and Umow observed each other, and then repeated the process the other was doing. They ended up synchronizing their counts (Dusi, Umow; 01/24; 1’25”) and arrived at the same conclusion. The fact that they each had their row and got coins to make their own set, and agreed to share to reach number equivalency showed that they understood the concept of number. When they failed to count together, in harmony, they waited until one of them was over with the counting to start counting on their own. They seemed to understand that they counted their own row, and just needed to compare the number they reached to conclude on the numeral equivalency.

In the following chapter the different ways of learning under each condition are investigated. The paths explored by the children are presented with the view of the concept of play and a wider relevance to early childhood education. The role language played in the math adventure is presented as well as the parent perspective on these experiments. Possible policy implications and future studies are presented as well.
CHAPTER FIVE

DISCUSSION

Introduction

The purpose of this study was to investigate the effects of manipulation and collaboration on the emergence of conservation of numbers with four- and five- year old children. Prior research points out the ways in which manipulation and collaboration may have affected the emergence and development of numeracy skills for the child developing from a pre-conservation stage to a conservation stage (Piaget, 1969). Thus, the goal of this study was to better understand the process by which children develop these important math skills and the implications this may have on future policies and program implementations concerning early numeracy skills in preschools.

Recent research shows that measures of mathematical skill acquisition before kindergarten constitute an indicator for academic success in elementary years (Duncan, 2011; U.S. Department of Education, 2008). Moreover, studies have shown that the mathematics skills of young children, when they finish second grade, is not as good in the US as it is in other countries (Fuson, 2004). How can we improve our educational policies and programs policies in early childhood education using a better knowledge of counting process development for the future of our children? The present discussion will bring forth elements pertaining to early mathematical skills acquisition that will be useful to practitioners and policymakers.
With a concentration on number skills, this study explored the works of Piaget’s notion of number conservation along with counting skills. Its purpose was to study how manipulation and collaboration can enhance the acquisition of such number skills. It included three conditions: Condition 1 was the child watching the experimenter move the coins; Condition 2 was represented by the child’s own manipulation of the coins; and Condition 3 included four dyads of children doing the manipulations. Overall, the number of trials was not a significant indicator of the operating differences between students placed in C1 conditions (no manipulation – no collaboration) from students placed in C2 (manipulation, no collaboration). Additionally, the very small sample size (N=4 in C1 and N=4 in C2) do not result in generalizable findings. It appeared though that children learned better in C2 and C3 conditions in the sense that they demonstrated more competences and discovered more concepts pertaining to the counting process when they were asked to manipulate, and even more when they were allowed to collaborate.

This chapter begins by exploring the overall findings. Then, various highlights that distinguished the three operating conditions and are specifically tied to developmental stages that are intermediate to number conservation while playing the penny game are analyzed. The role language played in these mathematical activities, how the use of videotaping the data collected allowed post experimental analysis, and the parental perception/interpretation are presented as well. There was also a surprise finding of the playfulness of the coins in the dyad setting – and this will be discussed briefly. Ideas for how to develop and stimulate early numeracy skills are presented, and – at last – ideas concerning policy implications and future research are discussed.
Review of the Findings

Allowing the children to do the manipulation did not lead to more successful conclusions on number conservation (see Tables 4.1 and 4.2, Chapter Four). Given the number of students operating in these conditions (N=4 in C1; N=4 in C2), based on the 1/3 rate of success in C1 condition and 1/4 in C2 condition the conclusion cannot be that either C1 condition or C2 condition affected the emergence of number conservation. However, it appeared that asking the students to count the pennies triggered the emergence of number conservation in one case, and apparently destabilized another student in one trial without shifting to the conclusion of conservation.

However, manipulation in the C2 condition triggered the conservation of number for two students on sporadic trials yet the conclusion to non-conservation reappeared on the next trials. Manipulation triggered definite access to conservation of number in one case (Tivj). Although it was expected that students would shift more from a non-conservation stage to a conservation stage when operating in the C2 condition – manipulating – than in the C1 condition – that is not manipulating, the data gathered in this research did not support the notion suggesting that manipulating solo would accelerate the emergence of the concept of conservation with regard to a control group that did not manipulate.

In the third condition, children who were allowed to manipulate and collaborate were strikingly more successful than those not allowed to collaborate, whether they were allowed to manipulate or not, and the number of trials before concluding to the number conservation was significantly less in the this condition, as compared to the other
conditions. Although these observations were made on small sized samples (groups of four students operating in C1 and C2 conditions, and four dyads operating in C3 conditions, totally 16 children), and thus, may hardly be generalized to induce a theory, it has been observed that collaboration had an effect on preventing misinterpretation of the meaning of spreading out the coins. The students’ collaborations (Kagan; 1990) seemed to have a positive impact on their success rate.

**Condition Highlights**

**Children Not Allowed to Manipulate or Collaborate**

When analyzing the data, there were less words and actions to interpret when children were not allowed to manipulate or collaborate than there were in the groups allowed to manipulate and to manipulate and collaborate. The children were merely spectators, and this did not seem to motivate them to express their thinking process. They did not perform much action compared to those allowed to manipulate and to those allowed to manipulate and collaborate. Some children operating in the C1 condition did not want to do the experiment again. For instance, Insu did not want to participate in the game on the fourth trial. This is perhaps because the experiment seemed repetitive to the child who became bored. The loss of control induced perhaps a loss of intrinsic motivation, as control, along with challenge, fantasy and curiosity are sources of intrinsic motivation (Santrock, 2009). Since the sequences of actions, dictated and paced by the experimenter did not offer anything new from one trial to the other, some students may have lost interest. The fact that children were mere spectator, rather than actors, may have contributed to the loss of interest in the activity.
Children who were not allowed to manipulate or collaborate could not follow their own pace and their own ways of doing. This may have implied that they could not adjust to the task or explore on their own schedule. It seems that the situation presented by the experimenter was too disconnected from the children’s knowledge and experience. They did not seem to be able to connect the current experiment to their past experiences, knowledge and skills (Piaget, 1969). In such condition, Jips’ answer to the question asking why he thought there were more coins in the spread out row on the sixth trial was “you have more because you spread them out” (Jips; 02/01; 0’50”) and, on seventh trial, “Because I say so” (Jips; 02/08; 1’00”). Although the experimenter explained that there were no coins added or taken away, Jips had not learned on his own schedule. He could not adjust the task to his capabilities. He seemed not to have been given the possibility to access intermediate stages of number sense development.

