San Fernando Valley State College

SPEECH ANALYSIS METHODS APPLIED TO BIOELECTRIC SIGNALS

A thesis submitted in partial satisfaction of the requirements for the degree of Master of Science in Engineering
by
Frank Ira Doyle

June, 1972
The thesis of Frank Ira Doyle is approved:

San Fernando Valley State College
June, 1972
DEDICATION

This thesis is dedicated to my wife Patsy and children for their kindness and consideration throughout the effort required for this work.

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<td>$s(t)$</td>
<td>bioelectric signal</td>
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<tr>
<td>$\hat{s}(t)$</td>
<td>estimate of $s(t)$</td>
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<tr>
<td>$a_j x_j$</td>
<td>linear combination of mutually orthogonal functions</td>
</tr>
<tr>
<td>$x_j$</td>
<td>orthogonal function</td>
</tr>
<tr>
<td>$S(\omega)$</td>
<td>Fourier transform of $s(t)$</td>
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<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>filter impulse response</td>
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<tr>
<td>$A$</td>
<td>amplitude of Gaussian envelope</td>
</tr>
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<td>$S$</td>
<td>time duration of Gaussian envelope</td>
</tr>
<tr>
<td>$C$</td>
<td>center in time of Gaussian envelope</td>
</tr>
<tr>
<td>$\phi$</td>
<td>phase of cosine wave with respect to $C$</td>
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<tr>
<td>$F$</td>
<td>frequency of cosine wave</td>
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<td>$G_n(t)$</td>
<td>Gaussian wavefunction</td>
</tr>
<tr>
<td>$H_n(t)$</td>
<td>family of Hermite polynomials</td>
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<tr>
<td>$u(t)$</td>
<td>wavefunction</td>
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<tr>
<td>$W_m(t)$</td>
<td>wavefunction $m$ in filter band</td>
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<tr>
<td>$U_n(t)$</td>
<td>filter band</td>
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<tr>
<td>$M_n$</td>
<td>number of GCM wavefunction elements in filter band $n$</td>
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<tr>
<td>$\hat{U}(t)$</td>
<td>estimate of $U(t)$</td>
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<tr>
<td>$U_m(j \cdot)$</td>
<td>Fourier transform of $W_m(t)$</td>
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<tr>
<td>$K$</td>
<td>number of octave partitions</td>
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<tr>
<td>$\hat{U}(j\omega)$</td>
<td>estimate of $U(j\omega)$</td>
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\begin{align*}
N &= \text{number of wavefunction elements in time interval} \\
E_{i}(M_{i}T_{i}) &= \text{extrema} \\
BF_{n}(f) &= \text{bandpass filter } n \\
f_{n1} &= \text{filter bandwidth} \\
f_{n2} &= \text{filter center frequency} \\
n &= \text{filter count number} \\
T &= \text{sampling interval} \\
R &= \text{sampling rate} \\
N &= \text{number of points in } BF_{n}(i) \\
M &= \text{number of points in } U(j) \\
\hat{D}(j) &= \text{decimated data} \\
CM &= \text{GCM wavefunction existence criteria} \\
S_{LB} &= \text{lower bounds on } S \\
S_{UB} &= \text{upper bounds on } S \\
r(t) &= \text{filter output} \\
H_j(f) &= \text{bandpass filter } j \\
n &= \text{number of octave partitions} \\
S(\omega) &= \text{Fourier transform of } s(t) \\
r_j(t) &= \text{filter } j \text{ output} \\
f_{\text{max}} &= \text{maximum end of octave partitions} \\
f_{\text{min}} &= \text{minimum end of octave partitions} \\
S &= \text{sensitivity of } n \text{ to } f \\
dT &= \text{sample interval for moving average filter} \\
Q &= \text{total points in } h_j(t), H_j(i) \\
P &= \text{total points in } S(k)
\end{align*}
\[ L_i(t) = \text{Lagrangian interpolating polynomial} \]
\[ h_H(t) = \text{Hamming window} \]
\[ E_{\text{wf}} = \text{wavefunction energy} \]
\[ p = \text{ASC} \& \text{F parameter set vector} \]
\[ T = \text{training or diagnosis vector} \]
\[ p = \text{number of training coefficients} \]
\[ q = \text{number of ASC} \& \text{F coefficients} \]
\[ V = \text{regression slope matrix} \]
\[ T_o = \text{diagnostic vector} \]
\[ P_o = \text{input parameter vector} \]
\[ N(j) = \text{number of } P_n \text{ ASC} \& \text{F parameters representing the category} \]
\[ N = \text{total population} \]
\[ t = \text{time} \]
\[ f_{jn}(P_n) = \text{probability density distribution function} \]
\[ r(t) = \text{wavefunction} \]
\[ R(t) = \text{rule of random process} \]
\[ f_R(r,t) = \text{random process waveform density distribution} \]
\[ R_r(\tau) = \text{autocorrelation function of } r(t) \]
\[ n = \text{subject in population } N \]
\[ g(t) = \text{summation of random waveforms} \]
\[ \bar{F}(t) = \text{mean value of output of linear system} \]
\[ E(xx) = \text{expected value of } xx \]
\[ X = \text{state vector representation of ASC} \& \text{F system} \]
\[ t_{jn} = \text{element in vector } T \]
\[ p_{kn} = \text{element in vector } P \]
\[ \alpha + \beta x = \text{linear estimate of } y \]
ABSTRACT

Speech Analysis Methods Applied to Bioelectric Signals

by

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Prior bioelectric signal representation and analysis techniques have relied mostly on the process of estimating the signal by linear sets of mutually orthogonal functions. This approach is considered as a baseline in proposing new speech analysis methods for the representation and analysis of bioelectric signals. The advantages of speech time and frequency domain techniques over prior signal representation methods are discussed.

The speech ASCOF analysis procedure is presented. The ASCOF analysis of bioelectric signals is then considered. The sufficiency of ASCOF functions as bioelectric descriptors is discussed. The frequency bounds, filtering, and MC kernal requirements of the speech analysis procedure are extended to bioelectric signal requirements.

Implementation of the analysis algorithms on selected bioelectric signals is then presented. The
algorithms are evaluated on an individual basis and then in an automatic ASCΦF analysis mode. Preprocessing, sampling, parameter calculation, extrema evaluation, time/frequency domain behavior and synthesis relative to bioelectric signals is presented and discussed. Analysis results with selected signals are presented and analyzed. Data compression and bioelectric event recognition is considered.

The use of ASCΦF parameters in clinical diagnosis is introduced. Regression analysis methods of diagnosis using the analysis sets are considered and discussed. An alternative to formulation of diagnostic vectors is given. Applications of the ASCΦF system in bioelectric phenomena modeling/simulation, random process analysis, and linear system representation are given. Statistical properties of the analysis procedure are given and considered.
Chapter 1

INTRODUCTION

Signal analysis and representation has seen and will see wide applications in the interpretation of bioelectric phenomena. Representation of the signal in terms of selected parameters which uniquely describe the waveform behavior has important applications in data reduction, diagnosis, waveform recognition and classification, behavior prediction and physiological systems analysis. The increasing use of computer systems by the bio-engineer enhances the availability of the tools by which analysis of biological signals may be performed.

Biological signals change with time, a variational pattern which lends itself to time domain, frequency domain and statistical analysis. Direct analysis of the time varying bioelectric waveforms by measurement techniques is complicated by the non-periodic or non-stationary nature of the signals and the fact that simultaneous measurement of the relevant parameters describing a particular biological function is difficult to achieve. Analysis by direct measurements such as peaks, slopes and so forth gives little insight into the behavior of the signal or its underlying physiological process.

Signal analysis and processing offers a more
versatile approach to the analysis of biological signals. By this method, important time, frequency and statistical characteristics of the signal provide an analysis tool by which the nature of the signal and the physiological system from which it is derived may be more readily understood. Additionally, proper processing of the signal leads to enhancement of significant signal characteristics which are otherwise lost in the signal noise or suppressed by the generating physiological system. Finally, modeling of the signal by parameters which uniquely describe its behavior yields discrete numerical values which can be readily processed by computing systems for the purpose of event recognition, diagnostics and physiological systems analysis. These are important to both clinical and research applications for the physician, hospital and biomedical engineer.

1.1 Basic Approaches

Signal theory has offered investigators a basic tool by which bioelectric signal analysis and representation may be accomplished. The majority of past approaches to signal representation has been through the method of approximating the bioelectric function of time by a set of mutually orthogonal functions. To illustrate, consider a set of m functions $x_1(t), x_2(t), \ldots, x_m(t)$ which are orthogonal to one another over the interval $\alpha$ to $\beta$, that is:
\( \int_{\alpha}^{\beta} x_i(t)x_j(t) = 0 \quad \text{if} \ j \neq j \)

Then, let an arbitrary bioelectric signal \( s(t) \) be approximated over the interval \( (\alpha, \beta) \) by a linear combination of these \( m \) mutually orthogonal functions:

\[
\hat{s}(t) = a_1 x_1(t) + a_2 x_2(t) + \ldots + a_j x_j(t) + \ldots + a_m x_m(t)
\]

or:

\[
\hat{s}(t) = \sum_{p=1}^{m} a_p x_p(t)
\]

In order to obtain the best estimate for \( s(t) \), a certain criteria must be chosen. If the mean-square error (m.s.e.) is the criteria, then the coefficients \( a_1, a_2, \ldots, a_j, \ldots, a_m \) is:

\[
e = s(t) - \hat{s}(t)
\]

and the m.s.e., is given by:

\[
\varepsilon = \mathbb{E} \left\{ [s(t) - \hat{s}(t)]^2 \right\} = \mathbb{E} \left\{ [s(t) - \hat{s}(t)]^2 \right\}
\]

or:

\[
\varepsilon = \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} [s(t) - \sum_{p=1}^{m} a_p x_p(t)]^2 dt
\]

To minimize \( \varepsilon \), it is required that:

\[
\frac{\partial \varepsilon}{\partial a_1} = \frac{\partial \varepsilon}{\partial a_2} = \ldots = \frac{\partial \varepsilon}{\partial a_i} = \ldots = \frac{\partial \varepsilon}{\partial a_m} = 0
\]

Using equations (1-3) and (1-5) this becomes:

\[
\frac{\partial}{\partial a_i} \int_{\alpha}^{\beta} [s(t) - \sum_{p=1}^{m} a_p x_p(t)]^2 dt = 0
\]
Upon expansion of equation (1-8), the terms of the form:

\(\int x_i(t)x_j(t)dt = 0\)  

for all \(i \neq j\) by their orthogonality defined in equation (1-1). Similarly, the derivative with respect to \(a_i\) of all terms that do not contain \(a_i\) are zero. Equation (1-8) then becomes:

\[
\frac{\partial}{\partial a_i} \int_a^\beta [-2a_is(t)x_i(t) + a_i^2x_i^2(t)]dt = 0
\]

from which is obtained:

\[
a_i = \frac{1}{c_i} \int_a^\beta s(t)x_i(t)dt
\]

where:

\[
c_i = \int_a^\beta x_i^2(t)dt
\]

Equation (1-11) represents the values of the coefficients which minimize the m.s.e. Utilizing the foregoing results in equation (1-6), the m.s.e. becomes:

\[
\epsilon = \frac{1}{\beta - \alpha} \int_a^\beta s^2(t)dt - [a_1^2c_1 + a_2^2c_2 + \ldots + a_m^2c_m]
\]

As an example of this representation, consider representing the signal shown in Figure 1.1 given by:

\[
s(t) = \frac{t}{\pi} - 1 \quad 0 < t < 2\pi
\]

by a single function:

\[
\hat{s}(t) = a_1 \sin t
\]
then, from equations (1-11) and (1-12), \( a_1 \) is given by:

\[
\begin{align*}
\text{(1-16)} \\
\int_0^{2\pi} \frac{(t - 1)\sin t}{\int_0^{2\pi} \sin^2 t} \, dt \\
= \frac{1}{\pi} \int_0^{2\pi} t \sin t \, dt - \int_0^{2\pi} \sin t \, dt \\
\int_0^{2\pi} \sin^2 t \, dt \\
or:
\quad a_1 = -\frac{2}{\pi}
\end{align*}
\]

The best estimate of \( s(t) \) in the m.s.e. sense then becomes:

\[
\text{(1-17)} \quad \hat{s}(t) = -\frac{2}{\pi} \sin(t)
\]

This is shown in Figure (1-1).

In the limit, as the number of terms in equation (1-13) approach infinity, the sum:

\[
\sum_{p=0}^{\infty} a_p c_p
\]
converges to:

\[(1-19)\]
\[\int_{\alpha}^{\beta} s^2(t) dt\]

and hence: \(\varepsilon \to 0\)

In this case, equation (1-3) may be written as:

\[(1-20)\]
\[s(t) = \sum_{p=1}^{\infty} a_p x_p(t)\]

Past investigators have utilized series representation for approximating bioelectric signals by the use of a closed set of mutually orthogonal functions. These are commonly known as trigonometric and Fourier series in which the signal is represented by a series of harmonic terms.

1.2 An Alternate Approach

There are many basic components, \(x_j(t)\)'s given by equation (1-2) which might serve useful in the representation of the signal \(s(t)\) by an infinite series. The most popular ones have been the Fourier exponential series given by:

\[(1-21)\]
\[s(t) = \sum_{j=-\infty}^{\infty} a_i e^{jn\omega_0 t}\]

where:

\[a_i = \frac{1}{T} \int_{-T/2}^{T/2} s(t) e^{-jn\omega_0 t} dt\]

and:

\[T = \text{interval}; \ \omega_0 = \frac{2\pi}{T}\]
and the Fourier trigonometric series given by:

\[(1-22)\]

\[s(t) = a_0 + \sum_{i=1}^{\infty} a_i \cos i\omega_0 t + b_i \sin i\omega_0 t\]

where:
\[
a_0 = \frac{1}{T} \int_{\alpha}^{\alpha+T} s(t) dt
\]
\[
a_i = \frac{2}{T} \int_{\alpha}^{\alpha+T} s(t) \cos i\omega_0 t dt
\]
\[
b_i = \frac{2}{T} \int_{\alpha}^{\alpha+T} s(t) \sin i\omega_0 t dt
\]

These representations are considered in the interval:

\[(1-23)\]

\[\alpha < t < \alpha + T\]

However, these representations assume that the bioelectric signal is periodic, i.e.,

\[(1-23)\]

\[s(t) = s(t + T)\]

In most, if not all, of bioelectric signals equation \((1-23)\) does not hold and series representation of the bioelectric signal outside the interval:

\[(1-24)\]

\[|t| < \frac{T}{2}\]

does not describe the signal and gives no information of the behavior of the signal outside this interval. Thus, the series expansion is not true for every \( t \), and the signal \( s(t) \) is not equal to the series representation outside of this interval.
Consider, for example, representing the ECG shown in Figure 1.2 by series representation on the interval $a < t < a + T$. The indication of ventricular premature contraction which is evident in the actual signal is not indicated by the series representation which assumes periodicity.

Additionally, attempts have been made to derive frequency information from the signal based on series representation. This produces a discrete frequency spectrum based on the strength of each harmonic of the fundamental frequency of the signal. This approach, however, again assumes periodicity of the signal, and does not provide an accurate frequency domain representation of the nonperiodic signal. What is required in this case is the derivation of the continuous frequency spectrum given by the Fourier
transform of the signal:

\[(1-25)\]

\[S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} \, dt\]

and its pair:

\[(1-26)\]

\[s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} \, d\omega\]

which are the Fourier transform pairs representing the relationship between the time and frequency domain behavior of the bioelectric signal.

With series representation of the signal, one is faced with the problem of determining the coefficients as depicted in equations (1-21) and (1-22). Approaches have been from graphical methods to filters matched to the discrete frequencies of interest. Estimating signals by orthonormal basis components permits equation (1-1) to be written:

\[(1-27)\]

\[\sum_{\alpha}^{\beta} x_\alpha(t)x_\beta(t) = \delta_{ij}\]

where \(\delta_{ij}\) is the Kronecker delta defined by

\[\delta_{ij} = 1, \quad i = j\]

\[\delta_{ij} = 0, \quad i \neq j\]

Using this approach, equation (1-11) becomes:

\[(1-28)\]

\[a_i = \sum_{\alpha}^{\beta} s(t)x_\alpha(t)\, dt\]

since:

\[\sum_{\alpha}^{\beta} x_\alpha(t)\, dt = 1\]
from the definition in equation (1-27) of orthonormality. This representation simplifies the estimation process, but still requires determination of the $a_i$'s of equation (1-28).

Using the sampling property of the impulse function, the $a_i$ of equation (1-28) may be represented by:

$$a_i(t) = s(t) * \delta(t) = s(t)*h(t)$$

(1-29)

Then, using a filtering scheme shown in Figure 1.3, and

![Figure 1.3](image)

Determining the $a_i$'s by Matched Filters

defining the filter impulse response as that matched to the orthonormal basis components by:

$$X_i(t) * h(t)$$

(1-30)

then, if the signal $s(t)$ is applied to the filter as shown in Figure 1.3, the output at the $i$th filter becomes:

$$a_i(t) = \int_{-\infty}^{\infty} s(t)x_i(t-\tau)d\tau$$

(1-31)

This approach, however, requires as many filters as there are $x_i$'s needed to accurately represent the signal. For example, consider using equation (1-22) to represent a bio-electric signal. If there is no DC term, and the signal
has a highest frequency component \( f_{\text{max}} \), then a limit may be placed on the series summation by:

\[
M = f(f_{\text{max}})
\]

then the number of terms, or coefficients required\(^6\) is given by:

\[
M = f_{\text{max}}T
\]

where \( T \) is the length of the signal trace. Consider using this approach in estimating the Electroencephalogram (EEG) traces. The frequency ranges of interest for the various EEG activity are shown in Table 1.\(^7\) In this case, for example, for an \( f_{\text{max}} \) of 30 Hz and a trace length required of 20 seconds representing a basic frequency of 0.05 Hz; then the number of coefficients, and subsequently the number of filters becomes:

\[
M = (30)(20) = 600
\]

which represents an obvious disadvantage to this approach.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>less than 4 Hz</td>
</tr>
<tr>
<td>Theta</td>
<td>4 to 8 Hz</td>
</tr>
<tr>
<td>Alpha</td>
<td>8 to 13 Hz</td>
</tr>
<tr>
<td>Beta</td>
<td>greater than 13 Hz</td>
</tr>
</tbody>
</table>
Additionally, since the $a_i$s given by equation (1-31) are time variant, this also requires selection of the time at which the coefficients are desired to be evaluated.

For the estimation of the signal by equation (1-3), any orthogonal set of basis functions may be used as long as they form a complete set. However, physical limitations place a restriction on $m$, and to minimize m.s.e. given by equation (1-6) some basis sets may provide a more efficient expansion than others. Also, the basis set should be chosen such that the $a_i$s given in equation (1-6) are statistically independent, and one must assume that a finite number of terms in the estimation are adequate to represent the signal. These restrictions, therefore, require an extensive amount of a-priori knowledge about the nature of the signal to be estimated.

The foregoing discussion indicates that a different, less restrictive and more general approach is required in the analysis and estimation of bioelectric signals.

1.3 Objectives of Thesis

The objective of the thesis is to investigate the application of a speech analysis technique to the analysis and interpretation of bioelectric signals. The speech analysis technique was developed by my close friend, Dr. Bernard J. Carey, and may be termed "ASCØF" analysis. The method will be described in the following section. The original intent of the thesis was to apply ASCØF analysis
to a specific bioelectric signal. However, upon research into the method, the power of the methods was realized, and the objective was changed to the application of ASCOF analysis as a general tool for bioelectric signals.

The thesis hopes to minimize the problems associated with past bioelectric signal analysis which rely on signal estimation by orthonormal/orthogonal basis functions. This is done by applying the ASCOF analysis method which:

- does not depend on a periodic model of the signal
- provides signal characteristics in the time and frequency domains
- is not restricted to a series summation in order to minimize the m.s.e.
- minimizes the number of elements in the parameter set required to adequately describe the signal
- requires a single filtering process in the analysis algorithm
- minimizes the amount of a-priori knowledge required for the signal
- provides for data compression
- provides for near real-time analysis

to the analysis of bioelectric signals.

Thus, the objectives of the thesis may be summarized as follows:

1. Investigate the validity of ASCOF analysis in representing a general class of bioelectric signals.
2. Apply the analysis technique to data traces of selected bioelectric signals.

3. Analyze and investigate the use of the ASCØF parameter set in clinical diagnosis.

4. Analyze the use of ASCØF analysis in the statistical evaluation of bioelectric signals.

5. Research and extend past bioelectric signal analysis methods to the ASCØF approach.
Chapter 2

THE CAREY ASCØF ANALYSIS PROCEDURE

This section outlines the fundamental concepts developed by Dr. Bernard J. Carey for use in the analysis of human speech. The method is based on a wavefunction representation of the filtered acoustic waveform of human speech. The analysis procedure defines the requirements for a general, high speed and accurate time domain analysis method based on speech representation by acoustic filter bands. The frequency range of interest in the analysis is 100-3200 Hz and the filtering process of this range of human speech is based on octave partitioning, interpretation, and condensation supplied by this writer to facilitate the extension of this powerful technique to the analysis of bioelectric signals. The ASCØF analysis procedure may be broken down into three basic elements: waveform representation, preprocessing, and determination of the ASCØF parameter set.

2.1 Waveform Representation

Filtered speech can be represented in the time domain by an ordered set of Gaussian wavefunction elements whose individual elements satisfy the expression:

\[(2-1) \quad \ddot{u}(t) + \left(\frac{2\pi}{5}\right)^2 (t-C) \ddot{u}(t) + \left(\frac{2\pi}{5}\right)^2 (N^2 - 1/2) u(t) = 0\]
with the initial conditions:

\[ u(C) = A \cos \theta \]
\[ \dot{u}(C) = A \frac{2\pi N}{S} \sin \theta \]

A Gaussian wavefunction is a Gaussian envelope multiplied by a \( n \)th order Hermite polynomial for the case where \( n \) is an integer, and for \( n \) a non-integer, a closed form solution does not exist for the wavefunction element. The general family of wavefunctions is given by equation (2-1) and an individual wavefunction is shown in Figure 2.1. Equation (2-1) describes the family of wavefunctions called the Gaussian wavefunction because the wavefunction envelope is of the form of a Gaussian probability density function.

![Figure 2.1](image)

**Gaussian Wavefunction Element**
(pdf) expressed as a function of time by:

\[(2-2)\]

\[G(t) = \frac{A}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(t-u)^2}{2\sigma^2} \right] \]

Letting:

\[\sigma = 1\]

\[u = 0\]

\[A = (2\pi)^{\frac{1}{4}}\]

then, equation (2-2) can be written as:

\[(2-3)\]

\[G(t) = \exp \left[ -\frac{t^2}{2} \right] \]

For \(n\) an integer, the wavefunction can be expressed in a closed form solution as:

\[(2-4)\]

\[G_n(t) = \frac{d^n[G_0(t)]}{dt^n} = H_n(t)G_0(t)\]

where: \(H_n(t) = \text{the family of Hermite Polynomials}\)

Any member of the family of wavefunctions described by equation (2-1) can be uniquely specified by five (5) parameters:

\[A = \text{amplitude of the Gaussian envelope}\]

\[S = \text{time duration of the Gaussian envelope}\]

\[C = \text{center point in time of Gaussian envelope}\]

\[\phi = \text{phase of wavefunction with respect to C}\]

\[N = \text{effective number of peaks of the wavefunction during the time interval S}\]

Due to the restriction of \(n\) being an integer, equation (2-4) can represent only a small subset of the total family
of Gaussian wave functions.

