California State University, Northridge

STUDY OF ADAPTIVE FILTERS

A thesis submitted in partial satisfaction of the requirements for the degree of Master of Science in

Engineering

by

Walter Gene Lee

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California State University, Northridge

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TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Acknowledgements</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables</td>
<td>v</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>ix</td>
</tr>
<tr>
<td>Abstract</td>
<td>xi</td>
</tr>
<tr>
<td>Chapter I, Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Chapter II, Adaptive Filter</td>
<td>4</td>
</tr>
<tr>
<td>Chapter III, Mechanization of Adaptive Filter</td>
<td>18</td>
</tr>
<tr>
<td>Chapter IV, Sensitivity to Parameter Errors</td>
<td>45</td>
</tr>
<tr>
<td>Chapter V, Conclusions</td>
<td></td>
</tr>
<tr>
<td>A. Test Results</td>
<td>50</td>
</tr>
<tr>
<td>B. Comparison of Adaptive Filter</td>
<td>57</td>
</tr>
<tr>
<td>and Conventional Filters</td>
<td></td>
</tr>
<tr>
<td>Bibliography</td>
<td>61</td>
</tr>
<tr>
<td>Appendix A</td>
<td>64</td>
</tr>
<tr>
<td>Appendix B</td>
<td>71</td>
</tr>
<tr>
<td>Appendix C</td>
<td>78</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - I</td>
<td>Output Characteristics of the General Purpose Filter</td>
<td>14</td>
</tr>
<tr>
<td>3 - I</td>
<td>Component Values of the Adaptive Filter</td>
<td>38</td>
</tr>
<tr>
<td>3 - II</td>
<td>$\text{dVAR7/dt as a Function of k}$</td>
<td>39</td>
</tr>
<tr>
<td>4 - I</td>
<td>Sensitivity to Parameter Errors</td>
<td>49</td>
</tr>
<tr>
<td>5 - I</td>
<td>Capability of Conventional and Adaptive Filters</td>
<td>59</td>
</tr>
<tr>
<td>5 - II</td>
<td>Cost and Size of Conventional and Adaptive Filters</td>
<td>60</td>
</tr>
<tr>
<td>FIGURE NO.</td>
<td>TITLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2 - 1</td>
<td>Multiple-Feedback RC Active Filter</td>
<td>5</td>
</tr>
<tr>
<td>2 - 2</td>
<td>Laplace Transformation of Multiple-Feedback RC Active Filter</td>
<td>6</td>
</tr>
<tr>
<td>2 - 3</td>
<td>Block Diagram of a Bandpass Filter</td>
<td>8</td>
</tr>
<tr>
<td>2 - 4</td>
<td>Root-Locus of Equation (2-1)</td>
<td>9</td>
</tr>
<tr>
<td>2 - 5</td>
<td>Characteristics of Bandpass Filter with Different Values of Quality Factor Q</td>
<td>13</td>
</tr>
<tr>
<td>2 - 6</td>
<td>Block Diagram of Adaptive Filter</td>
<td>17</td>
</tr>
<tr>
<td>3 - 1</td>
<td>Typical Output Characteristics of FET in the Triode Region</td>
<td>20</td>
</tr>
<tr>
<td>3 - 2</td>
<td>Low Level Bidirectional Output Characteristics for Typical N-Channel FET</td>
<td>21</td>
</tr>
<tr>
<td>3 - 3</td>
<td>V-I Characteristics of Fixed Resistors</td>
<td>22</td>
</tr>
<tr>
<td>3 - 4</td>
<td>Normalized RDS vs VGS for Typical FET</td>
<td>23</td>
</tr>
<tr>
<td>3 - 5</td>
<td>The Basic Adaptive Filter with its Two Buffer Amplifiers</td>
<td>32</td>
</tr>
<tr>
<td>3 - 6</td>
<td>The Phase Detector of the Adaptive Filter</td>
<td>33</td>
</tr>
<tr>
<td>3 - 7</td>
<td>The Voltage Waveforms of the Phase Detector at Tuned Frequency</td>
<td>34</td>
</tr>
<tr>
<td>3 - 8</td>
<td>Detailed Diagram of the Adaptive Filter</td>
<td>35</td>
</tr>
<tr>
<td>FIGURE NO.</td>
<td>TITLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3 - 9</td>
<td>Plot of dVAR7/dt vs Normalized Frequency</td>
<td>40</td>
</tr>
<tr>
<td>3 - 10</td>
<td>Detailed Block Diagram of the Adaptive Filter</td>
<td>41</td>
</tr>
<tr>
<td>3 - 11</td>
<td>Increase Resistance Range by Cascading of Amplifiers</td>
<td>42</td>
</tr>
<tr>
<td>3 - 12</td>
<td>Characteristics of Cascaded Bandpass Filters</td>
<td>44</td>
</tr>
<tr>
<td>4 - 1</td>
<td>Block Diagram of Finalized Adaptive Filter</td>
<td>46</td>
</tr>
<tr>
<td>5 - 1</td>
<td>Block Diagram of the Test Circuit</td>
<td>51</td>
</tr>
<tr>
<td>5 - 2</td>
<td>Test Result A</td>
<td>50</td>
</tr>
<tr>
<td>5 - 3</td>
<td>Test Result B</td>
<td>52</td>
</tr>
<tr>
<td>5 - 4</td>
<td>Test Result C</td>
<td>53</td>
</tr>
<tr>
<td>5 - 5</td>
<td>Response of Amplitude Modulated Signal</td>
<td>54</td>
</tr>
<tr>
<td>5 - 6</td>
<td>Test Results of the Filter Gain and Phase</td>
<td>55</td>
</tr>
<tr>
<td>A - 1</td>
<td>Block Diagram of A Lowpass Filter</td>
<td>66</td>
</tr>
<tr>
<td>A - 2</td>
<td>Root-Locus of Equation (A-1)</td>
<td>64</td>
</tr>
<tr>
<td>A - 3</td>
<td>Characteristics of Lowpass Filter with Different Values of Quality Factor Q</td>
<td>70</td>
</tr>
<tr>
<td>B - 1</td>
<td>Block Diagram of a Highpass Filter</td>
<td>73</td>
</tr>
<tr>
<td>B - 2</td>
<td>Root-Locus of Equation (B-1)</td>
<td>71</td>
</tr>
<tr>
<td>FIGURE NO.</td>
<td>TITLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>B - 3</td>
<td>Characteristics of Highpass Filter with Different Values of Quality Factor Q</td>
<td>77</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOLS</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_{ARX})</td>
<td>Gain of Amplifier X</td>
</tr>
<tr>
<td>(A_B)</td>
<td>Gain of the Buffer Amplifier</td>
</tr>
<tr>
<td>(A_{BP})</td>
<td>Gain of the Bandpass Filter</td>
</tr>
<tr>
<td>(A_{HP})</td>
<td>Gain of the Highpass Filter</td>
</tr>
<tr>
<td>(A_{LP})</td>
<td>Gain of the Lowpass Filter</td>
</tr>
<tr>
<td>(AR)</td>
<td>Operational Amplifier</td>
</tr>
<tr>
<td>(C)</td>
<td>Capacitor</td>
</tr>
<tr>
<td>(CR)</td>
<td>Diode</td>
</tr>
<tr>
<td>(f_o)</td>
<td>The Center (Tuned) Frequency of the Filter</td>
</tr>
<tr>
<td>(g_{max})</td>
<td>Maximum Trans-conductance of FET</td>
</tr>
<tr>
<td>(H(W))</td>
<td>Transfer Function of Filter</td>
</tr>
<tr>
<td>(I_D)</td>
<td>Drain Current of FET</td>
</tr>
<tr>
<td>(I_{DSS})</td>
<td>Drain Current of FET at Zero Gate Voltage</td>
</tr>
<tr>
<td>(K)</td>
<td>Gain in Root-Locus Diagram; Constant</td>
</tr>
<tr>
<td>(k)</td>
<td>Ratio of (f/f_o)</td>
</tr>
<tr>
<td>(n)</td>
<td>Number of Cascaded Bandpass Filters</td>
</tr>
<tr>
<td>(Q)</td>
<td>Quality Factor of Filter; Transistor</td>
</tr>
<tr>
<td>(R)</td>
<td>Resistor</td>
</tr>
<tr>
<td>(R_{D(ON)})</td>
<td>Drain-to-Source Resistance of FET at</td>
</tr>
<tr>
<td>SYMBOLS</td>
<td>DEFINITION</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>( R_{DS} )</td>
<td>Drain-to-Source Resistance of FET</td>
</tr>
<tr>
<td>S</td>
<td>Equivalent to ( j2\pi f )</td>
</tr>
<tr>
<td>T</td>
<td>Time</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>V</td>
<td>Voltage</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>Input Voltage</td>
</tr>
<tr>
<td>( V_{BP} )</td>
<td>Output Voltage of the Bandpass Filter</td>
</tr>
<tr>
<td>( V_{CQX} )</td>
<td>Collector Voltage of Transistor QX</td>
</tr>
<tr>
<td>( V_{DS} )</td>
<td>Drain-to-Source Voltage of FET</td>
</tr>
<tr>
<td>( V_{GS} )</td>
<td>Gate-to-Source Voltage of FET</td>
</tr>
<tr>
<td>( V_{HP} )</td>
<td>Output Voltage of the Highpass Filter</td>
</tr>
<tr>
<td>( V_{IN} )</td>
<td>Input Voltage</td>
</tr>
<tr>
<td>( V_{LP} )</td>
<td>Output Voltage of the Lowpass Filter</td>
</tr>
<tr>
<td>( V_O )</td>
<td>Output Voltage</td>
</tr>
<tr>
<td>( V_P )</td>
<td>Gate-to-Source Pinch-off Voltage of FET</td>
</tr>
<tr>
<td>W</td>
<td>Equivalent to ( 2\pi f )</td>
</tr>
</tbody>
</table>
ABSTRACT OF THE THESIS
STUDY OF ADAPTIVE FILTERS
by
Walter Gene Lee
Master of Science in Engineering
June, 1972

