CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

FACTORING TRINOMIALS

A Tutorial Program

A thesis submitted in partial satisfaction of the requirements for the degree of Master of Science in Mathematics

by

Etta Saperstein

January, 1973

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The thesis of Etta Saperstein is approved:

Committee Chairman

California State University, Northridge
December, 1972
To my husband Seymour
ACKNOWLEDGMENT

I wish to express my gratitude to Mr. John W. McGee for his invaluable help in the preparation of the manuscript, to Dr. V. Peter Hansen for his encouragement, and to Mr. Donald L. Raun for his suggestions concerning the tutorial program. Thanks are also due to Nancy Pickering who typed the final version.
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ABSTRACT

FACTORING TRINOMIALS

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A tutorial program for factoring trinomials has been written for use on a time-sharing system. It is intended for computer-assisted instruction in a first year algebra course. In addition to the program, a flow chart and sample run are included.
The student can often optimize his educational experiences in a given subject by being allowed to proceed at his own rate of learning. For this reason the concept of individualized instruction has been introduced and found to be quite successful. One way of implementing this concept is through the use of computer-assisted instruction (CAI). Although time-sharing systems have been used in junior and senior high schools to teach programming, they can also be used advantageously in a learning situation to allow each student to work at his own rate of comprehension.

The topic of factoring trinomials has been chosen for programming because it lends itself easily to the step-by-step approach. A group of twenty-eight tenth-grade high school students taking first year algebra was given a sample lesson (see Appendix A) in factoring trinomials similar to the one that would be programmed. All of these students could be considered below average in mathematical ability based on previous aptitude tests and past grades. The papers were carefully analyzed for errors, and an item analysis was constructed (see page 2). Errors included in the category "difficulty finding correct combination of numerical factors" were of two types: 1) failure to identify factors of the constant term, and 2) confusion of the necessity for a sum or product of the factors in the linear and constant terms.

In developing the program consideration was given to the types and frequencies of the errors found. A flow
<table>
<thead>
<tr>
<th>TYPE OF ERROR</th>
<th>NUMBER OF ERRORS</th>
<th>PERCENTAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Could not factor at all</td>
<td>16</td>
<td>16.5%</td>
</tr>
<tr>
<td>Difficulty finding correct</td>
<td>40</td>
<td>41.2%</td>
</tr>
<tr>
<td>combination of numerical factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confusion of signs</td>
<td>35</td>
<td>36.1%</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>6</td>
<td>6.2%</td>
</tr>
<tr>
<td>Total</td>
<td>97</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Chart showing the general organization of the program can be found on page 7. The program was divided into three main sections. The first section deals with trinomials having only positive factors, the second with trinomials having negative coefficients in the linear term and positive integers in the constant term, and the third section deals with trinomials having positive or negative coefficients in the linear term and negative integers in the constant term; the sections being called respectively Tutor 1, 2, and 3.

Each section instructs the student using a question-and-answer technique. The first basic principle introduced in this program is that the process of factoring trinomials may be thought of as the reverse of multiplication of the factors giving a particular trinomial. From this point the student learns that this procedure is commutative. The student is then given a sample trinomial to factor. If he answers incorrectly, the program branches back, asking him simpler questions in order to help him discover his error.
Where necessary further explanation is provided, and the student is gradually led back to the original problem. The provision is made for three trials on any particular question, and if the student is still in error, he is instructed to consult his teacher and the program terminates.

After the student has answered three or four problems correctly, he is quizzed. Depending on the results of this quiz, he may either proceed to the next group of questions or is instructed to consult his teacher. At the end of the entire program, the student is given a short quiz in the form of ten trinomials to factor in order to test his understanding of the entire program. This quiz is also useful from the point of view that it provides both the student and the teacher an immediate evaluation of the student's performance.

Because the item analysis demonstrated that many students had difficulty in recognizing that the coefficient of the linear term is the sum of the factors and the constant term the product of the factors, these points are emphasized both in the tutorial part of the program and in the error analysis when a student is unable to respond correctly. In Tutor 1, 2, and 3 much emphasis is also placed upon the correct sign of the linear and constant terms.

A sample run which was actually done by one of the tenth-grade students in a first year algebra class at James
Monroe High School is included. It clearly demonstrates the functioning of the program. For clarification, the student responses have been underlined. To provide flexibility in the program the trinomials are stored in a data file. The data file used for the sample run is shown on page 18.

One of the most notable of the student responses was the enthusiasm generated. Eleven students volunteered their time on a Saturday morning to try the program. These students had been introduced to factoring in one class lesson prior to using the program. The students enjoyed themselves, and many asked to be allowed to return the following week to try more of these problems. They commented that they could learn without the fear of making mistakes. They also liked the idea that they could often find their own mistakes instead of having to ask for assistance from the teacher, and this contributed to their understanding of the factoring process. Immediate feedback provided the necessary encouragement and reinforcement to help their learning. In addition to the author's students, another teacher from the same high school used this program with a group of tenth-grade students in a first year algebra course. She found that they needed very little help, confirming the idea that the program is largely self-explanatory.

Many of the students thought that it would be easier to learn other topics in their first year algebra course
through the use of programmed teaching, especially word
problems and other types of factoring.

This program can be supplemented with other programmed
drill and practice, especially in the case of the slower
students. Since all the trinomials are stored in a data
file, it is important to note that this file can be changed
at the teacher's convenience in order to provide more
extensive practice or a short quiz. The post test, which
may also be changed at the teacher's discretion, allows for
a final evaluation that is important to both the teacher
and the student. The student may call in any one of the
three sections, Tutor 1, 2, or 3; it is not necessary that
he complete the entire run in one session.

There are many advantages to be gained from using CAI.
This program can be used to help slower learners such as
tenth-graders in understanding algebraic concepts. It will
also reinforce their previous learning experiences and,
therefore, may serve as a good review. After a short
introduction in the classroom, this type of program can
also be used to teach students in an accelerated math
program who study first year algebra in the eighth grade.

Finally, it points the way to the fact that many other
topics are also suitable for CAI programming. These
include the solution of both linear and quadratic equations,
operations with exponents, and additional types of
factoring. Probably the most challenging type of program
would be that which could be used as an aid in helping the
student analyze and solve word problems.

Since the student-teacher ratio of thirty to one is a hindrance to individualizing instruction, this program with its ability to explain and correct student errors can increase the valuable time of the teacher for use in other needed areas. This program, therefore, is intended not to replace the teacher but to serve as a valuable adjunct to the teaching process.

Because students respond with enthusiasm to programmed instruction, it is important to consider the possibilities which this type of instruction suggests. First, both the teacher and the student save time; the teacher in explaining certain types of errors which the program can explain to the student and the student because he does not waste his time doing problems on a topic he already fully understands. Secondly, if the student really does not understand the topic, the teacher can often tell by looking at his responses where he is having difficulties and can then explain this particular area to him. This allows each student to proceed as rapidly as his own ability will permit. Lastly, the student who has learning experiences which are reinforced and which he enjoys is more apt to take an active interest in delving further into the world of mathematics and other learning situations.
FORTRAN PROGRAM