As an alternate approach, the Gaussian Cosine Modulated (GCM) family of wavefunctions, which closely approximate the Gaussian wavefunction family and given by equation (2-5) can

\[ u(t) + a(t-C)u(t) + \left[ \omega_0^2 + \frac{a^2}{2} + \frac{a^2(t-C)^2}{4} \right] u(t) = 0 \]

with initial conditions:

\[ u(C) = A \cos \varphi \]
\[ \dot{u}(C) = A \omega_0 \sin \varphi \]

be used to represent filtered speech. The closed form solution of equation (2-5) is given by:

\[ u(t) = A \exp \left( -\left( \frac{T}{S} (t-C) \right)^2 \right) \cos \left( 2\pi F(t-C) - \varphi \right) \]

In this case, an individual GCM wavefunction can be uniquely defined by the five (5) parameter set \( \{ A, S, C, \varphi, F \} \) where:

- \( A \) = Amplitude of Gaussian envelope
- \( S \) = Time duration of the Gaussian envelope
- \( C \) = Center in time of the Gaussian envelope
- \( \varphi \) = Phase of the cosine wave with respect to \( C \)
- \( F \) = Frequency of the cosine wave

Hence, the analysis element set consists of these five (5) parameters for each time segment in the interval:

\[ C - 3/4S < t < C + 3/4S \]

of each frequency partition of the acoustic waveform. The GCM wavefunction is shown in Figure 2.1.
Additionally, the Hanning Cosine Modulated (HCM) wavefunction family, where an individual HCM wavefunction is given by:

\[
(2-8) \quad u(t) = \frac{A}{2} \left\{ 1 + \cos \left[ \frac{2\pi F(t-C)}{S} \right] \right\} \cos \left[ 2\pi F(t-C) - \phi \right]
\]

provides a valid representation for filtered speech. Equation (2-8) defines a Hanning window multiplied by a cosine wave. Any given HCM element is defined by the (ASCØF) parameter set having the same definition as for the GCM element. Equation (2-8) is advantageous due to its mathematical simplicity. Thus, the GCM formulation is a significant improvement over the Gaussian wavefunction, and the HCM formulation provides an advantage over the GCM formulation by replacing the exponential envelope with a Hanning window.

2.1.1 Synthesis

Once the ASCØF parameter sets are computed from the original filtered speech, the generation of a synthetic version of the filtered data from the ASCØF parameter sets is a simple inversion of the analysis procedure. In this case, the synthetic GCM elements are sequentially determined from the parameter set. Thus, for a given wavefunction \( W_m(t) \) given by:

\[
(2-9) \quad W_m(t) = \frac{A}{m} \exp \left\{ -\frac{1}{2} \left( \frac{t - C_m}{S_m} \right) \right\} \cos \left[ 2\pi F_m(t - C_m) - \phi_m \right]
\]

a filtered band, \( U_n(t) \) of human speech is given by:
\[ U_n(t) = \sum_{m=1}^{M_m} W_m(t) \]

where: \( M_n = \) Number of GCM wavefunction elements in filter band \( n \)

Then, the entire original acoustic waveform signal is given by:

\[ \hat{U}(t) = \sum_{n=1}^{K} \sum_{m=1}^{M_n} W_m(t) \]

where: \( K = \) Number of filter bands or the number of contiguous partitions in the frequency range of interest

Thus, equation (2-11) represents an estimation of the signal based on a single basic wavefunction element and the ASCOF analysis parameter sets which represent that acoustic signal.

2.1.2 Frequency Domain Representation

The superposition property of the Fourier transform provides the basis for the frequency domain representation of the acoustic signal based on the ASCOF parameter sets, i.e.,

\[ x(t) + y(t) \leftrightarrow X(\omega) + Y(\omega) \]

The Fourier transform of a GCM wavefunction is given by:

\[ U_m(j\omega) = \frac{A_m S_m}{2\sqrt{\pi}} \exp \left\{ -[S_m(f-F_m)]^2 \right\} \exp \left\{ -j(\phi_m + 2\pi F_m C_m) \right\} \]
for the range: \( f_{\text{max}} - f_{\text{m}} \leq f \leq f_{\text{m}} + f_{\text{max}} \)

where: \( f_{\text{max}} = (3/4)/S_{\text{m}} \).

and, \( U_m(j\omega) \) is the Fourier transform of the GCM wavefunction defined by equation (2-9).

The Fourier transform of a HCM wavefunction defined by equation (2-8) is given by:

\[
U_m(j\omega) = \frac{A_m}{2} \frac{f_0}{\left| f_0^2 - (f-f_{\text{m}})^2 \right|} \sin \left[ \frac{\pi (f-f_{\text{m}})}{f_0} \right] \exp \left[ -j \left( \phi_{\text{m}} + 2\pi f_{\text{m}} C_{\text{m}} \right) \right]
\]

for the range: \( f_{\text{m}} - f_{\text{max}} \leq f \leq f_{\text{m}} + f_{\text{max}} \)

where: \( f_0 = 1/S_{\text{m}} \)

Since the time domain behavior of the speech acoustic waveform have the closed form solutions represented by equations (2-8) and (2-9), can be represented by equation (2-11); and from the superposition principle of the Fourier transform, the frequency spectrum of the entire signal can be described by:

\[
\hat{U}(j\omega) = \sum_{m=1}^{N} U_m(j\omega)
\]

where \( N \) is the number of wavefunction elements described by ASC\(\Phi\)F parameter sets in the interval:

\[
t_{\text{min}} \leq t \leq t_{\text{max}}
\]

and \( N \) is given by:
Thus, the acoustic signal which can be estimated in the time domain by equation (2-11) can also be described in the frequency domain by equation (2-15) using the ASCOF sets.

2.1.3 The Analysis Algorithm

Figure 2.2 shows the ASCOF analysis algorithm developed and implemented for the analysis of human speech. The basic steps in the procedure are as follows:

1. Low pass filter and then fast filter the data \( u_D(t) \) into \( k \) contiguous octave bandpass frequency bands to cover the frequency range of interest.

2. Locate and calculate extreme \( E_k(t) \) from filtered data \( u_k(t) \).

3. Store extrema \( E_k(t) \).

4. Detect existence of GCM or Sine wavefunction from extrema list \( E_k(t) \).

5. Calculate Normal GCM and Sine ASCOF parameters based on extrema centered around \( C,(ASCOF)_m \).

6. For GCM wavefunction, residue subtract present extrema magnitude \( M_i \) coupling into future to obtain independence in the calculation of the ASCOF parameters for the next wavefunction.

7. Store parameter set \( (ASCOF)_n \) for each filter.
Figure 2.2 ASCSF analysis procedure
8. Synthesize original data \( u(t) \) from parameters, \((\text{ASCQT})_n, k, M_n \) and \( W_m(t) \).

The procedure analyzes any arbitrary sample of human speech, provides high speed decomposition into wavefunction elements, gives highly accurate results, analyzes both GCM and sinusoid wavefunction elements, and provides high speed filtering algorithms. Additionally, the procedure is fast because it is non-iterative, requires minimum storage space from the computer system for implementation, and realizes minimal parameter calculation algorithms. Finally, it utilizes a closed form wavefunction solution to allow simple analysis algorithms, a closed form description of the acoustic waveform itself, and straightforward transforms between the time and frequency domain features of human speech.

2.2 Preprocessing

Figure 2.3 illustrates the basic steps in the preprocessing procedure. The speech signal is filtered into \( k \) contiguous octave filter, where the input/output relationship is given by:

\[
(2-17) \quad u(t) = u_1(t) + u_2(t) + \ldots + u_k(t)
\]

or:

\[
(2-18) \quad u(t) = \sum_{n=1}^{k} u_n(t)
\]
The extrema, $E_i$, for a wavefunction is shown in Figure 2.4.

The extrema is defined:

\[ E_i = (M_i, T_i) = \text{extrema} \]
where: \[ M_i = \text{signed magnitude of } E_i \]
\[ T_i = \text{time of occurrence of } E_i \]

### 2.2.1 The Filtering Algorithms

The filtering process involves a partitioning and filtering process which is described by letting:

(2-20) \[ K = \text{number of contiguous partitions in the frequency range of interest, } f \in [f_{\min}, f_{\max}] \]

and defining bandpass filters \( BF_n(f) \) by:

(2-21) \[
BF_n(f) = \begin{cases} 
1.0 & f \in [f_{\min}, f_{\max}] \\
0 & \text{elsewhere}
\end{cases}
\]

where: \[ n = 1, 2, 3, \ldots, K \]
\[ f_{\min} = f_{\min} \]
\[ f_{K_{\max}} = f_{\max} \]

The output of any filter band \( n \) in Figure 2.3 is given by:

(2-22) \[
u_n(t) = u(t) * BF_n(t) = \int_{-\infty}^{\infty} u(t) BF_n(t-\tau) d\tau
\]

where \( BF_n(t) \) is the inverse Fourier transform of \( BF_n(f) \) given by:

(2-23) \[
BF_n(t) = \frac{1}{2\pi} \int_{f_{\min}}^{f_{\max}} [BF_n(f) \exp(j\omega t)] df
\]

which is:

(2-24) \[
BF_n(t) = \frac{\sin(\pi f_{n1} t)}{(\pi f_{n1} t)} \frac{\cos(2\pi f_{n2} t)}{(\pi f_{n2} t)}
\]
where: \( f_{n1} = f_n \max - f_n \min \) = filter bandwidth

and: \( f_{n2} = f_n \max - f_n \frac{1}{2} \) = filter center frequency

Thus, equation (2-22) is the convolution of the \( \frac{\sin H}{H} \) bandwidth filter function \( B_{f_n}(t) \) with the acoustic signal \( u(t) \). Carey showed experimentally that any one octave frequency partition would correctly pre-filter speech and that a sufficient condition for defining a pre-filtering function \( B_{F_n}(j\omega) \) is:

(2-25)

\[
B_{F_n}(j\omega) = \begin{cases} 
1.0 & f \in \left[f_{\min}, 2f_{\min}\right] \\
0 & \text{elsewhere}
\end{cases}
\]

Equation (2-24) represents the impulse response of a zero phase, ideal bandpass filter. To realize this filter an approximation is made to the ideal \( \frac{\sin H}{H} \) filter function and techniques are employed\(^{10,11}\) to window the ideal impulse response and optimize the filter characteristic in the frequency domain so as to minimize the time domain truncation effect.

Octave filtering can be simulated by use of the discrete convolution equation given by:

(2-26)

\[
U_n(t) = \sum_{j=1}^{N} \sum_{i=N+1}^{2} B_{f_n}(i) U(j-1)
\]

and the ideal filter impulse response, designated a kernal by:
\[
B_{f_{n0}}(t) = \frac{\sin(\pi f_{n1} i T)}{(\pi f_{n1} i T)} \cos(2\pi f_{n2} i T)
\]

where: 
\(j = 1, 2, 3, \ldots, M\)
\(i = \frac{(-N+1), \ldots, 0, \ldots, (+N-1)}{2}\)
\(n = K, K-1, K-2, \ldots, 3, 2, 1\)
\(K = \text{number of octave partitions}\)
\(T = \frac{1}{R}\)
\(R = \text{sampling rate}\)
\(N = \text{discrete points in } B_{f_{n0}}(t) = \text{odd number}\)
\(M = \text{discrete points in } U(t)\)

2.2.1.1 High Speed Filtering

Carey developed a filtering algorithm based on previous work\(^1\) which:

1. Minimized the time required for time domain convolution filtering.
2. Performs a one-octave partitioning of the frequency domain.
3. Minimizes the truncation effect with a time domain windowing procedure.
4. Provides for only six times real time for \(K = 5\) one octave partitions of the frequency range of interest using the IBM 1800 system.

The characteristics of equation (2-27) are such that if:
\[(2-28) \quad f_{n1} = f_{n2} = \frac{R}{4}\]
three out of four points in the filter impulse response will be zero which gives a minimum coefficient (MC) kernal. Also, using the filter symmetry and limiting the sampling rate for each frequency partition to four times the upper cutoff frequency of the partition minimizes the number of arithmetic operations in the filtering process. This allows equation (2-26) to be written:

\[
\begin{align*}
U_n(t) & = jT \\
& = \sum_{j=1}^{M} \sum_{i=2,4}^{N-1} Bf_n(i) [U_j+U_{j+1}] \\
& = Bf_n(0)U_j \\
\end{align*}
\]

and requires frequency components greater than \(f_n\max\) of a given octave partition to be removed prior to filtering to avoid aliasing problems.

If the initial conditions are set such that:

\[
R = 4(f_k\max) = R_k
\]

where: \(f_k\max = \) maximum frequency of interest

then:

\[
R_k = 2R_{k-1} = 4f_n \ldots \ldots \ldots 2 \ R_1
\]

which says that the sampling rate can be halved for each decreasing octave partition of the original data, and can be accomplished by decimating every other sample data point in the original data list after each successive octave partition. Also, the higher frequency components must be subtracted from the data list prior to the next lower octave bandpass filtering sequence to avoid aliasing the lower
frequency data due to halving the sampling rate by decimation. These decimation and subtraction processes are given by:

\[ (2-32) \quad U_D(j) = U_D(m) - U_n(m) \]

where:
- \( U_D(m) \) = filter band \( n \) partition
- \( U_D(j) \) = original sample data list

\[ n = K, K-1, K-2, \ldots, 3, 2, 1 \]
\[ j = 1, 2, 3, \ldots, \frac{M}{2^{K-n+1}} \]
\[ m = 1, 3, 5, \ldots, \frac{M}{2^{K-n}} \]

which gives the number of data points in each filter band as:

\[ (2-33) \quad M = M_K = 2M_{K-1} = \ldots = 2^{K-1}M_1 \]

Thus, with decimation of the data between adjacent partitions the same minimized kernel can be used for all filtering operations, since as \( R \) is halved, \( f_{n1} \) and \( f_{n2} \) are halved. The center frequency of the MC kernel is placed at \( f_{K_{\text{max}}} \) by equation (2-28), which requires a low pass filtering operation before MC filtering. This is expressed as:

\[ (2-34) \quad f_{\text{LP}} = f_{K_{\text{max}}} = \text{low pass filter cutoff} \]

Table 21 shows the window compensated MC kernels for the octave and low pass filtering, where \( N = 81 \).
Table 2.1

MC Kernel Coefficients

<table>
<thead>
<tr>
<th>Octave</th>
<th>Low Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_f(0)$ = +0.500000</td>
<td>$B_f(0)$ = +0.500000</td>
</tr>
<tr>
<td>$B_f(2)$ = -0.315317</td>
<td>$B_f(1)$ = +0.317551</td>
</tr>
<tr>
<td>$B_f(6)$ = +0.097476</td>
<td>$B_f(3)$ = -0.103867</td>
</tr>
<tr>
<td>$B_f(10)$ = -0.050142</td>
<td>$B_f(5)$ = +0.060028</td>
</tr>
<tr>
<td>$B_f(14)$ = +0.028260</td>
<td>$B_f(7)$ = -0.040481</td>
</tr>
<tr>
<td>$B_f(18)$ = -0.015763</td>
<td>$B_f(9)$ = +0.029075</td>
</tr>
<tr>
<td>$B_f(22)$ = +0.008286</td>
<td>$B_f(11)$ = -0.021652</td>
</tr>
<tr>
<td>$B_f(26)$ = -0.003876</td>
<td>$B_f(13)$ = +0.016281</td>
</tr>
<tr>
<td>$B_f(30)$ = +0.001511</td>
<td>$B_f(15)$ = -0.012238</td>
</tr>
<tr>
<td>$B_f(34)$ = -0.000397</td>
<td>$B_f(17)$ = +0.009171</td>
</tr>
<tr>
<td>$B_f(38)$ = +0.000031</td>
<td>$B_f(19)$ = -0.006760</td>
</tr>
</tbody>
</table>

2.2.2 Extrema Detection and Calculation

Four extrema, grouped around C, provide sufficient information for accurate calculation of the five ASC0F parameters which specify a GCM wavefunction. If the sampling rate $R$ is equal to four times the maximum frequency of interest, i.e.,

$$R = 4 \ f_{\text{max}} = 4 \ f_{\text{Kmax}}$$

then a three point parabolic estimation of the extrema will provide sufficiently accurate extrema points for the ASC0F parameter calculation procedure. This also provides for
implementation of the high speed octave filtering process. The rate meets the Nyquist criteria of twice the maximum frequency of interest and guarantees two discrete points per half cycle of the filtered data, which allows for adequate parabolic estimation of the true extrema. Figure 2.5 shows the conditions in the filtered data for the existence of maxima or minima extrema.

Figure 2.5
Existence of Maxima and Minima Extrema

These are defined by:

(2-36) 

a Maxima occurs when \( u_j > u_{j+1} \)

a Minima occurs when \( u_j < u_{j+1} \)

To detect the extrema, \( E_n(i) \), the sample data list for a particular filter output, \( u_n(j) \) is searched alternately for a maxima or a minima. Fitting a parabola through the three points \( u(j-1), u(j) \) and \( u(j+1) \) allows an accurate estimate
of the true extrema. As derived by Carey, the extrema $E_i$ is given by:

\[(2-37) \quad E_i = [M_i, T_i]\]

and:

\[
M_i = u_j - \frac{(u_{j+1} - u_{j-1})^2}{8(u_{j+1} - 2u_j + u_{j-1})}
\]

\[
T_i = \left[ j - \frac{(u_{j+1} - u_{j-1})}{2(u_{j+1} - 2u_j + u_{j-1})} \right] T
\]

\[
T = 1/R
\]

The extrema detection and computational algorithm can be simplified by detection of the polarity of the first extrema, then letting:

\[(2-38) \quad M_i = |M_i|\]

and sign determine subsequent extrema by alternating signs. Also, the polarity of $M_i$ is only considered when computing $\emptyset$.

The sampling interval $T$ provides a basic time unit for fixed interval sample data list with absolute time given by:

\[(2-39) \quad t = (j-1)T \quad j = 1, 2, 3, 4, \ldots, M\]

and, for decimated data which give $t$ as an integer multiple of $T$ by:

\[(2-40) \quad t = (j-1)(2^k-n)T\]

where:

\[
j = 1, 2, 3, 4, \ldots, \frac{M}{2^k-n}\]
2.3 Wavefunction Parameter Calculation and Analysis

With the existence of an extrema list given by:

\[ E_n = [M_i, T_i]^n \]

with:

\[ i = E(\text{extrema})\text{list index} = 1, 2, 3, \ldots, E_{\text{max}} \]

the calculation of the ASCOF wavefunction parameters requires location of a region in the extrema list which exhibits a wavefunction behavior, calculate the ASCOF from extrema in the interval, and remove the coupling of the present wavefunction into the next wavefunction by residue process. This last requirement assures independent wavefunctions. Thus, the ASCOF analysis procedure is a continuation of this process for the filter band.

The existence and location of a GCM and HCM wavefunction in the extrema list is given by the criteria:

\[ CM = (M_{i+1} > M_{i+3}) \land (M_{i+2} \geq M_i) \]

where \( \land \rightarrow "\text{and}" \)

Figure 2.6 illustrates a possible GCM or HCM wavefunction

![Wavefunction Diagram](image)
represented by five extrema, \( E = [M_i, T_i] \). The five ASC\&F parameters needed to specify the wavefunction of Figure 2.6 can be computed from the four extrema grouped around \( C \) which are delineated in equation (2-42). The five ASC\&F parameters can be calculated as follows for the GCM wavefunction:

**Spread \( (S) \):**

\[
S = 2\pi(T_{i+2} - T_{i+1}) F[\zeta]
\]

where:

\[
F[\zeta] = \left[ \frac{-1}{2\ln (2\sqrt{\zeta} - 1)} \right]^{1/2}
\]

and:

\[
\zeta = \frac{(M_i + M_{i+1})(M_{i+2} + M_{i+3})}{(M_{i+1} + M_{i+2})^2}
\]

A lower bound must be placed on \( S \) which occurs when \( M_i \) and \( M_{i+6} \) equal zero, consequently the end points of the four extrema used to characterize the wavefunction approach zero; and, for an ideal GCM wavefunction, the condition implies only one cycle of the cosine wave is under the Gaussian envelope. In this case:

\[
S_{LB} = \text{lower bound on } S = 2(T_{i+2} - T_{i+1})
\]

Experimentally, an upper bound on \( S \) was determined prevented \( S \) from becoming too large and inhibited the present wavefunction \( W_m \) from coupling past the center of the next (future) wavefunction \( W_{m+1} \) and destroying the characteristics during the residue process. The upper bound is given by:

\[
S_{UB} = \text{upper bound on } S = 2(T_{UB} - C_m)
\]
where: \( T_{UB} = \) 2nd extrema to the left of \( C_{m+1} \)

**Frequency (F):**

\[
F = \frac{1}{2(T_{i+2} - T_{i+1})}
\]  \hspace{1cm} (2-46)

**Center (C):**

\[
C = T_{i+2} - \mu(T_{i+2} - T_{i+1})
\]  \hspace{1cm} (2-47)

where: \( \mu = \frac{1}{1 + \rho} \)

and:

\[
\rho = \frac{\beta}{\alpha} \hspace{1cm} \text{if } \alpha > \beta
\]

\[
\rho = \frac{\alpha}{\beta} \hspace{1cm} \text{if } \alpha < \beta
\]

and:

\[
\alpha = M_{i+2} - M_i
\]

\[
\beta = M_{i+1} - M_{i+3}
\]

Also, \( \mu \) may be written:

\[
\mu = \frac{T_{i+2} - C}{T_{i+2} - T_{i+1}}
\]  \hspace{1cm} (2-48)

**Phase \( \phi \):**

\[
\phi = \pi \mu \hspace{1cm} \text{if } M_{i+1} > 0
\]

\[
= \pi \mu + 1 \hspace{1cm} \text{if } M_{i+1} < 0
\]  \hspace{1cm} (2-49)

**Amplitude (A):**

\[
A = M_{i+1} \exp \left[ \frac{\pi}{S} (T_{i+1} - C) \right]^2
\]  \hspace{1cm} (2-50)

Using equations (2-48) and (2-43), equation (2-50) can be expressed in terms of \( \zeta \) and \( \mu \) by:

letting \( i = 1 \), then
\[
\frac{\pi}{S}(T_2 - C) = \frac{\pi[T_2 + \mu(T_3 - T_2) - T_3]}{2\pi F(\zeta)(T_3 - T_2)}
\]
\[
= \frac{-T_2 - \mu(T_3 - T_2) + T_3}{-(T_3 - T_2)2F(\zeta)}
\]
\[
= \frac{1}{2F(\zeta)} \left[ \frac{-T_2 - \mu(T_3 - T_2) + T_3}{-(T_3 - T_2)} \right]
\]
\[
= \frac{1}{2F(\zeta)} \left[ \frac{T_3 - T_2 - \mu(T_3 - T_2)}{(T_3 - T_2)} \right]
\]
\[
= \frac{1}{2F(\zeta)} \left\{ \frac{T_3 - T_2 - \mu(T_3 - T_2)}{T_3 - T_2} \right\}
\]
\[
= \frac{1}{2F(\zeta)} \left[ (1 - \mu) \right]
\]

Hence,

(2-51)

\[
A = M_{i+1} \exp \left[ \frac{(1 - \mu)}{2F(\zeta)} \right]^2
\]

For calculation of the HCM ASCOF parameters, the expressions for \(c\), \(\phi\) and \(F\) remain the same as for the GCM elements. The calculations for the HCM S and A are:

(2-52)

\[
S = \frac{(T_{i+2} - T_{i+1})}{F_h(\zeta)}
\]

where:

\[
F_h(\zeta) = \cos^{-1} \left[ \frac{1}{2} \left( 1 + (2\sqrt{\zeta} - 1)^{1/2} \right) \right]
\]

and, where \(\zeta\) is defined as before in equation (2-43). Then,

(2-53)

\[
A = \frac{2M_{i+1}}{1 + \cos \left[ \frac{2\pi(T_{i+1} - C)}{S} \right]}
\]

and, as for the GCM elements in terms of \(\mu\) and \(\zeta\):

(2-54)

\[
A = \frac{2M_{i+1}}{1 + \cos \left[ \frac{2(1 - \mu)F_h(\zeta)}{} \right]}
\]
2.3.1 The Residue Process

In order to achieve independence in the calculation of the parameters for the next wavefunction, the present wavefunction's coupling into the future must be subtracted from the extrema list. This is called the residue process and is performed by subtraction of calculated extrema magnitudes, based on the parameters of the present wavefunction, from the corresponding magnitude in the extrema list. This is defined as:

\[(2-55) \quad M_i + M_i - \hat{M}_i \quad \text{for } i = i+3, i+4, \ldots, i+i_{\text{max}}\]

which means the residue operation takes place until:

\[(2-56) \quad T_{i+i_{\text{max}}} > C + 0.5S \quad \text{for HCM wavefunctions}\]

\[(2-56) \quad T_{i+i_{\text{max}}} > C + 1.5S \quad \text{for GCM wavefunctions}\]

and,

\[(2-57) \quad \hat{M}_i = A_m \exp \left\{-\left[\frac{\pi}{S_m} (T_i - C_m)\right]^2\right\} \quad \text{for GCM}\]

\[(2-57) \quad \hat{M}_i = \frac{A_m}{2} \left\{1 + \cos \left[\frac{2\pi}{S_m} (T_i - C_m)\right]\right\} \quad \text{for HCM}\]
Chapter 3

ASCF ANALYSIS OF BIOELECTRIC SIGNALS

The electrochemical activity of the brain, nerves, heart, and muscles produce as a byproduct electric potentials which can be measured. These electric potentials are called "bioelectric signals." Generally, the signals are measured at the surface of the body, but in some cases measurements are made within the body. The signals occur as a consequence of electrochemical activity which plays an important role in the control and communication functions of the body. Analysis of the signals plays an important role in diagnosis and understanding the physiological systems from which the signals are generated. With modern measurement devices, the amplitudes may be standardized, leaving as analysis variables the time varying normalized amplitudes of the signal and its related frequency spectrum. These characteristics may be then utilized for time, frequency and statistical analysis of the signal.

