EXPERIMENTAL STRESS
ANALYSIS OF A HOLLOW BEARING BALL
WITH WEB REINFORCEMENT

A graduate project submitted in partial satisfaction of the requirements for the degree of Master of Science in Engineering

by

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LIST OF SYMBOLS

\( E \) = Modulus of Elasticity  
\( K \) = Constant  
\( R \) = Resistance, radius  
\( S_g \) = Strain Sensitivity of a Gage, Gage Factor  
\( \Theta, \Phi \) = Angles  
\( \varepsilon \) = Normal Strain  
\( \varepsilon_a, \varepsilon_b, \varepsilon_c \) = Normal Strain in Three-Element Rectangular Rosette Strain Gage  
\( \varepsilon_1, \varepsilon_2 \) = Principal Normal Strains  
\( \nu \) = Poisson's Ratio  
\( \sigma \) = Normal Stress Component  
\( \sigma_1, \sigma_2 \) = Principal Normal Stress  
\( P \) = Load
ABSTRACT

ANALYSIS OF A HOLLOW BEARING BALL
WITH WEB REINFORCEMENT

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Problems encountered with bearings operating in very high DN values (bearing bore in millimeters times shaft speed in rpm) have led to experiments on several types of hollowed ball bearings which improve bearing fatigue life due to mass reduction and lower centrifugal forces.

This study is an experimental stress analysis of a ball bearing with approximately 50% mass reduction made by drilling a concentric hole into the ball bearing leaving a web at the center. The web is presumed to improve flexural fatigue life over a no web configuration.
THE PROBLEM

Among the parameters that affect bearing design there is the DN value defined as bearing bore in millimeters times rotational speed in rpm.

Advancing technology has brought about higher DN value requirements, particularly in jet engine bearings. Predictions are made that DN values of 3 to 4 million will be required in the future.

Fatigue life from increased centrifugal force resulting in increased contact pressures between ball bearings and their bearing race at DN value of 1.5 million becomes prohibitively short. One proposal to increase the fatigue life at higher DN values was to reduce the centrifugal force of ball bearings by reducing their mass. A cylindrical hole concentric to the ball is one of the simpler solutions which effectively reduce the bearing mass.

A further idea of interest was to consider a built-in web at the center of the ball bearing with the cylindrical hole. The built-in web presumably reduces the flexural strain and in turn reduces the stress level in the ball, achieving increased fatigue life. Previous tests indicate flexural fatigue has been a problem with hollowed ball bearings.

Static tests with strain gages seemed the simplest and most appropriate method of analysis in view of time.
and cost limitations to this project.
TEST

Description of Test Article

Static test was accomplished on a scaled up acrylic bearing ball. The bearing ball diameter was 4.74 inches. A concentric hole was drilled through the ball of 2.89 inches in diameter which was equivalent to a 50% mass reduction.

A repeat of static testing was done after bonding a center web in the hole. The web was .370 inch thick which was equivalent to replacing 4.35% of solid ball mass. In other words, the ball with the concentric hole and a center web in the hole had an overall mass reduction of 45.65%.

Strain Gage Installation

A three-element rectangular rosette foil gage type FRG-3 manufactured by Tokyo Sokki Kenkyujo Co. was installed as shown in Figure 1 and Figure 2. Epoxy cement was used to bond the gages to the model bearing ball.

Strain Indicator and Switching and Balancing Unit

Budd Digital Strain Indicator Model P-350 and a Budd Switching and Balancing Unit were utilized.

The following simplified schematic diagram describes the arrangement of a null balance Wheatstone bridge.
FIGURE 2

VIEW A-A

GAGES

30°
The potentiometer is placed across the bridge from points "B" to "D". The center tap of the potentiometer is connected to point "C". A null-balance meter "G" is placed across the bridge between point "B" and "D". The potentiometer adjustment which is calibrated is proportional to the resistance change in the active gage.

\[ S_g \xi = \frac{\Delta R_{\text{active}}}{R_{\text{active}}} \]

Where \( S_g \) = Strain gage factor
\( \xi \) = Strain inch/inch
Gage Factor

\[ S_g = \frac{\Delta R}{R} \] when \[ \epsilon_{yz} = -2.285 \epsilon_{xz} \]

When a gage is mounted parallel to the X axis, and the test specimen subjected to a uniaxial stress the above biaxial state of surface strain results. The sensitivity determined to convert the change in electrical resistance to strain is called the gage factor.

