California State University, Northridge

RATIO, PROPORTION, PERCENT AND CONSEQUENTIAL
TOPICS IN GEOMETRY

A graduate project submitted in partial
satisfaction for the degree of Master of Arts in

Education

by

Shirley Jean Teter

May, 1973
The graduate project of Shirley Jean Teter is approved

California State University, Northridge

May 1973
PREFACE

This project proposes a method to teach percent to general mathematics students by materials that do not rely upon memorization of formulas but show the relationship among ratio, proportion, and percent. Although the word percent has been a part of students' mathematical education since junior high school, students in general mathematics do not have an adequate understanding of the concept, much less a facility for ease in computation.

A clarification of the distinction between the nouns percent and percentage may be welcomed. This project regards these words as follows: Percent is a mathematical process; and percentage is the result of that process. For example, in the problem "16 is what percent of 32?", one uses a mathematical process, percent, to find the percentage. For our problem, the answer, 50%, is the percentage found by percent. Another use of the word percentage is in the business world--percentages of increase or decrease.

This investigation of the teaching of percent to general mathematics classes begins with a review of the objectives of the course in conjunction with several state approved texts. Then follows a review of the literature which includes the history of general mathematics, the general mathematics student and his mathematical education, general mathematics programs and texts,
ratio and proportion in the secondary schools, teaching of percent, and proportional reasoning. Third, accepted learning theories are discussed. Finally, an overview of the materials of the project suggests that the materials fulfill course objectives, follow recommendations of the research, and use accepted learning theories.

Two sparse areas were found during the progress of this project: research related to general mathematics and research related to teaching percent. The last area, prior to this project, has been empty.

The materials were acquired from three major sources. The first source is original materials created by the investigator. The other two are texts, one of which has an accompanying test booklet. These texts were written especially for students who had had difficulties with the basic concepts and skills of elementary arithmetic and elementary problem solving. In addition, they were designed to stimulate students of limited interest or ability—in a word, the students explore a wide variety of mathematical situations and find that they can enjoy mathematics while learning.
# TABLE OF CONTENTS

Chapter

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>1</td>
</tr>
<tr>
<td>Definition of Terms</td>
<td>6</td>
</tr>
<tr>
<td>II. REVIEW OF THE LITERATURE</td>
<td>7</td>
</tr>
<tr>
<td>History of General Mathematics</td>
<td>7</td>
</tr>
<tr>
<td>Programs in General Mathematics</td>
<td>12</td>
</tr>
<tr>
<td>Ratio and Proportion in the Secondary Curriculum</td>
<td>18</td>
</tr>
<tr>
<td>Teaching of Percent and Proportional Reasoning</td>
<td>19</td>
</tr>
<tr>
<td>Summary</td>
<td>21</td>
</tr>
<tr>
<td>III. THE PROBLEM SOLVED</td>
<td>23</td>
</tr>
<tr>
<td>The Solution</td>
<td>23</td>
</tr>
<tr>
<td>Implications</td>
<td>24</td>
</tr>
<tr>
<td>Summary</td>
<td>29</td>
</tr>
<tr>
<td>IV. DESCRIPTION OF THE MATERIALS</td>
<td>30</td>
</tr>
<tr>
<td>Arrangement of Worksheets</td>
<td>30</td>
</tr>
<tr>
<td>Additional Comments about Materials</td>
<td>30</td>
</tr>
<tr>
<td>Acknowledgement of Sources for Materials</td>
<td>30</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>32</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>35</td>
</tr>
</tbody>
</table>
Chapter 1

THE PROBLEM

From the investigator's limited experience in teaching general mathematics, it was found that much of the material is based on computation and rote memory rather than material that is based on the thinking processes. With this in mind, a method to teach percent to general mathematics and the materials to be utilized in this teaching will be presented.

One problem that a teacher faces in general mathematics classes is selecting materials to teach percent which emphasize thinking rather than computation. This project centers around the possibility of introducing the students to the study of percent by showing the relationship of ratio, proportion, and percent, and to materials which emphasize thinking rather than computation. Percent is one concept that is part of the curriculum in general mathematics and has been a part of the students' study of mathematics in previous years. However, most of these students do not have a significant understanding of percent.

Since these students lack a significant understanding of percent, most curriculum developers feel that this lack works an insurmountable hardship on the young person attempting to advance himself toward an adult work role. Typically, when a student reaches the ninth grade, he is given an aptitude test in mathematics. From
the results of this test, he may take algebra or something vari-
ously called general mathematics. This division is not to be
compared with ability grouping in English or history, as the mate-
rial in algebra is very different from that taught in general
mathematics. The difference is not merely in standards, or in in-
tensity, but in actual content. One of the aims of general mathe-
matics is to develop significant learning in basic mathematical
skills.

The content for general mathematics emphasizes arithmetic
skills and computation. The teaching of these skills can be in-
terpreted in two different aspects. If the teacher regards mathe-
matics as essentially computation, then he will teach it one way.
If he looks at mathematics as a way of thinking, then he will teach
it another. The teacher's methods are guided by the objectives of
the course and by the materials and texts available.

The objectives of general mathematics emphasize computation.
For instance, the objectives in the California curriculum guides
for the Modesto City Schools, Modesto, Lynwood Unified Schools
District, Lynwood, and Pomona School District, Pomona, reveal this
fact. Among the objectives listed are (1) to provide more drill in
fundamental processes of arithmetic, (2) to develop accuracy and
speed by use of computational short cuts and drill, (3) to build a
solid foundation for understanding the basic principles in mathe-
matics, and (4) to develop understanding of word problems through
careful reading analysis and application of appropriate mathematical
processes. Note that three of the four objectives emphasize compu-
tation!
Many school districts use texts and materials that emphasize computation and repeat materials presented in previous years. In a study in 1963 by the National Council of Teachers of Mathematics (NCTM), the more popular text used in general mathematics was Stein's *Refresher Mathematics* (1962). This text is referred to as a conventional text. Its contents may be typified by the pattern shown on p.17: a few examples of a computational problem followed by a page of similar exercises. Most California State approved texts for general mathematics were found to repeat materials that were a part of elementary school texts. It would seem more reasonable that material for general mathematics should be presented in a different manner.