159 COMMON IA(35),IVAR(35),IR(35),IC(35),ICOUNT(35),LINO(35),J(35)
160 15+ J(N)=1,35
160 COMMON IA(35),IVAR(35),IR(35),IC(35),ICOUNT(35),LINO(35),J(35)
160 OPEN(3) INPUT DATA
160 FORMAT(/26x,12/**(13.3X,13.4),3*15X,13))
160 READ(3,5) NUM, J(N),1A(N),1V(N),1B(N),1C(N),1N=1, NUM
160 550 close (13)
160 DO 71 N=1,35
160 71 J(N)=0
160 FORMAT(/26X,12/**(13.3X,13.4),3*15X,13))
160 WRITE(1,0) 2 IS THIS THE FIRST TIME YOU ARE TRYING THIS TUTORIAL?
160 + 2 LESSON IN FACTORING2,2,2TRINOMIALS= ANSWER YES OR NO.?
160 READ(0,211) IANS
160 FORMAT(A3)
160 IF(IANS=3*YES) GO TO 14
160 IF(IANS=3*NO) GO TO 208
160 515 L=0
160 IF(L=2,1554,205)*12
160 203 WRITE(1,0) YOU MUST TYPE THE ANSWER YES OR NO.?
160 GO TO 204
160 204 WRITE(1,0) 200 YOU WISH TUTOR 1, 2 OR 3, JUST TYPE THE 2?
160 * 2 NUMERAL 2
160 READ(0,212) IANS
160 IF(IANS=2*YES) GO TO 221
160 IF(IANS=2*NO) GO TO 222
160 15525 L=0
160 IF(M=2,1554,205)*12
160 221 WRITE(1,0) * START TUTOR 1.
160 GO TO 14
160 222 WRITE(1,0) * START TUTOR 2.
160 GO 10 222
160 223 WRITE(1,0) * START TUTOR 3.
160 GO TO 601
160 490C START OF TUTOR 1
160 500 14 N=1
160 WRITE(1,0) THIS IS A LESSON TO HELP YOU LEARN TO FACTOR 2.
160 + 2 TRINOMIALS 2
160 WRITE(1,0) YOU WILL BE GIVEN INSTRUCTIONS AND ASKED TO TYPE ANSWERS 2
160 * EACH TIME YOU ARE ASKED FOR AN ANSWER YOU WILL HEAR A BELL.
160 + ALWAYS USE NUMERALS FOR YOUR ANSWERS, DO NOT SPELL OUT THE N
160 + NUMBER. IF YOU WISH TO ANSWER 545, THEN YOU WILL TYPE 545.2
160 + PAPER YOU INPUT YOUR ANSWER ALWAYS HIT THE RETURN KEY L
160 + OTHER INSTRUCTIONS WILL BE GIVEN AS YOU NEED THEM.
160 WRITE(1,0) * THE PROCESS OF FACTORING IS THE REVERSE OF
160 * MULTIPLICATION 2
160 WRITE (1,0) 2 SINCE (x+3)(x+2) = x*x + 5x + 6 WE SAY 2
160 IF (x+3) AND 2
160 TRINOMIAL FACTORS OF THE TRINOMIAL X*X + 5X + 6 ARE:
160 THE CAN WRITE x*x + 5x + 6 = (X+3)(X+2)
160 32*(35.31-1252,523,12
160 621 E22 WRITE(1,0) * WHEN THE BELL RINGS TYPE THE CORRECT ANSWER 2
160 670 223 READ (8*0) KA
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>599</td>
<td>IF (KA=2) GO TO 824</td>
</tr>
<tr>
<td>600</td>
<td>K(N)=K(N)+1</td>
</tr>
<tr>
<td>601</td>
<td>IF (K(N)=2) 834, 834, 12</td>
</tr>
<tr>
<td>710</td>
<td>WRITE (10,0) ?&gt;NO. LOOK AT THE EQUATION CAREFULLY.2</td>
</tr>
<tr>
<td>720</td>
<td>GO TO 19</td>
</tr>
<tr>
<td>730</td>
<td>WRITE (10,0) ?&gt;THAT'S RIGHT.2</td>
</tr>
<tr>
<td>740</td>
<td>WRITE (10,0) ?&gt;NOW SINCE WE KNOW THAT MULTIPLICATION IS 2.</td>
</tr>
<tr>
<td>750</td>
<td>+ 2COMMUTATIVE WE 2+++</td>
</tr>
<tr>
<td>760</td>
<td>+ 2CAN ALSO WRITE 2</td>
</tr>
<tr>
<td>770</td>
<td>WRITE (10,0)+2X+2 *SX + 6 = (X+3)(X+3)+2</td>
</tr>
<tr>
<td>780</td>
<td>READ (0,0) K</td>
</tr>
<tr>
<td>790</td>
<td>IF (KA=2) GO TO 79</td>
</tr>
<tr>
<td>860</td>
<td>K(N)=K(N)+1</td>
</tr>
<tr>
<td>810</td>
<td>IF(K(N)=2) 845, 845, 12</td>
</tr>
<tr>
<td>820</td>
<td>WRITE (10,0) ?&gt;NO. REMEMBER WE ARE STILL LOOKING AT THE SAME 2.</td>
</tr>
<tr>
<td>830</td>
<td>+ EQUATION 2+++</td>
</tr>
<tr>
<td>840</td>
<td>+ 2X+2 * SX + 6 = (X+3)(X+2). BECAUSE OF THE COMMUTATIVE 2+++</td>
</tr>
<tr>
<td>850</td>
<td>+ 2PROPERTY OF MULTIPLICATION (X+3)(X+2) = (X+2)(X+3)+2+++</td>
</tr>
<tr>
<td>860</td>
<td>+ 2FOR THAT REASON WE CAN SAY 2</td>
</tr>
<tr>
<td>870</td>
<td>GO TO 826</td>
</tr>
<tr>
<td>880</td>
<td>WRITE (10,0) ?&gt;GOOD, YOU UNDERSTAND THAT THE ORDER IN WHICH WE 2</td>
</tr>
<tr>
<td>890</td>
<td>+ 2WRITE THE2+++</td>
</tr>
<tr>
<td>900</td>
<td>+ 2TRINOMIALS DON'T MATTER 2</td>
</tr>
<tr>
<td>910</td>
<td>WRITE (10,0) ?&gt;CONSIDER ONCE AGAIN THE EQUATION X+2 + SX + 6 2+</td>
</tr>
<tr>
<td>920</td>
<td>+ 2 = (X+3)(X+2)+2. NOTE THAT THE PRODUCT OF 2</td>
</tr>
<tr>
<td>930</td>
<td>+ 2THE FIRST TERMS OF EACH BINOMIAL 2+++</td>
</tr>
<tr>
<td>940</td>
<td>+ 2IS THE FIRST TERM OF THE TRINOMIAL. THAT IS 2+++</td>
</tr>
<tr>
<td>950</td>
<td>+ 2X+2 * X+2. ALSO THE PRODUCT OF THE LAST 2+++</td>
</tr>
<tr>
<td>960</td>
<td>+ 2TERM EQUALS THE LAST TERM OF THE TRINOMIAL. THAT IS 3+X+6 2+++</td>
</tr>
<tr>
<td>970</td>
<td>+ 2WHILE THE SUM OF THE LAST TERMS IS PLUS 2 1. THE COEFFICIENT 2+</td>
</tr>
<tr>
<td>980</td>
<td>+ 2OF THE MIDDLE TERM OF OUR TRINOMIAL 2+++</td>
</tr>
<tr>
<td>990</td>
<td>+ 2CONSIDER FACTORING THE TRINOMIAL 2</td>
</tr>
<tr>
<td>1000</td>
<td>N=2</td>
</tr>
<tr>
<td>1010</td>
<td>CALL WRITE(N)</td>
</tr>
<tr>
<td>1020</td>
<td>WRITE (10,0) ?&gt;SINCE THE FIRST TERM FACTORS INTO X*X, WE WILL 2</td>
</tr>
<tr>
<td>1030</td>
<td>+ 2HAVE (X + -)(X + -)?</td>
</tr>
<tr>
<td>1040</td>
<td>WRITE (10,175,1C(N))</td>
</tr>
<tr>
<td>1050</td>
<td>050 75  FORMAT (2THT'S THE TIME WE ARE LOOKING FOR TWO INTEGERS WHOSE 2,</td>
</tr>
<tr>
<td>1060</td>
<td>+ 2PRODUCT IS 2+13+3+2 AND WHOSE SUM IS 15--- WHAT IS THE SUM 2?</td>
</tr>
<tr>
<td>1070</td>
<td>READ (0,0) K</td>
</tr>
<tr>
<td>1080</td>
<td>IF (KA=IB(N)) GO TO 74</td>
</tr>
<tr>
<td>1090</td>
<td>K(N)=K(N)+1</td>
</tr>
<tr>
<td>1100</td>
<td>IF(K(N)=3) GO TO 12</td>
</tr>
<tr>
<td>1110</td>
<td>WRITE (10,0) ?&gt;NO. REMEMBER THE SUM MUST AGREE WITH THE 2</td>
</tr>
<tr>
<td>1120</td>
<td>+ 2COEFFICIENT OF 2+++</td>
</tr>
<tr>
<td>1130</td>
<td>+ 2THE SECOND TERM OF THE TRINOMIAL. LOOK CAREFULLY.2</td>
</tr>
<tr>
<td>1140</td>
<td>CALL WRITE(N)</td>
</tr>
<tr>
<td>1150</td>
<td>GO TO 100</td>
</tr>
<tr>
<td>1160</td>
<td>74 WRITE (10,0) ?&gt;THAT'S RIGHT. THEREFORE THE TWO INTEGERS WE ARE 2+++</td>
</tr>
<tr>
<td>1170</td>
<td>+ 2LOOKING FOR MUST BE 2++2+++</td>
</tr>
<tr>
<td>1180</td>
<td>+ 2TYPE THE TWO NUMERALS SEPARATED BY A COMMA. AFTER YOU TYPE&gt;&gt;&gt;</td>
</tr>
<tr>
<td>1190</td>
<td>+ 2YOUR ANSWER MUST BE SURE TO HIT THE RETURN KEY.2</td>
</tr>
<tr>
<td>1200</td>
<td>83 CALL TEST(N+IRTN)</td>
</tr>
<tr>
<td>1210</td>
<td>820 GO TO (6N+101,36,12)+IRTN</td>
</tr>
<tr>
<td>1220</td>
<td>101 WRITE (10,0) ?&gt;NO. LOOK AT THE TRINOMIAL CAREFULLY.2</td>
</tr>
<tr>
<td>1230</td>
<td>CALL WRITE(N)</td>
</tr>
<tr>
<td>1240</td>
<td>WRITE (10,0)+11B(N)+, 1C(N)</td>
</tr>
<tr>
<td>1250</td>
<td>901 FORMAT (2THE SUM MUST EQUALS 13, 2 AND THE PRODUCT 2, 13)</td>
</tr>
</tbody>
</table>
WRITE(1,0) DON'T FORGET TO TYPE A COMMA BETWEEN THE TWO 2.
1270  * 1NUMERALS*2
1280  GO TO 83
1290 80  WRITE(1,0) EVERY GOOD NOW WE KNOW THAT 2
1300  CALL BRWRITE(N)
1310 87  WRITE(1,0) YOUR ANSWER WILL OF COURSE BE IN THE FORM OF TWO 2,
1320  * 2BINARY*2,2,2INPUT YOUR ANSWER IN THE FORM ( 1 ( ) ),2,2,2
1330  * 2WHEN USING A 1-ONE-DIGIT NUMERAL YOU MUST TYPE A ZERO 2,
1340  * 2(OR SPACE) IN FRONT OF THE NUMERAL. FOR EXAMPLE (X*03)*2;
1350  * 2FROM THIS POINT ON WHENEVER 2*
1360  * 2YOUR ANSWER IS A BINOMIAL USE THIS FORM. OK NOW YOUR 2
1370  * 2ANSWER*2
1380  CALL BRWRITE(N)
1390  CALL BTEST(N,IRTN)
1400  GO TO (85,94*12), IRTN
1410 88  WRITE(1,0) NOT QUITE THINK CAREFULLY 2
1420  GO TO 87
1430 96  WRITE(1,0) YOU ARE CATCHING ON NOW TRY ANOTHER 2
1440  GO TO 409
1450 96  IF(COUNT(N)) 95,95,96
1460 95  WRITE(1,0) NOW TRY A FEW MORE, REMEMBER 2,
1470  * 2TO HIT THE 2
1480  * 2RETURN KEY AFTER ANSWERING 2
1490  GO TO 409
1500  CALL BWRITE(N)
1510 302  N=11
1520 303  WRITE(1,0) SO FAR WE HAVE ONLY CONSIDERED TRINOMIALS WHOSE 2,
1530  * 2TERMS ARE ALL>=0>=POSITIVE, SUPPOSE THE MIDDLE TERM IS 2,
1540  * 2NEGATIVE, AS IN>
1550  CALL WRITE(N)
1560 41  IF(IC(N)) 40+34,34
1570 40  WRITE(1,20) IC(N)
1580 20  FORMAT(2*E ARE NOW LOOKING FOR TWO INTEGERS WHOSE PRODUCT
1590 45  IF((B(N)<0)) GO TO 43
1600 45  GO TO 44
1610 43  WRITE(1,21) IB(N)
1620 21  FORMAT(2*PUT WHOSE SUM IS 2,13)
1630 46  GO TO 39
1640 34  WRITE(1,37) IC(N)
1650 37  FORMAT(2*E ARE NOW LOOKING FOR TWO INTEGERS WHOSE PRODUCT
1660 37  * 2IS >=2,13)
1670 46  GO TO 45
1680 46  WRITE(1,38) IB(N)
1690 38  FORMAT(2*PUT WHOSE SUM IS >=2,13)
1700 39  WRITE(1,0) YOUR NUMBERS ARE --------
1710 39  CALL TEST(N,IRTN)
1720 7  GO TO (4+6*10+12),IRTN
1730 4  WRITE(1,0) RIGHT THEREFORE 2
1740 6  GO TO 8
1750 6  WRITE(1,0)=NO, THINK CAREFULLY AND TRY AGAIN 2
1760 6  GO TO 40
1770 10  WRITE(1,0) THASS BETTER, THEREFORE 2
1780 8  K(N)=K(N)+1
1790 10  J(N)=0
1800 469  CALL BRWRITE(N)
1810 36  CALL BTEST(N,IRTN)
1940  GO TO (11,13,12,300) *RTN
1950  13 IF (INJ=3) 300,35,35
1960  35 IF (N=61) 250,500,52
1970  52 IF (N=11) 400,600,53
1980  63 IF (N=14) 380,500,454
1990  454 IF (N=210) 400,600,455
2000  455 IF (N=24) 350,700,8
2010  380 WRITE(1,87) *LET'S TRY THE NEXT ONE *
2020  GO TO 8
2030  500 WRITE(1,12) *FOR A SHORT QUIZ ON WHAT YOU HAVE LEARNED*
2040  + 350 FOR 2
2050  GO TO 400
2060  600 WRITE(1,22) *WRONG
1970  22 FOR=2/V12 YOU MISSED 3
1980  IF (N=20) 23.28.28
1990  23 WRITE(1,23) *EXCELLENT YOU ARE READY TO ADVANCE TO 2*
2000  + 2 NEXT SECTION 2
2010  27 WRITE(1,24) *DON'T STOP
2020  20 WRITE(1,25) *DO YOU WISH TO CONTINUE NOW? ANSWER YES OR NO*
2030  25 READ(1,26) IANS
2040  26 FOR=V1(A3)
2050  IF (IANS=V3=YES) GO TO 602
2060  IF (IANS=V3=NO) GO TO 300
2070  20 WRITE(1,27) *DO YOU TYPE YOUR RESPONSE CORRECTLY?
2080  + 2 YES OR NO?
2090  GO TO 25
2100  602 IF (N=11) GO TO 303
2110  IF (.22) GO TO 603
2120  GO TO 25
2130  26 WRITE(1,28) *YOU HAVE DONE QUITE WELL AND IF YOU ARE SURE 2*
2140  + 2 THAT YOU UNDERSTAND 2,2, YOUR MISTAKE YOU MAY ADVANCE TO 3*
2150  + 2 THE NEXT SECTION 3*
2160  GO TO 27
2170  28 WRITE(1,29) *CHECK WITH YOUR TEACHER BEFORE ADVANCING TO 3*
2180  + 2 THE NEXT SECTION 3,3, IT MAY BE ADVISABLE FOR YOU TO TRY 2*
2190  + 2 THIS LESSON ONCE MORE 3*
2200  GO TO 300
2210  IGC START OF TUTOR 3
2220  601 A=27
2230  603 WRITE(1,30) *SUPPOSE THE SIGN OF THE LAST TERM IS 2*
2240  + 2 NEGATIVE AS IN 2
2250  CALL WRITE(N)
2260  WRITE(1,111) ICN
2270  1 FCA-V152.VE ARE NOW LOOKING FOR 2 INTEGERS WHOSE PRODUCT IS 2*
2280  + 13V, AND WHOSE SUM IS ---------2
2290  502 K=1M
2300  READ(1,4) K=KX
2310  4 COUNT(I1,1,11)
2320  IF (K=1AR(N) AND K=1H ) GO TO 508
2330  IF (KAR(N) AND K=1VAR(N)) GO TO 501
2340  I COUNT(N)=COUNT(N)+1
2350  IF (COUNT(N)=3) 504,503,503
2360  504 WRITE(1,120) *LOOK AT THE TRINOMIAL CAREFULLY, 2*
2370  CALL WRITE(N)
2380  WRITE(1,123) *THE SUM OF THE TWO INTEGERS MUST EQUAL THE 2*
2390  + 2 COEFFICIENT OF THE 2ND MIDDLE TERM 2*
2400  GO TO 3
2410  503 WRITE(1,124) *THE SUM IS EQUAL TO THE COEFFICIENT ONLY, 2*
12