When Children are Allowed to Manipulate Objects

Children allowed to manipulate the coins were given opportunities to evolve through various stages of number processing development that could not have been explored without manipulating, such as appreciating qualitative perception of small collections of objects that lead to the process of counting, i.e., subitizing (Clements, 2004) and establishing one-to-one correspondence (Piaget, 1969). Even if the children persisted on concluding that there were more coins in the spread out row (Djmu; 02/08; 1’25”), and the number of trials needed to place the coins in the desired position was higher than expected (Djmu, Seon, Teom), from a developmental perspective various procedures pertaining to number sense were initiated and used by the children, reflecting an improvement in their understanding of discreet quantities. For instance, in the group of...
children allowed to manipulate, Seon (02/06; 2'50") managed to make two rows of five coins, and matched two rows by making one-to-one correspondence between the two rows (Seon; 02/06; 3’05”; Piaget & Szeminska, 1964). In other words, exploring the materials seemed to help their thinking even if they could not necessarily execute a new set of solutions.

However, in direct instruction settings such as in the C1 and C2 conditions, an attitude was noticeable and seemed a hurdle in the understanding process. It was the fact that children tended to answer without bringing much thought, such as yes/no random answers. Insu for instance seemed to feel expected to give predetermined answers, and tended to give yes/no answers, and did not initiate or explore. At the question “can you…?” the answer was often “no.”

When Children are Allowed to Manipulate and Collaborate

It seems that the children operating in the C3 condition engaged naturally in the collaborating process. Children intuitively started to share information and observed each other’s actions. They also corrected each other (Eska, Neas; 01/13; 1’24”), completed each other’s initiated methods, such as counting the coins in each row rather than in both rows, sometimes synchronizing their counts (Dusi, Umow; 01/24; 1’25”), and, most importantly, discussed questions pertaining to the domain of number sense development they could not discuss when they were on their own, such as the notion of space between the coins (Eska, Neas; 02/01; 3’00”) which, according to Piaget (1969) is key to the understanding of number conservation. In this specific example of Eska and Neas, they were playing with how much bigger they could make their row (to the end of the edges of
the table), to conclude that no matter the size, the number of pennies stayed the same. The collaborating process seemed to have benefitted both parties (Kagan, 1990).

One interesting side finding among the dyads was that in addition to their ability to conserve number, they were also more playful with the objects. For example, one pair made shapes with the pennies, one team made a game with the experimenter as if each of the three people needed coins to begin, and one team noted physical traits of the coins (sorting by color and then noticing features of the engraving enough to identify one coin as Canadian!). One pair even made comments about what they were going to do to the next day! Thus it may be that the collaboration not only spawned better number sense, it also occasioned more creativity with math-related concepts such as grouping, time, and how math is featured in many games.

**Intermediary Stages of Conservation of Number**

*Qualitative Characteristics Versus Quantitative Characteristics*

Another point observed (perceived by Piaget & Inhelder, 1966) is the qualitative perception, as opposed to a quantitative perception of a collection of objects. The instruction might have been quite complex to understand in the first trials, and the children understood them after various trial and error, sharing ideas, observing each other. The first trial with Eska and Neas for instance, revealed two students that were rather hesitant and shy, but starting to warm up. The different areas explored are at first qualitative rather than quantitative: Neas, for instance, separated the dark pennies from the shiny ones. Then observed the engravers and tried to find differences and similarities. The researcher included a Canadian penny, which drew Neas’ attention. Still on this first
trial, Neas had his attention brought to qualitative aspects rather than quantitative aspects of the coins. As previously stated above, these may also have been brought about through the interactive playfulness inherent in working together versus working alone.

**Reversibility**

Among these intermediate stages leading to number conservation is also the concept of reversibility as described by Piaget (1969). After spreading out his coins, Seon stated “we have the same, because they are separated, and before they were like this”; he put them back together, then spread out one row and then the other and established one-to-one correspondence (Seon; 02/06; 3’50”) – it’s an important way to solve conservation problems. The act of spreading them out and putting them back together, is reaching to the concept of reversibility, which seems to be an intermediate step before the stage of conservation. Sometimes children were able to put the coins back together, and spread out again upon instruction to do so, but still concluded that the spread out set contained more.

**Concept of Identity**

On his fourth trial Teso said that the number of coins was the same “because you put the same” (Teso; 01/24; 1’10”). Here is identified the key concept of identity. Teso found this in the C1 conditions by just observing the experimenter move the coins. In the C2 conditions, Seon stated that the spread out row had more coins (Seon; 02/08; 1’20”), but when asked to bring back his coins together and to spread out the other row, he said “we have the same” and explained “because we move them like this”. The “action” – the movement back and forth is related to the concept of identify – nothing has been added or
subtracted. Seon spread out the compact row to match with one-to-one correspondence the spread out row (Seon; 02/08; 2’00”).

**Subitizing**

When children were allowed to manipulate and had in mind to make two rows with the same number of pennies, they demonstrated capabilities of subitizing with small quantities (Clement, 2004). They adapted the number of coins involved to their capabilities of subitizing. For instance, over the various trials, it seems that Seon felt comfortable in making equivalent groups with cardinal two (Seon; 02/06; 2’ 15”). At some other trials, Djmu decided as well to make two rows that were equal in number, but with only two or three pennies in each row, leaving the remaining pennies on the side. So it seemed that, instead of counting or measuring the difference between two cardinals, some children started by establishing two smaller sets of equal number that they evaluated qualitatively, i.e. subitized, usually putting two or three pennies in each row, which corresponded to their comfort zone in terms of identifying at one glance number equivalency (Teom; 02/14; 3’35”). Then they completed their action with another technique consisting in adding one penny at a time, the remaining pennies to each row.

**Quincunxes**

Some students explored new paths such as making a five, which is a quincunx (Teom; 02/08; 1’55”) and sometimes said “I can make a five”, (Neas and Eska; 01/13; 2’25”). Neas made the quincunx, counted the pennies to five, and concluded “we have the same”. It seemed, since the surface covered by the pennies influences the child’s perception of the quantity, exploring various constellations of five coins may have been a
way to overcome this influence. However, Richardson (2004) mentions in her classroom, the children could associate the dice pattern with the number, but when asked to reproduce that pattern on a card, the exact number of dots composing it was not the same. The children matched a square shape of collection of dots for the nine, without printing nine dots, or making an X shape set of dots that would not equal to five. In our example, we could see the child accomplishing two steps: first, to make a quincunx, second to count by pointing with the finger to five the number of dots composing the shape.