The gage factor for the gages used in this test was 2.15. The Budd Digital Strain Indicator provided for dialing the gage factor directly into it, taking the factor into account.

Gage Current and Its Associated Temperature Effects

It is not generally recognized that temperature effects from low gage current and its associated power dissipation may produce erroneous strain indications. However, in this test the acrylic ball has a lower thermal conductivity than steel which may have contributed to some erroneous strain measurements.

For static problems a strain indicator with a 3-volt RMS supply, which was the condition of this test, with 120-OHM gage, is often used. Then the gage current produced is 25 milliamps. This current is considered to be normal without producing any significant strain error due to temperature
change, but with the acrylic ball bearing of low heat conductivity, the heat from gage current is not dissipated and may produce erroneous strain data. The amount of error could not be determined. However, the objective of this test was the comparison between a hollowed ball with and without a center web, therefore, this error would not be a factor affecting this comparison.

**Test Load**

In order to prevent the test specimen from being fractured during the test a preliminary test on a smaller plastic ball bearing was made.

\[
R_S = \text{Radius of Small Ball} \\
= .97 \text{ inch} \\
R_L = \text{Radius of Large Ball} \\
= 2.37 \text{ inches} \\
P_{S, FRACTURE} = 1010 \text{ lbs.} \\
\sigma_{FRACTURE} = \text{Fracture Stress} \\
\sigma_{FRACTURE} = \frac{K P_{FRACTURE}}{R^2} \]

\[
P_{S, FRACTURE} = \frac{\sigma_{FRACTURE} R_S^2}{K} \quad \text{(small ball)} \\
P_{L, FRACTURE} = \frac{\sigma_{FRACTURE} R_L^2}{K} \quad \text{(large ball)} \\
P_{L, FRACTURE} = \frac{\sigma_{FRACTURE} R_L^2}{K} \times \frac{K}{\sigma_{FRACTURE} R_S^2} \\
P_{L, FRACTURE} = \frac{R_L^2}{R_S^2} \]
\[
P_{\text{fracture}} = \frac{4.74^2}{1.94^2} \cdot \frac{P_{\text{fracture}}}{P_{\text{fracture}}}
\]
\[
= \frac{22.5}{3.76} \times 10^{10}
\]
\[
= 6040 \text{ lbs.}
\]

A second criteria of approximately 500 micro inch/inch strain which was considered to be in the safe range that would preclude fracturing of the test specimen. 300 lbs. load did produce strain somewhat greater than 500 micro inch/inch but was considered to be safe from the above fracture load calculations.

Therefore, maximum load of 300 lbs. and half the load of 150 lbs. were listed as the two test loads.

**Modulus of Elasticity**

Tensile test of acrylic bars gave the following test results shown in Figure 3. From these results E of \(0.48 \times 10^6 \text{ lb/in}^2\) was considered the best estimation to be used for stress calculations. Poisson's ratio, \(v\), was considered to be 0.3.

**Load Positions and Stress Calculations**

Loading positions for the two bearing conditions are tabulated in Table 1. Stress calculations were accomplished at each rosette position for all the loading conditions by using...
a computer program.

The two principal state of stress is given by the following equation:

\[
\sigma_1 = E \left( \frac{e_A + e_C}{2 (1-\nu)} + \frac{1}{2 (1+\nu)} \sqrt{(e_A - e_C)^2 + (2e_B - e_A - e_C)^2} \right)
\]

\[
\sigma_2 = E \left( \frac{e_A + e_C}{2 (1-\nu)} - \frac{1}{2 (1+\nu)} \sqrt{(e_A - e_C)^2 + (2e_B - e_A - e_C)^2} \right)
\]

Stress results that were of interest in comparing the hollow ball with and without the center web were plotted in Figures 4, 5, 6, 7, and 8 for clearer illustration.
Modulus of elasticity is calculated as shown:

\[ E = \frac{P}{A} \times \frac{1}{\varepsilon} \]

