A STUDY OF ANTENNA NEAR-FIELDS
IN TIME DOMAIN

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By
Nicholas Quinn Muhlmeyer

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The project of Nicholas Quinn Muhlmeyer is approved:

________________________________________  ________________________________
Vaughn P. Cable, Ph. D.                     Date

________________________________________  ________________________________
Matthew M. Radmanesh, Ph. D.               Date

________________________________________  ________________________________
Sembiam R. Rengarajan, Ph. D. Chair        Date

California State University, Northridge
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ABSTRACT

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A 2D Finite-Difference Time-Domain grid is generated using a MATLAB code. The absorbing boundary of the grid is modeled using a Berenger PML. An E-Plane sectoral horn, designed to transmit a 1 GHz signal, is placed in the grid. The fields along the aperture of the horn are computed in the time domain and transformed to the far-field. Additionally, the time domain near-fields are also computed at distances from the aperture ranging from $0.75D^2/\lambda$ to $10D^2/\lambda$.

The overall goals were two fold. The first was to characterize far-field performance of the horn using near-field analysis by transforming the near-field results to the far-field using an FFT. The second objective was to observe the local aperture fields and correlate these directly with the far-field results. Only the first goal was accomplished, and the second goal will require further investigation.

This report describes several methods including how to setup up the FDTD analysis, as well as some of its shortcomings, and solutions to these shortcomings. Lastly, computed near-field results of on-axis fields as a function of distance from the horn aperture are presented.
INTRODUCTION

Finite-Difference time-domain is a numerical method for computing E and H fields. FDTD is based on the differential form of two of Maxwell's equations. FDTD is a robust computational algorithm useful for electromagnetic analyses ranging from near DC to visible light applications. Due to its method of solving in the time-domain, it can cover a wide range of frequencies in a single simulation. The fields are computed using a leapfrog scheme where all E-fields are computed in one time-step and all H field components are computed one-half-time-step later. The process repeats until either steady state has been reached or the desired transient has occurred. While this process (the grid mesh computation) can become computationally large, it avoids solving simultaneous equations and that makes this method easily implemented in an algorithm. This method allows any field inside or outside the radiating (or scattering) structure to be found easily.

One of the disadvantages of FDTD is fields are always computed at every cell in the grid, whether those fields are of interest or not. Also, the gridding requires a mesh be chosen sufficiently fine for the geometry, the frequency of interest, and for numerical stability; this is particularly a problem for long thin structures such as wires. FDTD also requires a numerical boundary condition be inserted outside of the computational grid to absorb outward waves leaving the grid, thus simulating an unbounded problem.

Before proceeding further, the reader should be advised that a basic understanding of electromagnetic theory is required for understanding the theory and procedure of this analysis. For those less familiar with the variables and terms, or those seeking a more in-depth review of the mathematics, see appendices A and B.
THEORY

FD-TD is a differential equation based method where Maxwell’s differential curl equations are solved by discrete numerical differentiation in time and space. A two-dimensional case has been used in this investigation and is operated in the TE mode. We note in passing that, by use of the duality principle of electromagnetics, a set of equations for TE mode can quickly give rise to an equivalent set of equations in the TM mode case.

Since this 2D investigation examines only the fields in the xz and yz plane, in the TE mode, the Ez, Hx, and Hy components can be dropped. That leaves equations (1) and (2).

\[
\Delta \times E = -\frac{\partial B}{\partial t}
\]

\[
\Delta \times H = J + \frac{\partial D}{\partial t}
\]

Rearranging variables and substituting \( D = \varepsilon E \) and \( B = \mu H \) these equations become (3) and (4).

\[
\frac{\partial E_{x,y}}{\partial t} = \frac{1}{\varepsilon} \left( \pm \frac{\partial H_z}{\partial t} - \sigma E_{x,y} \right)
\]

\[
\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \rho'H_z \right)
\]

Next these equations need to be compiled in a way that can be numerically computed using finite differences. This requires a well defined grid, and here we use a grid consisting of Yee cells. The Yee cell is a square for 2D (or cube for 3D) with the lengths of every edge being the same length. For TE analysis, each cell contains two components of the electric field as well as a single component of the magnetic fields, in this case Ex, Ey, and Hz. These E-fields are defined on the edges of the cell, while the H-field is defined in the center of the cell face and is directed normal to this cell face, as shown in Fig. 1.