Most texts were found to rely heavily on memorization of formulas to solve percent problems. The manner in which this memorization is presented can be illustrated by the examples taken from the textbook *Arithmetic Concept and Skills* (1967). This textbook introduces the idea of percent by showing the relationship between ratio and percent. This statement leads to a definition that percent is a ratio using 100, or percent means dividing by 100. The three types of computation problem are presented with exercises for practice by the student; at this point formulas are used which necessitate memorization. Furthermore, no explanation is given on how to set up a ratio nor how ratio and percent are related. Other texts that use this typical presentation are *School Mathematics II* (1967) and *Modern School Mathematics* (1967). The former text explores the relationship between proportion and
percent and explains that equivalent ratios and proportion are the same. The latter text does not show this relationship. Both texts use the definition of percent and exercises similar to those given in *Arithmetic Concept and Skills*. Stein’s *Refresher Mathematics* excludes the relationship between ratio or proportion and percent and strictly relies on the memorization of formulas. Therefore, these texts follow the spirit of the aforementioned objectives which emphasize computation rather than thinking.

General mathematics texts and materials must create interest before the students will make any effort to learn the skills they unfortunately learned to dislike in earlier years. By using thought-producing texts and materials, the teacher is charged with the task to stir these students out of their state of lethargy and shoot a breath of fresh air in them. The task is to stimulate them toward the possibility of discovering mathematical concepts.

No matter how formidable the task may be to teach these students, it cannot be too formidable with a broad and determined effort. Hence, it is proposed that percent must be taught by using materials that emphasize the relationship among ratio, proportion, and percent since most of the material in general mathematics rely heavily on memorizing formulas and do not emphasize this relationship. The materials in common use are ineffectual and inadequate to accomplish significant learning of percent.

This project is a collection of materials to present a total view of ratio, proportion, percent, and consequential topics in geometry. Some of the features of the materials are: (1) From
the beginning, the students will study ratio, proportion, and percent mainly by word problems, and will be involved in the process of solving word problems. A careful step-by-step process will be used to show how word problems are solved, thereby the solutions to these types of problems will become easier as the study progresses. Since all students will work in the same framework, the feeling of self-doubt in the ability to perform these operations will be minimized.

(2) The consequential topics treating geometrical interpretation of ratio and percent will be presented from a discovery approach.

(3) Computation problems and word problems on percent will use proportion as the method of solution. (4) The processes learned at one stage will be used repeatedly in subsequent stages; hence, there will be no need for memorizing.

All of the cognitive operations incorporated in the materials are in the categories of ratio, proportion, and percent. In the category of ratio, the student will interpret and solve single ratio problems that involve rate, geometrical interpretations of similar triangles, area, coordinates of a point, segments and sides of a triangle. In the category of proportion, the student will interpret and solve proportion in work problems, scale drawings, and unit measures. In the category of percent, the student will interpret and solve problems that require one to translate sentences from percent to ratio, to use proportion to find percent in work problems, to compare areas by percent, to calculate three typical percent problems, and to find percent of increase and decrease.
The materials assume that the students are proficient in basic arithmetic skills, especially multiplication, division, and working with fractions. The materials may be utilized in a geometry, pre-algebra, or beginning algebra course, whenever a better understanding or a review of percent and related topics is needed. All of the materials are presented in worksheet form which permits the teacher to use any part, be it section, topic, or concept, for individualized instruction.

The following terms used in this project will be recognized to have the following definition or interpretation: general mathematics, a course offered on the secondary level for students not taking algebra or geometry and a course that is used to satisfy graduation requirements in mathematics; ratio, comparing two quantities; proportion, two equal ratios; and secondary level, grades seven through twelve.
Chapter II

REVIEW OF THE LITERATURE

The reviewing of the related literature is distributed over four topics: history of general mathematics, research on general mathematics programs, research concerning ratio and proportion in the secondary curriculum, and research on teaching of percent and on proportional reasoning.

History of General Mathematics

An overview of the history of general mathematics gives this project perspective. Paige (February, 1964) discussed the nature of general mathematics, its history and future. He stated that general mathematics was introduced into the secondary curriculum during the first two decades of the twentieth century. The utilitarian motive was the vogue in public education at that time, and the existence of parallel curricula, college preparatory and vocational, called for a new mathematics course which spawned the multitude of courses known as general mathematics. The utilitarian motive which started this new mathematics course was augmented by the motive for mass education. Course offerings in this field continued to grow with no apparent direction or control. At that time, the publication of a new text would start a new course.
Toward the end of the first quarter of the present century, general mathematics was given some direction. One of the first directive forces was the 23rd Yearbook of the National Society of the Study of Education, Part II (1923). Half of this study was devoted to the vocational or utilitarian needs of students. This study was the first of three early studies which tended to unify the various offerings in the field now known as general mathematics. Another unifying force was NCTM during the depression years. The 17th Yearbook of the National Council of Teachers of Mathematics (publication date not given) considered everyday applications of mathematical skills. While NCTM recognized the value of general mathematics, Paige noted that many of its members felt insulted if they were asked to teach general mathematics since it had not been recognized as an acceptable course. A third unifying force was the first report published by the Commission of Post-War Plans (1944). The report suggested three general types of courses for high school mathematics: first, mathematics related to practical vocations; second, sequential mathematics for the college-bound student; and third, social mathematics for the consumer. The Commission's plans identified what should be accomplished in each of these, but differences in general mathematics did not end. More attacks and defenses continued against and for general mathematics than in any other area in the secondary school curriculum.