2420 * 3. NOT THE ENTIRE 21+2 MIDDLE TERM, TRY AGAIN.2
2430 3 WRITE(1,0) WHAT SHOULD THE SUM BE = ?
2440 GO TO 502
2450 501 WRITE(1,0) 1B(N) + IC(N)
2460 90 FORMAT(2NO, THE SUM MUST EQUAL ? + 13 + ? , WHILE THE PRODUCT ?
2470 + 2IS 2*13)
2480 GO TO 510
2490 508 WRITE(1,0) 2RIGHT=2
2500 510 WRITE(1,0) CONSIDER THE SAME TRINOMIAL AGAIN.2
2510 CALL WRITE(N)
2520 WRITE(1,0) 1FIRST THINK ABOUT THE PRODUCT, THERE ARE FOUR 2.
2540 + 2 INTERGERS=2
2550 50 WRITE(1,0) 1 1. (+15)(-1)
2560 WRITE(1,0) 2 2. (-15)(-1)
2570 WRITE(1,0) 3 3. (+5)(-3)
2580 WRITE(1,0) 4 4. (-5)(+3)
2590 IF(JV(N)=8) GO TO 51
2600 IF(JV(N)=7) GO TO 512
2610 51 WRITE(1,0) WHICH OF THESE FOUR WAYS IS THE CORRECT CHOICE 2
2620 + 2 FOR OUR 2+2 EXAMPLE OF X**2 + 2X - 15 - TYPE THE NUMBER 2.
2630 + 2 FOR THE CORRECT ANSWER: 2**2+1,2+3 OR 4*2
2640 512 READ(0,0) KA
2650 IF(KE=8) GO TO 514
2660 JV(N)=JV(N)+1
2670 IF(JV(N)-3) 515, 513, 513
2680 515 WRITE(1,0) 2NO. REMEMBER THE SUM MUST EQUAL 2 AT THE SAME.
2700 + 2 WAYS IS CORRECT=2
2710 GO TO 59
2720 513 WRITE(1,0) 2NO. THE ONLY POSSIBILITY IS NUMBER 3! (+5)(-3),
2730 + 2 SINCE THE SUM 2+2 OF 5 AND -3 IS 15, 2. THUS 2
2740 GO TO 58
2750 514 WRITE(1,0) 2OF COURSE=2, THUS 2
2760 GO TO 89
2770 700 WRITE(1,0) 2LETES NO SUMMARIZE WHAT WE HAVE LEARNED, IFZ=8
2780 + 2 THE SIGN OF THE LAST TERM OF THE TRINOMIAL IS POSITIVE, 2+2
2790 + 2 THE LAST TERMS OF THE TRINOMIAL FACTORS MUST BOTH HAVE 2+2
2800 + 2 THE -----SIG=2 ANSWER IS SAME OR DIFFERENT FOR OPPOSITE.
2810 59 READ(0,2) IANS
2820 2 FORMAT(4A)
2830 IF(IANS=6) GO TO 56
2840 IF(IANS=6) GO TO 54
2850 WRITE(1,0) 2DID YOU TYPE YOUR ANSWER CORRECTLY= TRY AGAIN, 2
2860 GO TO 59
2870 54 IF(=A)
2880 IF(M=2) 55, 55, 12
2890 55 WRITE(1,0) 2NO. THIS IS A VERY IMPORTANT POINT TO 2
2900 + 2 UNDERSTAND, 2+2 THE PRODUCT OF TWO NUMBERS HAVING THE 2.
2910 + 2 SAME SIGN IS2+2 ALWAYS POSITIVE, WHAT IS -2 * -3 = -2
2920 64 READ(0,0) IANS
2930 IF(IANS=6) 65, 66, 62
2940 60 WRITE(1,0) 2OF COURSE, 2
2950 IF(IANS=6) 65, 66, 62
2960 GO TO 61
2970 61 WRITE(1,0) 2NO. THIS IS THE SAME ANSWER ONCE AGAIN, ? - ? = -2
2980 GO TO 64
2990 65 WRITE(1,0) 2NOT -2 * -3 = +2