![Figure 1: The Yee Cell (from Stutzman and Thiele 501)](image)

Hence, each face will have two E field components and one H field component. This problem examines the bottom face containing Ex, Ey, and Hz. Each face in this diagram corresponds to a different TE configuration. By utilization of the Yee cell in 2D equations (3) and (4), the finite differencing equations become (5), (6), and (7).
Where the constants $C_a$, $C_b$, $C_P$ and $C_Q$ are defined as

$$C_a(m) = \frac{2\varepsilon_0\varepsilon_\infty + \beta_d - \sigma \Delta t}{2\varepsilon_0\varepsilon_\infty + \beta_d + \sigma \Delta t}; \quad C_P(m) = \frac{2\Delta t}{2\mu_0 - \sigma_m \Delta t}; \quad C_Q(m) = \frac{2\Delta t}{2\mu_0 + \sigma_m \Delta t}$$

Equations (5), (6) and (7) are the final equations used to in the computational algorithm for this study. The equations in (8) are stated separately because they are functions in terms of time and material properties of each cell. This will be useful in how the algorithm is setup.
NUMERICAL STABILITY AND DISPERSION

Numerical stability is primarily influenced by numerical dispersion. Dispersion is the spreading out of a signal due to different frequencies propagating at different velocities on the grid. To minimize dispersion, a careful selection of cell size and time step is required. In practice cell size is usually selected first. The decision for the size should be based on both the geometry of the materials and desired accuracy or fineness of the mesh. Usually, cell edges are chosen to be 1/10 wavelength or smaller at the highest frequency of interest in the cell with the highest density material (largest epsilon). Once the cell size is settled upon the time step duration must then be set accordingly. In time, any point of the wave must not exceed one cell per time step, including on a diagonal, else distortion arises. Equation (9) gives a good guideline as to how to select the time step and cell size to minimize dispersive effects in two-dimensions. Generally speaking, the effect of numerical dispersion can be reduced, not eliminated, by strategic choice of the cell size and step size.

\[ \Delta t \leq \frac{\Delta s}{c\sqrt{2}} \]  

In this equation, \( \Delta s \) is the length of one edge of the (square) cell, and \( \Delta t \) is the time step, which is the time it takes for the fields to travel one cell (on a diagonal). Since the E and H fields have been defined at different locations in each cell, E on the edges and H in the face, a leapfrog method may be used. To iterate the procedure, first the E fields are evaluated based on last E and H values, then half a time step later the H-fields are computed from the current E-field data. This procedure will continue until the maximum number of time steps has been reached.

ABSORBING BOUNDARY CONDITIONS

The next issue to consider is the absorbing boundary conditions (ABC). The purpose of the ABC is to simulate the wave propagating to infinity with no reflection back into the main grid. There are several types of ABCs that can all be implemented to good accuracy. An ABC is an outer boundary to the main grid, usually not more than 10 cells thick, whose purpose is to absorb all the outgoing waves that reach that region. This means that any stray waves will dissipate in this region. The ABC, in effect, creates isolation between the structure of interest inside the main grid, and any external effects, so only the interactions inside the main grid can influence the results. The ABC performs this function by gradually changing the intrinsic impedance seen by the E and H fields as they propagate into the outer boundary. The Berenger Perfectly Matched Layer (PML) is generally seen to be one of the most accurate and is the ABC used in this study (Taflove 288).