Nevertheless, the placement and content of general mathematics still retain one major problem—acceptability. Some mathema-
tics teachers still are most reluctant to teach even one section of this course because of the problem of acceptability. Paige noted that the problem of acceptability tends to stigmatize students, which, in turn, creates more problems than the guidance departments want to handle. Thus, in order to avoid the problem of stigmatized students, general mathematics must be recognized as an acceptable course.

Just who are the students in general mathematics and what has been their "mathematical education?" Weiss' report (October, 1967) on the research in the teaching of mathematics to the terminal students helps to answer that question. He stated that 40% of 100,000 terminal high school students enrolled in the New York City high schools were placed in general mathematics. These students, he noted, had one or more of these characteristics: (1) college-capable students whose mathematical talents had as yet not been discovered, (2) students who had been insufficiently challenged, (3) students who wanted to prepare for an occupation that did not require them to take algebra or geometry, and (4) students from economically depressed areas, lazy or reluctant students, unmotivated students, and students with low native ability. Because these students were preparing neither for college nor for work, they did not have a sense of direction or commitment. Thus they tended to drift aimlessly through school—many dropping out before graduation. This large segment of the secondary school population included students who ranged across the whole spectrum of ability, motivation, and stability. The distribution of these characteristics
differed from those of the college-bound group, and a wider difference was found within each characteristic of the terminal students as compared with the college-bound students.

In the past, schools have placed these students into a general math class, assigned the less-favored materials, often were given the least competent teacher—and hoped for the best. Often success with these students was measured inversely to the amount of misbehavior in the class. A class that was "under control" was every teacher's goal. What these students learned, or how they learned, was considered to be of minor importance. What effect such teaching had on their attitude toward mathematics seemed of even less importance.

A typical mathematics program for these students consisted of a review of the arithmetic skills they were supposed to have learned in previous grades, followed by practice in what had not been learned. Very often these classes bogged down under the "review" and never progressed beyond it. The less the students knew, the more examples they worked, on the theory that "practice makes perfect." This "practice" also served to keep the students busy and orderly.

The idea of introducing these students to a mathematics program that dealt with mathematics rather than applications, and of organizing such a program in a way that would enable them to achieve some degree of mathematical maturity and power, was rejected. It was felt that nonacademic students did not have the ability, the interest, or the readiness to participate in such a program.
The results of this kind of mathematics program are well known. It produces many students who never really acquire the basic skills and understanding to deal with number and form. Its students are never really exposed to a broad view of the world of elementary mathematics through which they could explore their interests and test their abilities. So they lose confidence, faith, and hope in their ability to succeed in mathematics. They develop such an intense dislike for mathematics that it becomes an obsession with them for the remainder of their lives. This was the picture of general mathematics before the late fifties.

Then came the revolution in mathematics. It began in the colleges right after World War II and was boosted by Sputnik in 1957. A chain of activities was triggered that led to the development of modern programs in mathematics which were introduced into the secondary schools. Unfortunately, these modern programs were aimed at the college-capable students and ignored the terminal student.

These two studies develop the history of the curriculum in general mathematics and the type of instruction and materials used. It must be emphasized that these studies are not a few particular cases in our nation, but rather they typify the picture of general mathematics everywhere. (NCTM, it may be noted, is currently developing better materials for these students.)
Programs in General Mathematics

Research examining programs in general mathematics are sparse. One gets the feeling that since this mathematics course has been in secondary schools for nearly half a century, it is hallowed beyond investigation. The following reports suggest otherwise; indeed that it is holey (!) rather than holy.

Peterson (June, 1963) conducted a study to find if the traditional ninth grade general mathematics course is necessary. Students were taught arithmetic skills in a planned class that was in some other area of instruction, that is, not in a mathematics class. One purpose was to observe the growth and maintenance of the skills between students at the same grade level who took a formal mathematics course in comparison with those students not taking a formal course. An example from Peterson's study was: if a homemaking teacher wanted students to change a recipe from serving six to serving four people, then it was the responsibility of this teacher to review the concepts of ratio and proportion. The results of the study indicated that the traditional required ninth grade general mathematics course is not necessary for the maintenance of arithmetic skills when these skills are taught in the manner described. The study also concluded that non-mathematics teachers perform adequately in teaching and using arithmetic skills in their courses.

A comparison of achievement of low ability ninth grade students assigned to classes in general mathematics, modified
algebra, and a reading control group which received no formal mathematics instruction was conducted by Sederberg (1964). The pre-tests and post-tests measured functional competence in arithmetic computation, algebraic skill and insight, problem solving, and mathematics skills and concepts of the three groups.

Sederberg found that the general mathematics approach was the best of the three methods in teaching arithmetic computation skills. Low ability students did not acquire algebraic skills and insights from the modified algebraic instruction. None of the groups could claim superiority in problem solving. The post-test gains by the reading control group indicated that factors other than formal mathematics instruction contributed to the acquisition of mathematical skills and concepts. Sederberg concluded that the general mathematics approach is the best for achievement in arithmetic skills by average or below-average students. He agreed with Peterson in suggesting that perhaps the average or below-average students should be taught mathematics in other subject matter fields.