WRITE(1,0)'THE PRODUCT OF TWO NEGATIVE NUMBERS IS 2 1*'
3110 + ZALWAYS A POSITIVE NUMBER, THE ONLY WAY YOU CAN GET A 2 1*'
3120 + |NEGATIVE PRODUCT OF TWO NUMBERS IS IF ONE FACTOR IS 2 1*'
3130 + |NEGATIVE AND THE OTHER IS POSITIVE. 2
3140 GO TO 700
3150 56 WRITE(1,0)'THAT'S RIGHT, AND IF THE SIGN OF THE LAST TERM 2 1*'
3160 + |IS NEGATIVE, 2 1* THE FACTORS MUST EACH HAVE THE --SIGN. 2 1*'
3170 + |ANSWER SSMES OR ZOPPS FOR OPPOSITE. 2
3180 READ(0,2) IANS
3190 IF(IANS=4)GO TO 57
3200 IF(IANS=4)GO TO 46
3210 46 IF(M=-2)47,47,12
3220 47 WRITE(1,0)"REMEMBER THE ONLY WAY TO OBTAIN A NEGATIVE 2 1*
3230 + |PRODUCT IS FOR ONE FACTOR 2 1* THE OTHER 2 1*'
3240 + |NEGATIVE. 2
3250 GO TO 700
3260 57 WRITE(1,0)'YES, YOUVE GOT IT!!'
3270 105 WRITE(1,0)'NOW FOR A FINAL REVIEW, THIS TIME YOU WILL 2 1*'
3280 + |BE GIVEN 10 TRU-OR-FALES TO FACTOR, YOU WILL BE TOLD HOW 2 1*'
3290 + |YOU DID AT THE END, TYPE YOUR ANSWERS CAREFULLY 2 1*'
3300 + |IN THE CORRECT FORM OF (1)X+2
3310 + |GO TO 112
3320 58 IF(NUM-N)900+N12=N12
3330 112 CALL WRITE(N)
3340 95 CALL BITEST(N,IRTN)
3350 GO TO (59+59+12,200),IRTN
3360 950 IPERCT = (10-IWRONG)*10
3370 WRITE(1,51)IWRONG,IPERC'T
3380 91 FORMAT(2YOU MISSED-213;1X;2OUT OF 10. YOUR SCORE IS 2
3390 + |142 PER CENT.)
3400 + |GO TO 300
3410 12 WRITE(1,0)"IT IS NECESSARY THAT YOU SEE YOUR TEACHER 2 1*'
3420 + |FOR ADDITIONAL HELP 2 1* BEFORE CONTINUING 2 1*
3430 + |GO TO 700
3440 300 END
3450 SUBROUTINE TEST(N,IRTN)
3460 COMMON IA(35),IVAR(35),IVR(35),ICT(35),ICOUNT(35), LINO(35), J(35),
3470 + |JV(35),K(35)
3480 + |INTEGER A,N,SUM,PROD
3490 READ(0,0)A,B
3500 IF(A+B=1N(N))AND* A*B=IC(N))GO TO 15
3510 ICOUNT(N)=ICOUNT(N)+1
3520 IF(ICOUNT(N)=3)14,55,55
3530 15 IRTN=2
3540 RETURN
3550 16 IF(ICOUNT(N)=0)GO TO 16
3560 GO TO 17
3570 IRTN=1
3580 RETURN
3590 IRTN=3
3600 RETURN
3610 55 IRTN=4
3620 RETURN
3630 END
3640 SUBROUTINE BITEST(N,IRTN)
3650 COMMON IA(35),IVAR(35),IVR(35),ICT(35),ICOUNT(35), LINO(35), J(35),
3660 + |JV(35),K(35),IWRONG
INTEGER P1,VAR1,P23,VAR2,P4