The gain and phase characteristics of an adaptive filter are studied both theoretically and experimentally. This filter with its multiple negative feedback loops offers a quality factor comparable to any active filters. It can be used as a lowpass, bandpass, or highpass filter. However, the bandpass filter is being analyzed in this study because it rejects all frequencies outside the passband.

The two poles in the transfer function of the filter are variables which are functions of the input frequency of the filter. The filter is a narrowband amplifier. When the filter receives an input signal, it changes the location of the two poles in the filter transfer function so that the bandpass filter has its center frequency equal to the input signal. Filters can be cascaded to form higher-than-second-order filters for better noise rejection.
This filter can be mechanized easily with no special devices or new technology. Alternative methods of mechanization are discussed with respect to capabilities and limitations. The filter circuit uses state variable techniques, its multiple negative feedback loops offers stable and reliable performance. It can be manufactured economically because of simplicity of circuitry and availability of components.
CHAPTER I
INTRODUCTION

An ideal filter is a filter that passes only the signal which is continuous and time variant and within its passband. It has properties of distortionless transmission network for the signal within its passband and has infinite attenuation for other signals. The transfer function $H(W)$ of this filter can be expressed as follows:

$$H_{\text{INSIDE PASSBAND}}(W) = -H \quad (1.1)$$

$$H_{\text{OUTSIDE PASSBAND}}(W) = 0 \quad (1.2)$$

Where $H$ is a constant for all frequencies within the passband of the filter and the minus sign represents a phase shift of $180^\circ$ with respect to the input signal. For an input signal $V_{\text{SIG IN}}(t)$ whose frequency lies within the filter capability, the output $V_{\text{SIG OUT}}(t)$ will be of the form

$$V_{\text{SIG OUT}}(t) = -H V_{\text{SIG IN}}(t) \quad (1.3)$$

and the shape of the waveform of $V_{\text{SIG OUT}}(t)$ is identical to the waveform of $V_{\text{SIG IN}}(t)$ with $180^\circ$ of phase shift.

There are two basic types of input signals; the
amplitude modulated signal (AM) and the frequency modulated signal (FM). Most conventional filters are designed for specific applications, such as AM or FM signals. For amplitude modulated signals, the filter has a fixed center frequency with a very high quality factor to reject noise. For frequency modulated signals, the filter has a wide bandwidth to cover the frequency range and attenuates the lower voltage portion of the incoming signal to reduce noise. These two types of filters are special designs for particular operating conditions and cannot be used as general purpose filters. A general purpose filter is approximately ten times as complex as a special designed conventional one. It will usually be very complex and expensive with discrete components.

In the last few years, low cost integrated circuit operational amplifiers have become available. Medium scale integration, large scale integration and hybrid techniques are also available. A filter with a complicated circuit requires no precision component and maintenance may not be worse nor cost more than a less complex circuit that requires precision components. New filters which make use of these new technologies are being designed with some success. A general purpose filter will be analyzed and mechanized. It will have a very
high quality factor to reject noise for the amplitude modulated signal, and will automatically tune its center frequency to the input of a frequency modulated signal. This adaptive filter shall provide stable, automatically tuned, and high quality factor signal processing for an amplitude modulated signal, a frequency modulated signal, or a combination of an amplitude and frequency modulated signals. It has the capability of lowpass and highpass filtering.

The adaptive filter will be fabricated and tested in the laboratory. The test results will be compared with the calculated values of the circuit.
CHAPTER II
ADAPTIVE FILTER

A lowpass or a bandpass filter has been widely used to improve the signal-to-noise ratio in instrumentation systems. If the input signal frequency varies over a wide frequency range, a wideband filter has to be used to reduce distortion of the signal. A wideband filter is not very effective in reducing noise. An ideal filter should have a narrow bandwidth to reject noise and let the signal passed through without distortion. Since the input signal frequency varies over a wide frequency range, the filter with a narrow bandwidth has to adjust its center frequency automatically to track the signal frequency to achieve the optimum signal-to-noise ratio.

A basic conventional bandpass filter shall be analyzed and then shown how it can be modified for adaptive filtering. A simple bandpass filter should have one zero and two poles in its transfer function. There are many active filters that have this property and can be used as adaptive filters. The simple bandpass filter shown in Figure 2-1 offers stable and reliable performance because of its multiple negative feedback loops. It provides great versatility, since its independent, low impedance lowpass, bandpass, and highpass outputs are
Figure 2-1. Multiple-Feedback RC Active Filter.
Figure 2-2. Laplace Transformation of Multiple-Feedback RC Active Filter.
Figure 2-3  (a) Original Block Diagram;
(b) Block Diagram after the simplification of the Feedback loops for a bandpass filter;
(c) Block Diagram after the reduction of the Feedback loop;
(d) Simplification of the Block Diagram;
(e) Final Block Diagram of a bandpass filter.
available simultaneously. The analysis will be concentrated at the bandpass filter portion to achieve an understanding of its characteristics; however, the same method can be used to understand the lowpass and the highpass filter portions.

The multiple-feedback RC active filter shown in Figure 2-1 has one zero and two poles in its transfer function. It appears that it is suitable to be modified for adaptive filtering. Figure 2-1 can be represented by laplace transformation as shown in Figure 2-2. This filter is, then, represented in block-diagram form in Figure 2-3. The final block diagram indicates that the bandpass filter has one zero and two poles in its transfer function with the zero located at the origin of the root-locus diagram. This filter is very stable because it has a phase margin of 90° or better and a unity gain at the center frequency.

Figure 2-4 Root-locus of Equation (2-1).
The transfer function of the bandpass filter is written below for analysis.

\[
\frac{V_{BP}(S)}{V_{1}(S)} = -\frac{R4(R2+R3)}{R2R5C1(R1+R4)S} + \frac{R3}{R2R5R6C1C2} - \frac{R1(R2+R3)}{R2R5C1(R1+R4)S} + \frac{R3}{R2R5R6C1C2}
\]

(2-1)

The standard form of the bandpass filter is

\[
\frac{V_{BP}(S)}{V_{1}(S)} = -\frac{KS}{S^2 + \frac{W_0}{Q}S + W_0^2}
\]

(2-2)

A comparison of Equations (2-1) and (2-2) reveals that the bandpass filter has the following characteristics:

\[A_{BP} = \frac{K}{W_0/\sqrt{Q}}\]

(2-3)

\[= -\frac{R4(R2+R3)}{R2R5C1(R1+R4)S} + \frac{R3}{R2R5R6C1C2} - \frac{R1(R2+R3)}{R2R5C1(R1+R4)S} + \frac{R3}{R2R5R6C1C2}\]

\[= -\frac{R4}{R1}\]

The center frequency of the filter

\[f_0 = \frac{1}{2\pi} \frac{W_0}{R3} \sqrt{\frac{R3}{R2R5R6C1C2}}\]
And

\[ Q = \text{The quality factor of the filter} \]

\[ Q = \sqrt{\frac{R3}{R2R5R6C1C2}} \]

\[ = \frac{R2(R1+R4)}{R1(R2+R3)} \sqrt{\frac{R3R5C1}{R2R6C2}} \]  \hspace{1cm} (2-5)

For conventional bandpass active filter, all component values of the filter are constants; therefore, it can be seen from Equations (2-3), (2-4), and (2-5) that the center frequency gain \( A_{BP} \), center frequency \( f_0 \), and quality factor \( Q \) of the bandpass filter are constants.