**Finding How Many More Objects a Collection has than Another**

The task of finding out “differences” is more demanding (Clements, 2004) has been observed on several occasions. For instance it seems difficult to find right away how many coins needed to be transferred from one row to the other (Efen, Moen; 01/17; 2’25”), (Eska, Neas; 01/11; 2’57”). Seon (02/06; 0’10”) adopts the technique of alternately giving one penny to each party. As if it were to make sure that number equivalency established by subitizing the two rows of three coins each, he adds one penny to each set alternatively, attentive to the two rows’ increase in size as he performed this action. It seemed that he had found the technique to grow out of his subitizing comfort zone, adding coins one at a time, until he got confused – at the end – and was not capable of seeing which row needed the last penny (Seon; 02/06; 0’10”). Seon had evolved a lot, from not being able to make two equivalent sets out of ten pennies, to having elaborated a method to achieve this goal (Seon; 02/15; 1’00”).
One-to-one Correspondence

Children operating under the C2 and C3 conditions could manipulate the coins, and when asked to make two rows with the same number of coins, or “to share” the coins, they sometimes initiated to place in one-to-one correspondence the two collections of pennies even when, later in the game, they did not conclude to the conservation of number (Piaget, 1964). For instance Seon and Teom discovered one-to-one correspondence, but hadn’t reached the conservation stage yet. Manipulation is used to establish the visual correspondence (Djmu; 02/03; 1’00”). It is hence difficult to conclude that manipulating “accelerates” the acquisition of conservation of number in terms of number of trials needed to reach the goal of number conservation. Manipulating did help children discover on their own various processes that were tied to important mathematical concepts leading eventually to number conservation with more trials or maybe with time for children to mature!

Ordinality

A frequently occurring theme, which seemed to reflect an intermediate stage in attempt to reach number conservation, was the propensity for the children to count all the pennies to ten, instead of counting each row to five. Seon counted each penny of the two rows, up to ten, with no conclusion. When asked again, he stated that he had more pennies (Seon; 01/11; 2’50”). This certainly has implications for pre-K curricula worthwhile in so far as teachers need to move beyond the simple counting concepts to the subset/dividing/subtracting concepts that include sets and subsets. However, some of the children caught their own mistake, and after counting the whole collection of ten pennies,
counted each set to five (Efen, Moen; 01/27; 2’10”). It might appear that the “schema” for number is fixed, at least initially, in a ‘count all the objects’ manner.

**Making Two Equivalent Rows**

To make two rows with the same number of coins was supposed to be a preparation phase. It is a phase that did not exist in the C1 condition and has contributed to making the experiments in C2 and C3 conditions longer than the ones in C1 conditions. However, it appeared to be a potentially critical phase. To make two rows numerically equivalent, starting from a stack of ten pennies involved some numeral skills. Whether it was subitizing, adding one penny at a time alternately, or establishing one-to-one correspondence, children in C2 and C3 condition had a challenge that children in C1 conditions did not have, which perhaps biased our project. Indeed, one have might just prepared the same initial condition for each of the three conditions, that is the experimenter to prepare the two equivalent rows of five pennies, and the children in C2 and C3 conditions to spread them out. Maybe the findings would have been different, and success would not have been so flagrant in C3 conditions without all the discussion that started from just making two equivalent rows of five pennies.

**Cardinality**

Some students seemed already familiar with the counting process. They knew how to assign each object of a row to a word from a well ordered list of number list of words, without repeating twice the same word or counting twice a same object. Moreover, the insistence on the last number word said, and repeated as if it were a shift from ordinality to cardinality. For instance, Tivj says there are more when they are spread
on the first trial but immediately has a doubt and decides to count them to conclude then that there is conservation. “1, 2, 3, 4, 5: 5, we have the same!” Tivj emphasizes on the last number word said (Tivj; 01/11; 2’15”). In this case the shift from “there is more” as a first impression, to a “wait a minute” rational thinking, as if the rational quantitative aspect of the mind pushed through a qualitative perception of the situation, was quite striking. At their first trials Iwem said to Tuqj: “How much do we have? I am going to count them so we can see if we have the same.” (Iwem, Tuqj; 01/10; 2’45”).

Particularly in the C3 condition, children did not always put the rows side by side which would have allowed them to use visual one-to-one correspondence; they instead made their own row too far from the other’s to use one-to-one correspondence. They could glance at each other’s row, trying to visually compare the two quantities, or to subitize, but the number of coins seemed to be too high to evaluate at a glance, and the children had to find another way to make sure they had the same. This situation sometimes triggered the counting process to start right from the start. It may have been necessary because the context of the experiment: “you have some pennies and your friend has some pennies – can you make it so you have the same number of pennies?” may have triggered the “your pile, my pile” and thus the pennies would be more aligned to the individual children (as in familiar games) than to a common area. Pointing at each coin to assign it a number word, and using the last number word as the cardinal. It seems that when children had to count their pennies before having to spread them out, they were inclined to count them again after having one of them spread them out. The spreading of the coins did not impact their conclusion on numeral equivalency because the process of counting stayed the same. In Eska’s and Neas’ fifth trial, the major steps were to make
two equivalent rows (note that they were not using the one-to-one correspondence), then to spread out one row. Efen and Moen, on the first trial (2’10”), exchanged coins, proceeded by trial and error, and counted in order to have their rows equivalent. The attention at this point where the rows were not in visual correspondence was on the discreet quantity of each row. To conclude on the equivalency between the spread out row and the other one did not seem any more difficult since the children used the same counting process as the one they used to make the rows they started with: point with their finger each coin, associating each object with a number word, and conclude on the cardinality.

**Disequilibrium**

However, in the three conditions, what Piaget refers to “disequilibrium” seemed to have been identified in many cases. In chapter four, turning points were identified as a time when children changed the conclusion of their observation or changed their manipulation or discussion. For instance, when asked to bring back together the coins, or/and to spread them out, Teom – in condition two – consistently concluded that the longest row contained more, and when they had the same length they were the same. She seemed to understand that something is wrong and was a little confused, demonstrating a likely state of disequilibrium (Teom; 02/14; 6’05”). On the 7th trial, the experimenter prompted Insu to count the coins in each row when one of the rows had been spread out, which she did and came with the answer 5. When the experimenter asked again, if there were more coins in one of the rows or if they were the same, Insu seemed confused for a while (Insu; 02/10; 1’15”), and did not conclude. She seemed in a state of disequilibrium between conservation and non-conservation. After counting the pennies in each row and
noticing that the number is the same, Djmu seems unsure, but still maintains that the spread out row has more. After counting the pennies in each row and noticing that the number is the same, Djmu seemed unsure, but still maintained that the spread out row had more (Djmu; 02/15; 8’05”), because “it’s bigger”. The experimenter pointed out that the number is the same, but still one row has more according to the conclusion. Djmu seemed bothered but maintained the difference in amount while accepting that the number was the same.