3.1 Sufficiency of ASCF Functions as Bioelectric Descriptors

To be sufficient as a bioelectric signal descriptor, the functions which describe signal behavior must do so in the time and frequency domains to within some acceptable
error budget. ASCOF analysis is basically an estimation process given by:

\[(3-1) \quad \hat{s}(t, \omega) = f(A, S, C, \emptyset, F, \hat{w})\]

where the bioelectric signal, \(s(t, \omega)\) is estimated from the reconstruction process based on the ASCOF elements derived from \(s(t)\). Consider an octave filtering scheme for an arbitrary bioelectric signal shown in Figure 3.1 which is

![Diagram](image)

**Figure 3.1**

Decomposition of Bioelectric Signal by Contiguous Octave Bandpass Filters

the basic preprocessing scheme in ASCOF analysis. Then define the filters as:

\[(3-2) \quad H_1(\omega) = \begin{cases} 
  k & \omega \in (\omega_{\text{min}}, \omega_1) \\
  0 & \text{elsewhere} 
\end{cases} \]

\[(3-2) \quad H_2(\omega) = \begin{cases} 
  k & \omega \in (\omega_2, \omega_3) \\
  0 & \text{elsewhere} 
\end{cases} \]
$H_3(\omega) = \begin{cases} \omega \in (\omega_4, \omega_5) \\ 0 \text{ elsewhere} \end{cases}$

$H_n(\omega) = \begin{cases} \omega \in (\omega_i, \omega_{\text{max}}) \\ 0 \text{ elsewhere} \end{cases}$

and, a contiguous octave bandpass filter scheme by:

(3-3) \[ \Delta \omega_j = \text{bandwidth} = 2^{j-1} (\omega_1 - \omega_{\text{min}}) \]

and:

(3-4) \[ \omega_{\text{max}} - \omega_{\text{min}} = \sum_{j=1}^{n} \Delta \omega_j \]

where: \( j = 1, 2, 3, \ldots, n \)

\( k = \text{constant} \)

The relationship:

(3-5) \[ s(t) \leftrightarrow S(\omega) \]

where: \( S(\omega) = \text{Fourier transform of } s(t) \)

shows that either representation uniquely specifies the bioelectric signal. Thus, if the signal is sufficiently defined in the frequency domain it is also sufficiently defined in the time domain. A sufficient condition for the existence of \( S(\omega) \) is that:

(3-6) \[ \int_{-\infty}^{\infty} |s(t)| \ dt < \infty \]

This is guaranteed if the arbitrary chunk of data is taken during a fixed time interval as shown in Figure 3.2.
Truncating Data for ASCØF Analysis

For ASCØF analysis to sufficiently estimate the bioelectric signal \( s(t) \) it is required that:

\[
(3-7) \quad [s(t) - \hat{s}(t)] < \varepsilon(t)
\]

and that:

\[
(3-8) \quad \hat{s}(t) = \sum_{j=1}^{n} \sum_{m=1}^{M_n} w_m(t) = \sum_{j=1}^{n} r_j(t)
\]

where: \( \varepsilon(t) \) = an acceptable error criteria

Equation (3-8) says two things. First, the ASCØF elements must adequately describe the filter outputs, and, second, the filter outputs must adequately describe the signal.

The adequacy of the filters can be verified by showing, for the ideal case:

\[
(3-9) \quad s(t) = \sum_{j=1}^{n} r_j(t)
\]

From the convolution and superposition property of the Fourier transform, we have from Figure 3.1:
\begin{equation}
R_j(\omega) = S(\omega)H_j(\omega)
\end{equation}

\[ R_1(\omega) + R_2(\omega) + R_3(\omega) + \ldots + R_n(\omega) = H_1(\omega)S(\omega) + H_2(\omega)S(\omega) + H_3(\omega)S(\omega) + \ldots + H_n(\omega)S(\omega) \]

and:

\[ R_1(t) + R_2(t) + R_3(t) + \ldots + R_n(t) = \sum_{j=1}^{n} r_j(t) \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ H_1(\omega)S(\omega) + H_2(\omega)S(\omega) + H_3(\omega)S(\omega) + \ldots + H_n(\omega)S(\omega) \right] e^{j\omega t} \, d\omega \]

or:

\[ \sum_{j=1}^{n} r_j(t) = \frac{1}{2\pi} \int_{\omega_2}^{\omega_1} kS(\omega)e^{j\omega t} \, d\omega + \frac{1}{2\pi} \int_{\omega_2}^{\omega_3} kS(\omega)e^{j\omega t} \, d\omega + \ldots + \frac{1}{2\pi} \int_{\omega_2}^{\omega_N} kS(\omega)e^{j\omega t} \, d\omega \]

then, letting \( k = 1 \) and noting that the filter arrangement provides for disjoint and contiguous octave filtering,

\[ \sum_{j=1}^{n} r_j(t) = \frac{1}{2\pi} \int_{\omega_{\min}}^{\omega_{\max}} S(\omega)e^{j\omega t} \, d\omega \]

But, from the definition of the Fourier transform:

\[ s(t) = \frac{1}{2\pi} \int_{\omega_{\min}}^{\omega_{\max}} S(\omega)e^{j\omega t} \, d\omega \]

Hence,

\[ s(t) = \sum_{j=1}^{n} r_j(t) \]

If the frequency range of interest, \( \Delta \omega_I \), covers the frequency spread for particular bioelectric signal(s) and is
included in the total octave frequency spread, i.e.,

\[ \omega_{\text{min}} \leq \omega I \leq \omega_{\text{max}} \]

Then the filter outputs will completely specify it.

The adequacy of the ASCOF parameter calculation algorithm can be verified by showing:

\[ \sum_{m=1}^{M_n} W_m(t) = r_j(t) \]

Carey showed experimentally that equation (3-11) adequately described the filter outputs by using audible and visual criteria. Visual criteria involved comparison of the original signal with the synthetic signal. This proved to give very accurate results. For the purpose of bioelectric signal representation, it suffices to show that:

\[ \sum_{m=1}^{M} W_m(t) - r_j(t) \]

for a particular criteria. This subject will be discussed later in this paper.

### 3.2 Specifying the Frequency Bounds

In order to adequately use the ASCOF analysis procedure or bioelectric signals, equation (3-11) must be satisfied. For a general bioelectric signal analysis procedure, this requires encompassing all the frequencies in the signal spectrum. In Figure 3.3 the frequency ranges for some typical signals are shown. The fast filtering
algorithms and equation (2-25) require that:

\[ f_{\text{min}} \neq 0 \]  

Figure 3.3 illustrates the low frequencies which are involved in bioelectric signal analysis. Many of them are appreciably lower than the \( f_{\text{min}} \) of 100 Hz used in speech filtering algorithms. We must, therefore, determine the minimum number of octave partitions required to cover the frequency range of interest since:

\[ \frac{f_{\text{max}}}{f_{\text{min}}} = 2^n \]  

and, the particular octave changes given by:

\[ \Delta f_{\text{min \, oct.}} = \frac{1}{2^n} f_{\text{max}} \]

\[ \Delta f_{\text{max \, oct.}} = 2^n f_{\text{min}} \]

where:

\[ f_{\text{min}} \leq f_{\text{max}} \]

which relates the octave frequency increments as a function of \( n \), or the number of octave partitions. This characteristic is shown in Figure 3.4. Thus, to cover a given frequency increment requires more octave partitions for \( \Delta f_{\text{min}} \) than for \( \Delta f_{\text{max}} \). This is an important consideration since the fast filtering time is related to the number of octave partitions required in the signal analysis.
Figure 3.3 Frequency Ranges of Biophysiological Signals
Figure 3-3 (continued)
Sensitivity of Octave Partitions

The sensitivity of \( n \) to \( \Delta f_{\text{min}} \) and \( \Delta f_{\text{max}} \) is given by:

\[
\frac{\Delta f_{\text{min}}}{f_{\text{max}}} = \frac{\Delta f_{\text{max}}}{f_{\text{min}}}
\]

which shows the negative sensitivity of \( \Delta f_{\text{min}} \) to \( n \) and the positive sensitivity of \( \Delta f_{\text{max}} \) to \( n \). From the foregoing discussion, it is evident that the optimum number of octave partitions, \( n \), required to cover a frequency range of interest from \( f_0 \) to \( f_1 \) satisfies the inequality:

\[
\frac{f_1}{2^n} < f_0
\]

From Figure 3.3, then taking these values to be:
\[ f_1 = 3000 \text{ Hz} \]
\[ f_0 = 0.1 \text{ Hz} \]

then,
\[ \frac{3000}{2^n} \leq 0.1 \]

from which: \( n = 15 \)

which sets: \( f_{\text{min}} = 0.1 \text{ Hz} \)

\( f_{\text{max}} = 3276.8 \text{ Hz} \)

However, since the majority of these signals range from 0.5 to 250 Hz, this gives:

\( n = 9 \)

and:
\( f_{\text{min}} = 0.5 \)

\( f_{\text{max}} = 256 \)

This range of range frequencies has proven satisfactory for the particular signals analyzed by this investigator using the ASCOF analysis procedure.

This permits the definition of nine filter functions to be defined as:

\[ H_1(\omega) = 1.0 \quad 0.5 < f < 1.0 \text{ Hz} \]
\[ H_1(\omega) = 0 \quad \text{elsewhere} \]
\[ H_2(\omega) = 1.0 \quad 1.0 < f < 2.0 \text{ Hz} \]
\[ H_2(\omega) = 0 \quad \text{elsewhere} \]
\[ H_3(\omega) = 1.0 \quad 2.0 < f < 4.0 \text{ Hz} \]
\[ H_3(\omega) = 0 \quad \text{elsewhere} \]
\[ H_4(\omega) = 1.0 \quad 4.0 < f < 8.0 \text{ Hz} \]
\[ H_4(\omega) = 0 \quad \text{elsewhere} \]
for the analysis of bioelectric signals.

### 3.3 Bandpass Digital Filter and MC Kernels for Bioelectric Signal Analysis

The low frequency characteristics of bioelectric signals make the use of digital filters very attractive, since these frequencies are too low to be realized by a practical passive analog filter. A filter useful in bioelectric signal analysis is the so-called moving average type filter.\(^\text{13}\) These filters are basically numerical algorithms which convolve discrete points of the input signal with discrete weighted points of the filter impulse response. They are called moving average filters since they function by summing convolved samples.\(^\text{14}\) An example of this moving average filter operation is illustrated in Figure 3.5. The moving average operation has the general

\[
\begin{align*}
H_5(\omega) &= 1.0 & 8.0 < f < 16.0 \text{ Hz} \\
H_5(\omega) &= 0 & \text{elsewhere} \\
H_6(\omega) &= 1.0 & 16.0 < f < 32.0 \text{ Hz} \\
H_6(\omega) &= 0 & \text{elsewhere} \\
H_7(\omega) &= 1.0 & 32.0 < f < 64.0 \text{ Hz} \\
H_7(\omega) &= 0 & \text{elsewhere} \\
H_8(\omega) &= 1.0 & 64.0 < f < 128.0 \text{ Hz} \\
H_8(\omega) &= 0 & \text{elsewhere} \\
H_9(\omega) &= 1.0 & 128.0 < f < 256.0 \text{ Hz} \\
H_9(\omega) &= 0 & \text{elsewhere}
\end{align*}
\]
Figure 3.5

Moving Average Numerical Filtering

form:

\[ r(t) = \sum_{i=0}^{\alpha} h_i S[(t + i)dT] \]

where:
- \( n \) = number of discrete points incremented
- \( dT \) = sample interval
- \( S \) = input signal
- \( h \) = weighting function or filter discrete impulse response
- \( \alpha \) = limits on weighting interval
- \( i \) = discrete weighting point
- \( r \) = output signal

The filtering algorithms used in the ASCŒF analysis preprocessing, and given by equations (2-26) and (2-29) are of the weighting average type and can find wide applications in the numerical filtering of biophysiological sig-
The filter function defined in equation (2-21) can be written as:

\[
H_j(f) = \begin{cases} 
1.0 & f_{j\min} < f < f_{j\max} \\
0 & \text{elsewhere}
\end{cases}
\]  

(3-22)

This is shown in Figure 3.6(a). To derive the impulse response \( h_j(t) \), use is made of the modulation theorem often used in communication theory. In this case, the frequency spectrum may be translated by multiplying the signal \( h(t) \) by a sinusoidal signal. The multiplication of the signal \( h(t) \) by a sinusoidal signal (modulation) will translate the whole frequency spectrum, since a sinusoidal signal of frequency \( \omega_0 \) can be expressed as the sum of ex-
ponentials. From Figure 3.6(a), it is required to translate the frequency spectrum an amount $\pm \omega_0$. This can be done by finding the impulse response $h'(t)$ for the non-translated case in Figure 3.6(b) and multiplying this by a $\cos \omega_0 t$ function. From Figure 3.6(b) we have:

(3-23) \[ h'(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df \]

\[ = \frac{1.0}{j2\pi t} \left[ e^{j2\pi ft} \right]_{f=-1/2}^{f=1/2} \]

\[ = \frac{1}{\pi t} \left[ \frac{e^{j\pi ft} - e^{-j\pi ft}}{2j} \right] \]

\[ = \frac{1}{\pi t} \sin(\pi ft) \]

\[ = \frac{\sin(\pi ft)}{\pi ft} \]

Then, using the identity:

(3-24) \[ h'(t) \cos \omega_0 t = \frac{1}{2} \left[ h'(t)e^{j\omega_0 t} + h'(t)e^{-j\omega_0 t} \right] \]

and, the frequency shifting theorem of the Fourier transform:

(3-25) \[ h'(t) \longleftrightarrow H'(\omega) \]

which leads to the results:

(3-26) \[ h'(t) \cos \omega_0 t = \frac{1}{2} [H(\omega + \omega_0) + H(\omega - \omega_0)] \]

Then, $\omega_0$ is given by:

(3-26) \[ \omega_0 = \frac{f}{j2} \]
Hence, the bandpass filter response is given by:

\[
(3-27) \\
\begin{align*}
    h_j(t) &= f_{j1} \sin(\pi f_{j1} t) \cos(2\pi f_{j2} t) \\
            &\quad + f_{j2} \pi f_{j1} t \\
\end{align*}
\]

which is the result given in equation (2-24), where:

- \( f_{j1} \) = bandwidth of the jth filter
- \( f_{j2} \) = center frequency of the jth filter

Equation (3-27) is a \( \sin(X) \) function, which is frequency domain translated by a \( \cos \) function. The zeros of the \( \sin(X) \) occur at:

\[
\pi f_{j1} t = \pi, 2\pi, 3\pi, 4\pi, \ldots \ldots
\]

Giving the filter symmetry by multiplication by the \( \cos \) factor is required to produce no phase shift. Thus, any weighting function which is symmetrical with respect to future and past time will produce no phase shift.

For the discrete representation of equation (3-27) we let:

- \( i \) = a discrete point in \( h_j(t) \)
- \( Q \) = total number of points in \( h_j(t) \) = odd for symmetry
- \( T \) = sampling time = \( 1/R \)
- \( R \) = sampling rate

then, from equation (2-27), equation (3-27) can be written as:

\[
(3-28) \\
H_j(t_{iT}) = \left. h_j(t) \right|_{t=iT} = f_{j1} \frac{\sin(\pi f_{j1} iT)}{\pi f_{j1} iT} \cos(2\pi f_{j2} iT)
\]
which is the impulse response of the bandpass filter in the discrete form.

Figure 3.7 illustrates a bandpass filter function which was used in the initial ASCOF analysis for the electrocardiogram. In this case:

\[ f_{j1} = 16.0 \text{ Hz} \]
\[ f_{j2} = 24.0 \text{ Hz} \]
\[ R = 125 \text{ samples/sec} \]
\[ T = 0.008 \text{ sec} \]
\[ Q = 69 \text{ points} \]

The \( \sin(0.128\pi t) \) function is shown in Figure 3.7(a), the \( \cos(0.384\pi t) \) in Figure 3.7(b), \( H_j(t) \) in Figure 3.7(c), and the filter frequency spectrum in Figure 3.7(d). Notice the GCM wavefunctions which are apparent in Figure 3.7(c).

To utilize the high-speed filtering algorithms in the ASCOF analysis of bioelectric signals, it is required to determine the MC kernel which can be used. For this purpose, it is required that:

(3-29) \[ f_{j1} = f_{j2} = R/4 \]

hence, \[ R = 4f_{j1} = 4f_{j2} \]

and:
\[ T = 1/R = \frac{1}{4f_{j1}} = \frac{1}{4f_{j2}} \]

Substituting these values into equation (3-28):

\[ H_j(t) \bigg|_{t=iT} = f_{j1} \sin \left[ \pi f_{j1} \frac{i}{4f_{j1}} \right] \cos \left[ 2\pi f_{j2} \frac{i}{4f_{j2}} \right] \]
Figure 3.7 Bandpass filter function used in the initial ASCOF analysis of the Electrocardiogram.
which gives:

\[
H_j(t) = \frac{\sin(\pi/4)\cos(\pi/2)}{(\pi/4)} e^{t(\pi i/4)}
\]

Equation (3-30) says that the MC bandpass kernal is independent of the particular bandpass, and dependent only on setting the initial conditions such that:

\[
R = 4f_{\text{max}}
\]

A few of the points in equation (3-30) are, for example,

<table>
<thead>
<tr>
<th>(i)</th>
<th>(t)</th>
<th>(H_j(t)/f_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>(T)</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(2T)</td>
<td>(-2/\pi)</td>
</tr>
<tr>
<td>3</td>
<td>(3T)</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>(4T)</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>(5T)</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>(6T)</td>
<td>(2/3\pi)</td>
</tr>
<tr>
<td>7</td>
<td>(7T)</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>(8T)</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>(9T)</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>(10T)</td>
<td>(-2/5\pi)</td>
</tr>
</tbody>
</table>

This sets the time for any particular octave partition to:

\[
T = T_j + T
\]

\[
R = R_j + R
\]

\[
T_j = \frac{1}{R_j}
\]

\[
T_j = \frac{1}{R} \cdot 2^{n-j} \quad \text{and} \quad R_j = \frac{R}{2^{n-j}}
\]

where: \(j = n, n-1, n-2, \ldots, 3, 2, 1\)
Similarly, for the low pass filtering required to avoid aliasing, we have:

\[ f_{j1} = f_{\text{max}} = R/4 \]
\[ f_{j2} = f_{j1}/2 = R/8 \]

and:

\[
H_{1p}(t) = (R/4) \sin \left( \frac{\pi R t i}{4 R} \right) \cos \left( \frac{2\pi R t i}{8 R} \right)
\]

which becomes:

\[
(3-32) \quad \frac{H_{1p}(t)}{(R/4)} = \sin \left( \frac{\pi i}{4} \right) \cos \left( \frac{\pi i}{r} \right)
\]

A few of the points in the equation (3-32) are, for example,

<table>
<thead>
<tr>
<th>(i)</th>
<th>(t)</th>
<th>(H_{1p}(t)/(r/4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>(T)</td>
<td>2/(\pi)</td>
</tr>
<tr>
<td>2</td>
<td>2(T)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3(T)</td>
<td>-2/3(\pi)</td>
</tr>
<tr>
<td>4</td>
<td>4(T)</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5(T)</td>
<td>2/5(\pi)</td>
</tr>
</tbody>
</table>

Thus, the MC kernels given in Table 2.1, as used in the preprocessing of speech, can be used directly in the analysis of bioelectric signals. The only requirement for their use is that the following criteria are met:

\[
(3-33) \quad H_j(f) = \begin{cases} 
1.0 & f_{\min} < f < 2f_{\min} \\
0 & \text{elsewhere}
\end{cases}
\]

\[ R = 4f_{\text{max}} = (4)^n f_{\text{min}} \]
These MC kernel coefficients are again listed in Table 3.1 below. These coefficients were used in the analysis of bio-electric signals, which will be discussed in the following chapters.

Table 3.1

MC Kernel Coefficients for Use in the Analysis of Bioelectric Signals

<table>
<thead>
<tr>
<th>Octave</th>
<th>( H_j(t) )</th>
<th></th>
<th>Low Pass</th>
<th>( H_{lp}(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+0.500000</td>
<td>0</td>
<td>+0.500000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.315317</td>
<td>1</td>
<td>+0.317551</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>+0.097476</td>
<td>3</td>
<td>-0.103867</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.050142</td>
<td>5</td>
<td>+0.060028</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>+0.028260</td>
<td>7</td>
<td>-0.040481</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>-0.015763</td>
<td>9</td>
<td>+0.029075</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>+0.008286</td>
<td>11</td>
<td>-0.021652</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>-0.003876</td>
<td>13</td>
<td>+0.016281</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>+0.001511</td>
<td>15</td>
<td>-0.012238</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>-0.000397</td>
<td>17</td>
<td>+0.009171</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>+0.000031</td>
<td>19</td>
<td>-0.006760</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>21</td>
<td>+0.004913</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
<td>23</td>
<td>-0.003464</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td>25</td>
<td>+0.002380</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td>27</td>
<td>-0.001556</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td></td>
<td>29</td>
<td>+0.000977</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td></td>
<td>31</td>
<td>-0.000549</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td></td>
<td>33</td>
<td>+0.000290</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>35</td>
<td>-0.000122</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td></td>
<td>37</td>
<td>+0.000056</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4

IMPLEMENTATION OF THE ANALYSIS
ALGORITHMS ON BIOELECTRIC SIGNALS

To test the validity of the foregoing discussion, it was required to implement the ASCOF analysis theory to the analysis of bioelectric signals. This was done, first, by using the individual segments of the procedure in an initial test and, then, by full implementation of the complete analysis algorithms. The procedure segments and the full algorithm were implemented on the CDC 3170 computer system. This will be covered in detail in subsequent sections of this paper.