For frequency modulated input signal, this bandpass active filter can be modified for adaptive filtering. The center frequency gain \( A_{BP} \) and quality factor \( Q \) shall be kept constant under normal operating condition. The center frequency \( f_0 \) can be varied by changing the values of resistors \( R5 \) and \( R6 \) without affecting the gain \( A_{BP} \) and quality factor \( Q \).

If the values of resistors \( R5 \) and \( R6 \) are equal and varied with the input frequency, Equations (2-3), (2-4), and (2-5) can be simplified:

\[ A_{BP} = - \frac{R4}{R1} \]

\[ = - K1 \]  \hspace{1cm} (2-6)
Where $K_1$, $K_2$, and $K_3$ are constants. From Equations (2-3), (2-4), and (2-5) the values of capacitors $C_1$ and $C_2$ may be changed to increase the frequency capability of the adaptive bandpass filter without affecting the gain $A_{BP}$ and quality factor $Q$. The characteristics of a bandpass filter with different values of quality factor $Q$ are shown in Figure 2-5.

The characteristics of the lowpass and highpass outputs of the filter are analyzed in appendixes A and B, respectively. The transfer functions of these two outputs are listed below.

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2C_2}} \sqrt{\frac{1}{R_5R_6}}$$

$$= K_2 \sqrt{\frac{1}{R_5R_6}} \quad (2-7)$$

$$Q = \frac{R_2(R_1+R_4)}{R_1(R_2+R_3)} \sqrt{\frac{R_3C_1}{R_2C_2}} \sqrt{\frac{R_5}{R_6}}$$

$$= K_3 \quad (2-8)$$

$$\frac{V_{LP}(s)}{V_{1}(s)} = \frac{R_4(R_2+R_3)}{s^2 + \frac{R_1(R_2+R_3)}{R_2R_5R_6C_1C_2}(R_1+R_4)} \frac{R_2R_5R_6C_1C_2}{s + \frac{R_3}{R_2R_5R_6C_1C_2}} \quad (2-9)$$

$$\frac{V_{HP}(s)}{V_{1}(s)} = \frac{R_4(R_2+R_3)}{s^2 + \frac{R_1(R_2+R_3)}{R_2R_5R_6C_1C_2}(R_1+R_4)} \frac{R_2R_5R_6C_1C_2}{s + \frac{R_3}{R_2R_5R_6C_1C_2}} \quad (2-10)$$
Figure 2-5 Characteristics of Bandpass Filter with Different Values of Quality Factor $Q$. 
Table 2-I  Output Characteristics of the General Purpose Filter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bandpass Output</th>
<th>Lowpass Output</th>
<th>Highpass Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>$A_{BP} = - \frac{R_4}{R_1}$</td>
<td>$V_{LP} = \frac{R_4}{R_1R_6C_2S}$</td>
<td>$V_{HP} = \frac{R_4R_5C_1S}{R_1}$</td>
</tr>
<tr>
<td>Tuned Frequency</td>
<td>$f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2R_5R_6C_1C_2}}$</td>
<td>$f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2R_5R_6C_1C_2}}$</td>
<td>$f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2R_5R_6C_1C_2}}$</td>
</tr>
<tr>
<td>Quality Factor</td>
<td>$Q = \frac{R_2(R_1+R_4)}{R_1(R_2+R_3)} \sqrt{R_3R_5C_1} / R_2R_6C_2$</td>
<td>$Q = \frac{R_2(R_1+R_4)}{R_1(R_2+R_3)} \sqrt{R_3R_5C_1} / R_2R_6C_2$</td>
<td>$Q = \frac{R_2(R_1+R_4)}{R_1(R_2+R_3)} \sqrt{R_3R_5C_1} / R_2R_6C_2$</td>
</tr>
</tbody>
</table>
The gains, tuned frequencies, and quality factors from the outputs of this filter are summarized in Table 2-1.

If the filter is mechanized in such a way that

\[ SR5 = K4 \] (2-11)

\[ SR6 = K5 \] (2-12)

\[ \frac{R5}{R6} = K6 \] (2-13)

Where \( K4, K5, \) and \( K6 \) are constants.

From Table 2-1,

\[ A_{BP} = -\frac{R4}{RI} \] (2-14)

\[ A_{LP} = \frac{R4}{RIC2K5} \] (2-15)

\[ A_{HP} = \frac{R4C1K4}{RI} \] (2-16)

The gains \( A_{BP}, A_{LP}, \) and \( A_{HP} \) are constants.

If the filter is properly tuned, from Table 2-1, there will be 180° of phase shift between the input and the bandpass output of the filter as indicated by the minus sign; there will be 90° of phase lag between the input and the lowpass output of the filter as indicated by the term \( 1/S \); and there will be 90° of phase lead between the input and the highpass output of the filter as
indicated by the term $S$. However, if the filter is not tuned to the input frequency, then the phase shift is not $180^\circ$ for the bandpass output, $-90^\circ$ for the lowpass output, and $90^\circ$ for the highpass output. A phase detector shall be used to generate an error signal for the returning of the filter by changing the values of resistors R5 and R6 proportionally. From an examination of Equations (2-9) and (2-10), or Figures A-3 and B-3, it appears that the outputs of the highpass and the lowpass cover the whole range of frequency from D-C to infinite with a $90^\circ$ phase shift with reference to the input signal. A phase detector shall be designed to compare these highpass and lowpass outputs with the input signal as the reference.

A block diagram of this adaptive filter is shown in Figure 2-6.
Figure 2-6 Block Diagram of Adaptive Filter.
CHAPTER III
MECHANIZATION OF ADAPTIVE FILTER

From Figures 2-1 and 2-12, the adaptive filter has the same form as the conventional active filter. The adaptive filter, however, will change its characteristics according to the input signal. One way to achieve the adaptability is to have resistor R5 and R6 made variables and controlled by the phase detectors as shown in Figure 2-12. The phase detector senses the phase relation of the output signal with respect to the input signal. It changes the tuning frequency $f_0$ of the filter by adjusting the values of resistors R5 and R6 until the phase of the output signal is in agreement with the input signal.

The variable resistors in this adaptive filter make the filter a general purpose type. These variable resistors can be made of thermistors controlled by heaters, resistance ladder network controlled by switches, or field-effect transistors (FET) controlled by gate voltages, the FET appears to be the best choice because it has a fast response time and a continuous resistance range. A set of typical FET output curves is shown in Figure 3-1. The drain-to-source voltage axis is expanded to show the detail. The zero-current channel
resistance at any gate bias voltage is equal to the reciprocal of the pinched-off transconductance at the same bias. The slope of the output curve at $V_{GS} = 0$ is equal to $1/R_{D(ON)}$ and it is also equal to $g_{max}$. The FET described by these curves is approximately a square-law device, the slope intersects the dotted constant-current line drawn from $I_{DSS}$ at $V_P/2$.

$$R_{DS} = \frac{1}{g_{max}}$$
$$= \frac{2I_{DSS}}{V_P} \left( \frac{-V_{GS}}{V_P} + 1 \right)$$
$$= \frac{V_P^2}{2I_{DSS} (V_P - V_{GS})} \quad (3-1)$$

Equation (3-1) is the A-C drain-to-source resistance, evaluated around $V_{DS} = 0$, that is controlled by D-C bias voltage $V_{GS}$ applied to the high-impedance gate terminal. Minimum $R_{DS}$ occurs when $V_{GS} = 0$ and, as $V_{GS}$ approaches the pinch-off voltage, $R_{DS}$ rapidly increases. A comparison of Figures 3-2 and 3-3, for $V_{DS} < \pm 0.1$ volts and $V_{GS} = \text{constant}$, the FET has a bilateral characteristic offset voltage, just like a fixed resistor. However, when $V_{DS} > \pm 0.1$ volts, the FET's characteristic has noticeable curvature.

The FET has the characteristic for applications where the drain-to-source voltage is a low-level A-C
Figure 3-1 Typical Output Characteristics of FET in the Triode Region.

\[ \frac{I_D}{I_{DSS}} = \frac{V_{DS}}{V_P} \left[ 2 \left( \frac{V_{GS}}{V_P} - 1 \right) - \frac{V_{DS}}{V_P} \right] \]
Figure 3-2 Low Level Bidirectional Output Characteristics for Typical N-Channel FET.
Figure 3-3 V-I Characteristics of Fixed Resistors.
\[ V_P^* = \text{value of } V_{GS} \text{ when } I_D/I_{DSS} = .001 \]

\[ R'_{DS} = \text{value of } R_{DS} \text{ when } V_{GS} = 0 \]

Figure 3-4 Normalized \( R_{DS} \) VS \( V_{GS} \) for Typical FET.
signal with no D-C component. Thus the FET's operating point will swing symmetrically around $V_{DS} = 0$. In the first quadrant, signal distortion depends on to what extent the FET output characteristic deviates from a straight line or linear relation. Besides the linearity problem in the third quadrant, when $V_{GS}$ is near zero and $V_{DS} > 0.5$ volts RMS, the gate-channel junction will become forwardbiased and cause additional curvature in the characteristic. Also, whenever the gate becomes forward biased due to any combination of $V_{GS}$ and $V_{DS}$, it ceases to be a high-impedance control terminal for the FET. In order to maintain the high-impedance characteristic of the gate, a diode is connected in series with the gate as shown in Figure 3-5.