**Language Roles in Math**

**Abstract Concept Acquisition through Language**

The language used in the experiment contained elements sometimes new to the children. Learning these words associated with concepts was part of the experience toward the mastery of numeracy skills. Numeracy skills are highly related to linguistic skills (Kleemans et al., 2011). Helping children understand concepts, instructions, and agree on the precise meaning of some words was an important part of operating in the C2 and the C3 conditions for which the task was more than just saying if a row had more coins or not. The process of “spreading out” the coins for instance was not understood by any student. However, the word “separate” or the phrase “move the pennies apart” (Djmu; 01/11; 1’30”) seemed to be understood. Access to this terms and meanings could be given most of the time by using different words, but ultimately by having the experimenter show with the meaning, through gestures most often, the task became clear. In our case, “to spread out”, and to “make a row”, were not merely language to acquire but also abstract concepts to grasp. It appeared to be a challenge to spread out the coins and to leave them on a row (Efen, Moen; 01/17; 4’30”), the experimenter used the word
“separate”, and spread became separate all the coins from one another. The word “row” appeared to be a challenge as well (Tivj; 01/11; 0’15”). The experimenter used the word “line” and simulated with the movement of his hands the wanted shape. (Iwem, Tuqj; 01/10; 3’40”); (Iwem; Tuqj; 01/17; 1’40”) Gaining in numeracy skills involved gaining in vocabulary and linguistic skills as well.

**Quality and Quantity of Questions**

Research has shown that the quality of the questions, as well as the quantity of questions on number conservation with four- and five- year-old children had an impact on the results (Rothenberg, 1969). For instance, the instructions to put the coins in two rows so that we have “the same” was sometimes understood as contradictory because the “same”, was understood as “together”, and therefore was in contradiction with “making two rows” which implied “separate rows” – move punctuation marks inside quote marks, please. Teom for instance concludes after putting all the pennies together: “This is both of our rows!” (Teom; 02/10; 2’05”) The instruction to make “two rows, one row for me, and one row for you”, although helpful in terms of identifying what is for me and what is for you, was sometimes interpreted as giving “two coins” to the experimenter and “two coins” to the child, which had the experimenter redirect by asking to add coins in each row in a way that keeps the same number of coins in each row. Furthermore, the questions “Do you have more?” versus “Do you have more pennies?” or “Do we have the same”, or “Do we have the same number of pennies?” turned out to be important in math understanding. Why? Because the visual appearance of the coins is NOT the same, it has changed and pre operational children are very perceptual. Thus the clarity of the question, “do we have the same NUMBER of coins” is all that much more important. All these
nuances may have some impact on the understanding of a child who is emerging from a stage where quality perspective and quantity perspective are not very distinct yet.

A Useful Tool: The Video Camera

Opportunity to see What had Escaped the Experimenter’s Attention

The videos taken of each experiment has allowed the experimenter to see a number of actions, and reactions that had escaped his attention at first hand. At some sessions (Eska, Neas; 01/13) the children confused the experimenter by taking him on unanticipated aspects of the experiment that would have been lost without the right documentation tool. This session turned out to be interesting after observation of the video. Indeed, with two students, the attention of the experimenter may be focused on one area such as child speaking, while the other thinks, touches, all in silence, but the movement of the eyes, and the expression of the face, as well as the sequence of thoughts and actions, seemed to indicate that progress is being made. There were more interpretations possible after watching the video. The action of the children is rich, and harder to catch at first hand, especially in the C2 and C3 condition.

A second element that induced different behaviors in the C3 condition was that children who did the activity together explored many directions. Sometimes, even the experimenter did not see their actions go toward what was asked it is only after reviewing the video that one could see that the children were building a bridge toward the expected notions, by first consolidating what they could do with the coins. Moving them around, making various shapes instead of the expected row only contributed to the building of strong discreet quantitative perception of the collection of pennies. Spreading the coins out in an exaggerated manner could seem to be only goofing around but was actually the
proof by the absurd that the space between the coins did not affect the cardinality of the set. Thus the video camera allowed a deeper, slower, and more detailed review of the actual nuances of the children’s behavior and conversation. Was the video camera a distraction to the children? At some points in the experiment (Neas, Eska), children were intrigued by the camera and wanted to go behind it to see what was filmed. It even became a game. However, when playing the game, the children did not seem to be distracted by the tripod and the flip camera.

**Parent Perceptions and Interpretations of the Project**

**Parents’ Expectations**

It appeared at the time of gathering authorizations from parents that some were intrigued about results of the study and eager to learn about their child’s cognitive level. They wanted to know what their children were capable of doing. Although the study was presented as a game, for some parents, it seemed to be perceived as a means to measure performance. The experimenter had to promise them to give the parents a full report. However, it was mentioned that the report would not compare children, or even evaluate them. The point of the study was to evaluate the impact of manipulation and collaboration on the emergence of a mathematical concept. Nonetheless, some parents wanted to see the videos of their child, and to also know if their child was behind or ahead what children typically achieve at that age.
Misunderstanding of Child Development Through Piaget’s Stages

The concept of Piaget’s stages of development may be here misinterpreted as “it is better if my child is advanced” instead of “my child is currently in this stage of development, and it is good to know this in order to interact accordingly with her”. However, it is difficult to argue with parents that there isn’t a need to rush our children through developmental stages, when the hypothesis of the study is that some factors may accelerate the emergence and the mastery of a concept that is typical of a developmental stage. The conclusion of the research though is of an improvement in the depth and quality of the understanding of a concept with amelioration of the experience’s context.

One cannot control what parents do once they receive the invitation for their child to participate in a mathematical game. Because this game takes place at school, and that school is often associated with performance, it was to fear that parents actually did play that game at home and taught their child to count. Iwem for instance mentioned that he talked to his parents about this game (Iwem, Tuqi; 01/24; 2’20”), a discussion on Djmu’s mother also revealed that Djmu told her mother that this was not a game, but work! This was the only indicator of parents’ involvement in this activity. Tuqi mentions it too but it is unclear if this is done to say the same thing as Iwem. However, that children talked about what they have done at school to their parents allows them to repeat the words that have been used during the experiment and to become more acquainted with them.

What might be relevant to parents’ interest in their children’s acquisition of math concepts is how to engage their children in their own constructed games, informally and not as ‘work.’ For example, parents might be interested in learning more about ordinality,
cardinality and subitizing and teachers could prepare a short one-page handout for this purpose. The parents who want to know even more could be directed to the Foundations themselves, easily accessible on line through the California Department of Education. In fact, games like those in the current study are a natural way to engage parents in the exploration of every day math activities with their children, such as when they are waiting in a restaurant for their meal, grabbing some coins might be the trick for many kinds of math-related conversations and experiments.