Price (1965) conducted an experiment on discovery and its effect on the achievement and the critical thinking of tenth grade general mathematics students. The control group was taught a normal course using traditional textbooks involving deductive reasoning and using the lecture method of instruction. The experimental group was taught materials similar to the control group but used discovery lessons requiring inductive reasoning. A third class, the transfer group, used the same materials as the experi-
mental group but in addition was given prepared transfer lessons. These lessons were designed to promote various aspects of critical thinking such as assessing evidence, evaluating arguments, and drawing inferences from materials which were non-mathematical in nature. The pre-tests and post-tests for all groups were to measure achievement in mathematics, mathematical reasoning, inductive reasoning, and critical thinking.

The two groups taught by techniques to promote a student's discovery of concepts showed no significant gain over the control group in achievement, but there was a greater increase in mathematical reasoning in comparison with the control groups, and a significant gain over the control group in inductive reasoning with a positive attitude change towards mathematics. The control group showed a negative change in attitude. The transfer group showed a significant increase in critical thinking. Price thought that students using transfer and/or discovery materials had greater interest, enthusiasm, and concern for the class. In addition, these students were more able to consider more material than the control group in the same period of time.

Research on texts used by general mathematics was conducted by Findley (1966). He evaluated the effectiveness of the textbook Advanced General Math against two traditional texts Mathematics to Use and Holt General Mathematics (dates of publication were not given). Group I used Advanced General Math and calculators, Group II used the traditional texts and calculators, and Group III (the control class) used the traditional texts in the usual manner.
At the end of one year, Group II performed better than Group I in arithmetic achievement. Findley suggested that the materials in *Advanced General Math* can be used with success as a traditional text without calculators up to a point, but after protracted and exclusive use of these materials, boredom tends to set in. He recommended that variety of materials and techniques should be utilized in general mathematics classes, and a complete mathematics laboratory would be most advantageous.

Brown (May, 1966) reported on an unnamed California District's improvement of instruction in problem solving for general mathematics students. He stated that the most vexing issues to face curriculum makers in the past quarter of a century are those presented by ninth grade general mathematics. The identification of the student, activities appropriate for the course, and means of evaluation have been the concern of those responsible. The research relating to this grade level has been almost nonexistent, and, in fact, any pertinent literature has been entirely too inadequate to aid in formulating improved programs.

Brown reported on a study that was initiated after several teachers attended mathematics institutes and state department sponsored conferences. In its attempts to set up a curriculum, the group explored some of the new mathematics experimental programs. Some teachers planned to follow one or more of these programs while others seemed content with the traditional program but wished to find ways to make it more effective. It was suggested that the teachers be permitted to teach the general mathematics classes
using any approach for which they could find materials and which they could justify in their own thinking. At the same time, they would evaluate their work. The objectives for all were: (1) to improve competence in the student's ability to work with measurement and percent; (2) to improve competence in the student's ability to solve problems; (3) to develop greater appreciation of and interest in mathematics; and (4) to develop greater insight in and understanding of mathematics processes and the nature of mathematics as a discipline.

At the end of the year, perhaps the most interesting conclusion was that many students made no progress in improving their computation skills, particularly ninth graders. The evidence indicated that the ninth grade programs were ineffective for one-fourth of the students. The same result was found in problem solving. Inasmuch as the problem solving objective was considered the major one, it was felt that the program should be analyzed critically. It was concluded that the ninth grade general mathematics course at the schools tested was comparatively ineffective. From personal experience, observation in other schools, and discussions with teachers, Brown concluded that similar results would be obtained in a majority of schools. The ninth grade general mathematics problem showed itself to be a perplexing one needing serious attention on a national level. He recommended that the complex process of solving problems should be broken down into a logical series of steps and should identify some ways to practice each step.
One of the most extensive reports on general mathematics was given by Schulte (1966). He presented all the research related to general mathematics at the secondary level. Like Brown, he found it surprising that there had not been more research studies at this grade level. Most of the research that he found had been done in a fragmentary manner, and no large-scale, long-term studies had been attempted.

The following are some of the reports to which Schulte referred: (1) The mathematical literacy of Louisiana's Negro high school general mathematics students was analyzed by Randall (1955) in order to determine the common errors made by students and the students' ability to solve word problems. A two-hour mastery test was devised and given to the best general mathematics students in select high schools. The most common errors were in addition, substraction, multiplication, division, positioning of the decimal point, and changing mixed numbers to fractions. Problem solving was very poor. (2) Bernstein (1955) did a similar study in clinics at Cody High School (Detroit) which gave individualized instruction to small groups. He found that 80% of the error patterns was similar to the common errors described by Randall. Bernstein's experiment showed that individualized instruction accomplished about twice as much as class instruction in much less time. Additionally, he observed that many students exhibited gains in overall school adjustment—self-confidence, self-understanding, and easing of character disturbances. (3) Holton (1964) investigated the relative effectiveness of instructional, motivational vehicles
on achievement in general mathematics. Except for the applications, each class used the same unit from a linear-response programmed booklet. Four types of applications were used: (a) automobile, which was chosen for student interest; (b) farming, since the study was carried out in a farming region; (c) social utility, such as insurance, taxation, social security, purchases of groceries and clothing, and saving accounts; and (d) intellectual curiosity, since this was related to the new programs at that time. The only significant differences found were between interest levels where students with high-interest level did better. This difference remained significant in the retention test.

Ratio and Proportion in the Secondary Curriculum

It is pertinent to ask: Does the general mathematics student need to learn ratio and proportion? Aside from practical applications such as in homemaking or wood shop, there would seem to be some need to study these concepts in preparation for other secondary subjects, such as the sciences. The following two articles report on the need for the study of ratio and proportion at the secondary level.