Where

\[ P = 300 \text{ lbs.} \]
\[ A = .2 \text{ in}^2 \]

\[ E = \frac{300}{.2} \times \frac{1}{.003365 - .00023} \]

\[ E = .477 \times 10^6 \text{ PSI} \]
TABLE 1

CONFIGURATIONS WITH AND WITHOUT CENTER WEB

INTERNAL GAGE MEASUREMENTS
ORIENTATION AND LOADING

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$0^\circ$</th>
<th>$30^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
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<tbody>
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<td>$0^\circ$</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$+20^\circ$</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$-20^\circ$</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$+40^\circ$</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$-40^\circ$</td>
<td>150</td>
<td>150</td>
<td>150</td>
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</table>

EXTERNAL GAGE MEASUREMENTS
ORIENTATION AND LOADING

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\theta$</th>
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<th>$90^\circ$</th>
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<tr>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$+20^\circ$</td>
<td>300</td>
<td>300*</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>150*</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$-20^\circ$</td>
<td>300</td>
<td>300*</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>150*</td>
<td>150</td>
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</tr>
<tr>
<td>$+40^\circ$</td>
<td>150</td>
<td>150*</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$-40^\circ$</td>
<td>150</td>
<td>150*</td>
<td>150</td>
<td></td>
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</table>

*Weights or loads are shown in lbs.
*No measurements taken for no web condition.
\[ \phi = 0^\circ \quad \text{LOAD} = 300 \text{ LB} \]

**INTERNAL SURFACE**

\[ \sigma_1 \]

\[ \sigma_2 \]

**EXTERNAL SURFACE**

\[ \sigma_1 \]

\[ \sigma_2 \]

**PRINCIPLE STRESS RESULTS**

**FIGURE 4**
$\phi = +20^\circ$ LOAD = 300 LB

**INTERNAL SURFACE**

$\sigma_1$

$\sigma_2$

**EXTERNAL SURFACE**

$\sigma_1$

$\sigma_2$

**PRINCIPLE STRESS RESULTS**

**FIGURE 5**
$\phi = -20^\circ$ LO\textspace AD = 300 LB

INTERNAL SURFACE

\[ \sigma_1 \]

\[ \sigma_2 \]

EXTERNAL SURFACE

\[ \sigma_1 \]

\[ \sigma_2 \]

PRINCIPLE STRESS RESULTS

FIGURE 6
\[ \phi = +40^\circ \quad \text{LOAD} = 150 \text{ LB} \]

**INTERNAL SURFACE**

- \( \sigma_1 \)
- \( \sigma_2 \)

**EXTERNAL SURFACE**

- \( \sigma_1 \)
- \( \sigma_2 \)

**PRINCIPLE STRESS RESULTS**

**Figure 7**
\( \phi = -40^\circ \)  \ LOAD = 150 \text{ LB} \\

**INTERNAL SURFACE**

\[ \sigma_1 \]

\[ \sigma_2 \]

**EXTERNAL SURFACE**

\[ \sigma_1 \]

\[ \sigma_2 \]

**PRINCIPLE STRESS RESULTS**

**FIGURE 8**
DISCUSSION

To discuss the direction of load application, φ will be defined as the tilt angle and θ as the roll angle.

Tilt angle is the angle between the axis of the cylindrical hole and a horizontal reference plane. Roll angle is the rotation about the axis of the cylindrical hole.

Figures 4 through 8 indicate consistently higher principle stress near the center of the bearing ball when no center web is installed.

However, at the outer rim of the cylindrical hole for a tilt angle of +40°, a significantly high principle stress for the webbed ball was indicated. This was the highest principle stress encountered when rolled at +40° tilt angle.
CONCLUSION

At tilt angles of ±40°, the outer rim becomes the highest stressed area indicating a limiting angle of some value between 20° and 40° should be recommended. With this limitation, the center web reinforcement does definitely produce a benefit in increasing flexural fatigue life of the bearing ball.

Further evaluations can be made to determine trade off values of mass reduction for bearing race stress and flexural stress in the bearing ball itself.
BIBLIOGRAPHY

Dally, James W., and Riley, William F.:
EXPERIMENTAL STRESS ANALYSIS, McGraw-Hill