**Concepts of Play and Additional Relevance of Math to Early Childhood Education**

**The Role of Manipulation and Language in Constructing Thought**

One important aspect of the thesis, this time joining Piaget’s and Vygotsky’s socio-constructivist points of views, concerns the role manipulation and language played in constructing thoughts along the quest for number conservation. Thought and language may be considered at the center of the C3 operating conditions analysis. Indeed, even though some episodes merely featured a child explaining an idea to another child – for instance Efen explaining what separate means to Moen (Efen, Moen; 01/25; 4’20”) – the fact that this child showed how to separate to the other, was the proof of a thorough understanding of the concept, but also a repetition which anchored the concept strongly into the child’s knowledge foundations. Collaboration in this episode, almost one minute with both children’s full attention, is an example of vocabulary scaffolding and of Efen using ‘theory of mind’ to help Moen (Efen, Moen; 01/17; 5’50”). Another example is Eska counting to demonstrate to Neas that no matter the configuration, the number of coins remained the same (Eska, Neas; 02/06; 2:30). By doing so, he showed the
reliability of understanding of the concept. Reliability tested by the researcher insisting on “but it is longer, there should be more...” The answer was “it is longer but we still have the same, and we can make our lines even bigger!” The eighth trial (Eska, Neas; 02/08) confirmed that the skills had been acquired through the collaboration between students.

**Comfort Zone in Peer Interaction**

Collaboration provides – besides the comforting presence of a friend which according to Kagan (1990) lowers anxiety – the opportunity to talk more, and to rephrase what one has learned. On her third trial, Teom who asked for her two-year old friend Nepf to come with her, is not disturbed by her presence and she does the whole activity while Nepf watches. At the end of the activity, she asks if she can play the game with her friend, and takes the role of the experimenter, distributing coins and making rows, and asking where there are more coins, changing configurations. This scene is highly worthwhile in so far as the child really wants to teach (and thus learn more) about the concept. It’s a very good way to practice math – by teaching it to others who did not interfere in the activity. This showcases how the social context again allows multiple existing schema to flourish. Thus noticing nuanced properties of the coins (color, engraving) or constructing a game by distributing the coins among the three (two children plus the experimenter) and in this case, seeming to play school or at least take the role of the teacher seems to demonstrate again how children relate their every day experiences to each other and to existing schema.
Ownership and Concept of Sharing

One aspect that has been observed only in the C3 condition, and which may have had an impact on the outcome of the activity, is that children engagement seemed to be reinforced when they could become the owner of one row and compare what they had with what others had. It was noticed that instructions were easily understood in this case, and much harder to understand when they were asked to make one row for both of them and one row for the experimenter. The concept of sharing seemed to be a concept one could rely on for the activity. At one point the children shared the ten coins between all parties involved in the C3 conditions, allotting three coins to each child and four coins for the experimenter which is a brilliant way to divide ten into three groups with the experimenter getting the extra penny and the children getting equal numbers (Iwem, Tuqj; 01/10; 1’00”). Neas and Eska shifted the process to having one row per child on the sixth trial, as if it were a natural attractor for all participants, although they were asked to make a row for the experimenter and a row for both children. At one point Eska wonders “are we missing some coins” and counts, but this shows a clear mental state of knowing what the result was supposed to be and understanding that insufficient materials are available to produce it.

Parallel Play and Joint Play

Moreover, when working together, the children often complemented each other in terms of initiating actions. While Eska seems keen on counting, in a rational way, he also seems to enjoy watching Neas making various configurations with the coins and commenting on them. Eska’s rationality seemed to complete well Neas creativity. Dusi and Umow acted in perfect symbiosis, observing each other and repeating what the other
did, to actually synchronize their counting. Iwem had the systematic initiative to count each row and conclude using the cardinal that the quantities in each row remained the same, when Tuqi had the vision of setting the rows of coins as wished – action Iwem had difficulties to do (Iwem, Tuqi; 01/18; 2’11”). The propensity of children to count all coins up to ten mentioned earlier was a couple of times countered by the other child in the C3 group.

Another element brought by Kagan, is that children feel more at ease when with a peer [find Kagan’s note about that]. It has been observed in our study when Insu did not want to come to participate to the test after three trials, but accepted to come with a friend who just remained seated behind her. And the only time Teom succeeded in finding that the two rows were equivalent even when one was spread out, was one she was accompanied by a friend from the two year old classroom. She, furthermore, wanted to play the game with her after our activity, time during which she managed to share the coins between her friend, myself and herself.

The Role of Control in Acquiring a New Concept

Manipulation allows children to have control over the displacements and they ultimately played with the displacement. For instance, when Neas exaggerates the displacement when separating the coins moving them apart in many directions, or moving them all the way to the neighboring table (Eska, Neas; 02/06; 4’30”), he actually plays with the intervals between the coins, which is key to number conservation according to Piaget (1964). Piaget explains that it is the negligence of these intervals that induces the misperception of the quantity. The child assimilates the surface covered to the quantity. When using extreme intervals, the child breaks that amalgam of coins and
intervals to dissociate one from the other. A behavior that could have been interpreted as erratic, actually contributed to making a big step in the emergence of concept.

**Play Ideas**

As stated earlier, children manipulate a wide variety of things, including materials in mass (sand, rice, water), but also discrete objects that may be sorted, classified, arranged in patterns, and counted (Copple, 2004). NAEYC (2009) emphasizes that teachers can help children “mathematize” their everyday encounters, practicing and learning math skills and concepts when playing with blocks or manipulatives, or doing movement activities, or even computer time where it is assumed that children are in control of the machine. Here again, it is not mentioned how really manipulating would allow for enhanced learning experience, but it demonstrates how children from age three to age five are naturally inclined to move, to touch, and to manipulate. Hence the reasons to believe that learning mathematical concepts and skills ought to be enhanced through manipulation.

According to the results of this thesis, collaboration seems to be an important success factor. Children benefit from working together on number sense activity. As a suggested play activity for children, we can imagine anytime of collaborative games involving numbers, such as dominoes, games with coins, cards, or other collection of objects. The games must involve exchange, with various values, such as one red coin is worth ten blue coins. It is important to allow board games that involve numbers and discussions. Piaget also writes about how important it is to consider the “rules of the
“game” as negotiating things that move from variable to fixed over time and this seems a precursor to the idea of ‘theory of mind’

To play with dice and dominoes or board games seems ideal for becoming acquainted with number concepts. Richardson (2004) refers to his students not drawing the exact number of dots making the configuration on a die or a domino, but rather a figure that has the same shape. This could be observed, Richardson states, when the child is asked to reproduce the configuration of dots on a paper. Children would draw a similar configuration that would not contain the same number of dots. There seems to be a need of an activity that would consist in allowing the children to develop number sense while appreciating the patterns to facilitate the transition from qualitative perception to quantitative perception. The game with dominos contributes to the child’s number sense development, as well as various board games with dice. We could also imagine, an age appropriate activity which would consist in rolling on the die and then represent that number in a different configuration with either cars, chips, or other counting objects. In some cases they could subitize, in other cases, finger count and use cardinality.