Schaff (October, 1965) sought to determine the scientific concepts that should be taught on the secondary level in mathematics. He observed that, while the curriculum in secondary mathematics can be constructed with little or no reference to the physical sciences, it is nearly impossible to teach physics, chemistry, and portions of general science meaningfully without introducing certain mathematical concepts.
Hence, he proposed the following topics which involve ratio and proportion in the mathematics curriculum on the secondary level:

1. The lever as a simple machine which includes the principles of moments;
2. Temperature, which includes temperature conversion of Fahrenheit to Centigrade degrees;
3. Density, which includes the symbolic expression of a rate rather than a ratio;
4. Elasticity, which measures the ratio of stress to strain; and
5. Force, which uses the horizontal and vertical components as ratios.

The topics of proportion and percent have presented difficulties to the upper secondary grades as reported by Parque (May 1966). He set out to determine the areas of mathematical deficiencies. The deficiencies, among others, lie in proportion and calculation of percentage error.

**Teaching of Percent and Proportional Reasoning**

Similar to research on programs in general mathematics, research examining approaches to teaching percent to secondary students is sparse. In fact, it is nonexistent for general mathematics. Again, one gets the feeling that since percent has been taught in secondary schools for many years, it is hallowed beyond investigation. Unfortunately, the following reports neither sustain nor reject that feeling.

McMahon (1959) compared the initial learning and retention of seventh grade students who were taught percent by the ratio method and by the conventional method that used formulas. At the end of five weeks of the investigation, a percent test was given.
Six weeks later a retention test was given. She found no difference between the two methods of instruction in developing the ability to interpret statements about percent. The ratio method resulted in a greater skill in computation involving percent and more permanent learning than the conventional method. Neither method was successful in teaching percent to low-ability students.

Parplus and Peterson (1970) investigated the student's ability to apply the concept of ratio to a problem requiring a change in the unit of length of measurement, which they called "proportional reasoning." It may be pointed out that this study did not teach anything about ratio and proportion. After a demonstration in measuring a figure with large paper clips, the students answered questions using smaller paper clips and their own "proportional reasoning." The longitudinal study dealt with the Piagetian formal operational thought. The students' answers were categorized and certain sequences measured Piaget's pre-operational developmental stage and formal thought developmental stage. Other sequences were not as easily identified. They were disappointed to find no successful proportional reasoning achieved earlier than the last years in high school, even though the concepts of ratio and proportion were in most mathematics programs in the junior high schools. They concluded that there is a serious gap between secondary school mathematics and science curricula and the students' ability to reason about proportion.
Summary

From the review of the history of general mathematics, the following points are emphasized. General mathematics has been a course having less favored materials and the least capable teachers. The curriculum has been more drill in arithmetic skills on the questionable theory that "practice makes perfect." This course never progressed beyond this arithmetic review and into a program to help increase the students' mathematical maturity and insight. General mathematics has not been an acceptable course, and even the modern mathematics programs ignore this area. This sorry state of affairs has stigmatized students, even today!

Under the heading of programs in general mathematics, there has been too little research. The few articles, however, have made positive suggestions. Noteworthy are the following: (1) Arithmetic skills can be taught in other subject matter areas. (2) Discovery materials stir up interest, create enthusiasm, contribute significantly to the development of inductive reasoning—in a word, they effect a positive attitudinal change in general mathematics students. (3) The teacher does well to use a variety of materials and techniques—a strong argument for a fully equipped mathematics laboratory. (4) Individualized instruction accomplishes more in less time than classroom instruction, not to mention student gains in overall school adjustment.

Under the rubrics: ratio and proportion in the secondary school, teaching of percent and proportional reasoning, three con-
conclusions stand out: (1) A better understanding of ratio and proportion is needed in secondary level physical sciences, which includes ninth grade general science. (2) The ratio method of instruction in teaching computation problems in percent is better than the conventional method that uses formulas. (3) No successful proportional reasoning is accomplished before the late high school grades.
Chapter III

THE PROBLEM SOLVED

The problem of teaching percent to general mathematics students is to select the proper materials. These materials should emphasize thinking rather than computation, with a new approach to ideas and concepts rather than repeating materials used in previous grades. These materials should promote mathematical maturity rather than emphasize memorizing. The solution to this problem is to teach the sequence of ratio and proportion which leads to percent.

The features of this project are:

1. The study emphasizes thinking by using word problems.

2. The study does not emphasize memorizing because what is studied at one stage is used over and over again in subsequent stages.

3. The study is a fresh approach to the relationship among ratio, proportion, and percent. Instead of concentrating on basic operations in arithmetic and their social application, the students are introduced to ideas in geometry. Some examples are: (a) in geometrical interpretations of ratio, the students use ratio to compare similar triangles by finding the perimeter, area, and lengths of corresponding sides of similar triangles; (b) fraction of a percent is introduced by showing a set of 100
squares to represent 100% (or 1), followed by showing a fractional percent, like ½, being a part of one of these squares; (c) the comparison of areas of two different rectangles and other geometrical figures shows the student how to find percents greater than 100% geometrically; and (d) showing percent on a number line by comparing two line segments of equal length but divided into different unit lengths demonstrates another aspect of percent.

4. The materials of the study are in worksheet form. This format affords the teacher the opportunity to use any part of the study for individualized instruction as well as having available materials of select topics for a non-mathematics teacher. This format permits flexibility such that some worksheets may be omitted. For example, although the geometrical treatment of ratio and of percent are from a discovery approach, these topics may be too advanced (or not needed) for some classes.

The use of individualized instruction, non-mathematics teachers for instruction in basic arithmetic skills, and discovery oriented materials are suggestions supported by different research reports (see Chapter II). These ideas suggest how different problems in teaching general mathematics have been handled with some degree of success.