4.1 ASCOF Parameter Sets and Wavefunction Synthesis

Determination of the ASCOF parameter set and wavefunction synthesis was initially performed using a filtered sample of the Electroencephalogram (EEG) which contained an Alpha rhythmic burst. This is a desirable approach since this type of bioelectric signal lends itself readily to ASCOF modeling and analysis. Alpha rhythm is a burst of around 10 Hz which occurs in a subject who is awake but has his eyes closed. The activity disappears when the person opens his eyes. It tends to appear in amplitude-modulated bursts, with the envelope rising to a maximum, and then
decaying towards zero.

Figure 4.1(a) shows an example of this type of activity which has been set to an arbitrary amplitude and zero time reference. The extrema are very evident as is the GCM characteristic wavefunction. There is only one GCM wavefunction element apparent, and its extrema are given by:

\[ E_j = [M_i, T_i] \]

or:

\[ M_i, T_i = 1.1, 0.20 = M_1, T_1 \]
\[ M_{i+1}, T_{i+1} = -1.2, 0.25 = M_2, T_2 \]
\[ M_{i+2}, T_{i+2} = 1.3, 0.30 = M_3, T_3 \]
\[ M_{i+3}, T_{i+3} = -1.1, 0.35 = M_4, T_4 \]

and, a GCM wavefunction exists, since:

\[ CM = (M_{i+1}, M_i+3) \land (M_{i+2}, M_i) \]

criteria is satisfied.

As an example, the ASCOF parameters can be computed as follows:

for S:

\[ \zeta = \frac{(M_1 + M_2)(M_3 + M_4)}{(M_2 + M_3)^2} = \frac{(1.1 + 1.2)(1.3 + 1.1)}{(1.2 + 1.3)^2} \]
\[ \zeta = 0.883 \]
\[ F[\zeta] = \left( \frac{-1}{2 \ln [2 \sqrt{\zeta} - 1]} \right)^{1/2} = 1.97 \]
\[ S = 2\pi (T_3 - T_2)F[\zeta] = 2\pi (0.3 - 0.25)(1.97) = 0.620 \]

for F:

\[ F = \frac{1}{2(T_3 - T_2)} = \frac{1}{2(0.3 - 0.25)} = 10.0 \]
Figure 4.1 ASCM Analysis of Filtered EEG Alpha Rhythm
for $C$:

$$\alpha = M_3 - M_1 = 1.3 - 1.1 = 0.2$$
$$\beta = M_2 - M_4 = 1.2 - 1.1 = 0.1$$

and:

$$\rho = \frac{\beta}{\alpha} \quad \text{if } \alpha > \beta$$
$$\rho = \frac{\alpha}{\beta} \quad \text{if } \alpha < \beta$$

therefore,

$$\rho = \frac{\beta}{\alpha} = \frac{0.10}{0.20} = 0.50$$
$$\mu = \frac{1}{1 + \rho} = \frac{1}{1 + 0.5} = 0.667$$
$$C = T_3 - \mu(T_3 - T_2) = 0.3 - 0.667(0.30 - 0.25) = 0.267$$

for $\varnothing$:

$$\varnothing = \pi\mu \quad \text{if } M_2 > 0$$
$$= \pi(\mu + 1) \quad \text{if } M_2 < 0$$

therefore, $\varnothing = \pi\mu = 0.667 = 2.095$

for $A$:

$$A = M_2 \exp\left[\left(\frac{\pi}{S}\right)(T_2 - C)\right]^2$$
$$= 1.2 \exp\left[\left(\frac{\pi}{0.620}\right)(0.250 - 0.267)\right]^2 = 1.22$$

alternatively,

$$A = M_2 \exp\left[\left(1 - \frac{\mu}{2F(\zeta)}\right)^2 = 1.2 \exp\left[\frac{(1 - 0.667)}{2(1.97)}\right]^2\right.$$
$$= 1.22$$

Thus, the calculations of the ASCOF parameter set based on the extrema data can be readily accomplished.

The time units are:

Time = seconds if $R = \text{Hz}$
Time = msec if $R = \text{KHz}$
Then, the parameters are:

\[
\begin{align*}
A &= 1.22 \\
S &= 0.620 \text{ seconds} \\
C &= 0.267 \text{ seconds} \\
\phi &= 2.095 \text{ radians} = 120 \text{ degrees} \\
F &= 10.0 \text{ Hz}
\end{align*}
\]

To test the adequacy of the ASCØF elements in representing the signal in Figure 4.1(a), it is required to utilize equation (2-9), i.e.,

\[
(4-3) \quad W_m(t) = A_m \exp\left(-\frac{\pi}{S_m} (t - C_m)^2\right) \cos[2\pi F_m (t - C_m) - \phi_m]
\]

to synthesis the original wavefunction. For the values listed above and for \(m = 1\), the wavefunction becomes:

\[
W_1(t) = 1.22 \exp\left(-\frac{\pi}{0.620} (t - 0.267)^2\right) \cos[2\pi 10.0 (t - 0.267) - 2.095]
\]

Algorithms were developed and programs written to implement the foregoing calculations for the ASCØF parameter set and for wavefunction synthesis. The source program and subroutines developed to implement the algorithms are listed in the Appendix. The source program flow chart is shown in Figure 4.2, the subroutine ASCØF flow chart in Figure 4.3 and the subroutine SYNTFB flow chart in Figure 4.4. The subroutine ASCØF solves for the ASCØF parameters based on equations (2-43) through (2-50). The data entered into the subroutine are the extrema satisfying equation (4-2). The outputs are the ASCØF elements and various computational quantities used in the computations for the
Figure 4.2 Source Program Flow Chart for ASC$^\Phi$F
Calculations and Wavefunction Synthesis
\[ A_{1A} = |A_1| A_{2A} = |A_2| A_{3A} = |A_3| A_{4A} = |A_4| \]
\[ T_B = (A_{1A} + A_{2A})(A_{3A} + A_{4A}), \quad D_T = T_3 - T_2 - (A_{2A} + A_{4A})^2 \]
\[ A_N = 2\sqrt{T_B - 1}, \quad F_{TB} = \sqrt{1 - T_B^2} \]
\[ T_B = 2\ln(A_N) \]

\[ S = 2\sqrt{D_T}(F_{TB}) \]
\[ F = 1/(2D_T) \]

\[ \alpha = A_{3A} - A_{1A} \]
\[ \beta = A_{2A} - A_{4A} \]

\[ \alpha < \beta ? \]

Yes:
\[ \rho = \frac{\alpha}{\beta} \]

No:
\[ \rho = \frac{1}{1 + \rho} \]

\[ \mu = \frac{1}{1 + \rho} \]

\[ C = T_3 - (U)(D_T) \]

\[ A_2 < 0 ? \]

Yes:
\[ \varphi = \mu U \]

No:
\[ \varphi = \mu U + \Pi \]

\[ \varphi = 57.29(\varphi), \quad T_{2C} = |(T_2 - C)|, \quad E_X = [(\Pi/S)(T_2C)]^2 \]

Output: TB, DT, AN, TBF, FTB, EX; \[ A = A_{2A}\exp(EX) \]

Figure 4.3 Flow Chart for Subroutine ASCSF
Figure 4.4 Flow Chart for Subroutine SYMTFB

\[ TC = \left( T - CT(I) \right) \]
\[ Wnt(I) = AT(I) \exp \left( \frac{\pi}{CT(I)} \right) \cos \left[ 2\pi \left( \frac{CT(I) \left( T - CT(I) \right)}{-\phi(T)} \right) \right] \]
elements. The subroutine SYNTFB utilizes equation (4-3) to produce a synthetic wavefunction based on the ASCOF parameter inputs.

Table 4.1 tabulates the various elements obtained by hand calculation and from the program ASCOF subroutine from the EEG sample:

Table 4.1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ASCOF Subroutine</th>
<th>Hand Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.209</td>
<td>1.22</td>
</tr>
<tr>
<td>S</td>
<td>0.620</td>
<td>0.620</td>
</tr>
<tr>
<td>C</td>
<td>0.267</td>
<td>0.267</td>
</tr>
<tr>
<td>ø(rad)</td>
<td>2.094</td>
<td>2.095</td>
</tr>
<tr>
<td>F</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>ø(deg)</td>
<td>119.98</td>
<td>120.0</td>
</tr>
<tr>
<td>ζ</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>T3-T2</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2√ζ - 1</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>2ln[2√ζ - 1]</td>
<td>-0.26</td>
<td>-0.256</td>
</tr>
<tr>
<td>F[ζ]</td>
<td>1.97</td>
<td>1.97</td>
</tr>
</tbody>
</table>

which indicates the excellent results.

Figure 4.1(b) shows the synthetic wavefunction derived from the ASCOF parameters and produced by the subroutine SYNTFB. It is shown in Figure 4.1(b) to compare
it against the original filtered EEG sample in Figure 4.1(a). This again indicates the excellent results obtainable by the method.

4.2 Frequency Domain Representation and the Fourier Transform with Spectral Densities

From equation (2-13), the Fourier transform of a GCM wavefunction is given by:

\[
W_m(\omega) = \frac{A_m S_m}{2\sqrt{\pi}} \exp \left[-\frac{(s_{Fm} (f-Fm))^2}{2} \exp[-j(\phi + 2\pi F_m C_m)]ight]
\]

and for the range: 
\[F_m - f_{\text{max}} \leq f \leq F_m + f_{\text{max}}\]

where: 
\[f_{\text{max}} = (0.75)/S_m\]

The imaginary term in equation (4-4) has the effect of translating the time domain representation by an amount:

\[\omega t = (\phi + 2\pi F_m C_m)\]

as is illustrated in Figure 4.5.

This can be derived as follows. For the time shifting property of the Fourier transform, we have:

\[a(t) \leftrightarrow A(\omega)\]

then: 
\[a(t - t_0) \leftrightarrow A(\omega) e^{-j\omega t_0}\]

This says that a function shifted in the time domain by an

Figure 4.5. Shifting a Wavefunction in the Time Domain
amount $t_0$ has its magnitude spectrum $|A(\omega)|$ unaffected, but the phase spectrum is changed by an amount $-\omega t_0$.

Similarly, we have:

$$w[\omega(t - t_o) + \theta] \leftrightarrow W(\omega) \exp[-j(\theta + 2\pi F_m C_m)]$$

since:

$$F[w[\omega(t - t_o) + \theta]] = \int_{-\infty}^{\infty} w[\omega(t - t_o) + \theta] e^{-j\omega t} dt$$

let:

$$\tau = \omega(t - t_o) + \theta$$

then:

$$\omega t = \tau + \omega t_o - \theta$$

$$\frac{d\omega t}{d\tau} = 1$$

hence:

$$F[w[\omega(t - t_o) + \theta]] = \int_{-\infty}^{\infty} w(\tau) e^{-j\tau} e^{-j(\omega t_o - \theta)}$$

$$= W(\omega) \exp[-j(\omega t_o + \theta)]$$

where:

$$t_o = C_m$$

$$W = 2\pi F_m$$

A normalized plot of $|W(\omega)|$ is shown in Figure 4.6. In this case for each value of $k$ indicated we have:

$$f = F_m \pm k/S_m$$

with $k$ set of 0.75 maximum per equation (4-4). Using Figure 4.6, the frequency spectrum of any wavefunction for a particular ASCOF parameter set can be found. The values for $|W_m(\omega)|$ for the values of $k$ are tabulated in Table 4.2, with the frequency spread given by equation (4-5).

To find the Fourier transform of the ASCOF parameter set previously determined in the EEG wavefunction analysis, we have:
\[
F_m = F_1 = 10.0 \\
S_m = S_1 = 0.620 \\
A_m = A_1 = 1.209
\]

Figure 4.6

Normalized Fourier Transform of GCM Wavefunction
Table 4.2

Normalized Parameters for Use in Determining the Fourier Transform Distribution for an Arbitrary ASCOF Parameter Set

| k    | \( \frac{|W_m(f)|}{A_m S_m} \cdot 2 \sqrt{\pi} | W(f)| \leq 0.121 |
|------|------------------------------------------|-------------------|
| 0.75 | 0.569                                    | 0.212             |
| 0.70 | 0.613                                    |                   |
| 0.60 | 0.697                                    |                   |
| 0.50 | 0.778                                    |                   |
| 0.40 | 0.852                                    |                   |
| 0.30 | 0.913                                    |                   |
| 0.20 | 0.961                                    |                   |
| 0.10 | 0.990                                    |                   |
| 0    | 1.000                                    |                   |

Then, using Table 4.2 and equation (4-5), the resulting Fourier transform spectrum for the EEG ASCOF parameter set is shown in Figure 4.7. The range of frequencies for which it is defined are:

\[ 8.79 \text{ Hz} \leq f \leq 11.21 \text{ Hz} \]

The amplitude of \( W(f) \) range from:

\[ 0.212 \leq |W(f)| \leq 0.121 \]

Thus, it can be seen that the ASCOF analysis method provides a ready transformation from the frequency domain to the time domain and vice versa. This indicates one of the powerful attributes of the method.
4.3 Discrete Convolution
Filtering and Extrema

Discrete filtering and extrema detection/calculation was initially performed utilizing a PQRS\textsuperscript{T} interval sample of the Electrocardiogram (ECG). The ECG sample is shown in Figure 4.8.\textsuperscript{3} It is an "average" ECG PQRS\textsuperscript{T} interval with the following times for the significant intervals:
Figure 4.8
Sample ECG PQRS Interval Used in
Discrete Filtering and Extrema
Analysis

\[ \tau_{PR} = 0.16 \text{ sec} \quad f_{PR} = 6 \text{ Hz} \]
\[ \tau_{QRS} = 0.08 \text{ sec} \quad f_{QRS} = 12 \text{ Hz} \]
\[ \tau_{QT} = 0.36 \text{ sec} \quad f_{QT} = 3 \text{ Hz} \]

Thus, one would expect that most of the energy out of the
bandpass filters would be concentrated in these ranges.

The signal was filtered using the discrete convolution
equation from equation (2-26) given by:

\[
R_j(t) = \sum_{k=1}^{P} \sum_{i=(-Q+1)/2}^{(Q-1)/2} H_j(i) S(k-i)
\]
where:

\[ H_j(i) \text{ is given by equation (3-27)} \]

\[ Q = \# \text{ discrete points in } H_j(i) = \text{ odd } \# \]

\[ P = \# \text{ discrete points in } S(kT) \]

\[ k = 1, 2, 3, \ldots, P \]

\[ j = n, n-1, n-2, \ldots, 3, 2, 1 \]

\[ n = \# \text{ of contiguous (octave) partitions} \]

\[ T = 1/R = \text{ sampling interval (time)} \]

\[ R = \text{ sampling rate} \]

\[ i = (-Q+1)/2, \ldots, 0, \ldots, (+Q+1)/2 \]

Equation (4-6) may be written:

\[ (4-7) \quad r(t) = h(t) * s(t) = s(t) * h(t) \]

Since \( H_j(i) \) is symmetrical about the zero time origin:

\[ (4-8) \quad h(t) = h(-1) \]

\[ H_j(i) = H_j(-i) \]

equation (4-6) may be mechanized without time reversing \( s(t) \). Thus, the discrete convolution process can take place as is shown in Figure (4-9).

Equation (4-6) was initially mechanized by forming the matrix:

\[ A(m,n) \]

with the moving average given by:

\[ (4-7) \quad R(t) \bigg|_{t=mT} = \sum_{n=1}^{P} A(m,n) \]

where:

\[ m = 1, 2, 3, \ldots, P+Q-1 \]
Figure 4.9

Discrete Convolution Mechanization Due to Symmetry of $H_j(i)$

with the individual elements in the matrix given by:

(4-8) \[ a(m,n) = S(P+1-n)H(m+1-n) \]

From equations (2-36) and (2-37), the expressions for the extrema become:

(4-9) \[
M_i = r_k - \frac{(r_{k+1} - r_{k-1})^2}{8(r_{k+1} - 2r_k + r_{k-1})}
\]

\[
T_i = k \left[ \frac{(r_{k+1} - r_{k-1})}{2(r_{k+1} - 2r_k + r_{k-1})} \right] T
\]

with:

Maxima $r_k > r_{k+1}$

Minima $r_k < r_{k+1}$

Algorithms were developed and programs written to implement the foregoing definitions for discrete convolu-
tion filtering and extrema analysis. The source program and subroutines used in this section of the analysis are contained in the Appendix. Figure 4.10 shows the flow chart for the source program.

Based on the frequency components of the ECG sample as referenced on Page 74, it was decided to select the following octave bands with which to implement the discrete convolution filtering scheme. Thus, we have:

\[
H_1(f) = \begin{cases} 
1.0 & 1 < f < 2 \text{ Hz} \\
0 & \text{elsewhere}
\end{cases}
\]

\[
H_2(f) = \begin{cases} 
1.0 & 2 < f < 4 \text{ Hz} \\
0 & \text{elsewhere}
\end{cases}
\]

\[
H_3(f) = \begin{cases} 
1.0 & 4 < f < 8 \text{ Hz} \\
0 & \text{elsewhere}
\end{cases}
\]

\[
H_4(f) = \begin{cases} 
1.0 & 8 < f < 16 \text{ Hz} \\
0 & \text{elsewhere}
\end{cases}
\]

\[
H_5(f) = \begin{cases} 
1.0 & 16 < f < 32 \text{ Hz} \\
0 & \text{elsewhere}
\end{cases}
\]

with which we have \( n = 5 \).

Additionally, another set of non-octave filter bands were tested, which were from 0 to 6, 6 to 12, 12 to 18, 18 to 24 and 24 to 30 Hz. These are essentially harmonic contiguous filter bands.

Figure 4.11 is the flow chart for the subroutine CONFIL which mechanizes equation (4-6) through the use of equations (4-7) and (4-8). The sampling rate used in
Figure 4-10 Source Program Flow Chart for Discrete Convolution Filtering and Extrema Analysis
Figure 4.10 (continued)
Figure 4.10 (continued)
Figure 4.11 Flow Chart for the Subroutine CONFIL
Figure 4.11 (continued)
this particular test was 125 samples/second which puts \( T \) at 0.008 seconds. Figure 4.12 is the flow chart for the subroutine EXTREMA which mechanizes equation (4-9).

For the octave filtering on Page 77, the following parameters were inputed:

\[
\begin{align*}
P &= 70 & f_{\text{max}} &= 32 \text{ Hz} \\
Q &= 69 & f_{\text{min}} &= 1 \text{ Hz} \\
n &= 5 \\
R &= 125
\end{align*}
\]

The sampled data was taken from Figure 4.8. For harmonic filtering on Page 77, the following parameters were inputed:

\[
\begin{align*}
P &= 70 & f_{\text{max}} &= 30 \text{ Hz} \\
Q &= 61 & f_{\text{min}} &= 0 \text{ Hz} \\
n &= 5 \\
R &= 125
\end{align*}
\]

with the sampled data also taken from Figure 4.8. In both cases this sets:

\[
T = 1/R = 0.008 \text{ seconds}
\]

Figure 4.13 shows the ECG PQRS interval filtered into five contiguous octave frequency partitions. The GCM wavefunctions are very apparent in the filtered specimens. Notice also the relative energy of each of the bandpasses. The high energy components are in the 4-16 Hz range, which agrees with the original prediction made on Page 74. There is also some strong energy around 2-4 Hz, which agrees with predictions. The extrema characteristics are also apparent.
Figure 4.12 Flow Chart for the Subroutine EXTREMA
Figure 4.13 Five Octave Frequency Partitions of a PQRST Interval of the Electrocardiogram
Since the filters are contiguous, we can use equation (3-9) to reconstruct, or synthesize the input signal, i.e.,

\[ s(t) = \sum_{j=1}^{5} r_j(t) \]

This is shown in Figure 4.14 below. Compare this to Figure 4.8 of the original signal. They are very close indeed. The differences due to machine truncation effects and the fact that higher frequencies were not removed prior to octave filtering at \( f_{\text{max}} \) in order to avoid aliasing. Figure 4.14 is produced by summing the filter outputs.
shown in Figure 4.13.

Figure 4.15 shows the same ECG interval filtered into harmonic partitions. Notice the distortion of the wavefunctions which agrees with Carey's results on human speech. However, the major energy components are contained in the proper frequency bands.

4.3.1 Extrema Algorithm and Detection

The extrema for the filtered ECG sample interval were detected and calculated utilizing equation (4-9) and implemented by the algorithm shown in Figure 4.12. Table 4.3 summarizes the extrema detected and computed by these algorithms for the octave filtered ECG data shown in Figure 4.13. To illustrate, we compute the 3rd extrema in the 1-2 Hz frequency partition. In this case:

$$E_3 = [M_3, T_3]$$

$$r_k = r_{48} = 4.030$$

$$r_{k-1} = r_{47} = 4.029$$

$$r_{k+1} = r_{49} = 4.004$$

$$T = 1/R = 1/125 = 0.008 \text{ seconds}$$

This defines an extrema maxima, since $r_k > r_{k+1}$. Then:

$$M_3 = r_{48} - \frac{(r_{49} - r_{47})^2}{8(r_{49} - 2r_{48} + r_{47})} = 4.030 - \frac{(4.004 - 4.029)^2}{8(4.004 - 2(4.030) + 4.029)}$$

$$= 4.030 + \frac{(0.025)^2}{8(0.027)} = 4.030 + 0.004 = 4.034$$
Figure 4.15 Five Harmonic Frequency Partition of a PQRSST Interval of the Electrocardiogram
Table 4.3
Extrema Data from Octave Filtered ECG PQ RST Interval

<table>
<thead>
<tr>
<th></th>
<th>1 - 2 Hz M</th>
<th></th>
<th>8 - 16 Hz M</th>
<th></th>
<th>16 - 32 Hz M</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>M</td>
<td>T</td>
<td>i</td>
<td>M</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>1.588</td>
<td>.048</td>
<td>1</td>
<td>-0.814</td>
<td>.021</td>
</tr>
<tr>
<td>2</td>
<td>-1.031</td>
<td>.107</td>
<td>2</td>
<td>-0.894</td>
<td>.068</td>
</tr>
<tr>
<td>3</td>
<td>4.033</td>
<td>.389</td>
<td>3</td>
<td>-0.688</td>
<td>.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>3.652</td>
<td>.146</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>-4.055</td>
<td>.189</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>8.615</td>
<td>.271</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>-16.275</td>
<td>.313</td>
</tr>
<tr>
<td>1</td>
<td>3.576</td>
<td>.085</td>
<td>8</td>
<td>19.733</td>
<td>.355</td>
</tr>
<tr>
<td>2</td>
<td>-8.342</td>
<td>.208</td>
<td>9</td>
<td>-16.062</td>
<td>.397</td>
</tr>
<tr>
<td>3</td>
<td>7.389</td>
<td>.361</td>
<td>10</td>
<td>8.391</td>
<td>.439</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>11</td>
<td>-1.013</td>
<td>.477</td>
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<td></td>
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<td>.496</td>
</tr>
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<td></td>
<td>13</td>
<td>-2.846</td>
<td>.528</td>
</tr>
<tr>
<td>2</td>
<td>-4.055</td>
<td>.189</td>
<td>14</td>
<td>-16.275</td>
<td>.313</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15</td>
<td>-19.141</td>
<td>.356</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td>-1.200</td>
<td>.516</td>
</tr>
</tbody>
</table>


\[ T_3 = \left[ 48 - \frac{(r_{49} - r_{47})}{2(r_{49} - 2r_{48} + r_{47})} \right] T = \left[ 48 + \frac{(0.025)}{2(0.027)} \right] T = 48.463 T \]

\[ = 48.463(0.008) = 0.388 \text{ seconds} \]

Thus,

\[ E_3 = [4.034, 0.388] \]

which agrees with Table 4.3.