Clements (2004) offers a set of recommendations for engaging young children in mathematics of which were to build mathematics on play and natural relationships. Clements assumes according to what one can observe when children play, that play involves touching, grabbing, moving. To follow this recommendation and involve manipulation it may be an incentive for teachers to mindfully plan for a variety of manipulatives in the classroom. Objects that symbolize number units may be part of the free play of children. Although Clements suggests distributing napkins to the table and counting the number of students in the room is not enough to a real construction of
number knowledge, according to Piaget, because this type of manipulation is necessary until at least the age of seven to build the foundation of abstract thinking.

**Policy Implications and Future Research**

**Policy Implications**

In order to engage children in activities pertaining to number sense development, it is required to have teachers who are knowledgeable of number sense development theory themselves. A teacher who does not know the stages of number sense development cannot guide or advise children to use a selected list of playing materials, engaging them in number sense activities. This could have an impact on their developing early numeracy skills. Sarama and DiBiase (2004) point that teachers who believe that mathematics is a series of facts, and that memorizing these facts is an appropriate route to learning mathematics, will be resistant to change and to explore evolutive paths to number sense development, involving, for instance, the skills that are listed in this chapter and which will contribute to number sense development. It seems important to educate teachers (Bredekamp, 2004) and allow them to envision the development of mathematical skills. They will be enabled to, in turn, engage children in pertinent activities.

Recent research shows that early numeracy skills are a predictor of academic success in the elementary years (Duncan, 2011). Hirsch (2011) presents the American preschool as being the anti-curriculum institution since 1939, when the progressive movement established that the child should acquire knowledge only in real situation following their needs and interests; the position of the teacher should be only to guide
and to advise and shall not try to implement subject matter or the planned organization of a curriculum. Perhaps, as stated in the literature review, an idea of a preschool curriculum emphasizing on numeracy and literacy skills would allow children to play with their academic skills. A way of marrying the free play and academic content would be through a careful selection of materials evolving according to a curriculum that teachers would use as guidelines to elaborate their activities.

Newly developed resources from the California Department of Education, specifically the Early Learning System that emphasizes both training of teachers through the Faculty Initiative Project (for pre-service teachers) and the California Preschool Instructional Network (for in-service teachers) along with the Learning Foundations, Curriculum Framework and the Desired Results Developmental Profile – Revised are one way to begin to appreciate the role of math within the daily preschool day. The current study also adds the important element of parents into the mix of how children want to learn about math – with others as scaffolds and as playmates. Without question, however, teachers should be aware of how math learning transpires and intentionally offer opportunities for children to explore a variety of materials in the many ways suggested in the Foundations and Curriculum Framework (CDE, 2008, 2006)

**Future Research**

Considering Piaget’s cognitive developmental stages and the three control groups used in this thesis, it would be interesting to further the study with a measure of the effect of manipulation, and peer interaction on the conservation of volume, seriation, conservation of length, conservation of order, conservation of matter, of weight, of
volume with group of students of different ages. The point in this thesis is to observe the cognitive attitude of a child facing a concept relatively new, and an aptitude emerging. The same setting used in this thesis could be used with fifth grade students, questioning the conservation of volume. Using a ball of play dough perfectly round, the question would be: “If we modify the shape of the play dough, would it change its volume?” Would collaborative work with manipulation have such an overt success over the individual spectator?

Another area of important future research would be to focus on teachers’ growing awareness of their professional development regarding math. What kinds of training (pre-service, in-service, internships, mentoring) work best? What are the best ways to encourage teachers to think about math throughout the curriculum? How can they be energized to see all the potential of math for young children?

Additional studies, including videotapes that resulted in a huge amount of data that would otherwise have been lost, are important for understanding the depth to which certain kinds of language, socialization, and collaboration are integral to math. Other ideas could be related to use of computers and math for young children. Ultimately, there are an infinite number of ways to engage children in math concepts, particularly problem solving because they are part of children’s natural world.
REFERENCES


APPENDIX A: Human Subjects Protocol Approval Form

HUMAN SUBJECTS PROTOCOL APPROVAL FORM
CALIFORNIA STATE UNIVERSITY, NORTHridge

This Protocol Approval Form must be completed for all California State University, Northridge faculty and student research which involves human subjects. Additional material(s), as described below, must be attached to this form at the time it is submitted to the Standing Advisory Committee for the Protection of Human Subjects (SACPHS) in the Office of Research and Sponsored Projects (UH 265, ext. 2901). In ALL cases, RESEARCH MAY NOT PROCEED until authorized by the Committee. You will be notified of the action of the Committee following the receipt of an original and nine copies of this form and all required supplementary information (see below) in the Office of Research and Sponsored Projects. ALL SIGNATURES MUST BE OBTAINED PRIOR TO SUBMISSION. Brief, excerpted definitions and guidelines regarding research involving human subjects appear on the attached instructions. For a copy of complete regulations, contact the Office of Research and Sponsored Projects. Read all instructions before completing the form. ONLY TYPEWRITTEN FORMS WILL BE ACCEPTED.

1. Title of research  NUMBER SENSE DEVELOPMENT WITH 4 YEAR OLDS: NUMBER CONSERVATION

2. Name of researcher(s)  STEPHANE PLANCKE
   Campus ext.  Major or Department  EPC/ECE.

3. Address  7306 Sale Ave
   West Hills, CA, 91307
   Home phone  323-481-2531
   Email Address  stephane.plancke.953@my.csun.edu

4. Name of Faculty Advisor  Carrie Rothstein-Fisch
   Faculty Advisor ext.

5. Period of Project (see pg. 1-Itemized Instructions) From November 15th, 2011 To March 15th, 2011

6. Check one:  ☑ Faculty Research  ☑ Student thesis  ☑ Other (specify)
   Course prefix and number  EPC 696
   Course title  Thesis

7. Check one:  ☑ Unfunded  ☑ Funded  ☑ Date (to be submitted)  February 4th, 2012

8. History of Protocol:  ☑ New  ☑ Renewal  Approval Date

9. Does this protocol contain modification(s) from a previously approved protocol?  ☑ Yes (explain)  ☑ No  ☑ N/A

10. Special procedures: (give detailed description on separate sheet)
    ☑ Radioactive materials  ☑ Drug(s), Specify: __________

11. Is a Subject Bill of Rights attached?  ☑ Yes  ☑ No

12. Are copies of any questionnaire(s), survey instrument(s) and/or interview schedule(s) referred to in this protocol statement attached?  ☑ Yes  ☑ No

13. Is draft Informed Consent Form(s) attached?  ☑ Yes  ☑ No  14. Is a letter of permission attached?  ☑ Yes  ☑ No

15. SIGNATURES: Refer to page 1, General Instructions–letter D, before signing.

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<th>Student Investigator's Signature (specify grad. or undergrad.)</th>
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Chair, SACPHS, or Director, RSCH  Date  Expedited Reviewer(s)  date reviewed

Revised 2/08
PROJECT INFORMATION

TYPE THIS FORM ON YOUR COMPUTER (HANDBRITTEN FORMS WILL NOT BE ACCEPTED).