The solution to the problem implies the following:

1. Limitation on learning is not the student's inability to acquire, but the teacher's inability to convey, ideas.

2. Students are in a process of developing ideas--they are not concentrating on arithmetic skills and their social application.
3. The study not only teaches mathematical concepts but also helps raise the students' reading level--they are learning to interpret written materials.

4. The use of materials that emphasize word problems necessitates careful teaching. The solution to word problems must be broken down into logical steps with adequate practice provided for each step; thereby, solving word problems becomes easier as the study progresses. In a sense, the students become acclimated to the language of word problems.

5. By using word problems as exercises, the students tend to mature mathematically--they are analyzing data and applying mathematical principles.

The first and second implications concerning limitations on learning and student involvement in a process of developing ideas are discussed by Weiss (1967). In Chapter II, he cites some work done on the innovations and research about the teaching of general mathematics. Germaine to the implications, Weiss points out how learning theory has changed and what effect this had on general mathematics. He emphasizes Bruner's position (1959) that any subject can be taught effectively in some intellectually honest form to any child in any stage of development. In other words, a good teacher can teach almost any subject to any child provided the teacher uses the right words, materials, and methods. This hypothesis implies that the limitations on learning are due, not to students' unreadiness to deal with the ideas, but to the teacher's inability to teach these ideas. If Bruner is correct,
then a general mathematics program can be built around concepts, principles, and ideas that are important in the world of mathematics. The task is to provide the teachers with the knowledge, words, materials, and methods to teach students these notions.

Bruner is not alone in his position. Closely allied with him is J. M. Hunt. In his book *Intelligence and Experience* (1961), he takes a definite stand. In the past, Hunt notes, people believed that a person's intelligence was immutable, that his IQ was fixed, and that the development of a person's basic responses and abilities were predetermined by heredity. Hunt found these assumptions untenable in the light of Piaget's studies. Relying on Piaget's observations on the development of intelligence in children and drawing upon other research, Hunt questioned the immutability of IQ and suggested that experience rather than heredity is the crucial influence in the development of intelligence. If Hunt is correct, then the crucial factor in developing the mathematical ability of any student is the right kind of mathematical experiences rather than the student's present IQ. Therefore, the possibilities of what can be taught to the general mathematics students are widened.

In summary, whether the students are the gifted or slow, college-bound or terminal, good teaching gives them the opportunity to do two things--to engage actively in the development of ideas and to assimilate them into their own thinking.
The third and fourth implications concerning the improvement of the students' reading level and the careful presentation of word problems are discussed by Perel and Vairo (1968). Their article stated that the prose of word problems in general mathematics texts tends to be long and involved. This is frustrating to the students who read poorly, for they can hardly tell what is desired or what they are supposed to find. It was proposed that the general mathematics course contain materials that will help raise the reading level of the students. Supporting this idea were Bassler and Kolb in their book Learning to Teach Secondary School Mathematics (1971). They proposed that materials for the general mathematics class should be written so that a teacher would be able to combine mathematics lessons with reading lessons. These lessons should give the students practice in interpreting the meaning of the written materials. Brown (1966) clarified how word problems should be presented. He stated that solving problems is a complex process, and growth in this process does not automatically occur as a result of daily assignments of word problems. He recommended that the general mathematics students need to examine this complex operation, break it down into simpler steps, and practice each step separately.

The last implication about problem solving is discussed by Polya (1967), a leading authority on heuristic teaching of secondary level mathematics. He observes that a problem is a "great" problem if it is very difficult, and it is just a "little" problem if
it is just a little difficult. Yet some degree of difficulty occupies a space in the domain of the problem; where there is no difficulty, there is no problem.

Polya notes that our knowledge about any subject has two aspects: information and knowledge. If one has a genuine experience of mathematical work on any level, he does not doubt that know-how is much more important than mere possession of information. Therefore, on the secondary level, teachers should impart, along with a certain amount of information, a certain amount of know-how. He defines know-how in mathematics as the ability to solve problems. He concludes that the first and foremost duty in teaching secondary level mathematics is to emphasize methodical work in problem solving. Furthermore, the most important task of the teacher is to teach how to solve word problems. In solving these types of problems, Polya continues, the students will translate the real situation into mathematical terms. In this manner, they will have an opportunity to experience that mathematical concepts may be related realities but that such relations must be carefully worked out.

According to Polya, this is the first opportunity afforded by the curriculum for this basic experience that mathematical concepts are related realities. Furthermore, this may be the last opportunity for this basic experience for a student who will not use mathematics in his profession. While Polya's philosophy is not wholly accepted by all mathematics teachers, it is the philosophy of this project.
In summary, the materials of this project are in line with recent research and accepted learning theory. A capsule look of the processes involved in this project are as follows: The students first translate word problems involving ratio into mathematical terms. Since a proportion is two equal ratios, word problems involving proportion follow immediately. After this translation is completed, techniques learned in proportion are used to solve problems involving percent. These processes emphasize a study of percent that requires thinking. At the same time, it uses a new approach that promotes mathematical maturity.
Chapter IV

DESCRIPTION OF THE MATERIALS

The materials are worksheets on ratio, proportion, and percent. They are coded at the top: RT (ratio), PR (proportion), and PC (percent). The order of the worksheets are RT, PR, and PC; and each topic has a sequence that may be followed by the numbering 1, 2, 3, etc. The geometrical treatment of ratio and percent are included under those topics. The list of topics for the worksheets are listed in tabular form. Each main topic is introduced by notes for the teacher. The key to all the worksheets and a list of evaluation questions appear at the end of the materials.