Figure 4.16 shows a plot of these extrema for the
Figure 4.16 Extrema Data of ECG Sample Interval
various octave bands. Notice the alternating signs of the extrema and the approximately equal time interval between the extrema, which confirms equations (2-38) through (2-40).

The time for a particular interval is determined from:

\[(4-10) \quad t_{\text{max}} = \frac{P}{R}\]

and, from the sampling theorem:

\[(4-11) \quad R_{\text{min}} = 2f_{\text{max}}\]

Then, for the case considered for which:

\[P = 70\]
\[R = 125\]

we have:

\[t_{\text{max}} = 0.560 \text{ seconds}\]
\[f_{\text{max}} = 62.5 \text{ Hz}\]
Chapter 5

AUTOMATIC ASCOF WAVEFUNCTION
ANALYSIS OF BIOELECTRIC SIGNALS

Once the individual ASCOF algorithms were tested on bioelectric signals, automatic wavefunction analysis was performed using a total integrated analysis system. This was implemented, as were the individual algorithms, on the Control Data Corporation system. The system is shown in Figure (5.1). The procedure implemented was that shown in Figure (2.2) and outlined on page 22 with modifications required to perform analysis on bioelectric signals in a simplified manner.

5:1 The Bioelectric ASCOF Analysis Procedure

The procedure implemented on the CDC 3170 system for the automatic wavefunction analysis of selected bioelectric signals is that shown in Figure (5.2). The source program algorithm for the procedure is shown in Figure (5.3), with the listing contained in the appendix. Table (5.1) shows the various subroutines used and their functions. The subroutines BICFL, BFLTR, BEXTR, PCALC, and GCMSY are those originally developed and utilized by Carey in the analysis of human speech; and, modified as required for the implementation of the bioelectric analysis procedure. The subroutine listings are contained in the appendix. Table 92
Figure 5.1 CDC 3170 System
Figure 5.2 Procedure for Automatic ASCSF wavefunction Analysis of Bioelectric Signals
(5.2) tabulates the various relevant parameters used in the program and their particular functions.

The basis for implementing the automatic analysis procedure is the mechanization of equations (2-30) through (2-35) and (3-33).

Table 5.1
Subroutines Used in Automatic Wavefunction Analysis Procedure

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLE</td>
<td>Samples $s(t)$ at $R = 4f_{\text{max}}$; sets value of $P$</td>
</tr>
<tr>
<td>BICFL</td>
<td>Removes end truncation by windowing; performs low pass filtering</td>
</tr>
<tr>
<td>BFLTR</td>
<td>Performs octave bandpass filtering; performs data list subtraction and decimation</td>
</tr>
<tr>
<td>BEXTR</td>
<td>Locates and computes extrema data</td>
</tr>
<tr>
<td>PCALC</td>
<td>Computes ASCØF parameters; performs residue subtraction; sets wavefunction count</td>
</tr>
<tr>
<td>GCMSY</td>
<td>Synthesizes original waveform from ASCØF list</td>
</tr>
</tbody>
</table>
### Table 5.2
Parameters Used in Wavefunction Analysis Program

<table>
<thead>
<tr>
<th>Name</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSK</td>
<td>Sampling rate, R</td>
</tr>
<tr>
<td>M</td>
<td>Original signal data list length</td>
</tr>
</tbody>
</table>
| X      | Array for: a) sampled signal data, $s_D(t)$  
|        | b) extrema list, $E_i(t)$  
|        | c) Synthesized signal, $\hat{s}(t)$ |
| XTAB   | Array for original signal data dependent variable, $s(t)$ |
| YTAB   | Array for original signal data independent variable, $t$ |
| LD     | Sampled signal data list length, $P$ |
| NFB    | # Octave partitions, $n$ |
| TMAX   | Time length of original signal data, $s(t)$ |
| Y      | Array for: a) filtered data, $r_j(t)$  
|        | b) ASCOF parameters, $(ASCOF)_m$ and wavefunction count, $(M_n)_m$ |
| F      | Arrays for storing the Lo Pass and Bandpass MC kernel coefficients, $H_j(t)$ |

This is one of the most important advantages of the method, *i.e.*, its simplicity. The only real basic requirement is to specify the frequency range of interest, and then sample at 4 times the maximum frequency of interest.

In this chapter are presented the results of automatic wavefunction analysis on a selected sample of bioelectric signals utilizing the procedure previously discussed and outlined by the algorithm of Figure (5.2)
5.2 **Sampling Requirements**

To meet the criteria established for ASCOF analysis and MC coefficient octave filtering, i.e.,

\[
(5-1) \quad R = 4f_{\text{max}}
\]

it was required to implement a sampling algorithm which would provide this capability.

This was accomplished by utilizing a Lagrangian 2nd degree interpolating polynomial of the form:

\[
(5-2) \quad s_2(t) = a_0 + a_1 + a_2 t^2
\]

to fit the sampling interval function:

\[
(5-3) \quad s(t) = F(T) = F(1/R)
\]

to a Lagrangian polynomial based on existing data points.

The interpolating scheme is shown in Figure (5.4a).

The interpolating polynomial is given by:

\[
(5-4) \quad s(t) = \sum_{i=0}^{n} L_i(t)s_i(t)
\]
Figure 5.3 Source Program Flow Chart for Automatic Bioelectric Wavefunction Analysis
\[ \text{Call BFLTR} \]
\[ \text{Call BEXTR} \]
\[ \text{Call PCALC} \]
\[ \text{Nf=Nfb?} \]
\[ \text{Nf=Nf+1} \]
\[ \text{Nf=Nfb?} \]
\[ \text{Nf=Nf+1} \]
\[ \text{Output Y(\(j_p\)), Y(\(k-1\)), Y(\(k-2\)), Y(\(k-3\)).} \]
\[ k=j_p-j \]
\[ j=j_{\text{max}}? \]
\[ j=j+5 \]
\[ N1=N_{\text{f}}b? \]
\[ N1=N1+1 \]

Figure 5.3 (continued)
Figure 5.3 (continued)
\[ S(t) = \sum_{i=0}^{n} (t - t_i) \frac{t_j - t_i}{t_j - t_i} \]

In order to eliminate \( n^2 \) multiplications and \( n^2 \) subtractions
by adding \( n+1 \) divisions, equation (5.5) can be written as:

\[
L_i(t) = \frac{K}{(t - t_i)} \prod_{j=0}^{n} \frac{(t_i - t_j)}{(t - t_j)}
\]

where:

\[
K = \prod_{j=0}^{n} (t - t_j)
\]

Implementation of the Lagrangian interpolation algorithm to provide for sampling of the bioelectric signal resulted in excellent sampling characteristics. An example is shown in Figure (5.4b), which shows a sampled interval of the ECG P wave. In each case, the absolute time is given by:

\[
t = t + T = t + \frac{1}{R}
\]

which gives the required sampling rate. Figure (5.5) shows the interpolating algorithm used to sample the data.

5.3 Preprocessing and Extrema

The fast filtering algorithms required removal of unwanted high frequency components for the bioelectric signal data in order to avoid aliasing problems. This involves two tasks:
\[ L_1 = \frac{K}{(t - t_1)} \frac{2}{\prod_{j=0}^{2} (t_1 - t_j)} \]

\[ s = \sum_{i=0}^{2} L_1 s_1 \]

\[ k = k + 1 \]
\[ t = t + dt \]

Figure 5.5 Data Sampling Algorithm
1. Removal of high frequency components due to truncating a data sample.

2. Removal of high frequency components by low pass filtering set at $f_{\text{max}}$

The first part is accomplished by windowing the data by an appropriate windowing filter. An example of this type of window is the Hanning lag window shown in Figure (5.6a). Extending this, we have:

\[
(5-8) \quad h_H(t) = \frac{1}{2} \left[ 1 + \cos \left( \frac{\pi t}{t_{\text{max}}} \right) \right]
\]

letting:

\[
\tau_{\text{max}} = \frac{PT}{2}, \tau = kT
\]

this becomes:

\[
(5-9) \quad h_H(t) \bigg|_{\tau=kt} = \frac{1}{2} \left[ 1 + \cos 2\pi \left( \frac{k}{P} \right) \right] \quad kT < \frac{PT}{2}
\]

\[
t=kt \quad \quad \quad \quad \quad \quad \quad k < \frac{P}{2}
\]

This is shown in Figure (5.6a)

The speech analysis algorithm used a window shown in Figure (5.6b) and given by:

\[
(5-9) \quad h_H(t) \bigg|_{\tau=kt} = \frac{1}{2} \left[ 1 + \cos 2\pi \left( \frac{10k}{P} \right) \right] \quad k < \frac{P}{20}
\]

This showed to be adequate in the particular samples of bioelectric signals analyzed.

Figure (5.7a) shows the ECG sample after window
multiplication to correct for truncation end effects. Also shown is the window applied at the end of the truncated data. Figure (5.7b) shows the ECG after low pass filtering where:

\[ f_{\text{low pass}} = 128 \text{ Hz} \]

This indicated the adequacy of the low pass filtering process by the minimum amount of frequency distortion in the low passed signal. It also indicated that the ECG has very low frequency energies above 128 Hz.

Figure (5.8) shows six octave partitions for filtered data of the ECG were:
\[ R = 250 \]
\[ f_{\text{max}} = 62.5 \text{ Hz} \]
\[ n = 6 \]

This sets the lower frequency, from equation (3-20) to:
\[ f_{\text{min}} = 0.965625 \text{ Hz} \]

Notice the GCM wavefunctions which are apparent.

---

Avoiding aliasing by removing high frequency components from the ECG

This establishes an important property of the fast octave filtering algorithm. This is:
Figure 5.8 Fast Filtering of the ECG at 
$R = 250$
Figure 5.8 (continued)
Equation (5-10) simply states that given a certain \( R \) and \( n \), then \( f_{\text{max}} \) and \( f_{\text{min}} \) are set. Conversely, given a certain \( f_{\text{max}} \) and \( f_{\text{min}} \), then \( R \) and \( n \) are set.

The experimental data results showed that the extrema for the filtered data were well defined and accurate. Figure (5.8) also verifies equation (2-33). If, initially

\[
(5-11) \quad t_{\text{max}} = \text{time length of truncated signal}
\]

then,

\[
P = \frac{t_{\text{max}}}{T} = t_{\text{max}} R
\]

hence,

\[
P_j = \frac{P}{2^{n-j}} \quad j = n, n-1, n-2, \ldots, 1
\]

This is immediately verified from Figure (5.8), since:

\[
P_1 = 5
\]
\[
P_2 = 10
\]
\[
P_3 = 20
\]
\[
P_4 = 40
\]

etc.

which shows that the sampling rate is being halved for each successive octave partition.

Another important consideration in his portion of the experiment was the determination of the data length, \( t_{\text{max}} \), required initially in order to detect a GCM wavefunction at the lowest frequency, \( f_{\text{min}} \). Sampling at a rate of 4
times the frequency of interest (maximum) guarantees two
data points per half cycle of data. This is shown in Fig-
ure (5.9). However, detection of extrema requires a mini-
imum of two data points per half cycle. Also, the existence
of a GCM wavefunction requires a minimum of four extrema.
This requires:

$$k(\text{CCM}) = 2 \times 4 = 8$$
data points contained in two cycles of filtered data to de-
tect the existence of a GCM wavefunction.

![Figure 5.9](image)

**Figure 5.9**
The Minimum Time to Detect a GCM Wavefunction

This consideration is important due to the nature
of the low frequency bioelectric signals. Thus, if \( f_{\text{min}} \) is
the lowest frequency of interest, then the minimum time
length of the data trace becomes:

$$t_{\text{min}} = 2 \frac{1}{f_{\text{min}}} \quad (5-12)$$
in order to detect a GCM wavefunction at that frequency.
For example, from Figure (3.3), the length of the trace to
detect a GCM wavefunction at 0.1 Hz for the Vibrocardio-
gram is:

\[ t_{\text{min}} = 2 \left[ \frac{1}{0.1} \right] = 20 \text{ seconds of traces} \]

Figure (5.10) illustrates the low pass filtering, subtraction, and decimation process required since:

\[ f_{j1} = f_{j2} = \frac{R}{4} \]

for a sample bioelectric frequency range of from 2-16 Hz.
Figure 5.10 Computer printout of data for ECG analysis
In this case $R = 64$ and we have the following breakdown:

<table>
<thead>
<tr>
<th>$j$</th>
<th>$f_{j1} = f_{j2} = R/4$</th>
<th>$R_j$</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>16</td>
<td>64</td>
<td>8-16</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>32</td>
<td>4-8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>16</td>
<td>2-4</td>
</tr>
</tbody>
</table>

Figure (5.10) shows a computer printout of the filtered data and extrema for the 2-4 Hz and 4-8 Hz bandpass partitions of the ECG. In Figure (5.11) the filtered data and extrema are plotted. Notice the good definition of the extrema and the GCM wavefunctions in the filtered data.

![Filtered Data and Extrema](image)

Figure 5.11
Filtered Data and Extrema for the 2-4 Hz and 4-8 Hz Bandpass Partitions Of the ECG

The extrema array of Figure 5.10 contains the data in the following sequence:
where:

\[ T_i = t_i - t_{i-1} \]

5.4 ASCŒF Parameter Sets For Bioelectric Signals

This section presents the results of automatic ASCŒF analysis and parameter calculations for various bioelectric signals. The signals included various time lengths of the ECG in Figure (4.8), the Fetal Electrocardiogram (FE CG) shown in Figure (5.12a),\(^{19}\) and the Femoral Artery Pulse (FAP) shown in Figure (5.12b).\(^{20}\)

5.4.1 Parameter List Organization

The CDC 3170 ASCŒF parameter list computer outputs are organized essentially the same as for speech analysis. This organization is shown below:
Figure 5-12 Bioelectric Signals for Automatic ASCOF Analysis and Parameter Representation
(5-14) \[
\begin{array}{cccccc}
M_j & A & S & C & \emptyset & F & j \\
xxxx & xxxx & xxxx & xxxx & xxxx & xxxx & n
\end{array}
\]

\[
f_{\text{max}} < f < \frac{f_{\text{max}}}{n-j+1} \quad \frac{f_{\text{max}}}{n-j}
\]

where equation (5-14) defines the octave partition for that group of ASC\(\text{OF}\) parameters, and where:

(5-15) \[M_j = \text{number of wavefunctions defined by the (ASC\(\text{OF}\)) sets for that particular octave partition.}\]

5.4.2 ASC\(\text{OF}\) Lists for Bioelectric Signals

Figure (5.13) presents the computer output showing the parameters computed from equations (2-43) and (2-47) and their (following) ASC\(\text{OF}\) parameters for:

- ECG: 0.5 to 1 Hz
  - 1 to 2 Hz
- FECG: 4 to 8 Hz

and:

\[f_{\text{max}} = 256 \, \text{Hz}\]
\[f_{\text{min}} = 0.5 \, \text{Hz}\]

where:

\[Zeta = \zeta \quad Fzeta = F[\zeta]\]
\[Alpha = \alpha \quad Beta = \beta\]
\[Rho = \rho \quad Mu = \mu\]
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<th>Pzeta</th>
<th>Alpha</th>
<th>Beta</th>
<th>Rho</th>
<th>Mu</th>
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<td>0.01</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.39</td>
<td>0.60</td>
<td>0.01</td>
<td>0.02</td>
<td>0.38</td>
<td>0.73</td>
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<tr>
<td></td>
<td></td>
<td>1.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.01</td>
<td>0.00</td>
<td>0.40</td>
<td>0.29</td>
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<td>7.22</td>
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<tr>
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<td>0.65</td>
<td>0.27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.72</td>
<td>1.18</td>
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<td>0.00</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>0.08</td>
<td>0.30</td>
<td>0.74</td>
<td></td>
<td>6.48</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.13  ASCGF Parameters and Computational Variables for the ECG and FECG
In the following Figures (5.14) through (5.19) are presented computer printouts of ASCØF parameter lists which resulted from analysis of bioelectric signals. Table (5.3) summarizes the various parameters involved with each parameter list associated with the various figures.

The list indicate exactly what was expected in that the higher energy components, as represented by the magnitude of A and the AS product, are contained at the lower frequencies of the frequency spread. Also notice that each value of F is uniquely contained in that octave segment, which is in agreement with equation (5-14). Also evident is that the number of wavefunctions per octave partition decreases as the bandwidth of the octave partition is decreased. The reason for this is that the total time needed to define a wavefunction increases with each decreasing octave partition, which is agreement with the discussion surrounding equation (5-12). In addition, the spread S is higher with increasing amplitude of A, which agrees with equations (2-43) and (2-50)-(2-51).
<table>
<thead>
<tr>
<th>A</th>
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<th>C</th>
<th>ϕ</th>
<th>F</th>
</tr>
</thead>
<tbody>
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<td>91.3572</td>
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<tr>
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<td>97.6830</td>
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<tr>
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<tr>
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<tr>
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</tbody>
</table>

(a) ECG: 1-128 Hz, $M_{min} = 0.0002$

(b) ECG: 0.965625-62.5 Hz, $M_{min} = 0.02$

Figure 5.14 ASCϕF Parameter Sets
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>G</th>
<th>F</th>
</tr>
</thead>
<tbody>
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<td>0.0194</td>
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</table>

(a) ECG: 965625-62.5 Hz, M_min = 0.00002

<table>
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<th>G</th>
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</thead>
<tbody>
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<tr>
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(b) FAP: 0.5-256 Hz, M_min = 0.001

Figure 5.15 ASCQF Parameter Sets
<table>
<thead>
<tr>
<th>A</th>
<th>S</th>
<th>NEXT</th>
<th>C</th>
<th>G</th>
<th>F</th>
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Figure 5.16 ASC/OF Parameter Sets for ECG, 1-256 Hz, $M_{min} = 0.00002$
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(a) ECG: 1-256 Hz, $M_{\text{min}} = 0.001$

Figure 5.17 ASCOF Parameter Sets
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(b) ECG: 0.965625–31.25 Hz, $M_{\text{min}} = 0.02$

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Figure 5.18 ASCG@ Parameter Sets for FECG, .5-256 Hz, $M_{\text{min}} = 0.001$
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Figure 5.19 ASCØF Parameter Sets for EGG, 5-256 Hz, 
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<td>0.2593</td>
<td>0.6147</td>
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<td>8.9274</td>
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<td>0.2817</td>
<td>0.8369</td>
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<tr>
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<td>0.1856</td>
<td>19.7895</td>
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<td>0.5527</td>
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<td>53.9877</td>
<td>5.9739</td>
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<tr>
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<td>1.4488</td>
<td>59.9710</td>
<td>6.2393</td>
</tr>
<tr>
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<td>0.3719</td>
<td>2.0888</td>
<td>62.0136</td>
<td>5.9764</td>
</tr>
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<td>0.7223</td>
<td>0.3021</td>
<td>226.4038</td>
<td>3.1973</td>
</tr>
<tr>
<td>0.1129</td>
<td>1.1826</td>
<td>1.1869</td>
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<td>3.0355</td>
</tr>
<tr>
<td>0.0790</td>
<td>0.9702</td>
<td>0.6738</td>
<td>96.4774</td>
<td>1.5354</td>
</tr>
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<td>0.0122</td>
<td>1.2664</td>
<td>1.3060</td>
<td>130.6509</td>
<td>0.5962</td>
</tr>
</tbody>
</table>

Figure 5.19 (continued)
Table 5.3

Experimental Criteria for Automatic ASCOF Analysis of Bioelectric Signals

<table>
<thead>
<tr>
<th>Figure</th>
<th>Signal</th>
<th>n</th>
<th>f\text{max}</th>
<th>f\text{min}</th>
<th>t\text{max}</th>
<th>R/P</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.14a</td>
<td>ECG</td>
<td>7</td>
<td>128</td>
<td>1</td>
<td>.640</td>
<td>512/325</td>
<td>0.00002</td>
</tr>
<tr>
<td>5.14b</td>
<td>ECG</td>
<td>6</td>
<td>62.5</td>
<td>.965625</td>
<td>.640</td>
<td>250/160</td>
<td>0.02</td>
</tr>
<tr>
<td>5.15a</td>
<td>ECG</td>
<td>6</td>
<td>62.5</td>
<td>.965625</td>
<td>.640</td>
<td>250/160</td>
<td>0.00002</td>
</tr>
<tr>
<td>5.15b</td>
<td>FAP</td>
<td>9</td>
<td>256</td>
<td>0.5</td>
<td>.900</td>
<td>1024/921</td>
<td>0.001</td>
</tr>
<tr>
<td>5.16</td>
<td>ECG</td>
<td>8</td>
<td>256</td>
<td>1</td>
<td>.1280</td>
<td>1024/1306</td>
<td>0.00002</td>
</tr>
<tr>
<td>5.17a</td>
<td>ECG</td>
<td>8</td>
<td>256</td>
<td>1</td>
<td>.2560</td>
<td>1024/2617</td>
<td>0.001</td>
</tr>
<tr>
<td>5.17b</td>
<td>ECG</td>
<td>5</td>
<td>31.25</td>
<td>.965625</td>
<td>.1280</td>
<td>125/320</td>
<td>0.02</td>
</tr>
<tr>
<td>5.18</td>
<td>FECG</td>
<td>9</td>
<td>256</td>
<td>0.5</td>
<td>1.1500</td>
<td>1024/1182</td>
<td>0.001</td>
</tr>
<tr>
<td>5.19</td>
<td>ECG</td>
<td>9</td>
<td>256</td>
<td>0.5</td>
<td>.2560</td>
<td>1024/2617</td>
<td>0.001</td>
</tr>
</tbody>
</table>

5.4.3 Data Compression

Examination of the ASCOF parameter lists indicated that the higher amplitude wave functions were condensed in the lower frequencies, which is what might be expected. Thus, if some of these components were removed, it would compress the size of the ASCOF list but at the same time still retain adequate descriptors of the signal.

To eliminate wavefunctions of amplitudes less than
If Extrema $M_i \leq M_{\text{min}}$

then: $M_i = M_{i+1}$

and: $T_i = T_{i+1}$

where: $M_{\text{min}} = \text{threshold}$

This essentially eliminates wavefunctions with:

$$A \leq M_{\text{min}}$$

To illustrate, compare the number of parameter sets in Figure (5.14b), with an $M_{\text{min}}$ of 0.02, with Figure (5.15a) having an $M_{\text{min}}$ of 0.00002.

To test the validity of this approach, we note that the QRS complex in the ECG is the interval with the highest frequency content.\(^{21}\) In this case, amplitude wavefunction compression would have more, if any, effect on the QRS complex. Figure (5.20) shows the synthesized QRS complexes with $M_{\text{min}}$s of 0.00002 and 0.02. There is basically no distinguishable difference between the two complexes. Further investigation showed that an $M_{\text{min}}$ of 0.001 provides adequate amplitude data compression for this class of bioelectric signal.