Number of subjects involved: 16  Age of subjects: 4  Will a control group be used? □ Yes □ No

Source of human subjects Pre-Kindergarten class - Preschool (If subjects will be obtained at a non-CSUN institution, a letter of permission from a representative of that institution must be attached.)

1. Provide a detailed description of your project, including a clear statement of your hypothesis and all procedures and methodology:

What is the best ways to teach young children math (Clements, Sarama & DiBlase, 2004)? In the current experiment, a popular concept, "conservation of number" (that objects can be moved around spatially, but the number of objects remains the same) will be observed in three different conditions: 1) with the teacher manipulating the objects, 2) with the child manipulating the objects, and 3) with two children working together and manipulating the objects. Often math is taught by the teacher, where he or she demonstrates something and the children are expected to learn it. This is represented by the first condition in the study where the teacher manipulates the objects, in this case 10 pennies. However, emphasis on hands-on-learning can be highly effective, as is proposed in the second condition when the child manipulates the pennies. The third condition, working in pairs, derives from a social-constructivist approach that posits learning with another can be beneficial for both and in this condition, the two children would work together manipulating the coins as they solved the problem. It is hypothesized that children will learn the concept of "conservation of number" best in pairs and least well (as measured by the number of trials in the 'game' needed to understand the concept) when they merely watch the teacher manipulate the pennies.

Specifically to the current study:

Sixteen four-year old children from the same preschool class will be grouped randomly into three conditions C1, C2, and C3. There will be four students in C1, four students in C2, and eight students randomly paired to result in four groups in C3. The math game will be part of the normally occurring free-choice activities offered to the children for 40 minutes each morning.

Parents in the four-year old classroom will be given a recruitment flyer (Attachment A) describing the study and asking for their children’s participation along with a photo permission form (Attachment B) to allow for the use of videotapes that can be used for educational training purposes.

In the current study, children will be invited by the researcher to participate in the 'math game' twice/week for up to six weeks or until they have successfully determined that the number of pennies has not changed despite their visually altered appearance (by being lined up edge to edge or spread apart) as part of their normal free-choice time as mentioned earlier. They will likely enjoy the game, but if they do not, they can move to another area of interest and perhaps be invited later in the same day if they seem willing. The entire game time will be likely to take about 5 minutes. Child will sit at the math table, the video camera will be switched on, and the conversation/interaction will occur. See Appendix C for the wording of the experimenter’s conversation and questions to the children.

2. Describe the projected outcomes and how they relate to your hypothesis:
My hypothesis is that the typical milestone of number conservation is not reached yet at age four generally. At age four, which lies in psychologist Jean Piaget's pre-operational stage, I expect students to state that there are more coins in a row where they have been spread out. However, after a certain number of trials, I expect them to understand number conservation, and state that even if the coins are spread out, their number remains the same. Most of the students will fail to state in the first trials that the numbers of coins in each row remain equal after spreading the coins out. Children will hopefully realize after a certain number of trials, that the there is the same number of coins in each row, no matter the spreading. Moreover, children trying this experiment without themselves manipulating the coins will need more trials to understand the conservation than the children who are allowed to manipulate the coins. Furthermore, children allowed to manipulate the coins and to share this experiment with a classmate will understand the conservation in a less number of trials. Again, it is hypothesized that control over the objects will reduce the time needed to learn 'conservation of number' and that working with peers is likely to speed up the process because they will scaffold the materials and use language to facilitate their thinking together.

3. List any potential risks to subjects participating in project and what steps if any have been or will be taken to minimize or remove these risks:

<table>
<thead>
<tr>
<th>Polling students out of instruction time with their teacher presents a risk of having them lose important interactions with the class. However this will not be the case in the current experiment because:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The experiment will take place in the class or very close to the entrance of the class with a glass door as a separation, so that the subject will not lose contact with whatever happens in the class and will not feel pulled out.</td>
</tr>
<tr>
<td>- The experiment should be relatively short – about 5 minutes. I hope it will not impact on the program the teacher has prepared for the students in other choice areas.</td>
</tr>
<tr>
<td>- Students may feel ostracized if they are pulled out of a group, or if others have a treatment that they do not have, but the majority of the children will be participants and the other children can also play the 'math game' if they wish without the video camera on.</td>
</tr>
<tr>
<td>- Many students will be called consecutively. Moreover, the teacher will let the students know that everyone will get to &quot;play that game&quot;, because the test will definitely be presented as a game.</td>
</tr>
<tr>
<td>- Parents and teachers may want to know the outcome of the experience and interpret it as they would interpret an intelligence test. They may over generalize and draw conclusions on the development of the child and this may impact in turn the development of the child. However, the researcher will explain that 'non-conserving' responses are normal and that most math and counting games are a way to support understanding number concepts.</td>
</tr>
<tr>
<td>- Parents and teachers will be informed of the outcomes of the students as a whole with aggregated findings reported-- after the test – and notified that there is no implication of any kind on the development of the individual per se.</td>
</tr>
</tbody>
</table>

4. Significance of project to your discipline, department, school, university, community, etc.:
- If I can demonstrate that 4 year old children learn better when they can manipulate the objects themselves, then teachers may be more likely to provide hands-on lessons along with current methods.

- If I can demonstrate that 4 year old children learn better when they can communicate with each other while working on the same project, it will show schools and communities that when a new concept is studied, working in groups and communication between students is valuable.

- If I can demonstrate that number conservation may be typically reached at age four, it will allow schools to be aware that this is a concept to be considered as new in pre-kindergarten and should be tackled as new and not taken for granted in daily activities. This is fortified by the concepts of ‘intentional’ teaching as suggested by the National Association for the Education of Young Children.

5. Summary of your qualifications to conduct project (include prior research—resumés may be attached):

I am currently the director of an elementary school which includes a preschool. I have been assigned to this position since July 2006. I have completed all required California Early Childhood Education Units at Pierce College, including CD1, CD2, CD3, CD 38, and CD 11, totalling 15 units.

I have been assisting teachers with their teaching activities with children from age two through age twelve at many occasions. I have been leading daycare activities occasionally as well. I hold French teaching credentials (C.A.P.E.S.) to teach mathematics from middle school through high school. I have taught mathematics in all grade from 6th through 12th in French Programs and in International Baccalaureate programs for ten years.
Dear Pre-K Parents,

Stéphane Plancke, a graduate student at California State University of Northridge, is conducting research as part of his Masters of Art Educational Psychology and Counseling Program with an emphasis on Early Childhood Education.