Figure 3-4 represents a typical plot of normalized $R_{DS}$ versus $V_{GS}$, evaluated at $V_{DS} = 0$ with a low level of A-C drain-to-source voltage. For applications that require matched pairs, or close tracking from one unit to another, avoid operating at high resistance because, as $V_{GS}$ approaches $V_P$, $R_{DS}$ increases very rapidly so that unit-to-unit matching is almost impossible. An alternative method of matching resistances of two FET's is to control them independently.

When the input signal to the filter is a frequency modulated signal with the amplitude of 0.4 volts peak-
to-peak and its frequency range is 50 HZ to 5 KHZ, from Figure 3-1, it is possible to obtain $I_D$ and $V_{DS}$ at any $V_{GS}$ with a low level of deviation from a straight line. For a maximum deviation of not more than 10%,

$$I_D = 0.750 \ IDSS \ (1 - \frac{V_{GS}}{V_P})^2 \quad (3-2)$$

$$V_{DS} = 0.375 \ V_P \ (1 - \frac{V_{GS}}{V_P}) \quad (3-3)$$

A typical FET has the following characteristics:

$$IDSS = -100 \ mA \quad (3-4)$$

$$V_P = -6 \ volts \quad (3-5)$$

From Equations (3-2) and (3-4),

$$I_D = 0.750 \ IDSS$$

$$= 0.750 \times (-100 \times 10^{-3})$$

$$= -75 \ mA \ @ \ V_{GS} = 0 \quad (3-6)$$

From Equations (3-3) and (3-5),

$$V_{DS} = 0.375 \ V_P$$

$$= 0.375 \times (-6)$$

$$= -2.25 \ volts \ @ \ V_{GS} = 0 \quad (3-7)$$
From Equations (3-1), (3-6), and (3-7),

\[ R_{\text{DS(ON)}} = \frac{(-6)^2}{2(-100 \text{m}) (-6-0)} = 30 \text{ Ohms} \]

\[ = R_{\text{DS}} \bigg|_{V_{GS} = 0} \]

(3-8)

Let \( R_{\text{DS}} \big|_{f_0 = 5 \text{ K Hz}} \)

\[ = R_{\text{DS}} \bigg|_{V_{GS} = 0} = 30 \text{ Ohms} \]

(3-9)

For tuned frequency \( f_0 = 50 \text{ Hz} \), the drain-to-source resistance \( R_{\text{DS}} \big|_{f_0 = 50 \text{ Hz}} \) is increased as follows:

\[ R_{\text{DS}} \big|_{f_0 = 50 \text{ Hz}} = 100 \times R_{\text{DS}} \bigg|_{f_0 = 5 \text{ K Hz}} = 100 \times 30 \]

\[ = 3 \text{ K Ohms} \]

(3-10)

From Equation (3-1),

\[ V_{GS} = V_P - \frac{V_P^2}{2IDSS \cdot R_{\text{DS}}} \]

(3-11)

And,

\[ V_{GS} \bigg|_{f_0 = 50 \text{ Hz}} = -6 - \frac{(-6)^2}{2(-100 \text{m}) \cdot 3 \text{K}} \]

\[ = -5.94 \text{ volts} \]

(3-12)
With $V_{GS} = -5.94$ volts into Equations (3-2) and (3-3)

$$I_D = 0.750 \times IDSS \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$= 0.750 \times (-100 \times 10^{-3}) \left(1 - \frac{5.94}{6}\right)^2$$

$$= -7.5 \text{\,ua} \quad (3-13)$$

$$V_{DS} = 0.375 \times V_P \left(1 - \frac{V_{GS}}{V_P}\right)$$

$$= 0.375 \times (-6) \times \left(1 - \frac{5.94}{6}\right)$$

$$= -22.5 \text{\,mv} \quad (3-14)$$

Therefore, the maximum allowable voltage for a maximum waveform distortion of 10% is 22.5 millivolts. If the actual input signal is 0.40 volts peak-to-peak, buffer amplifiers with gains have to be used. The gain of the buffer amplifier at the input is

$$A_{B1} = \frac{-V_{allow}}{V_{IN \text{ Actual}}}$$

$$= \frac{-22.5 \times 10^{-3}}{0.40 \times 1/2}$$

$$= -112 \times 10^{-3}$$

$$A_{B1} = -0.01 \quad (3-15)$$

used
The gain of the output buffer amplifier is

\[ A_{BP} \frac{1}{A_{B1 \text{ used}}} = \frac{1}{-0.01} = -100 \]

\[ A_{BP} = -100 \quad \text{(3-16)} \]

It is very important to design the filter with integrated circuit operational amplifiers which will operate at low input voltage and low input current as indicated by Equations (3-13) and (3-14).

If FET's are used as variable resistors R5 and R6, and controlled independently, the maximum usable frequency range of the filter is between 10 Hz to 10 K Hz. If the two FET's are controlled by the same phase detector as shown in Figure 2-12, the practical usable frequency range of the filter is less than 10 Hz to 10 K Hz because of resistance deviation from typical FET characteristics as shown in Figure 3-4. Matching of the two transistor drain-to-source resistances will maximize the usable filter frequency range.

The basic adaptive filter with its two buffer amplifiers is shown in Figure 3-5. The values of resistors R1, R2, R3 and R4 are chosen in such a way that they will
provide sufficient input current to the integrated circuit operational amplifiers without loading down the outputs of those amplifiers.

Resistors R5A and R6A limit the maximum values of the variable resistors; so that, the integrated circuit operational amplifiers will not be saturated under normal operating conditions. Diodes CR1 and CR2 are used to protect transistors Q1 and Q2 to become forward biased. Resistors R9 and R10 provide biased current for Q1 and Q2 respectively. The value of resistance is not critical for these two resistors. Buffer amplifiers AR4 and AR5 have gain of -0.01 and -100 respectively.

Resistors R7, R8, R13, and R16 are used to reduce the amplifier output voltage offset because of input bias currents. The values of these resistors are not important because the bias currents are much less than the input signal currents.

The phase detector as shown in Figure 2-12 is the controlling circuit of the adaptive filter. It compares the phase relationship of the output voltage to that of the input voltage. Should the phase of the output voltage deviate from its normal value, a new voltage is developed at the gates of transistors Q1 and Q2. The drain-to-source resistances of these two transistors will change to new values until the phase of the output
voltage recovers to its normal state. A phase detector is shown in Figure 3-6 for analysis.

The input signal $V_{IN}$ is being amplified 100 times and used as the reference signal for chopper transistors $Q_3$ and $Q_4$. The output voltages of $AR_1$ and $AR_3$ are being amplified 100 times also. $AR_8$ and $AR_{10}$ are unity gain inverting amplifiers to invert the voltages of $AR_9$ and $AR_{11}$ respectively. Transistors $Q_3$ and $Q_4$ chop the output voltages from amplifiers $AR_8$, $AR_9$, $AR_{10}$ and $AR_{11}$. If the adaptive filter is operating at tuned frequency, there will have no DC voltage component at the chopper transistors. If the filter is not operating at tuned frequency, there will have DC voltage at the chopper transistors. The integrator consists of $AR_7$, $C_3$, $R_{21}$, $R_{23}$ and $R_{25}$, and it will integrate the DC voltage and change the variable resistors $R_5$ and $R_6$ to their new values until the adaptive filter is at tuned frequency again. All component values of this phase detector are not critical because they are used for the returning of the filter only.

The voltage waveforms of the phase detector at tuned frequency is presented in Figure 3-7.

Mechanization of the adaptive filter is now completed and is shown in Figure 3-8. This filter contains all basic elements of an adaptive filter. With some modifications in the circuit, one may change the parameters of the filter to achieve a specific requirement. All com-
ponent values of this filter are listed in Table 3-1 for a quality factor of two.