5.4.4 Bioelectric Event Recognition

Often it is desirable to detect the occurrence of certain events in bioelectric signals. The ASCOF parameters provide a useful tool towards this goal. As an example,
consider the FECG shown in Figure (5.12a) and its corresponding ASCOF parameter list shown in Figure (5.18). The highest power density of the R-wave of the mother (M) lies between 10 and 30 Hz; additionally, that of the child (F) is in the range 15-40 Hz.\textsuperscript{21}

From Figure (4.6) it is evident that when:

\begin{equation}
(5.18) \quad f = F_m
\end{equation}

then:

\begin{equation}
W_m = \frac{A_m S_m}{2\sqrt{\pi}}
\end{equation}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure520.png}
\caption{Effect of Wavefunction Amplitude Suppression on Synthesized ECG QRS Complexes}
\end{figure}
In this case the $A_m S_m$ product may be considered a measure of the wavefunction spectral energy density at $F_m$. Or:

\begin{equation}
E_{wf}^{f=F_m} = A_m S_m
\end{equation}

In Figure (5.21), the energies of the AS products are plotted as a function of time for the frequency range 4-32 Hz. It is very evident by comparing Figure (5-21) with Figure (5.12), that the maximum energies occur at the onset of the Maternal (M) and Fetal (F) QRS complexes. Thus, the AS product can readily be used to recognize an event.

**Figure 5.21**

Wavefunction Energies in the FECG for the 4-32 Hz Frequency Range
In addition, the parameter $C_m$ provides information as to what time the event is occurring.

Figure (5.18) and (5.21) are marked with the letters F and/or M at the various times these events are occurring. A similar approach can be used to recognize other bioelectric events using the ASCOF parameter set.

5.4.5 Synthesis of Bioelectric Signals from ASCOF Parameter Lists

The next consideration is the synthesis of the original bioelectric signal from the parameter lists. This is simply an inversion of the analysis process in which equation (3-8) is used to reconstruct the original signal.

First, the filter band is reconstructed by:

\[
(5-20) \quad r_j(t) = \sum_{m=1}^{M_j} W_m(t)
\]

then, the original signal is reconstructed by:

\[
(5.21) \quad \hat{s}(t) = \sum_{j=1}^{n} r_j(t)
\]

Figure (5.22) shows the synthesized FECG which was reconstructed using equations (5-20) and (5-21) for the ASCOF parameters listed in Figure (5.18). Compare this to the original signal in Figure (5.12a). It is an excellent reproduction. Figure (5.23) shows the wavefunction distribution for the 32-64 Hz frequency partition of the FECG. The time duration for each wavefunction is given by:
Figure 5.22 Synthetic FECG
Figure 5.23 Wavefunction distribution for the 32-64
Frequency partition of the FECG
The synthesized ECG QRS complexes are shown in Figure (5.20). Figure (5.24) shows the synthesized first three wavefunctions in the 64-128 frequency partition of the ECG. Figure (5.25) shows the amplitude versus frequency of the 1 to 128 Hz frequency range of the ECG.

Figure (5.26) shows a computer printout of the synthesized seventh wave function in the 64-128 frequency range of the ECG. The variables:

\[ \begin{align*}
(5-22) & \quad t = kT \\
(5-23) & \quad kC \quad kS/2 \quad k\text{min} \quad k\text{max} \\
\end{align*} \]

specify the location of the wavefunction in the filtered octave partition. The time of occurrence is computed by using equation (5-22), and where:

\[ \begin{align*}
(5-24) & \quad k\text{min}T = C - (3/4)S \\
& \quad k\text{max}T = C + (3/4)S \\
(2) & \quad kS/2(T) = S \\
& \quad kC(T) = C \\
\end{align*} \]

The frequency distribution shown in Figure (5.25) verifies what was suspected, i.e., that most of the frequencies in the ECG fall below 256 Hz.

5.4.6 Processing Time and Requirements

The programs for ASCOF analysis, as mentioned before, were performed on the CDC 3170 system, using a 12 bit I/O channel. Generally, 36K words of core had to be
Figure 5.24 Synthesized wavefunctions in the 64-128 frequency partition of the ECG

Figure 5.25 Amplitude versus frequency distribution for the 1-128 Hz range of the ECG
<table>
<thead>
<tr>
<th>$kC$</th>
<th>$\text{NEXT}$</th>
<th>$kS/2$</th>
<th>$k\text{min}$</th>
<th>$k\text{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>249</td>
<td></td>
<td>15</td>
<td>238</td>
<td>260</td>
</tr>
</tbody>
</table>

$W_m(t)_{\text{NEXT}}$  
-0.0000  
0.0000  
0.0000  
-0.0000  
-0.0001  
-0.0001  
0.0001  
0.0003  
-0.0000  
-0.0005  
-0.0003  
0.0003  
0.0005  
0.0000  
-0.0004  
-0.0002  
0.0001  
0.0001  
0.0000  
-0.0000  
-0.0000  
0.0000  
0.0000

-0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000

$W_m(t)$  
$-0.0215$  
$-0.0195$  
$-0.0176$  
$-0.0156$  
$-0.0137$  
$-0.0117$  
$-0.0098$  
$-0.0078$  
$-0.0059$  
$-0.0039$  
$-0.0020$  
$-0.0000$  
$0.0020$  
$0.0039$  
$0.0059$  
$0.0078$  
$0.0098$  
$0.0117$  
$0.0137$  
$0.0156$  
$0.0176$  
$0.0195$  
$0.0215$

Figure 5.26 Synthesized wavefunction in the 64-128 frequency partition of the ECG
scheduled to accommodate the automatic ASCOF analysis program. This was required, mainly, because of the long data lists caused by the low frequency characteristics of the bioelectric signals. The length of the data lists is given by:

\[ P = \frac{t_{\text{max}}}{T} = t_{\text{max}} = t_{\text{max}} = t_{\text{max}} \frac{4f_{\text{max}}}{1/R} \]

where:

- \( t_{\text{max}} \) = time length of signal

but, from equation (5-12), \( t_{\text{min}} \) defines a minimum \( t_{\text{max}} \) based on the minimum frequency partition requirements. This defines:

\[ P_{\text{min}} = (2) \frac{1}{f_{\text{min}}} \frac{4f_{\text{max}}}{f_{\text{min}}} = 8 \frac{f_{\text{max}}}{f_{\text{min}}} \]

which, for octave partitioning becomes:

\[ P_{\text{min}} = (8) 2^n \]

For example, for 7 octave partitions in the analysis of the ECG, the minimum number of data points becomes:

\[ P_{\text{min}} = (8) 2^7 = 1024 \]

which is greater than the data list length of 640 used in speech analysis.

In Table 5.4 are summarized the CDC 3170 CPU and IOC processing times for each of the experiments listed in Table (5.3).

5.4.7 Sinusoid ASCOF Analysis

The synthesis of the FAP ASCOF parameters listed in Figure (5.15b) was not successful and produced only
Table 5.4
Processing Time on the
CDC 3170 for Automatic
ASCØF Waveform Analysis
of Bioelectric Signals

<table>
<thead>
<tr>
<th>Figure Experiment</th>
<th>CPU Time</th>
<th>IOC Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.14a: ECG</td>
<td>00/01/37.060</td>
<td>00/00/21.913</td>
</tr>
<tr>
<td>5.15a: ECG</td>
<td>00/00/57.437</td>
<td>00/00/16.898</td>
</tr>
<tr>
<td>5.14b: ECG</td>
<td>00/00/47.959</td>
<td>00/00/16.283</td>
</tr>
<tr>
<td>5.17b: ECG</td>
<td>00/00/46.694</td>
<td>00/00/15.824</td>
</tr>
<tr>
<td>5.17a: ECG</td>
<td>00/01/46.587</td>
<td>00/00/20.022</td>
</tr>
<tr>
<td>5.15b, 5.18, 5.19: ECG, FECG, FAP</td>
<td>00/03/10.393</td>
<td>00/00/22.135</td>
</tr>
</tbody>
</table>

sinusoidal waveforms. Due to the characteristic nature of the pulse shown in Figure (5.12b), it was assumed that the filtered data from the FAP was sinusoidal in nature, although the filtered data was not available from the computer output. Thus, it is possible, that this type of signal may be analyzed by the sinusoidal portion of the analysis procedure covered in Chapter 2.
Chapter 6

APPLICATION OF ASCØF PARAMETER MODELING TO DIAGNOSIS

An important consideration for any set of parameters which model or describe a bioelectric signal is the usefulness of the parameters as a tool in diagnosis. Physicians generally perform diagnosis by classifying a waveform to a particular pathological category based on comparison to known waveforms. If an analysis set can be said to uniquely describe a signal, then the analysis set can also be used in diagnosis by comparison of the set to known sets describing a particular pathological category. In this aspect, the ASCØF analysis elements provide the basis for a useful tool in clinical diagnosis.

6.1 Backgrounds in Diagnosis

Diagnosis is based primarily on the utilization of a set of parameters or measurements which uniquely describe the bioelectric waveform behavior. Previous work in this area has generally fallen into two areas.

The first of these is diagnosis by means of regression analysis. In general an unknown parameter, $p_1$, which describes an arbitrary signal and is related to another parameter, $p_2$, may be estimated in terms of the past sta-
tistics of the two parameters by:

\[ (6-1) \quad \hat{p}_1 = \alpha + \beta p_2 \]

Or, in vector form this may be written as:

\[ (6-2) \quad \hat{p}_1 = \alpha + \beta \mathbf{p}_2 \]

Young and Huggins\(^2\) outlined a computer program for
diagnosis of the ECG using a linear regression technique.
In this approach there are \( j \) parameters which describe the
waveform and the corresponding \( i \) diagnosis categories for
the parameter set. In this case:

\[ (6-3) \quad \begin{bmatrix} p_1 \\ \vdots \\ p_i \\ \vdots \\ p_n \end{bmatrix} \Rightarrow \begin{bmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1i} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} p_2 \\ \vdots \\ p_i \\ \vdots \\ p_n \end{bmatrix} \Rightarrow \begin{bmatrix} p_{21} \\ p_{22} \\ \vdots \\ p_{2j} \end{bmatrix} \]

where:

\[ n = 1, 2, \ldots, i = 1, 2, \ldots, j \]

Their parameter set representing the ECG was the numerical
coefficients of an exponential series defined by equation

\[ (1-31) \]

In this approach, they assigned an "attribute vector" \( \mathbf{p}_1 \) to each parameter set \( \mathbf{p}_2 \) according to the likelihood of its belonging to certain pathological categories.
For simplicity, they assigned:

\[ (6-4) \quad p_{1i} = 1 \text{ for the corresponding parameter set } p_2 \]

\[ p_{1i} = 0 \text{ otherwise} \]

The considered six pathological diagnostic categories:

1. Normal
2. Left-ventricular hypertrophy
3. Right-ventricular hypertrophy

4. Left-ventricular conduction defect

5. Right-ventricular conduction defect

6. Myocardial infarction

For example, in the case of right-ventricular conduction defect, the attribute vector becomes:

\[
P_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
\]

In their experiments they achieved greater than 75% accuracy in correct diagnosis. Their basis number of parameter measurements were 45 to describe the generalized signal vector.

The second of these areas of diagnosis is the use of statistical decision theory to decide which diagnostic category a particular set of measurements describing a bio-electric waveform belongs. Sprecht outlined a procedure for utilizing the Bayes strategy and pattern recognition techniques in vectorcardiographic diagnosis. In this approach it is required to decide whether a waveform belongs to a particular diagnostic category based on a set of \( n \) measurements, or parameters, represented by the vector:

\[
P^T = [p_1 \ p_2 \ p_3 \ \ldots \ p_n]
\]

In this case, the accepted criteria for decision rules or strategies which are used to classify patterns is that they do so in such a way as to place the expected risk at
where:

\[ n = \text{number of measurements (parameters)} \]
\[ m = \text{number of training patterns} \]
\[ \sigma = \text{smoothing factor} \]
\[ l = \text{training vector number} \]

\[ P_{kl} = lth \text{ training vector from category } C_k \]

where:

\[ (6-9) \quad P_k^T = \begin{bmatrix} P_{k11} & P_{k12} & \cdots & P_{k1n} \end{bmatrix} \]

6.2 ASCOF Parameters and Regression Analysis

The ASCOF parameters provide a strong foundation for the application of regression analysis. Consider the correlated data parameters shown in Figure 6.1. Let us estimate \( y \) by a linear function of \( x \) by:

\[ (6-10) \quad \hat{y} = \alpha + \beta x \]

where:

\[ y = f(\text{ASCOF}) \]
\[ x = f(\text{ASCOF}) \]

Equation (6-10) is called the regression line. Assuming a Gaussian distribution about the regression line, \( y_n \) becomes a deviation from the mean of \( \alpha + \beta x \); having a constant variance \( \sigma^2 \). In this case, probability density distribution for a particular \( y_j \) becomes:

\[ (6-11) \quad f_{Y_j}(y_j) = \frac{1}{\sigma(2\pi)^{1/2}} \exp \left[ -\frac{(y_j - \bar{y}_j)^2}{2\sigma^2} \right] \]
a minimum. The Bayes strategies are such approaches and can be applied to problems containing any number of categories.

A useful extension of this pattern classification procedure is in diagnosis utilizing a set of parameters or measurements which describe the bioelectric waveform. In this case, the "category" actually becomes a "diagnosis" in that it places an undiagnosed set of parameters into a diagnostic category. The Bayes strategy leads to the decision rule for the general case:

\[(6-7) \quad d(P) = C_i \quad \text{(category } i)\]

\[
\text{if:} \quad p_i l_i f_i(P) \geq p_j l_j f_j(P) \quad \text{for all } j \neq i
\]

where:

- \(p_{i,j}\) = a priori probability of occurrence from (diagnostic) category \(C_{i,j}\)
- \(l_{i,j}\) = loss associated with making the incorrect decision
- \(f_{i,j}\) = probability density distribution function for categories

Sprecht formulated that the probability density distribution function for the \(k\)th category is given by:

\[(6-8) \quad f_k(P) = \frac{1}{\sigma^n (2\pi)^{n/2} } \frac{1}{m} \]

\[
= \sum_{1 = 1}^{m} \exp \left[ -\frac{(P_{kl} - P_k)T(P_{kl} - P)_k}{2\sigma^2} \right]
\]
Regression Analysis on ASCOF Data Parameters

assuming that when \( y_j \) is measured it is located within a small interval \( \Delta y_j \). The regression line shown is the best representation of the data points if the total probability of all the values of \( y \) occurring simultaneously is a maximum. This happens when the mean square error (m.s.e.) is minimized.

The error is given by:

\[
\epsilon = y - \hat{y} \tag{6-12}
\]

and the m.s.e. by:

\[
\overline{\epsilon^2} = E[(y - \hat{y})^2] = E[(y - \alpha - \beta x)^2] \tag{6-13}
\]

expanding:

\[
\overline{\epsilon^2} = E(y^2) - 2\alpha E(y) - 2\beta E(xy) + \alpha^2 + 2\alpha\beta E(x) + \beta^2 E(x^2)
\]
To find the values of $\alpha$ and $\beta$ which minimize the m.s.e.,
we have:

(6-14) \[ \frac{\partial \varepsilon^2}{\partial \alpha} = \frac{\partial \varepsilon^2}{\partial \beta} = 0 \]

which becomes:

(6-15) \[ -\bar{y} + \alpha + \beta \bar{x} = 0 \]
\[ \bar{xy} - \alpha \bar{x} - \beta \bar{x}^2 = 0 \]

Solving simultaneously equation (6-15), we obtain:

(6-16) \[ \alpha = \bar{y} - \beta \bar{x} \]
\[ \beta = \frac{\bar{xy} - (\bar{x})(\bar{y})}{\bar{x}^2 - (\bar{x})^2} \]

Noting that:

(6-17) \[ \bar{x}^2 = \frac{\Sigma x^2}{N}, \bar{xy} = \frac{\Sigma xy}{N} \]
\[ (\bar{x})(\bar{y}) = \frac{\Sigma x \Sigma y}{N^2}, (\bar{x})^2 - \frac{\Sigma x \Sigma x}{N^2} \]

the expression of $\beta$ becomes:

(6-18) \[ \beta = \frac{\frac{\Sigma}{N} \sum_{n=1}^{N} (x_n - \bar{x})(y_n - \bar{y})}{\frac{\Sigma}{N} \sum_{n=1}^{N} (x_n - \bar{x})^2} \]

Using the value for $\alpha$ in equation (6-16) into equation

(6-10) produces:

\[ \hat{y} = \bar{y} - \beta \bar{x} + \beta x \]

or:

(6-19) \[ \hat{y} - \bar{y} = \beta (x - \bar{x}) \]

where:

\[ \bar{x} = \frac{1}{N} \sum_{n=1}^{N} x_n ; \bar{y} = \frac{1}{N} \sum_{n=1}^{N} y_n \]
6.2.1 Regression on ASC0F Parameters

As an example of applications of the foregoing discussion, consider the ASC0F parameter elements from the 128-32 Hz bandpass for the ECG shown in Table 6.1. It is desired to obtain a regression of A on S for values of A less than 0.001, to determine if there is a relationship and to use this relationship in predicting other values of A given S for that frequency range.

Table 6.1

Parameter Sets With A 0.001 in the Frequency Range 128-32 for the ECG

<table>
<thead>
<tr>
<th>A</th>
<th>S</th>
<th>C</th>
<th>ϕ</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0009</td>
<td>0.0438</td>
<td>0.0764</td>
<td>190.9979</td>
<td>83.5009</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0428</td>
<td>0.2774</td>
<td>86.0758</td>
<td>59.2834</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0526</td>
<td>0.3750</td>
<td>235.7435</td>
<td>84.9860</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.0351</td>
<td>0.4158</td>
<td>241.1136</td>
<td>98.8735</td>
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<td>0.0007</td>
<td>0.0562</td>
<td>0.4442</td>
<td>154.1533</td>
<td>93.4990</td>
</tr>
<tr>
<td>0.0006</td>
<td>0.0295</td>
<td>0.4844</td>
<td>54.0156</td>
<td>101.7542</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0401</td>
<td>0.5095</td>
<td>353.1908</td>
<td>113.7255</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.0331</td>
<td>0.5374</td>
<td>248.1264</td>
<td>99.7041</td>
</tr>
<tr>
<td>0.0009</td>
<td>0.0673</td>
<td>0.4084</td>
<td>69.9715</td>
<td>43.9400</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.0759</td>
<td>0.4571</td>
<td>256.5495</td>
<td>51.7595</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0722</td>
<td>0.5362</td>
<td>316.9508</td>
<td>42.2865</td>
</tr>
</tbody>
</table>

In this case, equation (6-19) can be written as:

\[ \hat{A} - \bar{A} = \beta(S - \bar{S}) \]  

The first ten data points from Table 6.1 are plotted in Figure 6.2. In this case, \( N = 10 \). Then:
Figure 6.2

Regression of $A$ on $S$ for $A < 0.001$ in the 128-32 Frequency Range of the ECG

\[ \bar{A} = \frac{1}{10} \sum_{n=1}^{10} A_n = 0.00068 \]

\[ \bar{S} = \frac{1}{10} \sum_{n=1}^{10} S_n = 0.04183 \]

\[ \frac{\sum_{n=1}^{10} (S_n - \bar{S})(A_n - \bar{A})}{\sum_{n=1}^{10} (S_n - \bar{S})^2} = \frac{144.537 \times 10^{-7}}{1801.8 \times 10^{-6}} = 8.02 \times 10^{-3} \]

hence, \[ \hat{A} - 0.00068 = (8.02 \times 10^{-3})(S - 0.04183) \]
This regression line is shown in Figure 6.2. The results were obtained by completing the following table:

<table>
<thead>
<tr>
<th>A</th>
<th>A-\bar{A}</th>
<th>S</th>
<th>S-\bar{S}</th>
<th>(A-\bar{A})(S-\bar{S})</th>
<th>(S-\bar{S})^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Then, taking S = 0.0722 from Table 6.1, from equation (6-21) we obtain:

\[ \hat{A} = 0.00087 \]

which agrees very closely with the A of 0.008 in Table 6.1. This is an example of the usefulness of the linear regression techniques in analyzing the ASC\&F parameter sets, and as a tool in diagnosis.

6.3 Using ASC\&F Parameters in Multidimensional Regression Analysis for Use in Diagnosis

Previous work in regression diagnosis\(^2\) provides a starting point in the application of the ASC\&F parameters to clinical diagnosis. In vector notation, equation (6-19) can be written as:

\[ \hat{Y} - \bar{Y} = \beta (X - \bar{X}) \]

Note: In the following analysis, the upper case of the letter shall denote a vector, i.e.,

upper case \( y = Y = \text{vector} = \bar{Y} = \bar{Y} \)
Let us formulate p training, or diagnostic, coefficients $t_{jn}$ and q ASCOF parameter set data measurements $p_{kn}$ for each subject $n$ in a population $N$ where:

(6-23) \[ j = 1, 2, \ldots, p \]
\[ k = 1, 2, \ldots, q \]
\[ n = 1, 2, \ldots, N \]

Then, for the $n$th patient, the $t_{jn}$ training coefficients form a $p$-dimensional training vector $T_n$ and the $p_{kn}$ ASCOF parameters set form a $q$-dimensional vector $P_n$. Then, from equation (6-22), the linear estimate of the training vector in terms of the previously known ASCOF parameter sets is given by:

(6-24) \[ \hat{T}_o - T = V [P_o - P] \]

A two dimensional analogy of this multidimensional linear regression equation is shown in Figure 6.3. Equation (6-24) represents the regression of a $p \times 1$ vector, $T_1$ on a $q \times 1$ vector, $P$. Where:

(6-25) \[ P = \text{ASOF parameter vector} \]
\[ T = \text{corresponding training or diagnostic vector} \]

The elements of equation (6-24) are given by:

(6-26) \[ T_o = \begin{bmatrix} t_{o1} \\ t_{o2} \\ \vdots \\ t_{oj} \\ \vdots \\ t_{op} \end{bmatrix} \quad P_o = \begin{bmatrix} p_{o1} \\ p_{o2} \\ \vdots \\ p_{ok} \\ \vdots \\ p_{oq} \end{bmatrix} \]
(p x 1 vector for training in diagnosis)

\[ T - \overline{T} = V \left[ p_{o} - \overline{p} \right] \]

Figure 6.3

Using ASCØF Parameters in Multidimensional Regression Analysis Diagnosis

\[(6-26) \text{ continued} \]

\[ T = \begin{bmatrix} t_{1} \\ t_{2} \\ \vdots \\ t_{j} \\ \vdots \\ t_{p} \end{bmatrix} = \frac{1}{N} \sum_{n=1}^{N} T_{n} ; \quad T_{n} = \begin{bmatrix} t_{1n} \\ t_{2n} \\ \vdots \\ t_{jn} \\ \vdots \\ t_{pn} \end{bmatrix} \]

and
\[
\bar{p} = \begin{bmatrix}
\bar{p}_1 \\
\bar{p}_2 \\
\vdots \\
\bar{p}_k \\
\vdots \\
\bar{p}_q
\end{bmatrix} = \frac{1}{N} \sum_{n=1}^{N} p_n ; \quad p_n = \begin{bmatrix}
p_{1n} \\
p_{2n} \\
\vdots \\
p_{kn} \\
\vdots \\
p_{qn}
\end{bmatrix}
\]

and:
\[
\bar{\xi}_j = \frac{1}{N} \sum_{n=1}^{N} t_{jn} ; \quad \bar{p}_k = \frac{1}{N} \sum_{n=1}^{N} p_{kn}
\]

finally:
\[
V = \begin{bmatrix}
v_{11} & v_{12} & \cdots & \cdots & v_{1q} \\
v_{21} & v_{22} & v_{23} & \cdots & v_{2q} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
v_{p1} & v_{p2} & v_{p3} & v_{p4} & \cdots v_{pq}
\end{bmatrix}
\]

The \( V \) matrix is derived in a similar manner as equation (6-16), i.e., to minimize the m.s.e. and is given by\(^2\)

\[
(6-27) \quad V = \left[ \sum_{n=1}^{N} (T_n - \bar{T})(\bar{P}^T_n - \bar{P}^T) \right]^{-1} \left[ \sum_{n=1}^{N} (P_n - \bar{P})(P^T_n - \bar{P}^T) \right]
\]

The matrix reduction to a \( p \times 1 \) training or diagnostic matrix proceeds as follows:
As an example of this type of approach, suppose we had:

\[ q = 5 = (A, S, C, \emptyset, F) \]
\[ p = 3 = (a, b, c) \]
\[ n = 2 \]

This means that the training or diagnostic matrix consisting of elements a, b, and c is weighted according to the particular known diagnosis for the signal modeled with the parameter set A, S, C, \emptyset, and F. Then

\[ P_1 = \begin{bmatrix} A_{11} \\ S_{21} \\ C_{31} \\ \emptyset_{41} \\ F_{51} \end{bmatrix} \leftrightarrow T_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} ; \quad P_2 = \begin{bmatrix} A_{12} \\ S_{22} \\ C_{32} \\ \emptyset_{42} \\ F_{52} \end{bmatrix} \leftrightarrow T_2 = \begin{bmatrix} a_{12} \\ b_{22} \\ c_{32} \end{bmatrix} \]

which leads to an important relation in regression diagnostics:
which says that every ASCOF vector has a corresponding diagnostic vector.