SOURCES

There are two classes of sources, hard and soft. Hard sources effectively force the field value(s) at a particular (feed) location to be whatever the applied source is at every time step. Note, if the applied source goes to zero at any point in time, that location becomes a short circuit for a voltage-type source or an open circuit for a current-type source and thus cause reflections. A soft source, on the other hand, is influenced by both the exciting source and the neighboring fields. A soft source is the summation of the fields that would exist at that location were no source present combined with the impressed field. Mathematically, a hard source is defined by equation (10) and a soft source is defined by equation (11). In order to have the most accurate results, a soft source is implemented in this work.
\[ E_z \mid \_n = f(t) = E_o \sin(2\pi f_o n\Delta t) \]  \hspace{1cm} (10)

\[ E_z \mid \_n = E_z \mid \_n - 1 + \frac{\Delta t}{\varepsilon_0 \Delta x}(H_y \mid \_n - 1/2 - H_y \mid \_n + 1/2) + f(t) \]  \hspace{1cm} (11)

The applied (source) signal used for this study is “soft” version of a differentiated Gaussian pulse following equation (12).

\[ f(t) = E_o e^{-[(n - n_0)/n_{\text{decay}}]^2} \]  \hspace{1cm} (12)

FAR-FIELD

There are two ways to obtain the far-field pattern of an antenna using the FDTD method. The first is to draw an imaginary boundary near the outer edge of the main computational grid, adjacent to the ABC, and perform a spatial Fourier transform of the data. The second method is to make the entire computational grid large enough such that the equivalent physical distance extends into the far-field region, and calculate the fields in that region. Equation (13) shows the accepted far-field criterion known as the Fraunhofer distance, observed by Joseph von Fraunhofer.

\[ d = 2D^2/\lambda \]  \hspace{1cm} (13)

Here, \( d \) is the separation between the aperture and the observed point and \( D \) is the aperture size. This is an approximation at which the \( E \) and \( H \) fields can be assumed to have a simple relationship; that they are orthogonal to one another and orthogonal to the direction of propagation. In the near-field, the relationship between \( E \) and \( H \) becomes much more complex in the sense that all components of \( E \) and \( H \) are present and the field contains evanescent as well as propagating waves.

THE FDTD PROCEDURE

To summarize the FDTD procedure used in this study, we first determine cell size and allocate space to the PML and main grid. We then create the geometry of any structures in the grid. Next, we compute constants such as time step, and the electrical material properties, including evaluation of equation (8) and functional form of the source. Once these preliminaries are finished, we zero out all fields in all the cells and begin the time stepping. The \( E_x \) fields in the main grid and in the PML region are then computed using equation (6) and a similar operation is then performed for \( E_y \) using equation (7). Finally, \( H_z \) is calculated in all the cells using (5). Then we increment the time step and re-evaluate \( E_x \), \( E_y \), and \( H_z \) and this procedure continues until all time steps have been processed. Note that because we reuse computer memory for \( E \) and \( H \) at each new time step, the desired field data must be, explicitly, stored at each time step; e.g., on the near-to-far-zone transform surface.

MATLAB was used as the computational engine to perform this analysis. The original base code used is a 2D FDTD code written by Susan Hagness and published with the second edition of the book “Computational Electrodynamics: The Finite-Difference Time-Domain Method.” This code only implements the non-time varying parameters of the initial analysis. It also computed the \( E \) and \( H \) field components for a simple cylindrical structure and a hard point source and the default output was a real-time display of the \( E_x \), \( E_y \), and \( H_z \) fields in the grid that updated every time step. For this study several functions were added to this base code. The geometry was changed to an E-plane sectoral horn. The source excitation became a soft Gaussian
pulse applied on a plane originating inside the horn. The far-field conversion was also added including all methods of data analysis.
RESULTS

The first task is to compare current calculated results to the existing examples provided by others. Since the specific size or frequency is inconsequential, the same parameters from Stutzman and Thiele's example in their book Antenna Theory and Design can used (Stutzman 524-531). Fig. 2 shows the general geometry and the computational space, including the overall size of the grid excluding the (ABC) boundary region.