The rate of change of output voltage $V_{AR7}$ will determine the capturing time of the filter.

$$\frac{dV_{AR7}}{dt} = 0.425 \frac{1+k^2}{(1-k^2) + j0.5k} \cos \left[ \tan^{-1} \frac{k}{2(1-k^2)} \right]$$

(3-17)

The relationship between $dV_{AR7}/dt$ and $k$ is tabulated in Table 3-II and is plotted in Figure 3-9. Under normal operating condition, when the input signal is at a fixed frequency, the output voltage of the integrator will have a fixed value. When the adaptive filter is not in tune with the input frequency, the phase detector will change the gate voltages of the two variable transistors until a new equilibrium is achieved. The maximum rate of change of voltage is at $f_{\text{in}}=0.76f_0$ or $f_{\text{in}}=1.24f_0$ as shown in Figure 3-9.

for $k = 0.76$,

$$dV_{AR7}/dt = 0.870 \angle -42^\circ 50'$$

(3-18)

The minimum time for the input signal moved from one end of the frequency range to the other with a 3 dB loss of the input signal is

$$\int_0^T dt = \frac{1}{0.870} \int_0^6 dV_{AR7}$$

(3-19)
Figure 3-5  The Basic Adaptive Filter With Its Two Buffer Amplifiers.
Figure 3-6 The Phase Detector of the Adaptive Filter.
Figure 3-7 The Voltage Waveforms of the Phase Detector at Tuned Frequency.
Figure 3-8  Detailed Diagram of the Adaptive Filter.
Therefore,

\[ T = 6.9 \text{ second} \]  \hspace{1cm} (3-20)

The maximum capturing time for the filter to get in tune with the input signal is

\[ \text{dt} = \frac{1}{0.425} \, \text{d}V_{AR7} \]  \hspace{1cm} (3-21)

Therefore,

\[ T_{\text{CAP}} = 14.2 \text{ seconds} \]  \hspace{1cm} (3-22)

A detailed block diagram of the adaptive filter is presented on Figure 3-10. The adaptive filter consists of the bandpass filter and the phase detector. The phase detector is used to control the resistances of resistors R5 and R6 to keep the filter in tune with the input signal. The relationships of these blocks are indicated by arrows for clarity. Amplifier AR4 is used to provide high input impedance and attenuation of input signal. Amplifier AR5 is used to provide low output impedance for the load and high gain for the input signal. Amplifiers AR1, AR2 and AR3 are the basic parts of the band-pass filter. Amplifier AR6 is used as a buffer for the input signal with a high gain and is used as the reference for chopper transistors Q3 and Q4. Amplifiers AR9
and AR11 are high gain amplifiers to increase the signals for further processing. Most of the amplifier offsets can be reduced by individual amplifier adjustments. However, the offsets due to the drift of amplifiers can not be reduced by simple adjustments. Amplifiers AR8 and AR10 are unity gain amplifiers and are used to compensate for the offsets from amplifiers AR9 and AR11. Transistors Q3 and Q4 form a demodulator. Input from transistors Q3 and Q4 is than integrated by amplifier AR7 to change the resistances of R5 and R6 until the adaptive filter is in tune with the input signal.

The frequency range of the filter can be increased by increasing the resistance ranges of variable resistors R5 and R6. Suppose the usable resistance range of a FET is from 60 Ohms to 6 K Ohms, it is possible to increase the range by cascading amplifiers as shown in Figure 3-11.

For one FET input stage,

\[ I_{R5} = \frac{V_{AR1}}{R5} \]

\[ = \frac{V_{AR1}}{6K} \quad \text{Minimum Current} \quad (3-23) \]

\[ I_{\text{max}} = \frac{V_{AR1}}{60} \quad \text{Maximum Current} \quad (3-24) \]
Table 3-I Component Values of the Adaptive Filter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1 K</td>
<td>R22</td>
<td>100 K</td>
</tr>
<tr>
<td>R2</td>
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</tr>
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<td>1 M</td>
<td>R27</td>
<td>5.1 K</td>
</tr>
<tr>
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<td>10 K</td>
<td>R28</td>
<td>10 K</td>
</tr>
<tr>
<td>R8</td>
<td>10 K</td>
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<td>10 K</td>
</tr>
<tr>
<td>R14</td>
<td>100</td>
<td>R35</td>
<td>10 K</td>
</tr>
<tr>
<td>R15</td>
<td>10 K</td>
<td>R36</td>
<td>5.1 K</td>
</tr>
<tr>
<td>R16</td>
<td>100</td>
<td>R37</td>
<td>10 K</td>
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<tr>
<td>R17</td>
<td>1 K</td>
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<td>R18</td>
<td>100 K</td>
<td>R39</td>
<td>100</td>
</tr>
<tr>
<td>R19</td>
<td>1 K</td>
<td>C1</td>
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</tr>
<tr>
<td>R20</td>
<td>100 K</td>
<td>C2</td>
<td>.5</td>
</tr>
<tr>
<td>R21</td>
<td>51 K</td>
<td>C3</td>
<td>1.5</td>
</tr>
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All Resistances are in Ohms.

All Capacitances are in uF.
Table 3-II  \( \frac{dV_{AR7}}{dt} \) as a Function of \( k \)

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \frac{1+k^2}{\sqrt{(1-k^2)^2+(0.5k)^2}} )</th>
<th>( \tan^{-1} \frac{k}{2(1-k^2)} )</th>
<th>( \cos \left[ \tan^{-1} \frac{k}{2(1-k^2)} \right] )</th>
<th>( \frac{dV_{AR7}}{dt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0°</td>
<td>1</td>
<td>0.425</td>
</tr>
<tr>
<td>.1</td>
<td>1.02</td>
<td>2°53′</td>
<td>0.999</td>
<td>0.433</td>
</tr>
<tr>
<td>.2</td>
<td>1.14</td>
<td>5°36′</td>
<td>0.995</td>
<td>0.482</td>
</tr>
<tr>
<td>.4</td>
<td>1.34</td>
<td>13°23′</td>
<td>0.973</td>
<td>0.553</td>
</tr>
<tr>
<td>.6</td>
<td>1.93</td>
<td>29°39′</td>
<td>0.869</td>
<td>0.712</td>
</tr>
<tr>
<td>.7</td>
<td>2.41</td>
<td>34°30′</td>
<td>0.824</td>
<td>0.842</td>
</tr>
<tr>
<td>.8</td>
<td>3.05</td>
<td>47°59′</td>
<td>0.669</td>
<td>0.867</td>
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<tr>
<td>.9</td>
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<td>0.613</td>
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<td>0</td>
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<td>-0.569</td>
</tr>
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<td>-0.637</td>
</tr>
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<td>1.24</td>
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<td>1.12</td>
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<td>-0.472</td>
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<td>5</td>
<td>1.08</td>
<td>174°4′</td>
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<td>-0.457</td>
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<td>6</td>
<td>1.06</td>
<td>175°6′</td>
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<td>-0.448</td>
</tr>
<tr>
<td>10</td>
<td>1.02</td>
<td>177°0′</td>
<td>-0.999</td>
<td>-0.433</td>
</tr>
<tr>
<td>∞</td>
<td>1</td>
<td>180°</td>
<td>-1</td>
<td>-0.425</td>
</tr>
</tbody>
</table>
Figure 3-9 Plot of $\frac{dv_{C3}}{dt}$ vs Normalized Frequency.
Figure 3-10 Detailed Block Diagram of the Adaptive Filter.
Figure 3-11 Increase Resistance Range by Cascading of Amplifiers.
For three FET's input stage,

\[ I_{R5c} = \frac{V_{AR1}}{R_{5a} R_{5b} R_{5c}} \]

\[ = \frac{V_{AR1}(6K)}{(6K)(6K)(6K)} \]

\[ = \frac{V_{AR1}}{6K} \text{ Minimum Current} \quad (3-25) \]

\[ I_{R5c} = \frac{V_{AR1}(6K)}{(60)(60)(60)} \]

\[ = \frac{V_{AR1}}{60} \times 10^4 \text{ Maximum Current} \quad (3-26) \]

It can be seen that the minimum currents for both cases are identical; however, the one with three FET's has a much higher maximum current as compared with the one with only one FET.