Continuing the example, we have:

\[
P_n \leftarrow T_n
\]

This, then is all the statistics required to formulate the diagnostic regression equation given by equation (6-24).

6.3.1 A Diagnostic Algorithm

Figure (6-4) shows a diagnostic algorithm using the ASCOF parameter with the foregoing analysis and discussion. It should be noted that the form of the diagnostic vector, \( T_0 \), is arbitrary, and subsequently the form of the statistical vector, \( T_n \), is also arbitrary. The only requirement is that it be suitably weighted to represent the particular physiological category. Equation (6-5) represents the weights given in reference \(^2\). Another approach is discussed in the following section.

6.4 Formulating the Diagnostic Vector

The diagnostic vector, \( T_n' \), formulated by Young and Huggins\(^2\) and represented by equation (6-4) is undesirable in that it assumes complete independence between each parameter vector, \( P_n \). In fact, there may be
Perform ASC\(\Phi\)F analysis on bioelectric signal. Formulate \(P_n\) ASC\(\Phi\)F vector

Store \(P_n\)

Formulate a suitably weighted diagnostic vector \(T_n\)

Store \(T_n\)

\(n = n_{\text{max}}?\)

\(\text{Yes}\)

\(N = n_{\text{max}}\)

\[\bar{p} = \frac{1}{N} \sum_{n=1}^{N} P_n\]

\[\bar{T} = \frac{1}{N} \sum_{n=1}^{N} T_n\]

\[V = \left[ \sum_{i=n}^{N} \left( (T_n - \bar{T})(P_n - \bar{P})^T \right) \right]^{-1} \left[ \sum_{i=n}^{N} (P_n - \bar{P})(P_n - \bar{P})^T \right]\]

Input clinical ASC\(\Phi\)F vector \(P\)

\[\hat{T}_0 = V \left[ P_0 - \bar{P} \right] + \bar{T}\]

Search \(T_0\) for to \(j_{\text{max}} = \) diagnosis

end of diagnosis?

\(\text{Yes}\)

Stop

\(\text{No}\)

Figure 6.4 Algorithm for Diagnosis using ASC\(\Phi\)F Parameters
correlation between the parameters of one signal to another. This may lead to errors in the diagnostic procedure. This problem can be minimized by using pattern classification methods represented by equations (6-8) and (6-9), and, the Bayes strategy. This method has proved successful in the clinical diagnosis of the vectorcardiogram.23

In the case of regression analysis previously discussed, the number of diagnostic categories for a particular sample population N is:

\[(6-30) \quad \# \text{diagnostic categories} = 1, 2, \ldots, j, \ldots, p\]

Then, for the jth category, there are N(j) number of Pn vectors representing that category. In this case let:

\[(6-31) \quad N(j) = \text{number of } P_n \text{ ASCOF vectors representing that category}\]

such that:

\[(6-32) \quad \sum_{j=1}^{p} N(j) = N\]

This provides a basis for utilizing equation (6-8) to determine the weights of each element in the Tn diagnostic vector. Equation (6-8) gives an indication of the amount of correlation there is between a given vector, P, and the corresponding training categories, Pkl. It is evident that the stronger the correlation, the larger will be the value of the density function \(f_k(P)\). This is the basis for the Bayes strategy given by equation (6-7). It is also evident that the less the correlation, the smaller will be
the value of \( f_k(P) \).

Applying this to the formulation of the diagnostic vector, we can write:

\[
(6-33) \quad t_{jn} \longleftrightarrow f_{jn}(P_n)
\]

and:

\[
T_n \longleftrightarrow P_n
\]

then, the jth diagnostic weight for the nth parameter vector is given by:

\[
(6-34) \quad t_{jn} = f_{jn}(P_n) = \frac{1}{q (2\pi)^{q/2}} \cdot \frac{1}{N(j)} \cdot \sum_{i=1}^{N(j)} \exp \left\{ -\frac{\frac{1}{2} (P_{N(j)i} - P_n)^T(P_{N(j)i} - P_n)}{2\sigma^2} \right\}
\]

As an example, consider the following categories and the number of \( P_n \) vectors in each:

<table>
<thead>
<tr>
<th>Category</th>
<th># ( P_n ) vectors</th>
<th>( T_n ) vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. left-ventricular hypertrophy</td>
<td>3</td>
<td>( t_{1n} )</td>
</tr>
<tr>
<td>2. right-ventricular hypertrophy</td>
<td>2</td>
<td>( t_{2n} )</td>
</tr>
<tr>
<td>3. normal</td>
<td>4</td>
<td>( t_{3n} )</td>
</tr>
</tbody>
</table>

Then, for vector \( P_1 \) having a corresponding diagnostic vector \( T_1 \), we have the following diagnostic weights:

\[
t_{11} = K \left( \frac{3}{3} \right) \exp \left\{ -\frac{\frac{1}{2} (P_{3i} - P_1)^T(P_{3i} - P_1)}{2\sigma^2} \right\}
\]

\[
t_{21} = K \left( \frac{2}{2} \right) \exp \left\{ -\frac{\frac{1}{2} (P_{2i} - P_1)^T(P_{2i} - P_1)}{2\sigma^2} \right\}
\]
\[
t_{31} = K \frac{1}{4} \sum_{i=1}^{4} \exp \left( -\frac{(P_{4i} - P_1)^T(P_{4i} - P_1)}{2\sigma} \right)
\]

Equation (6-34) permits the following definitions:

\[
q = \text{number of ASCOF measurements}
\]

\[
N(j) = \text{number of } P_n \text{ ASCOF vectors representing the } j\text{th category of diagnosis}
\]

\[
P_N(j) = \text{the } P_n \text{ vectors representing category } j
\]

\[
P_N(j)_i = \text{the } i\text{th } P_n \text{ vector in category } j
\]

This means, that given a particular vector, \( P_n \), the diagnostic weight of that vector with respect to its own category and the other categories can be measured. Figure (6-5) shows an algorithm for formulating the diagnostic vector using the pattern classification approach.

The foregoing approach permits the output diagnostic vector to be written:

\[
(6-35) \quad T_o = \begin{bmatrix}
t_{01} \\
t_{02} \\
\vdots \\
t_{0i} \\
\vdots \\
t_{0j} \\
\vdots \\
t_{op}
\end{bmatrix}
\]

From this the Bayes strategy can be used per equation (6-7) or simply as:
Figure 6.5 Algorithm for formulating diagnostic vector using pattern classification techniques
(6-36)

\[ d(P) = t_{oi} \]

if \( t_{oi} > t_{oj} \) for all \( i \neq j \)

to determine which of the diagnostic categories the particular P clinical vector belongs. Experiment\(^2\) in diagnosis using the pattern classification scheme has shown that a soothing factor, \( \sigma \), of 4 provides the highest accuracy in classifying patterns.
Chapter 7

EXTENSIONS OF THE ASCØF SYSTEM TO THE MODELING AND ANALYSIS OF BIOELECTRIC PHENOMENA

The ASCØF system provides the basis for work in the modeling and analysis of bioelectric phenomena. Its statistical characteristics, properties as a linear system, and fundamental mathematical structure lend themselves to further investigative possibilities in the study of bioelectric signals. In this chapter are presented some of those possibilities.

7.1 Modeling and Simulation

Equation (5-20) represents the filtered signal as a summation of the wavefunctions, \( W_m(t) \). Equation (5-21) equates the original bioelectric signal as a summation of all the filter band outputs. Thus, the total signal, \( s(t) \), is composed of GCM wavefunctions which individually are solutions to equation (2-5) which can be written as:

\[
(7-1) \quad \frac{d^2 r}{dt^2} + a(t-C) \frac{dr}{dt} + \left[ \omega_0^2 + a + \frac{a^2(t-C)^2}{4} \right] r = 0
\]

with initial conditions:

\[
\begin{align*}
  r(C) &= A \cos \phi \\
  \frac{dr}{dt}(C) &= A \omega_0 \sin \phi
\end{align*}
\]

Equation (7-1) is a nonautonomous, linear ordinary
differential equation. It is nonautonomous because time appears explicitly. It can be modeled by state variables by:

(7-2) let:

\[
\begin{align*}
X_1 &= r \\
X_2 &= \frac{dr}{dt} = \dot{r} \\
\dot{X}_1 &= \dot{r} = X_2 \\
\dot{X}_2 &= \frac{d^2r}{dt^2} = \ddot{r} \\
&= -a(t-C)X_2 - \left[\omega_0^2 + \frac{a}{2} + \frac{a^2(t-C)^2}{4}\right]X_1
\end{align*}
\]

which becomes in state vector form:

\[
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\left[\omega_0^2 + \frac{a}{2} + \frac{a^2(t-C)^2}{4}\right] & -a(t-C)
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
A \cos \phi \\
A \omega_0 \sin \phi
\end{bmatrix}
\begin{bmatrix}
X_1(C) \\
X_2(C)
\end{bmatrix}
\]

The model of equation (7-1) represented by equation (7-3) can be easily programmed for simulation either on the digital or analog computer. To derive the analog simulation model, it is first required that the analog of the \(t\) and \(t^2\) terms be developed. This is performed by using an implicit differential to solve for them. Then, letting:
we have:

\[ p = t^2 \]
\[ \dot{p} = 2t \]
\[ \ddot{p} = 2 \]

which can be easily programmed using two integrators and an inverter. Then, equating \( \frac{d^2r}{dt^2} \) in equation (7-1) to the balance of the expression, and expanding, we obtain:

\[ \dot{r} = -ar + \alpha Cr - r \left[ \omega_0^2 + \frac{a}{2} + \frac{a^2}{4} C^2 \right] \]

\[ \frac{-a^2 t^2 r}{4} + \frac{\alpha^2 t Cr}{c} \]

we obtain the desired form for analog simulation.

Thus, the computer program for simulating equation (7-1) is readily developed and is shown in Figure (7.1). Using the simulation model in Figure (7.1), investigations can be performed in the behavior of the various wavefunctions that compose the bioelectric signal. Also, the effect of the various wavefunction parameters on the solution of equation (7-1). The model represented by equation (7-3) can be readily programmed on a digital computer using any number of numerical methods available for solving state vector differential equations. Additionally, equation (7-3) can be easily used in phase plane analysis of the system, since solutions of equation (7-3) are curves in the phase space with \( t \) a parameter on the curve.
Figure 7.1 Analog Simulation of ASCPF Wavefunction System
Chapter 7.2: The ASCOF System as a Random Process

Consider the wavefunction elements which are outputs of the octave bandpass filters shown in Figure (7-2). To each sample point we then assign a filtered waveform, \( r_j(t) \) according to the rule:

\[
(7-6) \quad R(t) = A \exp \left\{ -\frac{\pi^2}{S^2} (t-C)^2 \right\} \cos \left[ 2\pi F(t-C) - \phi \right]
\]

The ensemble, then, is the total collection of all the
filterband outputs. If a probability distribution is assigned to the sample functions over the ensemble, then this can be modeled as a random process.¹

This approach offers some interesting possibilities in the interpretation of biophysiological phenomena which is represented by the filtered outputs. For example, any, a few, or all of the five ASCØF parameters could be considered as random variables in equation (7-6). Additionally, if the probability distributions of the ASCØF parameters are known or can be assumed, then the first-order probability-density function, \( f_R(r,t) \) for the random process \( R(t) \) can be determined. From this can be determined the mean and mean-squared value of \( R(t) \). Subsequent higher order statistics can also be determined from the basic expression:

\[
E(R^m) = \int_{-\infty}^{\infty} r^m f_R(r) \, dr
\]

Additionally, by examination of the first and second order statistics of the process, the stationary or non-stationary nature of the process can be determined.

Consider, for example, the random process given by equation (7-6). Assume that \( A, S, C, \) and \( F \) are constants and that \( \varnothing \) is a random variable. A reasonable assumption is that \( \varnothing \) is uniformly distributed over the interval 0 to 2π. This is shown in Figure (7.3). Then, the first order probability density function for the process is given by:
\[ f_R(r,t) = f_\varnothing(\varnothing) \]

which, after some calculations, gives:

\[ f_R(r,t) = \left[ \pi \left\{ \left[ \text{Aexp} - \left\{ \frac{\pi}{S} (t-C) \right\}^2 - R^2 \right] \right\}^{1/2} \right]^{-1} \]

\[ f_\varnothing(\varnothing) \]

Figure 7.3

Probability Density Distribution for ASC0F Parameter

Investigation of equation (7-10) shows the probability distribution is dependent upon time. For the case where \( t = C \), this reduced to:

\[ f_R(r,t) = \frac{1}{(A^2 - R^2)^{1/2}} \]
This distribution is shown in Figure 7.4. Utilization of

![Probability Distribution](image)

Figure 7.4

Probability Distribution for \( R(t) \)
Evaluated at \( t = C \)

equation (7-8) shows:

\[
\overline{R} = 0
\]

which means that for this particular example, the expected value is not dependent upon time.

Similar analysis can be performed using the system modeled as a random process. This provides researchers in bioelectric signal analysis with valuable tools with which to perform statistical evaluations on the signals.

7.3 The ASCζF Model as a Linear System

Some interesting properties are offered by the ASCζF system modeled as a linear system. Consider the filter arrangement shown in Figure 3.1. From the property of linearity, we have:

\[
(7-12) \quad r(t) = \int_{-\infty}^{\infty} h(\tau)s(t-\tau)d\tau
\]
and, the mean value of the output is:

\[ \bar{r} = \int_{-\infty}^{\infty} h(\tau) s(t-\tau) d\tau = \bar{s} \int_{-\infty}^{\infty} h(\tau) d\tau \]

hence:

(7-13) \[ \bar{r}_j = \bar{s} H(0) \]

But, for bandpass filters, \( H(0) = 0 \), hence

(7-14) \[ \bar{r}_j = 0 \]

Also, consider the bandpass filter arrangements for bioelectric signals shown on page 50. Since the filters are disjoint, i.e., they do not overlap, then their outputs are orthogonal. This means that the cross correlation between the filter outputs vanish, i.e.,

(7-15) \[ R_{r_i r_j}(\tau) = 0 \quad \text{for all } j \neq i \]

Since, the filters are disjoint, i.e.,

(7-16) \[ H_i(f) H_j(f) = 0 \quad \text{for all } j \neq i \]

then:

(7-17) \[ S_{r_i r_j} = 0 \]

which defines the power spectral density.

Then to determine the autocorrelation function for the total output, we define a new process:

(7-18) \[ g(t) = r_n(t) + r_{n-1}(t) + r_1(t) \]

The autocorrelation becomes:
but, from equation (7-15), due to the orthogonality of the outputs of the filters, the cross correlation terms in equation (7-19) go to zero. In this case equation (7-19 becomes:

(7-20) \[ R_G(\tau) = R_{\tau_1}(\tau) + R_{\tau_2}(\tau) + \ldots + R_{\tau_n}(\tau) \]

Equation (7-20) basically states, that for the ASCØF method of utilizing octave disjoint filters, the autocorrelation function of the total output signal is just the sum of the individual filter output autocorrelations.
Chapter 8

SUMMARY AND CONCLUSIONS

Summaries and conclusions are only meaningful in relation to the objectives of the work. In this respect, then, this chapter will consider the results relative to the objectives.

The ASCOF analysis procedure can be a valid method for representing a general class of bioelectric signals. If the contiguous octave filtering scheme covers the frequency range sufficiently to represent the signal, then the filter outputs are a valid representation of that signal. The frequency bounds of bioelectric signals are perfectly adaptable to the ASCOF preprocessing scheme. One disadvantage, and this is perhaps minimal, is that the number of partitions to represent low frequency bioelectric signals is greater than for speech samples. The same MC kernels which were used in speech analysis are perfectly adequate for filtering bioelectric signals. The Fourier transform of the GCM wavefunction provides a great deal of information relative to the continuous frequency spectrum of bioelectric signals.

The analysis algorithms provide an adequate method for performing ASCOF analysis on actual bioelectric data. Although the sample traces used in this investigation were small in number, it is felt that just as valid results would
have been obtained from numerous samples. The analysis procedure provided an extremely simplified method for analysis. This from the fact that the only requirement was to sample at four times the maximum frequency of interest. This was easily performed on the signals selected for analysis. Investigation of the parameter lists gave good insight into the behavior and structure of the signal from which they were derived. Additionally, a minimal amount of a-priori knowledge was needed concerning the signals prior to analysis. The signals could be reconstructed from the parameter lists which leads to the conclusion that the procedure is valid for bioelectric signals.

Some disadvantages are encountered from the long trace time and data list required due to the low frequency characteristics of the bioelectric signal. However, this is easily overweighed by the speed and ease of implementation of the algorithms. Some high frequency distortion was noticed when analyzing the ECG, however, this was probably due to truncation errors.

The analysis procedure is comprehensive in that the time, frequency, and statistical domain structure of the signal can be readily modeled using a single wavefunction element. This gives a tremendous advantage over traditional methods of analysis. The parameter sets can be used directly in diagnostic schemes discussed. Additionally, the modeling and statistical properties of the system
lend themselves readily to comprehensive analysis of any given bioelectric signal.
REFERENCES


APPENDIX A

ANALYSIS PROGRAM LISTINGS
PROGRAM TO DETERMINE ANALYSIS SET FOR FILTERED WAVEFUNCTION AND TO GENERATE THE SYNTHETIC WAVEFORM

DIMENSION AT(10), ST(10), CT(10), OT(10), FT(10), UNT(500)

C ENTER DATA EXTREMA
READ 5, A1, T1, A2, T2, A3, T3, A4, T4
FORMAT (*F20.2)

C GET ASCOF PARAMETER SET
CALL ASCOF (A1, T1, A2, T2, A3, T3, A4, T4, A, S, C, O, F, O, D, U)

C OUTPUT DATA
PRINT 10, A, S, C, O, F, O, D

FORMAT (1H10, 5X, F7.3, 5X, F7.3, 5X, F7.3, 5X, F7.3, 5X, F7.3)

C GENERATE SYNTHETIC WAVEFORM
READ 15, TMAX, DT, MN
FORMAT (*F20.2, I20)

AT(1) = A
ST(1) = S
CT(1) = C
OT(1) = U
FT(1) = F

CALL SYNTFB (AT, ST, CT, OT, FT, TMAX, DT, MN, UNT, NO)
PRINT 20
FORMAT (1H1)
DO 25 I = 1, NO
PRINT 30, UNT(I)
CONTINUE

PRINT (*F20.2, 15X, F7.2)
END

USASI FORTRAN DIAGNOSTIC RESULTS FOR FTNMAIN

NO ERRORS
SUBROUTINE TO COMPUTE THE ASCOF ANALYSIS SET

SUBROUTINE ASCOF(A1,T1,A2,T2,A3,T3,A4,T4,A5,T5,A6,T6,A7,T7,A8,T8,A7,F0,F0D,U)

A*S = MAGNITUDE FROM EXTREMA LIST
T*S = CORRESPONDING TIMES
A = AMPLITUDE OF GAUSSIAN ENVELOPE
S = TIME DURATION OF GAUSSIAN ENVELOPE
C = CENTER IN TIME OF GAUSSIAN ENVELOPE
O = PHASE OF COSINE WAVE WITH RESPECT TO C
On = PHASE OF COSINE WAVE WITH RESPECT TO C IN DEGREES
F = FREQUENCY OF THE COSINE WAVE

SOLVE FOR S
A1A = ABS(A1)
A2A = ABS(A2)
A3A = ABS(A3)
A4A = ABS(A4)
DT = T3 - T2

PI = 3.1415927
AN = 2.0*SQRT(TB) - 1.0
TRF = 2.0*ALOG(AN)
FTB = SQRT(-1.0/TFB)
S = 2.0*PI*DT*FTB

SOLVE FOR F
F = 1.0/(2.0*PI*DT)

SOLVE FOR C
ALPHA = A3A - A1A
BETA = A2A - A4A
IF (ALPHA GT BETA) RHO = BETA/ALPHA
IF (ALPHA LT BETA) RHO = ALPHA/BETA
U = 1.0/(1.0 + RHO)
C = T3 - U*DT
LN 0033  C  SOLVE FOR O
LN 0034  IF(A2 .LT. 0.0) O = PI*U
LN 0035  IF(A2 .GT. 0.0) O = PI*U + PI
LN 0036  OD = 57.29*O
LN 0037  C  SOLVE FOR A
LN 0038  T2C = ABS(T2-C)
LN 0039  EX = ((PI/S)*T2C)**2.0
LN 0040  PRINT 1,TB,DT,AN,TBF,FTB,EX
LN 0041  1  FORMAT (I1H1,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2)
LN 0042  A = A2A*EXP(EX)
LN 0043  RETURN
LN 0044  END

USASI FORTRAN DIAGNOSTIC RESULTS FOR ASCOF

NO ERRORS

OBJ,LGO
SUBROUTINE TO GENERATE SYNTHETIC FILTER BAND

SUBROUTINE SYNTFB (AT, ST, CT, OT, FT, TMAX, DT, MN, UNT, J)

AT = AMPLITUDE OF GAUSSIAN ENVELOPE ARRAY
ST = TIME DURATION OF GAUSSIAN ENVELOPE ARRAY
CT = CENTER IN TIME OF GAUSSIAN ENVELOPE ARRAY
OT = PHASE OF COSINE WAVE WITH RESPECT TO C ARRAY
FT = FREQUENCY OF COSINE WAVE ARRAY
TMAX = MAX TIME FOR SYNTHESIS
DT = TIME INCREMENT
MN = NUMBER OF GCM ELEMENTS IN FILTER BAND
UNT = SYNTHETIC FILTER BAND ARRAY
DIMENSION AT(10), ST(10), CT(10), OT(10), FT(10), UNT(500), WMT(10)

FORM FAMILY OF WAVEFORMS

T = 0.0

J = 1

PI = 3.1415927

DO 5 I = 1, MN

TC = ABS(T - CT(I))

WMT(I) = AT(I) * EXP( - ((PI/ST(I)) * TC)**2.0))

1

CONTINUE

CONTINUE

SUM FAMILY OF WAVEFORMS

DO 10 I = 1, MN

UNT1 = WMT(I)

UNT2 = UNT2 + UNT1

10 CONTINUE

UNT(J) = UNT2

J = J + 1

T = T + UT

IF (T - TMAX) 15, 20, 20

CONTINUE

RETURN

END
SOURCE PROGRAM FOR FIXED FILTERING OF BIOELECTRIC POTENTIAL

DIMENSION UK(140),BN(70),UT(70),EM(50),ET(50),ST(200)