Your child is invited to participate in a math game!

The math game consists of a 5-minute activity, twice a week, for a total of 4 to 6 weeks, in collaboration with your child’s pre-kindergarten teacher Michèle Lhuillier. The researcher will sit at a table in the classroom and invite your child to play with pennies – counting them in different ways during normal free-choice time. The entire encounter will take about 5-minutes and will be videotaped. The attached information provides more details specifically about the research.

The goal of the project is to answer the research question...

*What helps children learn the concept of number consistency best: watching, sorting, or working with a friend?*

Please review the materials attached and return completed forms to Mr. Plancke by Monday, December 12th, 2011.
APPENDIX C: Letter of Informed Consent

CALIFORNIA STATE UNIVERSITY, NORTHRIDGE
NUMBER SENSE DEVELOPMENT WITH FOUR YEAR OLDS: NUMBER
CONSERVATION PROJECT
PARENTAL INFORMED CONSENT FORM

The Number Sense Development With Four Year Olds: Number Conservation Project, conducted by Stéphane Plancke as part of the requirements for the M.A. degree in Educational Psychology, with emphasis in Early Childhood Education, is designed to provide teachers, parents, and administrator with important information concerning children’s development of number sense.

The research will add to the literature about how children learn math concepts best: by watching the teacher, by trying out math problems themselves, or by working with a friend.

Each child of the 16 pre-kindergarteners will be in the study for up to six weeks. The risks from participating in this study include stress related to participating on a one on one activity with an adult but these do not differ from the normal activities found in the classroom. Neither you nor your child will receive monetary compensation for participation in this study.

Any information that is collected in this study that can be identified specifically with your child will remain confidential and will be disclosed only with your written permission or if required by law. The cumulative results of this study will be published, but the names or identity of subjects will not be made known. All data/documentation collected as part of this project will be stored in a locked cabinet by the researcher at the conclusion of the study.

However, there may be specific benefits which your child can expect as a result of participation in this study, including gaining a better sense of number conservation (to know that objects can be moved but the number of objects stays the same unless new objects are added or if others are taken away) and other math, and social skills. Your child may benefit from enhanced teaching methods as a result of teacher interaction and play with a math-related game.

If you wish to voice a concern about the research, you may direct your question(s) to Research and Sponsored Projects, 18111 Nordhoff Street, California State University, Northridge, Northridge, CA 91330-8232, and by phone at 818-677-2901. If you have specific questions about the study you may contact Dr. Carrie Rothstein-Fisch, faculty advisor, 18111 Nordhoff Street, Northridge, CA 91330, and by phone at 818-677-2529.

You should understand that approval for your child to participate in this study is completely voluntary, and you may decline to allow your child to participate or withdraw your child from the study at any time without jeopardy. Your child may also discontinue the math game if he or she wishes at any time, as it is customary in our preschool. Likewise, the researcher may cancel this study at any time.

During the course of the project participants may be videotaped. Your initials here ____ signify your consent to allow your child to be videotaped. The purpose of the videotaping is to allow material for discussion on validity of the study. All tapes collected as part of this project will be destroyed by the researcher at the conclusion of the study unless a separate photo release is signed allowing us to use the video for teaching purposes.
I have read the above and understand the conditions outlined for participation in the described study. I have been provided with a copy of this consent form to keep and I give informed consent for my child, named below, to participate in the study.

Child's Name

Last    First    MI

Age _______ Years _________ Months

Parent/Legal Guardian Printed Name

Last    First    MI

Signature ___________________________ Date ________________

Witness/P.I. signature ___________________________ Date ________________

Give this form to Lourdes Harris, administrative assistant.

Keep one copy of this consent form for your records.
Monday, September 26, 2011

California State University, Northridge
Standing Advisory Committee for the Protection of Human Subjects
18111 Nordhoff Street
Northridge, CA 91330-8232

Dear Committee Members:

Stéphane Plancke has permission to conduct the project entitled “Development of Number Sense: Conservation of Numbers” at the Lycée International de Los Angeles, West Valley Campus. We have reviewed the project and are aware of all the activities involved in the project including the video taping of children taking the test he has prepared.

Elizabeth Chaponot
Head of School
Lycée International de Los Angeles
APPENDIX E: Video Permission and Release Form

Video/Photo Permission and Release

I hereby irrevocably grant to Stéphane Plancke and any person or company authorized by Stéphane Plancke as he may designate from time to time, the absolute right and permission to use in perpetuity my child’s likeness, photograph(s) and video, in whole or in part, without additional written or spoken agreement for the Number Sense Development Project, the Master of Arts in Early Childhood Education at CSUN, or any other advertising, trade or any other lawful purpose whatsoever, in any media now known or hereafter developed.

I hereby waive any right that I may have to inspect and approve the finished product or such written or spoken copy that may be used in connection therewith or the use to which it may be applied.

I hereby release, discharge and agree to save harmless Stéphane Plancke and all other persons using my child’s likeness, photograph(s) and video from any liability whatsoever, in accordance with the terms hereof, included but not limited to any liability for what might be deemed to be misrepresentation or defamation of my child, my child’s character or person due to distortion, alteration, optical illusion or faulty reproductions which may occur with the use of my child’s likeness.

Name_____________________________________________

Address ___________________________________________

Signature ___________________________ Date___________
APPENDIX F: Bill of Rights

CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

EXPERIMENTAL SUBJECTS
BILL OF RIGHTS

The rights below are the rights of every person who is asked to be in a research study. As an experimental subject I have the following rights:

1) To be told what the study is trying to find out,

2) To be told what will happen to me and whether any of the procedures, drugs, or devices is different from what would be used in standard practice,

3) To be told about the frequent and/or important risks, side effects or discomforts of the things that will happen to me for research purposes,

4) To be told if I can expect any benefit from participating, and, if so, what the benefit might be,

5) To be told the other choices I have and how they may be better or worse than being in the study,

6) To be allowed to ask any questions concerning the study both before agreeing to be involved and during the course of the study,

7) To be told what sort of medical treatment (if needed) is available if any complications arise,

8) To refuse to participate at all or to change my mind about participation after the study is started. This decision will not affect my right to receive the care I would receive if I were not in the study.

9) To receive a copy of the signed and dated consent form.

10) To be free of pressure when considering whether I wish to agree to be in the study.

If I have other questions I should ask the researcher or the research assistant, or contact Research and Sponsored Projects, California State University, Northridge, 18111 Nordhoff Street, Northridge, CA 91330-8232, or phone (818) 677-2901.

X

Signature of Subject         Date

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