If maximum attenuation in the stopband is required, it is only necessary to select the number of adaptive filters and connect them in series to achieve the desired performance. The greater the number of filters used, the closer the filter approaches an ideal response. However, cost and size of the complete filter will increase proportionally, so a tradeoff may be necessary. Figure 3-12 shows the characteristics of the cascaded bandpass filter.
Figure 3-12 Characteristics of the Cascaded Bandpass Filters.
CHAPTER IV
SENSITIVITY TO PARAMETER ERRORS

A block diagram of the adaptive filter is presented in Figure 4-1 for analysis. Its important characteristics are covered by Equations (4-1) and (4-2). All important parameters of the filter can be generated from these two basic equations. Since all component values contribute to the performance of the filter, they are studied individually to understand the affect to the filter parameter errors.

\[
\frac{V_{BP}}{V_{IN}} = - \frac{R_{12}R_{15}}{R_{11}R_{14}} \frac{R_{4}(R_{2}+R_{3})}{R_{2}R_{5}C_{1}(R_{1}+R_{4})} \frac{S}{S^{2} + \frac{R_{1}(R_{2}+R_{3})}{R_{2}R_{5}C_{1}(R_{1}+R_{4})} \frac{S}{R_{3}}} \frac{R_{3}}{R_{2}R_{5}R_{6}C_{1}C_{2}}
\]

(4-1)

\[
\frac{dV_{AR7}}{dt} = \frac{V_{IN}}{\pi C_{3}} \left\{ \cos \left[ \tan^{-1} \left( \frac{k}{1-k^{2}} \frac{R_{1}(R_{2}+R_{3})}{R_{1}+R_{4}} \left( \frac{R_{6}C_{2}}{R_{2}R_{3}R_{5}C_{1}} \right) \right] \right\} \times \frac{R_{4}(R_{2}+R_{3})(R_{5}C_{1})^{\frac{1}{2}}}{(1-k^{2})R_{3}(R_{1}+R_{4})(R_{2}R_{5}C_{1})^{\frac{3}{2}} + jkR_{1}(R_{2}+R_{3})(R_{3}R_{6}C_{2})^{\frac{1}{2}}} \times \left\{ \left[ \frac{R_{33}}{R_{21}R_{33} + R_{22}(R_{21} + R_{33})} + \frac{R_{32}}{R_{23}R_{32} + R_{24}(R_{23} + R_{24})} \frac{R_{26}}{R_{26}} \right] + \frac{R_{30}R_{12}}{R_{29}R_{11}} (R_{2})^{\frac{1}{2}} \right\} \times \left[ \frac{R_{22}R_{34}}{R_{21}R_{33} + R_{22}(R_{21} + R_{33})} \frac{R_{35}}{R_{35}} \right]
\]
Figure 4-1. Block Diagram of Finalized Adaptive Filter.
\[ + \frac{R_{24}}{R_{23}R_{24}+R_{32}(R_{23}+R_{24})} \left( \frac{R_{37}R_{12}}{R_{38}R_{11}} k^2 R_3 \left\{ \frac{1}{R_2^2} \right\} \right) \]  

(4-2)

From Equation (4-1), the following parameters may be obtained:

\[ A_{BP} = -\frac{R_{12}R_{15}R_4}{R_{11}R_{14}R_1} \]  

(4-3)

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2 R_5 R_6 C_1 C_2}} \]  

(4-4)

\[ Q = \frac{R_1+R_4}{R_1(R_2+R_3)} \sqrt{\frac{R_2 R_3 R_5 C_1}{R_6 C_2}} \]  

(4-5)

By partial differentiation, the sensitivity of the above parameters may be derived.

From Equation (4-3),

\[ \frac{A_{BP}}{A_{BP}} = \frac{R_4}{R_4} + \frac{R_{12}}{R_{12}} + \frac{R_{15}}{R_{15}} - \frac{R_1}{R_1} - \frac{R_{11}}{R_{11}} - \frac{R_{14}}{R_{14}} \]  

(4-6)

From Equation (4-4),

\[ \frac{f_0}{f_0} = \frac{1}{2} \left[ \frac{R_3}{R_3} - \frac{R_2}{R_2} - \frac{R_5}{R_5} - \frac{R_6}{R_6} - \frac{C_1}{C_1} - \frac{C_2}{C_2} \right] \]  

(4-7)

From Equation (4-5),

\[ \frac{Q}{Q} = \frac{1}{2} \left[ \frac{R_4}{R_4} + \frac{R_5}{R_5} - \frac{R_1}{R_1} - \frac{R_6}{R_6} + \frac{C_1}{C_1} - \frac{C_2}{C_2} \right] \]  

(4-8)

The results of parameter errors because of component deviations as shown in Equations (4-6), (4-7) and (4-8)
are listed in Table 4-1. The sensitivity of parameter \( \frac{dV_{AR7}}{dt} \) will not be calculated because it involves only the response time of the filter and it has enough margin of speed for most applications. Furthermore, the complexity of the terms involved in this parameter will not yield a meaningfully generalized picture for design purposes. It provides useful information for specific operating conditions as mentioned in Figure 3-9 and Equations (3-20) and (3-22).
Table 4-I  Sensitivity to Parameter Errors

<table>
<thead>
<tr>
<th>COMPONENTS</th>
<th>PARAMETER ERRORS (%)</th>
<th>( \frac{A_{BP}}{A_{BP}} )</th>
<th>( \frac{f_0}{f_0} )</th>
<th>( \frac{Q}{Q} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>R2</td>
<td>1</td>
<td>1</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>R3</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>R4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>R5</td>
<td>1</td>
<td>0</td>
<td>-0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>R6</td>
<td>1</td>
<td>0</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>R11</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>R12</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>R14</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>R15</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C1</td>
<td>1</td>
<td>0</td>
<td>-0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>0</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
CHAPTER V
CONCLUSIONS

1. TEST RESULTS:
The adaptive bandpass filter is breadboarded as shown in Figure 5-1 for testing. Amplifier AR12 is used as summing point for the input signal and injected noise. The test results are presented below.

The frequency of input signal $V_{IN}$ is 500 Hz with injected random noise of 500 K Hz range.

Figure 5-2. Test Result A
Figure 5-1. Block Diagram of the Test Circuit.
The frequency of input signal $V_{IN}$ is 500 Hz with injected random noise of 20 KHz range.

Figure 5-3. Test Result B.
The frequency of input signal $V_A$ is 500 Hz.

Figure 5-4. Test Result C.
Figure 5-5. Response of Amplitude Modulated Signal.
Figure 5-6 Test Results of the Filter Gain and Phase.
From Figure 5-6, it can be seen that the phase shift of the bandpass filter occurs at both ends of the frequency range. At the high frequency end the phase shift is caused by the saturation of the two field-effect transistors. At the low frequency end the phase shift is caused by the stability of the gate-to-source voltage $V_{GS}$. To reduce the phase shift at the low frequency end of the bandpass range, it is necessary to increase the integrator time constant for the improvement of $V_{GS}$ stability. However, a long integrator time constant will increase the time required for the tuning of the filter to an input signal. A best solution for this problem is to use a variable integrator time constant in the same way as the integrators used in the basic bandpass filter.
2. COMPARISON OF ADAPTIVE FILTER AND CONVENTIONAL FILTERS:

From Table 5-1, it can be seen that most of the conventional filters are designed for specific applications. For amplitude modulated signal, the filter must have a fixed center frequency with a very high quality factor to reject noise. For frequency modulated signal, the filter must have a wide bandwidth to cover the frequency range and attenuate the lower voltage portion of the incoming signal to reduce noise. The two types of filters are special designs for particular operating conditions and cannot be used as general purpose filters.

The adaptive filter, however, provides stable, automatically tuned, and high quality factor signal processing for an amplitude modulated signal, a frequency modulated signal, or a combination of an amplitude and frequency modulated signal. This filter has also the capability of lowpass and highpass filtering.

Table 5-II indicates the relative cost and size between a conventional and an adaptive filters. The cost of an adaptive filter will decrease in the future because the filter can be produced in large quantities with no high cost of circuit adjustment required. The use of an adaptive filter is the best choice if circuit adjustment is expensive or impossible inside a system, or minimiza-
tion of inventory of filter types is a requirement.
### Table 5-I Capability of Conventional and Adaptive Filters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simple Amplitude Modulated Signal Filter</th>
<th>Simple Frequency Modulated Signal Filter</th>
<th>Adaptive Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capability to filter frequency modulated signal</td>
<td>Not Capable. This filter does not have the bandwidth for the F.M. signal.</td>
<td>Good. This filter does not have the high quality factor to reject noise in the F.M. signal.</td>
<td>Excellent. This filter has the high quality factor to reject noise and the frequency swing for the F.M. signal.</td>
</tr>
<tr>
<td>Capability to filter amplitude modulated signal</td>
<td>Good. Stability of the center frequency of this filter depends on the stability of many circuit components.</td>
<td>Not Capable. This filter does not have the high quality factor to reject noise and it tends to distort the input A.M. signal.</td>
<td>Excellent. This filter has the high quality factor to reject noise and is self tuning to maintain the stability of the center frequency.</td>
</tr>
<tr>
<td>Capability of Highpass or lowpass filtering</td>
<td>No.</td>
<td>No.</td>
<td>Yes.</td>
</tr>
</tbody>
</table>
Table 5-II  Cost and Size of Conventional and Adaptive Filters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Cost and Size</th>
<th>Cost and Size</th>
<th>Adaptive Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simple Amplitude Modulated Signal Filter</td>
<td>Simple Frequency Modulated Signal Filter</td>
<td>Adaptive Filter</td>
</tr>
<tr>
<td>Cost of Filter</td>
<td>Low. This filter is very simple; however, it requires some high stability components and circuit adjustment.</td>
<td>Low. This Filter is more complex than the A. M. signal filter; however, it requires no high stability component and circuit adjustment.</td>
<td>High. This filter is about four times as expensive as the other types because of circuit complexity; however, it requires no high stability component and circuit adjustment.</td>
</tr>
<tr>
<td>Size of Filter</td>
<td>Small. This filter is very simple.</td>
<td>Small. This filter is larger than the A. M. signal filter.</td>
<td>Large. This filter is about four times as large as the A.M. signal filter.</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