3600 9 FORMAT(2(A1),13,A2,A1,13,A1)
3600 31 READ(*,9) P1,VAR1,P23,VAR2,P4
3610 76 IF(P1(.NE.1M).OR.(P23,.NE.2M).OR.(P4,.NE.1M))) GO TO 75
3620 + GO TO 75
3630 IF(VAR1,.NE.IVAR(N)) + OP. VAR2,.NE.IVAR(N) GO TO 70
3640 IF((I+IV=IB(N)) .AND. (I+IV=IC(N))) GO TO 22
3650 IF(N=6) 701,400,780
3660 IF(N=11) 400,701,791
3670 781 IF(N=14) 701,400,732
3680 782 IF(N=20) 400,701,733
3690 793 IF(N=24) 701,400,400
3700 400 IF(N=24) 1H0R(N)=1H0R(N)+1
3710 GO TO 402
3720 791 IF(K(N)-3) 72,72,95
3730 72 WRITE(1,0)"NO, REMEMBER"
3740 ICOUNT(N)=0
3750 JRTN=1
3760 RETURN
3770 75 J(N)=J(N)+1
3780 IF(J(N)-3) 23,35,95
3790 25 WRITE(1,0)"SEX YOU USE THE CORRECT FORM TO ENTER 2?"
3800 +"THE RINOMIALS,2,2,TRY AGAIN, 2,"
3810 + USE THE FORM ( ) ( )
3820 GO TO 31
3830 79 IF(N=6) 708,400,771
3840 771 IF(N=11) 400,702,772
3850 772 IF(N=14) 702,400,773
3860 773 IF(N=20) 400,702,774
3870 774 IF(N=24) 702,400,400
3880 72 J(V(N))=J(V(N)+1
3890 IF(J(V(N))-1) 225,25,85,85
3900 225 WRITE(1,0)"YOU INPUT THE CORRECT VARIABLE, TRY AGAIN, 2,"
3910 CALL 2HIT(I)
3920 GO TO 31
3930 22 IF(N=6) 222,432,223
3940 223 IF(N=11) 402,222,224
3950 224 IF(N=14) 222,402,225
3960 225 IF(N=20) 402,222,227
3970 227 IF(N=24) 222,402,492
3980 222 WRITE(1,0)G00002
3990 402 KEN=1
4000 JRTN=2
4010 RETURN
4020 85 JRTN=3
4030 RETURN
4040 END
4050 SUBROUTINE 2HIT(N)
4060 COMMON IA(35),IVAR(35),IR(35),IC(35)
4070 IF (IA(N)==1 .AND. IC(N)<0 .AND. IB(N)<1) GO TO 42
4080 IF (IA(N)==1 .AND. IC(N)<0 .AND. IB(N)=1) GO TO 20
4090 IF (IA(N)==1 .AND. IC(N)<0 .AND. IB(N)=1) GO TO 22
4100 IF (IA(N)==1 .AND. IC(N)>0 .AND. IB(N)=1) GO TO 37
4110 IF (IA(N)==1 .AND. IC(N)>0 .AND. IB(N)=0) GO TO 24
4120 IF (IA(N)==1 .AND. IC(N)=0 .AND. IB(N)=1) GO TO 25
4130 IF (IA(N)==1 .AND. IC(N)=0 .AND. IB(N)>0) GO TO 39
4140 IF (IA(N)==1 .AND. IC(N)>0 .AND. IB(N)<0) GO TO 41
4150 IF (IA(N)==1 .AND. IC(N)>0 .AND. IB(N)<0 .AND. IC(N)<0) GO TO 43
4160 IF (IA(N), .NE., 1, AND, (IB(N) = 1, AND, IC(N) < 0)) GO TO 2
4170 IF (IA(N), .NE., 1, AND, (IB(N) = -1, AND, IC(N) < 0)) GO TO 3
4180 IF (IA(N), .NE., 1, AND, (IB(N) = 1, AND, IC(N) > 0)) GO TO 4
4190 IF (IA(N), .NE., 1, AND, (IB(N) = -1, AND, IC(N) > 0)) GO TO 5
4200 IF (IA(N), .NE., 1, AND, (IB(N) = 1, AND, IC(N) < 0)) GO TO 6
4210 IF (IA(N), .NE., 1, AND, (IB(N) = -1, AND, IC(N) > 0)) GO TO 7
4220 IF (IA(N), .NE., 1, AND, (IB(N) = 1, AND, IC(N) > 0)) GO TO 8
4230 20 WRITE(1,21) IVA(N), IVA(N), IC(N)
4240 21 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4250 RETURN
4260 22 WRITE(1,23) IVA(N), IVA(N), IC(N)
4270 23 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4280 RETURN
4290 43 WRITE(1,19) IA(N), IVA(N), IB(N), IC(N)
4300 19 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4310 RETURN
4320 24 WRITE(1,26) IVA(N), IVA(N), 1C(N)
4330 26 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4340 RETURN
4350 25 WRITE(1,27) IVA(N), IVA(N), IC(N)
4360 27 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4370 RETURN
4380 32 WRITE(1,33) IVA(N), IB(N), IVA(N), IC(N)
4390 33 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4400 RETURN
4410 2 WRITE(1,41) IA(N), IVA(N), IVA(N), IC(N)
4420 41 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4430 RETURN
4440 3 WRITE(1,51) IA(N), IVA(N), IVA(N), IC(N)
4450 51 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4460 RETURN
4470 37 WRITE(1,36) IVA(N), IB(N), IVA(N), IC(N)
4480 36 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4490 RETURN
4500 6 WRITE(1,61) IA(N), IVA(N), IVA(N), IC(N)
4510 61 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4520 RETURN
4530 7 WRITE(1,71) IA(N), IVA(N), IVA(N), IC(N)
4540 71 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4550 RETURN
4560 39 WRITE(1,40) IVA(N), IB(N), IVA(N), IC(N)
4570 40 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4580 RETURN
4590 41 WRITE(1,42) IVA(N), IB(N), IVA(N), IC(N)
4600 42 FORMAT (/A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4610 RETURN
4620 44 WRITE(1,45) IA(N), IVA(N), IB(N), IVA(N), IC(N)
4630 45 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4640 RETURN
4650 47 WRITE(1,47) IA(N), IVA(N), IB(N), IVA(N), IC(N)
4660 47 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4670 RETURN
4680 48 WRITE(1,49) IA(N), IVA(N), IB(N), IVA(N), IC(N)
4690 49 FORMAT (/13, A1, 3*9*2, 1*X, 1*M, 1*A, 1*X, 13)
4700 RETURN
4710 END
4720 SUBROUTINE WRITE(N)
4730 COMMON IA(35), IVA(35), IB(35), IC(35)
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>4740</td>
<td>IF (IA(N)=1) .AND. (IC(N)&lt;0 .AND. IB(N)&lt;0) GO TO 32</td>
</tr>
<tr>
<td>4750</td>
<td>IF (IA(N)=1) .AND. (IC(N)&lt;0 .AND. IB(N)=1) GO TO 20</td>
</tr>
<tr>
<td>4760</td>
<td>IF (IA(N)=1) .AND. (IC(N)&lt;0 .AND. IB(N)=-1) GO TO 22</td>
</tr>
<tr>
<td>4770</td>
<td>IF (IA(N)=1) .AND. (IB(N)&gt;1 .AND. IC(N)&gt;0) GO TO 37</td>
</tr>
<tr>
<td>4780</td>
<td>IF (IA(N)=1) .AND. (IB(N)=1 .AND. IC(N)&gt;0) GO TO 24</td>
</tr>
<tr>
<td>4790</td>
<td>IF (IA(N)=1) .AND. (IB(N)&gt;1 .AND. IC(N)&lt;0) GO TO 25</td>
</tr>
<tr>
<td>4800</td>
<td>IF (IA(N)=1) .AND. (IB(N)&lt;1 .AND. IC(N)&gt;1) GO TO 39</td>
</tr>
<tr>
<td>4810</td>
<td>IF (IA(N)=1) .AND. (IB(N)&gt;1 .AND. IC(N)&lt;0) GO TO 41</td>
</tr>
<tr>
<td>4820</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)&lt;1 .AND. IC(N)&lt;0) GO TO 43</td>
</tr>
<tr>
<td>4830</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)=1 .AND. IC(N)&lt;0) GO TO 2</td>
</tr>
<tr>
<td>4840</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)=1 .AND. IC(N)&lt;0) GO TO 3</td>
</tr>
<tr>
<td>4850</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)&lt;1 .AND. IC(N)&gt;0) GO TO 44</td>
</tr>
<tr>
<td>4860</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)&lt;1 .AND. IC(N)&lt;0) GO TO 6</td>
</tr>
<tr>
<td>4870</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)=1 .AND. IC(N)&lt;0) GO TO 7</td>
</tr>
<tr>
<td>4880</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)&lt;1 .AND. IC(N)&lt;0) GO TO 46</td>
</tr>
<tr>
<td>4890</td>
<td>IF (IA(N),NE.1 .AND. (IB(N)=1 .AND. IC(N)&gt;0) GO TO 48</td>
</tr>
<tr>
<td>4900</td>
<td>WRITE(1,21) IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>4910</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>4920</td>
<td>RETURN</td>
</tr>
<tr>
<td>4930</td>
<td>WRITE(1,22) IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>4940</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>4950</td>
<td>RETURN</td>
</tr>
<tr>
<td>4960</td>
<td>WRITE(1,19) IVAR(N),IB(N),IC(N)</td>
</tr>
<tr>
<td>4970</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>4980</td>
<td>RETURN</td>
</tr>
<tr>
<td>5000</td>
<td>WRITE(1,26) IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5010</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5020</td>
<td>RETURN</td>
</tr>
<tr>
<td>5030</td>
<td>WRITE(1,27) IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5040</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5050</td>
<td>RETURN</td>
</tr>
<tr>
<td>5060</td>
<td>WRITE(1,33) IVAR(N),IB(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5070</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5080</td>
<td>RETURN</td>
</tr>
<tr>
<td>5090</td>
<td>WRITE(1,14) IA(N),IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5100</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5110</td>
<td>RETURN</td>
</tr>
<tr>
<td>5120</td>
<td>WRITE(1,5) IA(N),IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5130</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5140</td>
<td>RETURN</td>
</tr>
<tr>
<td>5150</td>
<td>WRITE(1,38) IVAR(N),IB(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5160</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5170</td>
<td>RETURN</td>
</tr>
<tr>
<td>5180</td>
<td>WRITE(1,8) IA(N),IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5190</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5200</td>
<td>RETURN</td>
</tr>
<tr>
<td>5210</td>
<td>WRITE(1,9) IA(N),IVAR(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5220</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5230</td>
<td>RETURN</td>
</tr>
<tr>
<td>5240</td>
<td>WRITE(1,40) IVAR(N),IB(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5250</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5260</td>
<td>RETURN</td>
</tr>
<tr>
<td>5270</td>
<td>WRITE(1,42) IVAR(N),IB(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5280</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
<tr>
<td>5290</td>
<td>RETURN</td>
</tr>
<tr>
<td>5300</td>
<td>WRITE(1,43) IVAR(N),IB(N),IVAR(N),IC(N)</td>
</tr>
<tr>
<td>5310</td>
<td>FORMAT(15HTHE FACTORS OF IA,3H*2,IX,1H=AI,IX=13)</td>
</tr>
</tbody>
</table>
FORMAT(15THE FACTORS OF 13,A1,3**2,B1,13,A1,13,13)