Figure 2: Horn Geometry and Computational Grid (from Stutzman and Thiele)

A 1 GHz modulated Gaussian pulse as shown in equation (12) was used as a source. This source was placed in the horn located $\lambda/4$ from the back wall of the waveguide. It is also implemented as a soft source. The computed results are shown in Fig. 3 and they very closely match the expected result from Stutzman and Thiele.
Figure 3: Computed Modulated source pulse (left), Reference Source (from Stutzman et al.) (right)

The above source is applied to the structure and the fields begin to exit the aperture at roughly a time step of \(n=400\). The first peak of the radiated pulse is also diffracted at the edge of the aperture and propagates outside to the back end of the horn by approximately \(n=800\). The time-domain data is translated to the far-field pattern at this point \((n=800)\) by Stutzman, et al., and their far-field result is shown below.
Since the horn boresight is pointed to the right in the grid, we attempt to capture the far-field main beam by storing and transforming just the time domain fields along the right-hand boundary. Two transforms are needed; the time domain data transformed to the frequency domain, then the frequency domain data transformed to the far-field, or vice versa. The far-field pattern was calculated using equation (14),

\[ E_\theta = -j\omega[A_\theta + \eta F_\phi], \]  

(14)

where \( \omega \) is the angular frequency the electric and magnetic vector potentials \( F \) and \( A \), respectively, are given by

\[ A_\theta = A_x \cos(\theta) \cos(\phi) + A_y \cos(\theta) \sin(\phi) - A_z \sin(\theta), \]  

(15)

\[ F_\phi = -F_x \sin(\phi) + F_y \cos(\phi). \]  

(16)

Since only the Az component exists in this 2D problem, equation (15) can further simplify. Also, in this case, a sinusoid is used as the source for the horn and the simulation is run until steady state has been reached (at the right hand boundary), where two sets of the field data are recorded. The two sampled data sets are taken at a time-spacing of \( \lambda/4 \) apart, or 90°. This, in effect, gives us the real and imaginary components of the steady-state fields, directly, without having to take the time-to-frequency transform. Then, by taking the near-to-far transform, we obtain the magnitude and phase of the pattern. The end result for the computed far-field pattern is shown in Fig. 5. \( E_y \) is the main component in this case due to the geometry of the problem and \( E_x \) is in the direction of propagation and, thus, adds nothing to the far-zone result.
There are some deviations between figures 4 and 5, although, the similarity of the notches is clear as well as in the shapes of the main beams. Also, we note that the E/H ratio of the main beams ($\phi=0^\circ$) in Fig. 5 is the expected 51.5 dB (or 377 ohms) as it should be. We also point out that Stutzman and Thiele (Fig. 4) appear to have used a 3D result to scale their 2D far-field pattern since the 20dBi range is typical for a horn.

Another goal of this work was to compute far-field data, directly, using FDTD. In order to do this, the grid was greatly expanded; whereas, the original size of the grid for the above calculations was 200x260 cells, the far-field criterion ($2D^2/\lambda$) drives the grid size to 200x750 cells. This new grid enables the distances between the aperture of the horn and the sampled field point to be $0.75D^2/\lambda$, $D^2/\lambda$, $1.5D^2/\lambda$, and $2D^2/\lambda$ where D is the length of the aperture and $\lambda$ is the free space wavelength.

Fig. 6 below shows a second identical horn antenna has been placed to the right and facing the first horn. Here, the right horn is considered the receiver and the left horn the transmitter. And the ABC boundary is also shown outside the main computational grid. The separation between the two antennas is $d$, and $D$ is the aperture size (same for both). The following figures show results of the on-axis computed fields for distances $d=0.75D^2/\lambda$ (Fig. 7), $d=D^2/\lambda$ (Figure 8), $d=1.5D^2/\lambda$ (Figure 9), and $d=2D^2/\lambda$ (Fig. 10) for modulated pulse excitation. The upper plot in each case is the input pulse at the feed point and the bottom plot is the on-axis $E_y$ field at the respective distance.
Figure 6: Geometry of two horn setup (not to scale)

Figure 7: Computed in time-domain at \( d = 0.75D^2/\lambda \)
Figure 8: Computed in time-domain at $d = \frac{D^2}{\lambda}$.

Figure 9: Computed in time-domain at $d = 1.5D^2/\lambda$. 
The first figure (Fig. 6, \( d=D^2/\lambda \)) shows pulse spreading has already begun due to grid dispersion, though it is relatively minor. This grid dispersion appears to continue to increase slightly in the next 3 figures.