APPENDIX A
LOWPASS FILTER

From Figure 2-3 the block diagram can be re-arranged to obtain the transfer function of the lowpass filter as shown in Figure A-1. The final block diagram indicates that the lowpass filter has two poles in its transfer function. For stable operation of the filter, the phase margin of the frequency response of the circuit has to be considered. The transfer function of the lowpass filter is written below for analysis and its root-locus is shown in Figure A-2.

\[
\frac{V_{lp}(S)}{V_{1}(S)} = \frac{R_4(R_2+R_3)}{S^2 + \frac{R_1(R_2+R_3)}{R_2R_5R_6C_1C_2(R_1+R_4)} S + \frac{R_3}{R_2R_5R_6C_1C_2}}
\]

\[\text{(A-1)}\]

Figure A-2. Root-Locus of Equation (A-1).
Figure A-1  (a) Original Block Diagram;
(b) Block Diagram after the reduction of
the inner Feedback loop for a lowpass
filter.
(c) Block Diagram after the reduction of
the outer Feedback loop;
(d) Simplification of the Block Diagram;
(e) Final Block Diagram of a lowpass
filter.
The standard form of the lowpass filter is

\[
\frac{V_{LP}(s)}{V_1(s)} = \frac{K_7}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}
\]  \hspace{1cm} (A-2)

A comparison of Equations (A-1) and (A-2) reveals that the lowpass filter has the following characteristics:

\begin{align*}
A_{LP} & = \text{The gain of the lowpass filter at the tuned frequency} \\
& = \frac{R_4(R_2+R_3)}{R_2R_5R_6C_1C_2(R_1+R_4)} \frac{R_1(R_2+R_3)}{R_1R_6C_2} \frac{1}{s} \\
& = \frac{R_4}{R_1R_6C_2} (A-3)
\end{align*}

\begin{align*}
f_0 & = \text{The tuned frequency of the lowpass filter} \\
& = \frac{1}{2\pi} \omega_0 \\
& = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2R_5R_6C_1C_2}} (A-4)
\end{align*}

And

\begin{align*}
Q & = \text{The quality factor} \\
& = \frac{\sqrt{R_3}}{\sqrt{R_2R_5R_6C_1C_2}} \frac{\sqrt{R_3R_5C_1}}{R_1(R_2+R_3)} \frac{1}{\sqrt{R_2R_5C_1(R_1+R_4)}} \\
& = \frac{R_2(R_1+R_4)}{R_1(R_2+R_3)} \frac{\sqrt{R_3R_5C_1}}{\sqrt{R_2R_6C_2}} (A-5)
\end{align*}
For conventional lowpass active filter, all component values of the filter are constants; therefore, it can be seen from Equations (B-3), (B-4), and (B-5) that the tuned frequency $f_0$, gain $A_{LP}$, and quality factor $Q$ of the lowpass filter are constants.

For frequency modulated input signal, this lowpass active filter can be modified for adaptive filtering. The gain $A_{LP}$ and quality factor $Q$ shall be kept constant under normal operating condition. The tuned frequency $f_0$ can be varied by changing the resistances of resistors $R_5$ and $R_6$ without affecting the D. C. gain $A_{LP}$ and quality factor $Q$.

Let

$$R_5 = R_6 \quad (A-6)$$

Then

$$A_{LP} = \frac{R_4}{R_1 R_6 C_2 S}$$

$$= K_8 \quad (A-7)$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2 C_1 C_2}} \sqrt{\frac{1}{R_5 R_6}}$$

$$= K_2 \sqrt{\frac{1}{R_5 R_6}} \quad (A-8)$$

$$Q = \frac{R_2 (R_1 + R_4)}{R_1 (R_2 + R_3)} \sqrt{\frac{R_3 C_1}{R_2 C_2}} \frac{R_5}{\sqrt{R_6}}$$

$$= K_3 \quad (A-9)$$
Where $K_2$, $K_3$, and $K_8$ are constants.

The characteristics of a lowpass filter with different values of quality factor $Q$ are shown in Figure A-3.
Figure A-3 Characteristics of Lowpass Filter with Different Values of Quality Factor Q.
From Figure 2-3 the block diagram can be re-arranged to obtain the transfer function of the highpass filter as shown in Figure B-1. The final block diagram indicates that the highpass filter has two zeros and two poles in its transfer function. The two zeros are located at the origin of the root-locus diagram as shown in Figure B-2. This highpass is stable because it has a phase margin of 90° or better. The transfer function of the highpass filter is written below for analysis.

\[
\frac{V_{HF}(S)}{V_1(S)} = \frac{\frac{R4(R2+R3)}{R2(R1+R4)} S^2}{S^2 + \frac{R1(R2+R3)}{R2R5C1(R1+R4)} S + \frac{R3}{R2R5R6C1C2}} \quad \text{(B-1)}
\]

Figure B-2. Root-Locus of Equation (B-1)
Figure B-1  (a) Original Block Diagram;
(b) Block Diagram after the simplification of the Feedback loops for a Highpass Filter;
(c) Block Diagram after the reduction of the Feedback loops;
(d) Simplification of the Block Diagram;
(e) Final Block Diagram of a Highpass Filter.
The standard form of the highpass filter is

\[
\frac{V_{HP}(S)}{V_1(S)} = \frac{K_0 S^2}{S^2 + \frac{\omega_0}{Q} S + \omega_0^2}
\]  

(B-2)

From Equations (B-1) and (B-2),

\[A_{HP} = \text{The gain of the highpass filter at the tuned frequency}\]

\[= \frac{R_4(R_2+R_3)}{R_2(R_1+R_4)} \frac{S^2}{R_1(R_2+R_3)} \frac{S}{R_2R_5C_1(R_1+R_4)}
\]

\[= \frac{R_4R_5C_1}{R_1} \]  

(B-3)

\[f_0 = \text{The tuned frequency of the highpass filter}\]

\[= \frac{1}{2\pi} \omega_0
\]

\[= \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2R_5R_6C_1C_2}} \]  

(B-4)

And

\[Q = \text{The quality factor}\]

\[= \sqrt{\frac{R_3}{R_2R_5R_6C_1C_2} \frac{R_1(R_2+R_3)}{R_2R_5C_1(R_1+R_4)}}
\]

\[= \frac{R_2(R_1+R_4)}{R_1(R_2+R_3)} \sqrt{\frac{R_3R_5C_1}{R_2R_6C_2}} \]  

(B-5)
For conventional highpass active filter, all component values of the filter are constants; therefore, the tuned frequency $f_0$, gain $A_{HP}$, and quality factor $Q$ of the highpass filter are constants.

For frequency modulated input signal, this highpass active filter can be modified for adaptive filtering. The gain $A_{HP}$ and quality factor $Q$ shall be kept constant under normal operating condition. The tuned frequency $f_0$ can be varied by changing the resistances of resistors $R_5$ and $R_6$ without affecting the gain $A_{HP}$ and quality factor $Q$.