RETURN
WRITE(1,48) IA(N),IVAR(N),IB(N),IVAR(N),IC(N)
FORMAT(15THE FACTORS OF 13,A1,3**2,B1,13,A1,13,13)
RETURN
WRITE(1,49) IA(N),IVAR(N),IB(N),IVAR(N),IC(N)
FORMAT(15THE FACTORS OF 13,A1,3**2,B1,13,A1,13,13)
RETURN
END
DATA FILE

90C 101-105 ARE EXAMPLES FOR TUTOR1; 106-110 QUIZ TUTOR1;
91C 111-113 EXAMPLES TUTOR2; 114-119 QUIZ TUTOR2; 120-123
92C EXAMPLES TUTOR3; 124-133 QUIZ TUTOR3-
93C FORMAT (13,3X,13,A1,3X,13,A1,13)
940 NUMBER OF LINES = 33
951
101 1X**2+ 5X+ 6
102 1X**2+ 7X+ 6
103 1X**2+ 5X+ 4
104 1A**2+ 7A+12
105 1X**2+ 6X+ 8
106 1X**2+ 4X+ 3
107 1X**2+ 7X+10
108 1X**2+ 8X+12
109 1X**2+ 2X+ 1
110 1X**2+ 8X+15
111 1X**2+ 6X+ 8
112 1X**2+ 3X+ 2
113 1A**2- 1A+2A
114 1A**2+ 4A+ 7
115 1P**2+11P+2A
116 1X**2- 4X+2A
117 1P**2-10P+2A
118 1C**2- 9C+18
119 1X**2+ 9X+28
120 1X**2+ 2X-15
121 1Y**2+ 3X-10
122 1X**2- 3X-10
123 1P**2- 4P-12
124 1X**2- 4X-21
125 1A**2- 40-12
126 1P**2+ 8P+15
127 1X**2- 1X-26
128 1P**2-13P+42
129 1S**2+18S-24
130 1A**2-10A+25
131 1P**2+11P+28
132 10**2+ 70-30
133 1P**2- 4P-32
IS THIS THE FIRST TIME YOU ARE TRYING THIS TUTORIAL LESSON IN FACTORING TRINOMIALS? ANSWER YES OR NO.