Some of the worksheets may be omitted and others need additional comments. The geometrical treatment of ratio (RT 4-8) and the geometrical treatment of percent (PC 9 and 15) may be omitted as they are somewhat advanced for low ability students. It is suggested that no worksheets on proportion be omitted. There are not many exercises on each of the worksheets on the A, B, C's of Percent (PC 10-14) because most general mathematics texts have an adequate amount of these types of computation problems.

The worksheets RT 1 and 2, PR 1-4, PR 6, and PC 1-9 are materials taken from the Holt, Rinehart, Winston textbook Trouble-Shooting Mathematical Skills (1969), pp. 231-280. The
worksheets RT 3, RT 4-6, RT 7, 8, PR 5, and PC 15 are materials taken from the Houghton Mifflin textbook *Patterns in Mathematics* (1970), pp. 260, 243-251, 261-264-265, and 272, respectively. The evaluation questions 1-6 on ratio were taken from the progress tests for *Patterns in Mathematics* (1970), pp. 73-78. The remaining evaluation questions were taken from *Trouble-Shooting Mathematical Skills* on the pages mentioned above.
BIBLIOGRAPHY

A. BOOKS


B. PERIODICALS


Parque, Richard A. "An Experimental Study to Investigate the Mathematical Needs of Students in a Traditional Physics Course," School Science and Mathematics, LXVI (May, 1966), 405-408.

Peterson, Orval L. "Is the Traditional Ninth Grade Mathematics Course Needed?" School Science and Mathematics, LXXIII (June, 1963), 477-479.


C. PUBLICATIONS OF LEARNED SOCIETY


D. UNPUBLISHED WORKS


E. CURRICULUM GUIDES

Pomona Unified School District, Superintendent of Schools, Pomona, California, 1964.


Modesto City Schools, Superintendent of Schools, Modesto, California, 1966.
**TABULAR LIST OF TOPICS**

Below is a list of the 30 worksheets by number and title. Starred items are suitable for a geometry class.

<table>
<thead>
<tr>
<th>Worksheet Nos.</th>
<th>Title or Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT 1</td>
<td>Ratio</td>
</tr>
<tr>
<td>RT 2</td>
<td>Rate</td>
</tr>
<tr>
<td>RT 3</td>
<td>Agreement in Measurements</td>
</tr>
<tr>
<td>* RT 4</td>
<td>Similar Triangles</td>
</tr>
<tr>
<td>* RT 5, 6</td>
<td>Comparing Similar Triangles</td>
</tr>
<tr>
<td>* RT 7</td>
<td>Ratios and Segments</td>
</tr>
<tr>
<td>* RT 8</td>
<td>Ratios and Sides of Triangles</td>
</tr>
<tr>
<td>PR 1</td>
<td>Check cross products in Proportion</td>
</tr>
<tr>
<td>PR 2</td>
<td>Box Statements</td>
</tr>
<tr>
<td>PR 3, 4</td>
<td>Solving Proportions</td>
</tr>
<tr>
<td>PR 5</td>
<td>Scale Drawings</td>
</tr>
<tr>
<td>PR 6</td>
<td>Unit Measures</td>
</tr>
<tr>
<td>PC 1</td>
<td>Meaning of Percent</td>
</tr>
<tr>
<td>PC 2</td>
<td>Showing Percent on a Number Line</td>
</tr>
<tr>
<td>PC 3, 4, 5</td>
<td>Using Proportion to Find Percent</td>
</tr>
<tr>
<td>PC 6</td>
<td>Comparison Using Percent</td>
</tr>
<tr>
<td>PC 7</td>
<td>Fraction of a Percent</td>
</tr>
<tr>
<td>PC 8</td>
<td>Percents Greater than 100%</td>
</tr>
<tr>
<td>* PC 9</td>
<td>Comparing Areas</td>
</tr>
</tbody>
</table>
PC 10, 11, 12, 13, 14  
A, B, C's of Percent

* PC 15  
Geometrical Interpretations

PC 16  
Percent of Increase and Decrease
Notes to Teacher on Ratio

It is important to note that much of the operational work with ratios and many of the concepts are directly tied to the work with fractional numbers. Simplifying a ratio such as 50 parts water to 15 parts alcohol is operationally the same as reducing the fraction 50/15. This tie-in is important; however, it is equally important to avoid stating that the work with ratios and fractions is always the same. For example, consider the statement, "The school won 5 out of the first 12 games and 7 out of the next 11 games." We may write, as a result: (1) The ratio of the games won to the first 12 games played is 5/12. (2) The ratio of the number of games won to the next 11 games played is 7/11. (3) The ratio of the number of games won to the total number of games played is 12/23. While everything is stated correctly, we clearly cannot write the following: 5/12 + 7/11 = 12/23. Ratios cannot be added like common fractions. A very good article about this and how set theory is applied to percent and ratios is, "Percent: a Rational Number or a Ratio" by Jeane Nelson in Arithmetic Teacher, February, 1969, pages 105-9.

Worksheet RT 6 contains some difficult geometric problems. (Do not attempt too much at one time.) In problems 1, 2, 6, 7, and 9, there are two possible points for each solution.
A ratio is a comparison of two numbers that expresses a certain relationship between the measures of two quantities.

1. The directions for mixing orange juice from a can of frozen concentrate calls for 3 cans of water to each can of concentrate. The ratio of water to concentrate is 3 to 1, which may be written as 3:1 or as 3/1.

2. The low range gear ratio in many cars is about 2 1/2 to 1, which means that 2 1/2 revolutions of the engine cause the rear wheels to turn once or the engine turns 2 1/2 times faster than the rear wheels. That ratio may be written as 2 1/2:1 or as 2 1/2/1.