READ IN FILTER DATA

READ 6,FN1,FN2
FORMAT(2F30.10)

READ 5,R,N,M

NT = N * M

ZERO SYNTHESIS ARRAY

DO 50 I = 1,NT
ST(I) = 0.0

CONTINUE

READ IN SAMPLE BIOELECTRIC DATA

READ 10,(UT(I),I=1,M)

PERFORM DISCRETE CONVOLUTION FILTERING

CALL CONFIL(UK,BN,UT,R,N,M,FN1,FN2,NT)

SYNTHESIZE SET OF FILTER BANDS

DO 60 I = 1,NT
ST(I) = ST(I) + UK(I)

CONTINUE

OUTPUT FILTER RESPONSE

DO 20 I = 1,N
PRINT 20
FORMAT(1H1,10X,9HINTERVAL,5X,15HFILTER RESPONSE//)

DO 25 I = 1,N
PRINT 30*I,BN(I)

CONTINUE

FORMAT(1H0,13X,13,7X,F7.3)

OUTPUT FILTER INPUT,INCREMENT AND OUTPUT

PRINT 35

FORMAT(1H1,10X,9HINCREMENT,6X,

OUTPUT FILTER OUTPUT)
LN 0034       PRINT 45,J,UK(J)
LN 0035        CONTINUE
LN 0036        FORMAT (1H0,12X,I3,12X,F8.3)
LN 0037        C FIND EXTREMA
LN 0038        CALL XTRMA(UK,M,N,R,EM,ET,NE)
LN 0039        C OUTPUT EXTREMA
LN 0040        PRINT 46
LN 0041        FORMAT(1H1)
LN 0042        DO 47 K = 1,NE
LN 0043        PRINT 48,EM(K),ET(K)
LN 0044        CONTINUE
LN 0045        FORMAT (1H0,10X,FR,3*10X,F8.3)
LN 0046        C READ NEW FILTER PARAMETERS
LN 0047        READ 6,FN1,FN2
LN 0048        IF (FN1 .EQ. 0.0) GO TO 55
LN 0049        GO TO 15
LN 0050        CONTINUE
LN 0051        C OUTPUT SYNTHETIC WAVEFORM
LN 0052        PRINT 69
LN 0053        FORMAT (1H1)
LN 0054        DO 70 I = 1,NT
                  
LN 0055       PRINT 71,ST(I)
LN 0056        CONTINUE
LN 0057        FORMAT (1H0,10X,F10.3)
LN 0058        END

USASI FORTRAN DIAGNOSTIC RESULTS FOR FTN.MAIN

NO ERRORS
SUBROUTINE TO PERFORM DISCRETE CONVOLUTION FILTERING

SUBROUTINE CONVIL (UK, BN, UT, R, N, M, FN1, FN2, NT)

DIMENSION UK(140), BN(70), UT(70), A(140, 70)

UK(J) = FILTER OUTPUT
BN(I) = BANDPASS KERNELS
UT(J) = FILTER INPUT
R = SAMPLING RATE
N = DISCRETE POINTS IN BN
M = DISCRETE POINTS IN UT
FN1 = BANDWIDTH OF FILTER
FN2 = CENTER FREQUENCY OF FILTER

SET VALUES AND LIMITS

NL = (-N+1)/2
NU = (N-1)/2
NR = 2*NU + 1
NMID = NU + 1
T = 1.0/R
PI = 3.1415927
F1 = PI*FN1*T
F2 = PI*2.0*FN2*T

FIND KERNELS

A1 = NL
A1 = A1
DO 5 I = 1, NR
D1 = F1*AI
D2 = SIN(D1)
D3 = COS(F2*AI)
D4 = D2/V1

BN(I) = FN1*D4*D3
PRINT 6, V1, O2, D3, D4, AI
AI = AI + 1.0

CONTINUE
LN 0034   6 FORMAT (1H0,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2)
LN 0035   C BN(NMID) = FN1
LN 0036   C COMPUTE OUTPUT VALUES
LN 0037   C CLEAR MATRIX
LN 0038   DO 8 I = 1,140
LN 0039   DO 8 J = 1,70
LN 0040   A(I,J) = 0.0
LN 0041   8 CONTINUE
LN 0042   J = 1
LN 0043   MC = M
LN 0044   ND = 0
LN 0045   NT = M+N
LN 0046   C FORM MATRIX
LN 0047   9 ND = ND+1
LN 0048   NC = ND
LN 0049   DO 10 I = 1,N
LN 0050   A(NC,J) = UT(MC)*RN(I)
LN 0051   NC = NC+1
LN 0052   10 CONTINUE
LN 0053   MC = MC-1
LN 0054   J = J+1
IF (MC *EU* 0) GO TO 15
GO TO 9
SUM TERMS
15 LL = 1
J = 1
16 UK2 = 0.0
DO 20 MM = 1*M
UK1 = A(LL*MM)
UK2 = UK2 + UK1
20 CONTINUE
UK(J) = UK2
LL = LL + 1
J = J + 1
IF (J *EQ* NT) GO TO 21
GO TO 16
CONTINUE
21 RETURN
END
SURROUTINE TO COMPUTE EXTREMA LIST
SUBROUTINE XTRMA (UK, M, N, R, EM, ET, K)
DIMENSION UK(140), EM(50), ET(50)

C = FILTER OUTPUT
M = DISCRETE POINTS IN UT(T)
N = DISCRETE POINTS IN BN(T)
R = SAMPLING RATE
EM = MAGNITUDE OF EXTREMA
ET = TIME OF MAGNITUDE
K = NUMBER OF EXTREMA POINTS
T = 1.0/K
J = 2
JMAX = M+N-1

IF(UK(J)) = 5, 4, 8
J = J+1
IF(J < JMAX) = 1, 12
FIND MINIMUM EXTREMA

IF(UK(J) = UK(JP)) = 7, 6, 6
J = J+1
IF(J < JMAX) = 5, 5, 12
L = -1
GO TO 11

FIND MAXIMUM EXTREMA

IF(UK(J) = UK(JP)) = 9, 9, 10
J = J+1
IF(J < JMAX) = 8, 12, 12
L = +1

FIND EXTREMA TIME AND MAGNITUDE BASED C
PARABOLIC FIT

JN = J-1
AJ = J
TIME = AJ*T
C1 = (UK(JN) - 2.0*UK(J) + UK(JP))
C2 = UK(JP) - UK(JN)
XC = (C2*C2) / (8.0*C1)
TC = C2 / (2.0*C1)
EM(K) = UK(J) - XC
ET(K) = TIME - TC*T
K = K+1
IF(L) = 9, 9, 6
CONTINUE
RETURN
END

USASI FORTRAN DIAGNOSTIC RESULTS FOR XTRMA
SOURCE PROGRAM TO PERFORM ASCOF ANALYSIS OF BIOELECTRIC

DIMENSION X(640),Y(1360),XTAB(640),YTAB(640)
FORMAT(1H0*20X,4HNEXT)
READ 60,5,6
IF(M) 99,99,65
LD = NUMBER OF DISCRETE POINTS IN U(T)
FSK = SAMPLING RATE
NFB = NUMBER OF FILTER BANDS
READ 63*YTAB(I)*I=1,M
FORMAT (120,4F10.2)
INPUT DATA LIST
READ 63*YTAB(I)*I=1,M
FORMAT (8F10.3)
T = 0.0
DO 70 I = 1,M
XTAB(I) = T
T = T+DTM
CONTINUE
PRINT 200
FSK = 4.0*FMAX
K = 1
KD = 2**K(K-1)
DF = FMAX/DK
IF(DF LE. FMIN) GO TO 73
K = K+1
GO TO 72
NFB = K-1
CALL SAMPLE (X,XTAB,YTAB*M,FSK,I)
LD = I-1
PRINT 200
PRINT 71*LD,FSK,NFB
FORMAT (1H0*10X,120,10X,F10.2,10X,I20)
SET UP INITIAL CONDITIONS-DO LOW PASS FILTERING
CALL BICFL(X*Y*LD)
DO WAVEFORM ANALYSIS ON REQUIRED NUMBER OF FILTER BANDS
KK = 1
LF = 1280
DO 50 NF = 1*NFB
DO FAST FILTERING
CALL BFLTR(X*Y*LD*KK)
DO DETECT EXTREMA AND MAKE CALCULATIONS FROM FILTERED DATA
CALL BEXTR(X*FSK2LD*KK*MM*KMAX)
DO DETERMINE ASCOF PARAMETERS FROM EXTREMA LIST
MFAC = 2**(KK-2)
KP = (LF*B0)-((MFAC-1)*LF)/MFAC
CALL PCALC(X*Y*KMAX,MM,KP)
CONTINUE
50
C OUTPUT THE PARAMETERS
PRINT 200
DO 51 NL = 1*NFB
NFAC = 2**(NL-1)
JP = 80*LF/NFAC
NWF = Y(JP)
PRINT 100,NWF
FORMAT(515)
JMAX = 5*NWF
DO 201 J=1,JMAX,5
K = JP-J
PRINT 101,Y(K),Y(K-1),Y(K-2),Y(K-3),Y(K-4)
FORMAT(5F10.4)
CONTINUE
SYNTHESIZE ORIGINAL WAVEFORM FROM PARAMETER LIST
DO 52 J=1,LD
X(J) = 0.0
NSY = 1
CALL GCMSY(X,Y,FSK,NSY,LD)
NSY = NSY + 1
IF(NSY>NFB) 54,54,53
CONTINUE
OUTPUT SYNTHETIC WAVEFORM
PRINT 200
DO 97 J = 1,LD
PRINT 96,X(J)
CONTINUE
FORMAT (10F10.4)
GO TO 61
CONTINUE
END

USASI FORTRAN DIAGNOSTIC RESULTS FOR FTN,MAIN

NO ERRORS
SUBROUTINE SAMPLE (X,XTAB,YTAB,M,FSK,K)
DIMENSION X(640),XTAB(640),YTAB(640)
DT = 1.0/FSK
XARG = DT
JL = 1
K = 1
20
COE = 1.0
JU = JL + 2
JC = JL + 1
DO 5 J = JL+JU
IF(XARG,NE,XTAB(J)) GO TO 5
X(K) = YTAB(J)
GO TO 30
5
COE = COE*(XARG-XTAB(J))
YVAL = 0.0
DO 10 I = JL+JU
EVAL = YTAB(I)*COE/(XARG-XTAB(I))
DO 15 J = JL+JU
IF(I,NE,J) EVAL = EVAL/(XTAB(I)-XTAB(J))
YVAL = YVAL + EVAL
10
X(K) = YVAL
30
PRINT 35,XARG,X(K)
35
FORMAT (1H0,10X,F10.4,10X,F10.4)
K = K+1
XARG = XARG + DT
IF(XARG,GT,XTAB(JC)) JL = JL+1
IF(XARG,GT,XTAB(M-2)) JL = M-2
IF(XARG,GT,XTAB(M)) GO TO 25
GO TO 20
25
CONTINUE
RETURN
END
SUBROUTINE BICFL(X,Y,LD)

DIMENSION X(640),Y(720),F(20)

FORMAT(1H0,20X,4HEX)

PI = 3.1415927

C COEFFICIENTS FOR LOW PASS KERNEL

F(1) = 0.500000
F(2) = 0.317551
F(3) = -0.103867
F(4) = 0.060028
F(5) = -0.040481
F(6) = 0.029075
F(7) = -0.021652
F(8) = 0.016281
F(9) = -0.012238
F(10) = 0.009171
F(11) = -0.006760
F(12) = 0.004913
F(13) = -0.003464
F(14) = 0.002380
F(15) = -0.001556
F(16) = 0.000977
F(17) = -0.000549
F(18) = 0.000290
F(19) = -0.000122
F(20) = 0.000056

TRANSFER X(J) TO Y(J) AND SET ZEROES ON ENDS

X(J) FOR FILTERED DATA

Y(J) FOR DATA LIST

DO 1 J = 1,40

K = LD+40+J

Y(J) = 0.0

Y(K) = 0.0
JMAX = LD + 40
DO 2 J = 41, JMAX
K = J-40
Y(J) = X(K)

SET END CONDITIONS BY HAMMING WINDOW MULTIPLICATION
PRINT 200
JH = LD/20 + 41
HA = LD/10
DO 3 J = 41, JH
K = LD+81-J
W = 0.5*(1.0+COS(2.0*PI*(J-JH)/HA))
PRINT 9,W
Y(J) = Y(J)*W
Y(K) = Y(K)*W
PRINT 200
DO 8 JJ = 41, JMAX
PRINT 9,Y(JJ)
CONTINUE
FORMAT (1H0,10X,F10.4)
LOW PASS FILTERING OF DATA PRIOR TO HIGH SPEED FILTERING
JMAX = LD+40
DO 5 J = 41, JMAX
L = J-40
X(L) = F(I)*Y(J)
DO 6 K = 2, 20
M = J-2*K+3
N = J+2*K-3
X(L) = X(L) + F(K)*Y(M) + Y(N))
6 CONTINUE
5 CONTINUE
MOVE X TO Y AND ZERO X
DO 7 J = 1, LD
K = J-40
Y(K) = X(J)
X(J) = 0
7 CONTINUE
PRINT 200
DO 10 JJ = 41, JMAX
PRINT 9, Y(JJ)
10 CONTINUE
RETURN
END

USASI FORTRAN DIAGNOSTIC RESULTS FOR BICFL
USASI FORTRAN DIAGNOSTIC RESULTS FOR RFLTR

NO. ERRORS
SUBROUTINE REXTR (X, FSK, LD, KK, MM, KMAX)

DIMENSION X(640)

FORMAT (1H0, 20X, 4HNEXT)

SET IC FOR DECIMATED DATA

LFAC = 2**((KK-2)

T = IFAC/FSK

JMAX = LD/LFAC

J = 2

K = 1

TOLD = 0

FIND FIRST NONZERO POINT

IF (X(J)) 2*4*3

MM = -1

GO TO 5

MM = +1

GO TO 8

J = J + 1

IF (J = JMAX) 1*I, 12

MINIMUM LOOP

IF (X(J) = X(J+1)) 7*6*6

J = J + 1

IF (J = JMAX) 5, 12, 12

L = -1

GO TO 11

MAXIMUM LOOP

IF (X(J) = X(J+1)) 9*9*10

J = J + 1

IF (J = JMAX) 8, 12, 12

L = +1

EXTREMA CALCULATION: 3 POINT PARABOLIC FIT

MAG AND DELTA TIME FOR MAX OR MIN

TIME = (J-1)*T

C1 = X(J-1)+2.0*X(J)+X(J+1)

C2 = X(J+1)-X(J-1)

XC = (C2*C2)/(8.0*C1)

TC = C2/(2.0*C1)

X(K) = ABS(X(J)-XC)

TNEW = (TIME = TC*T)

X(K+1) = TNEW - TOLD

TOLD = TNEW

K = K + 2

IF (L) 9, 9, 6

KMAX = K - 2

PRINT 200

DO 13 JJ = 1, KMAX

PRINT 14, X(JJ)

CONTINUE

FORMAT (1H0, 10X, F10.4)

RETURN

END
SUBROUTINE PCALC(X,Y,KMAX,MM,KP)

DIMENSION X(640),Y(720),Z(5)

FORMAT(1H0,20X,4HNEXT)

PI = 3.1415927

SET IC FOR WAVEFUNCTION ANALYSIS OF EXTREMA LIST

J = 1

TIME = X(J+1)

KMAX = KMAX-6

KP$ = KP

KP = KP=1

THRES = 0.00002

THRESHOLD TEST

IF(X(J)=THRES)2,2,3

INCREMENT INDEXES

J = J+2

TIME = TIME+X(J+1)

MM = =MM

IF(J=KMAX)1,1,100

CHECK FOR LESS THAN 400 HZ

IF(X(J+5)=1.25)4,4,20

GCW WAVEFUNCTION TEST

IF(X(J+2)-X(J+6))2,2,5

IF(X(J+4)-X(J))2,6,6

DETERMINE MAXIMUM SPREAD

TIMES = TIME

JSAVE = J

TIME = TIME+X(J+3)+X(J+5)+X(J+7)

TIMEN = TIME

J = J+6

IF(J=KMAX)7,7,11

IF(X(J)=THRES)8,8,9

J = J+2
IF(J=KMAX)50,50,11
TIME = TIME+X(J+1)
GO TO 7
IF(X(J+2)-X(J+6))8,8,10
IF(X(J+4)-X(J))8,11,11
TNEXT = TIME
J = JSAVE
TIME = TIMES
KGCM = +1
C
CALCULATE F(GAMMA) AND U,VARIABLES USED TO
DETERMINE S,C, AND PHASE
GAMMA = (X(J)+X(J+2))*(X(J+4)+X(J+6))/((X(J+2)
+X(J+4))^2)
CHECK LOWER BOUND ON GAMMA
IF(GAMMA > 0.26) 12,13,13
GAMMA = 0.26
13 FG = SQRT(-1./(2.*ALOG(2.*SQRT(GAMMA)-1.*)))
ALP = X(J+4)-X(J)
BET = X(J+2)-X(J+6)
IF(ALP=BET)14,14,15
RHO = ALP/BET
U = i / (1.0*RHO)

GO TO 16

RHO = BET/ALP

U = RHO/(1.0+RHO)

CALCULATE THE ASCOF PARAMETERS FOR GCM OR

FOR SINUSOID CONDITION

PRINT 200

FORMAT(1H0,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,

1 5X,F10.2)

TDIFF = 2.*X(J+5)

PRINT 101,GAMMA,FG,ALP,BET,RHO,U

FIND C

Z(3) = TIME*X(J+3)*(1.0-U)*X(J+5)

IF(MM)17.17.18

FIND O

Z(4) = U*180.*180.
<table>
<thead>
<tr>
<th>Line No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN 0071</td>
<td>GO TO 19</td>
</tr>
<tr>
<td>LN 0072</td>
<td>Z(4) = U<em>180</em></td>
</tr>
<tr>
<td>LN 0073</td>
<td>FIND F</td>
</tr>
<tr>
<td>LN 0074</td>
<td>Z(5) = 1*TDIFF</td>
</tr>
<tr>
<td>LN 0075</td>
<td>SMAX = 2**((TNEXT-Z(3))</td>
</tr>
<tr>
<td>LN 0076</td>
<td>FIND S</td>
</tr>
<tr>
<td>LN 0077</td>
<td>Z(2) = PI<em>TDIFF</em>FG</td>
</tr>
<tr>
<td>LN 0078</td>
<td>CHECK UPPER BOUND ON S</td>
</tr>
<tr>
<td>LN 0079</td>
<td>IF(Z(2)-SMAX)39,39,38</td>
</tr>
<tr>
<td>LN 0080</td>
<td>Z(2) = SMAX</td>
</tr>
<tr>
<td>LN 0081</td>
<td>FIND A</td>
</tr>
<tr>
<td>LN 0082</td>
<td>Z(1) = X(J+2)EXP(((1-U)/(2*FG))**2)</td>
</tr>
<tr>
<td>LN 0083</td>
<td>CHECK FOR GCM VERSUS SINUSOID</td>
</tr>
<tr>
<td>LN 0084</td>
<td>IF(KGCM)37,37,30</td>
</tr>
<tr>
<td>LN 0085</td>
<td>SINUSOID TEST</td>
</tr>
<tr>
<td>LN 0086</td>
<td>SL1 = X(J)*X(J+2)</td>
</tr>
<tr>
<td>LN 0087</td>
<td>SL2 = X(J+2)*X(J+4)</td>
</tr>
<tr>
<td>LN 0088</td>
<td>SL3 = X(J+4)*X(J+6)</td>
</tr>
<tr>
<td>LN 0089</td>
<td>IF(SL1-SL2)21,21,27</td>
</tr>
<tr>
<td>LN 0090</td>
<td>TEST1 = SL1/SL2</td>
</tr>
<tr>
<td>LN 0091</td>
<td>IF(SL2-SL3)22,22,25</td>
</tr>
<tr>
<td>LN 0092</td>
<td>UPSWEEP TEST</td>
</tr>
<tr>
<td>LN 0093</td>
<td>IF(J+4=KMAX)28,23,23</td>
</tr>
<tr>
<td>LN 0094</td>
<td>SL4 = X(J+6)*X(J+8)</td>
</tr>
<tr>
<td>LN 0095</td>
<td>IF(SL3-SL4)24,24,28</td>
</tr>
<tr>
<td>LN 0096</td>
<td>SL5 = X(J+8)*X(J+10)</td>
</tr>
<tr>
<td>LN 0097</td>
<td>IF(SL4-SL5)29,29,28</td>
</tr>
<tr>
<td>LN 0098</td>
<td>TEST2 = SL3/SL2</td>
</tr>
<tr>
<td>LN 0099</td>
<td>NORMAL SINE TEST</td>
</tr>
<tr>
<td>LN 0100</td>
<td>TEST = TEST1*TEST2</td>
</tr>
</tbody>
</table>
LN 0101  IF (TEST = 81) 4, 29, 29
LN 0102  C  DOWNSWEEP TEST
LN 0103  27  TEST1 = SL2/SL1
LN 0104  IF (SL2 = SL3) 28, 29, 29
LN 0105  28  TEST? = SL2/SL3
LN 0106  GO TO 26
LN 0107  C  INVENT U AND FG8GCM VARIABLES
LN 0108  29  U = 5.

LN 0109  FG = 1./PI
LN 0110  TNEXT = TIME*X(J+3)+X(J+5)+X(J+7)
LN 0111  KGCM = -1
LN 0112  GO TO 16
LN 0113  C  RESIDUE FOR GCM WAVEFUNCTION
LN 0114  30  TMAX = 0.75*Z(2)
LN 0115  J = J+6
LN 0116  JSAVE = J
LN 0117  IF (J = KMAX) 31, 40, 40
LN 0118  31  TD = TDIFF/2.*
LN 0119  T = U*TD*TD
LN 0120  32  IF (T = TMAX) 33, 33, 36
LN 0121  33  W = Z(1) * EXP(- (T*PI/Z(2))**2)
LN 0122  X(J) = X(J) * W
LN 0123  IF (X(J)) 34, 35, 35
LN 0124  34  X(J) = 0.*
LN 0125  35  T = T*TD
LN 0126  J = J+2
LN 0127  IF (J = KMAX) 32, 40, 40
LN 0128  C  END CONDITIONS FOR GCM
LN 0129  C  J = JSAVE
LN 0130  TIME = TIMEN
USASI FORTRAN DIAGNOSTIC RESULTS FOR PCALC

NO ERRORS

OBJ,LGO
SUBROUTINE GCMSY(X,Y,FSK,NSY,LD)
DIMENSION X(640),Y(720)
FORMAT(1H0,20X,4HNEXT)
PI = 3.1415927
LF = 640
NFAC = 2**(NSY-1)
KS = 80 + LF/NFAC
NWF = Y(KS)
KMAX = 5*NWF
DO 1 KK=1,KMAX,5
K = KS-KK
PHI = Y(K-3)*PI/180*
WO = 2.0*PI*Y(K-4)
JC = FSK*Y(K-2) + 1.0
JW = Y(K-1)*FSK
JMIN = JC-JW*3/4
JMAX = JC+JW*3/4
PRINT 200
DO 2 J = JMIN,JMAX
FORMAT(1H0,5X,I10,5X,I10,5X,I10,5X,I10,5X,I10)
IF(JMIN) 3,3,4
JMIN = 1
IF(JMAX-LD) 6,6,5
JMAX = LD
CONTINUE
PRINT 7,JC,JW,JMIN,JMAX
PRINT 200
DO 2 J = JMIN,JMAX
USASI FORTRAN DIAGNOSTIC RESULTS FOR GCMSY

NO ERRORS