YES.

THIS IS A LESSON TO HELP YOU LEARN TO FACTOR TRINOMIALS.

YOU WILL BE GIVEN INSTRUCTIONS AND ASKED TO TYPE ANSWERS.

EACH TIME YOU ARE ASKED FOR AN ANSWER YOU WILL HEAR A BELL.

ALWAYS USE NUMERALS FOR YOUR ANSWERS. DO NOT SPELL OUT THE NUMBER. IF YOU WISH TO ANSWER '4', THEN YOU WILL TYPE '4'.

AFTER YOU INPUT YOUR ANSWER ALWAYS HIT THE RETURN KEY.

OTHER INSTRUCTIONS WILL BE GIVEN AS YOU NEED THEM.

THE PROCESS OF FACTORING IS THE REVERSE OF MULTIPLICATION. SINCE (X+3)(X+2) = X^2 + 5X + 6, WE SAY THAT (X+3) AND (X+2) ARE THE BINOMIAL FACTORS OF THE TRINOMIAL X^2 + 5X + 6.

WE CAN WRITE X^2 + 5X + 6 = (X+3)(X+2).

WHEN THE BELL RINGS, TYPE THE CORRECT ANSWER.

THAT'S RIGHT.

NOW SINCE WE KNOW THAT MULTIPLICATION IS COMMUTATIVE WE CAN ALSO WRITE:

X^2 + 5X + 6 = (X+2)(X+3).

GOOD, YOU UNDERSTAND THAT THE ORDER IN WHICH WE WRITE THE BINOMIALS DOESN'T MATTER.

CONSIDER ONCE AGAIN THE EQUATION X^2 + 5X + 6 = (X+3)(X+2).


CONSIDER FACTORING THE TRINOMIAL

X^2 + 7X + 6

SINCE THE FIRST TERM FACTORS INTO X*X, WE WILL HAVE (X + ?)(X + ?).

THIS TIME WE ARE LOOKING FOR TWO INTEGERS WHOSE PRODUCT IS 6 AND WHOSE SUM IS ? ? ? WHAT IS THE SUM?

THAT'S RIGHT. THEREFORE THE TWO INTEGERS WE ARE LOOKING FOR MUST BE -- -- -- ?

TYPE THE TWO NUMERALS SEPARATED BY A COMMA. AFTER YOU TYPE YOUR ANSWER BE SURE TO HIT THE RETURN KEY.

GOOD.

6+1

VERY GOOD. NOW WE KNOW THAT THE FACTORS OF X^2 + 7X + 6 ARE -- -- ?.

YOUR ANSWER WILL BE IN THE FORM OF TWO BINOMIALS.

INPUT YOUR ANSWER IN THE FORM ( )  ( )

WHEN USING A ONE-DIGIT NUMERAL YOU MUST TYPE A ZERO (OR SPACE) IN FRONT OF THE NUMERAL. FOR EXAMPLE (X+03). FROM THIS POINT ON WHENEVER YOUR ANSWER IS A BINOMIAL USE THIS FORM. OK, NOW YOUR ANSWER.

THE FACTORS OF X^2 + 7X + 6 ARE -- -- ?

GOOD.
NOW TRY A FEW MORE. REMEMBER TO HIT THE RETURN KEY AFTER ANSWERING.

THE FACTORS OF \(x^2 + 5x + 4\) ARE ---?

\((x+1)(x+4)\)

GOOD

LET'S TRY THE NEXT.

THE FACTORS OF \(x^2 + 7x + 12\) ARE ???

\((x+3)(x+4)\)

GOOD

LET'S TRY THE NEXT.

THE FACTORS OF \(x^2 + 6x + 8\) ARE ???

\((x+4)(x+2)\)

GOOD

NOW FOR A SHORT QUIZ ON WHAT YOU HAVE LEARNED SO FAR.

THE FACTORS OF \(x^2 + 4x + 3\) ARE ???

\((x+3)(x+1)\)

THE FACTORS OF \(x^2 + 7x + 10\) ARE ???

\((x+2)(x+5)\)

THE FACTORS OF \(x^2 + 8x + 12\) ARE ???

\((x+6)(x+2)\)

THE FACTORS OF \(x^2 + 2x + 1\) ARE ???

\((x+1)(x+1)\)

THE FACTORS OF \(x^2 + 8x + 15\) ARE ???

\((x+5)(x+3)\)

BE SURE YOU USE THE CORRECT FORM TO ENTER THE BINOMIALS.

TRY AGAIN. USE THE FORM \((x+\_)(x+\_)\).

\((x+5)(x+3)\)

YOU MISSED --

EXCELLENT! YOU ARE READY TO ADVANCE TO THE NEXT SECTION.

DO YOU WISH TO CONTINUE NOW? ANSWER YES OR NO.

YES

SO FAR WE HAVE ONLY CONSIDERED TRINOMIALS WHOSE TERMS ARE ALL POSITIVE. SUPPOSE THE MIDDLE TERM IS NEGATIVE, AS IN

\(x^2 - 6x + 8\)

WE ARE NOW LOOKING FOR TWO INTEGERS WHOSE PRODUCT IS + 8 BUT WHOSE SUM IS -6

OUR NUMBERS ARE ---,---?

\(-2,-4\)

RIGHT. THEREFORE

THE FACTORS OF \(x^2 - 6x + 8\) ARE ???

\((x-2)(x-4)\)

ERROR IN FORMATTED READ--ILLEGAL CHARACTER ON INPUT

\(x+b\)

BE SURE YOU USE THE CORRECT FORM TO ENTER THE BINOMIALS.

TRY AGAIN. USE THE FORM \((x+\_)(x+\_)\).

\((x-5)(x-2)\)
ERROR IN FORMATTED READ--ILLEGAL CHARACTER ON INPUT
-2)

BE SURE YOU USE THE CORRECT FORM TO ENTER THE BINOMIALS.
TRY AGAIN, USE THE FORM ( )( ).

GOOD
LET'S TRY THE NEXT.
The factors of \( x^2 - 3x + 2 \) are ---?

\((x-2)(x-1)\)

GOOD
LET'S TRY THE NEXT.
The factors of \( a^2 - 10a + 24 \) are ---?

\((a-6)(a-4)\)

GOOD
Now for a short quiz on what you have learned so far.
The factors of \( a^2 + 8a + 7 \) are ---?

\((a+1)(a+7)\)
The factors of \( r^2 + 11r + 24 \) are ---?

\((r+3)(r+8)\)
The factors of \( x^2 - 14x + 24 \) are ---?

\((x-2)(x-12)\)

ERROR IN FORMATTED READ--ILLEGAL CHARACTER ON INPUT
( )

\((x-2)(x-12)\)

ERROR IN FORMATTED READ--ILLEGAL CHARACTER ON INPUT
( )

\((x-2)(x-12)\)

BE SURE YOU USE THE CORRECT FORM TO ENTER THE BINOMIALS.
TRY AGAIN, USE THE FORM ( )( ).