As an additional check, the fields at both the top and bottom of the transmit aperture are plotted in Fig. 11. Due to symmetry, they should be identical. However, on close examination, they are not and Fig. 12 shows the difference between these two fields. Clearly some symmetrical discrepancy exists, however, there is only about a two percent difference.
Since the above results were not quite as expected, another simulation was performed using cells 1/2 the previous size, thus going from $\lambda/20$ to $\lambda/40$; i.e., the 200x750 grid became 400x1500. The expectation was that this smaller cell size should reduce dispersion and thus improve the convergence and accuracy, but at the expense of simulation time. The two plots below show the resulting time-domain fields and their Fourier transforms, respectively, for both the $\lambda/20$ and $\lambda/40$ cell size cases and the same $d=2D^2/\lambda$. 

![Figure 12. Computed Aperture Error](image1)

![Figure 13: Far field plot at $\lambda/20$ and $\lambda/40$ in time domain](image2)
Fig. 13 and 14 clearly show the smaller cell size results is less dispersion and a broader frequency spectrum, as would be expected. This data shows that to obtain accurate results, the $\lambda/40$ cell size must be used. However, simulations under the large grid conditions causes some concerns. Firstly, since the grid is so large, the RAM requirements for the computer to solve the problem increases vastly. Secondly, due to the same RAM limitation, the height of the grid cannot be made to match the length. Ideally the grid should be square funneling the computed fields through a long narrow corridor throughout the grid puts undue stress on the ABC as well as on the near-far transform.

The ABC, for instance, performs best when the computed waves (fields) approach normal to the boundary. Hence, grids with high aspect ratios, e.g., 3500 cells by 200 cells, should be avoided because skew incidence on the ABC does not provide good absorption and the resulting reflections can cause distortions.
We give here an example of the expected distortion due to a direct on-axis wave and an indirect (reflected) wave arriving at the receive aperture. Consider two horns with on-axis spacing \( d \) between apertures and grid boundaries parallel to this axis spaced \( h \) apart, then with knowledge of the speed of the wave, an estimated arrival time can be determined. The Pythagorean theorem yields the distance from the grid mid-line axis to the top middle of the grid. If a reflection is expected, the total distance traveled by the indirect (reflected) wave is then given by equation (14) below.

\[
d_{tot} = 2\sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{h}{2}\right)^2}
\]

(14)

Simplifying yields

\[
d_{tot} = \sqrt{d^2 + h^2}
\]

(15)

Incorporating the time component, the number of time steps can then be found from

\[
n_{\text{reflected}} = \frac{1}{r}\sqrt{d^2 + h^2}
\]

(16)

We will compare this to the number of time steps for the direct path which would be

\[
n_{\text{line}} = \frac{d}{r}
\]

(17)

where \( r \) is the rate at which the wave travels along the main grid; in this case, \( r = 0.707/\Delta t \) cells per time step.

In the case of an aperture separation of \( d = 10D^2/\lambda \), this amounts to \( 160\lambda \) separation, or 6400 cells with \( \lambda/40 \) cell size. This equates to 9050 time steps for the direct path. The total distance travelled for the indirect wave(s) for a total grid height of 400 cells is found to be about 6412 cells, with an expected arrival time of approximately 9067 time steps, i.e., only 17 time steps after the incident wave. When compared with the pulse source, 17 time steps equates to roughly \( 1/3 \) of a wavelength at 1 GHz. This would interfere with the direct wave since it would arrive approximately \( 108^\circ \) out of phase.
CONCLUSION

A 2D FDTD computation was performed on a Yee grid with PML boundaries. First one horn antenna, then two horn antennas (facing each other), were placed in the grid and aperture fields and far-fields were computed. Two Fourier Transforms (FFTs) were used, when a modulated pulse source was used, to convert the FDTD results into frequency and far-field results, respectively. The grid size was then increased to include the far field region and thus allow FDTD to compute the far-fields directly in the time domain. Again, these time domain field were converted into the frequency domain and a spatial Fourier Transform used to produce the far-field pattern.