Let

$$R_5 = R_6$$  \hspace{1cm} (B-6)

Then

$$A_{HP} = \frac{R_4 R_5 C_1 S}{R_1}$$

$$= K_{10}$$  \hspace{1cm} (B-7)

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{R_3}{R_2 C_1 C_2}} \sqrt{\frac{1}{R_5 R_6}}$$

$$= K_2 \sqrt{\frac{1}{R_5 R_6}}$$  \hspace{1cm} (B-8)

$$Q = \frac{R_2 (R_1 + R_4)}{R_1 (R_2 + R_3)} \sqrt{\frac{R_3 C_1}{R_2 C_2}} \sqrt{\frac{R_5}{R_6}}$$

$$= K_3$$  \hspace{1cm} (B-9)

Where $K_2$, $K_3$, and $K_{10}$ are constants.
The characteristics of a highpass filter with different values of quality factor Q are shown in Figure B-3.
Figure B-3  Characteristics Of Highpass Filter With Different Values of Quality Factor.
APPENDIX C
DEMODULATOR

For voltage waveforms of Figure 3-7, the D-C voltage of the chopped signal can be calculated in the following steps:

\[ \frac{V_{O, AR9}}{V_{IN}} = (-A_{AR9}) \frac{V_{LP}}{V_I} (-A_{AR4}) \]  \hspace{1cm} (C-1)

\[ \frac{V_{O, AR8}}{V_{IN}} = (-A_{AR8})(-A_{AR9}) \frac{V_{LP}}{V_I} (-A_{AR4}) \]  \hspace{1cm} (C-2)

\[ \frac{V_{O, AR11}}{V_{IN}} = (-A_{AR11}) \frac{V_{HP}}{V_I} (-A_{AR4}) \]  \hspace{1cm} (C-3)

\[ \frac{V_{O, AR10}}{V_{IN}} = (-A_{AR10})(-A_{AR11}) \frac{V_{HP}}{V_I} (-A_{AR4}) \]  \hspace{1cm} (C-4)

\[ V_{CQ3} = V_{CQ3} \bigg| V_{O, AR10}=0 + V_{CQ3} \bigg| V_{O, AR9}=0 \]

\[ = \left[ \frac{R_{21}}{R_{21}+R_{33}} V_{O, AR9} + \frac{R_{22}}{R_{22}+R_{33}} V_{O, AR10} \right] \times \frac{1}{2\pi} \int_{0}^{\theta_2} \sin \theta \, d\theta \]

\[ = \frac{1}{\pi} \left[ \frac{R_{21}R_{33}V_{O, AR9}}{R_{21}R_{33}+R_{22}(R_{21}+R_{33})} + \frac{R_{21}R_{22}V_{O, AR10}}{R_{21}R_{22}+R_{33}(R_{21}+R_{22})} \right] \times \cos \theta_1 \]  \hspace{1cm} (C-5)
From Table 2-1 and Equations (C-1) and (C-5),

\[
\begin{align*}
V_{CQ3} & = V_{IN} \left| V_0 = A R_{10} = 0 \right.
\end{align*}
\]

\[
- \frac{1}{n} \frac{R_{21} R_{33}}{R_{21} R_{33} + R_{22} (R_{21} + R_{33})} \frac{R_{30} R_{12}}{R_{29} R_{11}}
\]

\[
x \left[ \frac{R_4 (R_2 + R_3)}{R_2 R_5 R_6 C_1 C_2 (R_1 + R_4)} \frac{R_1 (R_2 + R_3)}{R_3} \frac{R_1 (R_2 + R_3) S + R_3}{R_2 R_5 R_6 C_1 C_2} \right] \cos \theta_1
\]

(C-6)

Let

\[
S^2 = - k^2 \frac{R_3}{R_2 R_5 R_6 C_1 C_2}
\]

(C-7)

Then

\[
S^2 + \frac{R_3}{R_2 R_5 R_6 C_1 C_2}
\]

\[
= (1 - k^2) \frac{R_3}{R_2 R_5 R_6 C_1 C_2}
\]

(C-8)

And

\[
\cos \theta_1
\]

\[
= \cos \left[ \tan^{-1} \frac{k}{1 - k^2} \frac{R_1 (R_2 + R_3)}{R_1 + R_4} \frac{R_6 C_2}{R_2 R_3 R_5 C_1} \right]
\]

(C-9)

Substitute Equations (C-7), (C-8), and (C-9) into Equation (C-6),
\[
\frac{V_{CQ3}}{V_{IN}} \bigg|_{V_0 \ AR10=0} = \frac{1}{w} \frac{R21R33}{R21R33+R22(R21+R33)} \frac{R30R12}{R4R22} \\
= x \left[ \frac{R4(R2+R3)(R2R5C1)}{(1-k^2)R3(R1+R4)(R2R5C1)^{\frac{1}{2}}+jR1(R2+R3)(R3R6C2)^{\frac{1}{2}}} \right] \\
= x \cos \left[ \tan^{-1} \frac{k}{l-k^2} \frac{R1(R2+R3)}{R1+R4} \left( \frac{R6C2}{R2R3R5C1} \right)^{\frac{1}{2}} \right]
\]

(C-10)

With similar methods,

\[
\frac{V_{CQ3}}{V_{IN}} \bigg|_{V_0 \ AR9=0} = \frac{1}{w} \frac{R21R22}{R21R22+R33(R21+R22)} \frac{R34R37R12}{R4R35R38R22} \\
= x \left[ \frac{k^2R3R4(R2+R3)(R5C1/R2)}{(1-k^2)R3(R1+R4)(R2R5C1)^{\frac{1}{2}}+jR1(R2+R3)(R3R6C2)^{\frac{1}{2}}} \right] \\
= x \cos \left[ \tan^{-1} \frac{k}{l-k^2} \frac{R1(R2+R3)}{R1+R4} \left( \frac{R6C2}{R2R3R5C1} \right)^{\frac{1}{2}} \right]
\]

(C-11)
And

\[ V_{CQ4} \]
\[ \frac{V_{OUT}}{V_{IN}} \]
\[ V_0 = 0 \]

\[ = \frac{1}{\pi} \frac{R_{23R24} R_{37R12}}{R_{23R24} + R_{32}(R_{23} + R_{24}) R_{38R11} R_{23R24}} \]

\[ x \left[ \frac{k^2 R_{3R4}(R_{2} + R_{3}) \left( \frac{R_{5C1}}{R_{2}} \right)^{\frac{1}{2}}}{(1-k^2)R_{3}(R_{1} + R_{4})(R_{2}R_{5C1})^{\frac{1}{2}} + jkR_{1}(R_{2} + R_{3})(R_{3}R_{6C2})^{\frac{1}{2}}} \right] \]

\[ x \cos \left[ \tan^{-1} \left( \frac{k}{1-k^2} \frac{R_{1}(R_{2} + R_{3}) \left( \frac{R_{6C2}}{R_{2}R_{3}R_{5C1}} \right)^{\frac{1}{2}}}{R_{1} + R_{4}} \right) \right] \]

(C-12)

And

\[ V_{CQ4} \]
\[ \frac{V_{OUT}}{V_{IN}} \]
\[ V_0 = 0 \]

\[ = \frac{1}{\pi} \frac{R_{23R32} R_{26R30R12}}{R_{23R32} + R_{24}(R_{23} + R_{24}) R_{28R29R11} R_{23R32}} \]

\[ x \left[ \frac{R_{4}(R_{2} + R_{3})(R_{2}R_{5C1})^{\frac{1}{2}}}{(1-k^2)R_{3}(R_{1} + R_{4})(R_{2}R_{5C1})^{\frac{1}{2}} + jkR_{1}(R_{2} + R_{3})(R_{3}R_{6C2})^{\frac{1}{2}}} \right] \]

\[ x \cos \left[ \tan^{-1} \left( \frac{k}{1-k^2} \frac{R_{1}(R_{2} + R_{3}) \left( \frac{R_{6C2}}{R_{2}R_{3}R_{5C1}} \right)^{\frac{1}{2}}}{R_{1} + R_{4}} \right) \right] \]

(C-13)
From Equations (C-10), (C-11), (C-12), and (C-13).

\[
\frac{dV_{AR7}}{dt} = \frac{I_{R21} + I_{R23}}{C3}
\]

\[
= \frac{V_{IN}}{\pi C3} \left( \cos \left[ \tan^{-1} \frac{k}{1-k^2} \frac{R1(R2+R3)}{R1+R4} \left( \frac{R6C2}{R2R3R5C1} \right)^{j1} \right] \right)
\]

\[
x = \frac{R4(R2+R3)(R5C1)^{j1}}{(1-k^2)R3(R1+R4)(R2R5C1)^{j1} + jKR1(R2+R3)(R3R6C2)^{j1}}
\]

\[
x \left\{ \frac{R33}{R21R33+R22(R21+R33)} + \frac{R32}{R23R32+R24(R23+R24)} \frac{R26}{R28} \right\}
\]

\[
x \left\{ \frac{R30R12}{R29R11} (R2)^{j1} + \left[ \frac{R22}{R21R33+R22(R21+R33)} \frac{R34}{R35} \right] \frac{R37R12}{R38R11} k^2R3 \left( \frac{1}{R2} \right)^{j1} \right\}
\]

(C-14)

Substitute values of Table 3-I into Equation (C-14)

\[
\frac{dV_{AR7}}{dt} = 0.425 \frac{l+k^2}{(1-k^2)+j0.5k} \cos \left[ \tan^{-1} \frac{k}{2(1-k^2)} \right]
\]

(C-15)