\((x-2)(x-12)\)

The factors of \( b^2 - 10b + 24 \) are ---?

\((b-6)(b-4)\)
The factors of \( c^2 - 9c + 18 \) are ---?

\((c-6)(c-3)\)
The factors of \( m^2 + 9m + 20 \) are ---?

\((m+5)(m+4)\)

YOU MISSED 0
EXCELLENT! YOU ARE READY TO ADVANCE TO THE NEXT SECTION.
DO YOU WISH TO CONTINUE NOW? ANSWER YES OR NO.

YES.
SUPPOSE THE SIGN OF THE LAST TERM IS NEGATIVE, AS IN

\[ x^2 + 2x - 15 \]

WE ARE NOW LOOKING FOR 2 INTEGERS WHOSE PRODUCT IS -15
AND WHOSE SUM IS ------?

2

RIGHT!
CONSIDER THE SAME TRINOMIAL AGAIN.

\[ x^2 + 2x - 15 \]
FIRST THINK ABOUT THE PRODUCT. THERE ARE FOUR WAYS IN WHICH
WE CAN EXPRESS -15 AS THE PRODUCT OF TWO INTEGERS:

1. \((-15)(+1)\)
2. \((-15)(+1)\)
3. \((+5)(-3)\)
4. \((-3)(+5)\)
Which of these four ways is the correct choice for our example of \( x^2 + 2x - 15 \)? Type the number of the correct answer:

1, 2, 3 or 4.

No. Remember the sum must equal +2 at the same time that the product equals -15. Which of the four ways is correct?

1. \((+15)(-1)\)
2. \((-15)(+1)\)
3. \((+5)(-3)\)
4. \((-5)(+3)\)

Of course, thus the factors of \( x^2 + 2x - 15 \) are ---?

\((x+5)(x-3)\)

good

Let's try the next.

The factors of \( x^2 + 3x - 10 \) are ---?

\((x-2)(x+5)\)

Error in formatted read--illegal character on input.

Be sure you use the correct form to enter the binomials.

Try again. Use the form \((x\ )\).

\((x-2)(x+5)\)

Good

Let's try the next.

The factors of \( x^2 - 3x - 10 \) are ---?

\((x-2)(x+5)\)

Good

Let's try the next.

The factors of \( b^2 - 4b - 12 \) are ---?

\((b-6)(b+2)\)

Good

Let's now summarize what we have learned. If the sign of the last term of the trinomial is positive, the last terms of the binomial factors must both have the same sign. Answer 'same' or 'opp' for opposite.

Same

That's right, and if the sign of the last term is negative, the factors must have the same sign?

Answer 'same' or 'opp' for opposite.

opp

Yes, you've got it!

Now for a final review. This time you will be given 10 trinomials to factor. You will be told how well you did at the end. Type your answers carefully in the correct form of \((x\ )\).

\(x^2 - 4x - 21\)

\((x-7)(x+3)\)

\(a^2 + 2a - 12\)

\((a+6)(a-2)\)
\( x^2 - x - 20 \\
(x-4)(x+5) \)

\( x^2 - 13x + 32 \\
(x-4)(x-8) \)

\( s^2 \cdot 105 - 2s \\
(5-2)(s-102)(s+12) \)

ERROR IN FORMATTED READ--ILLEGAL CHARACTER ON INPUT

\( A^2 - 10A + 25 \\
(A-5)(A-5) \)

\( P^2 + 11P + 28 \\
(P+4)(P+7) \)

\( q^2 + 7q - 30 \\
(q-3)(q+10) \)

\( r^2 - 4r - 32 \\
(r+4)(r-8) \)

YOU MISSED 2 OUT OF 10. YOUR SCORE IS 80 PER CENT.
Appendix

Sample Lesson

Name______________________________

Factoring Trinomials

The process of factoring is the reverse of multiplication. Since 
\((x + 3)(x + 2) = x^2 + 5x + 6\), we say that 
\((x + 3)\) and \((x + 2)\) are the binomial factors of the 
trinomial \(x^2 + 5x + 6\). We can write 
\(x^2 + 5x + 6 = (x + 3)(x + 2)\). 1. Ans. ________
or 
\(x^2 + 5x + 6 = (x + 2)(x + 3)\). 2. Ans. ________
The order in which we write the binomials doesn't matter.
Consider again the trinomial \(x^2 + 5x + 6 = (x + 3)(x + 2)\).
Note that the product of the first terms of each binomial, 
x\(\cdot x\), is the first term of the trinomial, \(x^2\). Notice that 
the product of 3\(\cdot 2\) is 6, the last term of our trinomial, 
while the sum of 3 and 2 is 5, the coefficient of the 
middle term of our trinomial.

Consider factoring the trinomial \(x^2 + 7x + 6\). Since 
the first term factors into \(x\cdot x\), we will have 
\((x + \_)(x + \_)\). This time we are looking for two integers 
whose product is 6 and whose sum is \_(3. Ans. ________
The numbers are \_ and \_. 4. Ans. ___ and ___
Therefore the factors of \(x^2 + 7x + 6\) are __________

5. Ans. __________
Now see if you can factor $a^2 + 5a + 4$.

6. Ans. __________

Try a few more:  
$x^2 + 6x + 8 = 7. Ans. __________$
$b^2 + 7b + 12 = 8. Ans. __________$
$a^2 + 9a + 18 = 9. Ans. __________$
$a^2 + 11a + 18 = 10. Ans. __________$
$14 + 9x + x^2 = 11. Ans. __________$
$x^2 + 8xy + 7y^2 = 12. Ans. __________$

So far we have only considered trinomials whose terms are all positive. Suppose the middle term is negative, as in $x^2 - 6x + 8$. Now we are looking for two integers whose product is +8 but whose sum is -6. Our numbers are ___ and ___.

13. Ans. ___ and ___

Thus the factors of $x^2 - 6x + 8$ are __?__.

14. Ans. __________

In factoring a trinomial, if the sign of the last term is positive, the signs of both factors must be the same; either both positive or both negative. To be sure you understand, try these:

$x^2 - 3x + 2 = 15. Ans. __________$

$a^2 - 10a + 24 = 16. Ans. __________$

Now suppose the sign of the last term is negative, as in $x^2 + 2x - 15$. We are now looking for two integers whose product is -15 and whose sum is ___.

17. Ans. __________

First think about the product. There are four ways in which we can express -15 as the product of two integers:
Which of these four ways is the correct choice for our example of \( x^2 + 2x - 15 \)? Answer with the correct number: 1, 2, 3, or 4.

1. \((+15)(-1)\)
2. \((-15)(+1)\)
3. \((+5)(-3)\)
4. \((-5)(+3)\)

Thus we know that the factors of \( x^2 + 2x - 15 \) are ___.

Let's now summarize what we have learned so far. If the sign of the last term is positive, the binomial factors must both have the ____?____ sign. Answer "same" or "opposite".

If the sign of the last term is negative, the binomial factors must each have ____?____ signs. Answer "same" or "opposite".

Now for a final review let's try these:

\[
\begin{align*}
x^2 + 6x + 8 &= 22. \text{ Ans.} \\
x^2 - 6x + 5 &= 23. \text{ Ans.} \\
x^2 + 3x - 10 &= 24. \text{ Ans.} \\
x^2 - 4x - 21 &= 25. \text{ Ans.} \\
x^2 + 2x + 1 &= 26. \text{ Ans.} \\
x^2 - 15x + 26 &= 27. \text{ Ans.} \\
x^2 - x - 42 &= 28. \text{ Ans.} \\
x^2 + 5x - 50 &= 29. \text{ Ans.} \\
x^2 + 6xy + 7y^2 &= 30. \text{ Ans.}
\end{align*}
\]