Recommendations based on these results include the following; 1) for the most accurate results, it is desirable to observe the fields on the outer edge, a few cells in from the boundary region all around the grid, thus forming a complete box around the whole computational grid; 2) for particular types of antennas that are focused in one direction, the above is not always necessary; 3) a square grid (2D) or cube shaped grid (3D) is always desired since the ABC boundaries do not properly absorb most of the outgoing fields in narrow rectangular grids; and 4) effective solution convergence depends on the resolution of the grid, i.e., the maximum cell size with respect to wavelength of highest frequency being considered.

One of the original goals of this investigation was to observe the correlation if any between the near-fields and far-field data. If the near-field information is known, can we predict the far-field results? Based on results computed here, the answer is yes. A second and related goal was to obtain far-field results directly from the aperture data. This was not fully examined, but there is good information to compare on-axis fields as a function of distance from the aperture, including results in the far-field.

Further research is recommended in the area of predicting far-field data from the aperture data. Once a relationship between the aperture and far fields is found, if any, this should then be extended to three-dimensions and observed if a similar pattern exists. Since far-field data is of prime importance for essentially all antenna applications, the ability to know something about the far-field response is key. Also, the concept of absorbing boundaries almost always needs to be considered for any far-field application. Finally, any extension of this work should include computations to examine the accuracy (and dispersion) which becomes even more important in larger problems with multiple objects and several material types, especially in 3D problems.
REFERENCES


APPENDIX A: GLOSSARY

ABC (absorbing boundary condition) – A group of cells outside of the main grid. It's primary purpose is to absorb any stray fields to prevent reflections back into the main grid. The ABC is design to simulate an infinite amount of distance at which field will naturally decay in magnitude.

Cell – The smallest sub-region of the main or ABC regions in which a single Ex, Ey, and Hz component is defined for any given time-step.

Cell size – The length of each cell. The length and width may vary depending on desired cell type used.

Dispersion – A wave with multiple frequencies where the velocity of the wave is a function of it's frequency causing an apparent error to the output.

E-plane sectoral horn – A type of rectangular horn antenna with a fan shaped beam in the E plane while having a broad band beamin the H plane.

Far-field – The region a large distance away from the antenna at which the E and H fields have a simple relationship to each other. The diffraction have become out of phase and have little influence over the fields.

FDTD (finite difference time domain) – A numerical method for solving the differential versions of Maxwell's equations to obtain E and H fields.

Fraunhofer distance – The distance away from the antenna at which the E and H fields have more understandable relationship. This distinguishes the near-field from the far-field.

Main grid – The group of cells inside the ABC that contains the structure and all relevant data and computations are performed for analysis.

Maxwell's Equations – The most fundamental equations that define how fields behave in any arbitrary space.

Near-field – The region near an antenna at which the E and H fields have a complex relationship with each other. This is caused by coupling and diffraction of antenna and structures near the antenna.

Numerical stability – The ability of a numerical method to converge and provide an accurate result.

PML (perfectly matching layer) – A type of ABC with exceptional wave impedance independant of the outgoing wave's frequency. It simulates a anisotropi absorbing medium adjacent to the outer boundary.

Time-step – The discrete duration between one iteration of the field computation with the next iteration of the same fields computation.

Yee cell – A type of cell in which each side of the cell is of equal length. This is the optimal configuration for a cell balancing computational requirements with accuracy.
APPENDIX B: VARIABLES

\( \lambda \) – wavelength
\( f_0 \) – fundamental frequency of the source, related to wavelength
\( \mu \) – Permeability, material property
\( \varepsilon \) – Permittivity, material property
\( \sigma \) – conductance, material property
\( E \) – Electric field intensity usually denoted with a subscript to indicate which plane the field is in.
\( H \) – Magnetic field intensity usually denoted with a subscript to indicate which plane the field is in.
\( D \) – dielectric field or electric displacement, related to \( E \) field.
\( B \) – Magnetic induction, related to \( H \) field.
\( \Delta t \) – Time step
\( \Delta s \) – Cell size (\( \Delta s = \Delta x = \Delta y \) for a Yee cell)
\( J \) – Current density
\( D \) – aperture size
\( d \) – distance
\( n \) – specific time step
\( \omega \) – angular frequency