Let us consider the following team scores:

<table>
<thead>
<tr>
<th>School</th>
<th>Won</th>
<th>Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Lady High School</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>St. James High</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

The ratio of wins to the number of losses for each school is as follows:

Our Lady H. S.: \( \frac{\text{wins}}{\text{losses}} = \frac{10}{6} = \frac{5}{3} \) (reduced form) 5 wins for 3 losses

St. James H. S.: \( \frac{\text{wins}}{\text{losses}} = \frac{20}{16} = \frac{5}{4} \) 5 wins for 4 losses

Notice that comparing the wins to the losses by subtraction shows:

Our Lady won 4 more games than it lost.
St. James won 4 more games than it lost.
It is easy to see that the ratio gives a better comparison of the two schools than the subtraction does in this case.

In each exercise, give the ratio in reduced form.

1. A class has 28 juniors and 7 seniors.
   a. What is the ratio: juniors to seniors?
   b. What is the ratio: seniors to juniors?
   c. What is the ratio: juniors to the entire class?
   d. What is the ratio: seniors to the entire class?

2. During target practice Pete scored 22 hits and 10 misses.
   a. What was his ratio: hits to misses?
   b. What was his ratio: hits to total shots?
   c. What was his ratio: misses out of total shots?

3. A 15-foot steel beam weights 450 pounds. What is the ratio pounds to feet?

4. Use the word MISSISSIPPI to answer the following:
   a. What is the ratio: the number of times the letter I appears to the number of times the letter S appears?
   b. What is the ratio: I to P?
   c. What is the ratio: S to P?

5. A punch mixture consists of 4 cups of crushed strawberries, 2 cups of lemonade, 3 cups of ginger ale, and 2 cups of orange juice.
   a. What is the ratio: orange juice to lemonade?
   b. What is the ratio: ginger ale to strawberries?
   c. What is the ratio: lemonade to strawberries?
6. Joe missed 9 questions on a test of 50 questions.
   a. What is the ratio of the number wrong answers to the number of questions on the test?
   b. What is the ratio of the number of wrong answers to the number of correct answers on the test?

7. A school of 1,200 pupils has 48 teachers. What is the pupil-teacher ratio?

8. A building 144 ft. tall casts a shadow 16 ft. long. What is the ratio of the height of the building to the shadow?

9. $A = \{\circ \circ \circ \}$ $B = \{\circ \circ \circ \circ \}$
   What is the ratio of the number of elements in set $A$ to the number of in set $B$?

10. A clothing section of a department store had the following stock:
    - 1800 shirts
    - 3200 pairs of socks
    - 800 neckties
    - 450 sets of pajamas
    - 200 hats
    - 120 lounging robes

    a. What is the ratio of the number of shirts to the number of neckties?
    b. What is the ratio of the number of hats to the number of sets of pajamas?
    c. What is the ratio of the number of neckties to the number of hats?
    d. What is the ratio of the number of neckties to the number of pairs of socks?
    e. What is the ratio of the number of lounging robes to the number of shirts?
    f. How are the ratios obtained in (a) and (b) related? In (c) and (d)?
RATE

When a ratio is used to compare unlike measures, it is called a rate.

Example: 3 grapefruit for 50¢

20 miles per hour

8 inches of elastic to 1 foot of fabric

17 miles for each gallon of gas (usually expressed 17 miles per gallon)

12 points in 3 games (sometimes express as 4 points per game)

Express each rate in simplest (reduced) form.

1. 50 years per 10 seconds
2. 3 quarts for 90¢
3. 4 years for $8.00
4. 10 hits in 2 games
5. 150 miles in 3 hours
6. 14 inches in 182 yards
7. 35 passes in 25 games
8. 42 walks in 168 games
RT 3

To express the ratio of the first measurement to the second measurement, the kinds of measurements must be the same.

Example: To find the ratio: one foot to one inch

First change 1 foot to 12 inches, then express the ratio 12:1; or first change 1 inch to 1/12 foot, then express the ratio 1:1/12.

Express each ratio with whole numbers, when possible.

1. 1 foot, 1 yard
2. 1 inch, 1 yard
3. 3 inches, 1 yard
4. 1 ounce, 1 pound
5. 1 ounce, 1 ton
6. 1 day, 1 year
7. 3 ounces, 2 pounds
8. 1 centimeter, 1 meter
9. 1 square foot, 1 square yard
10. 2 feet and 6 inches, 1 yard
11. 1 pint, 1 quart
12. 1 quart, 1 gallon
13. 1 pint, 1 gallon
14. 1 foot, 1 mile
15. 5 feet, 1 mile
16. 15 feet, 3 miles
17. 35 feet, 7 miles
18. Divide 4 yards into two parts whose ratio is 2:1. Express the answer in feet.
19. Divide 1 yard into two parts whose ratio is 5:7. Express your answer in inches.

20. Divide 1 pound into two parts whose ratio is 3:5. Express your answer in ounces.

21. Divide 1 square yard into two parts whose ratio is 1:2. Express the answer in square feet.
For the triangles $ABC$ and $A'B'C'$ the following are true:

$\angle A = \angle A', \angle B = \angle B', \text{ and } \angle C = \angle C'$. Because of the above equalities, we say triangle $(A)ABC$ is similar to triangle $(A)A'B'C'$ where the $\sim$ stands for "is similar to." Do you agree that the two triangles have the same shape?

The angles that have the same measure are called corresponding angles. Thus, $\angle A$ and $\angle A'$ are corresponding angles, $\angle B$ and $\angle B'$ are corresponding angles. What is another pair of corresponding angles?

The sides opposite corresponding angles are called corresponding sides. Thus, segments $BC$ and $EF$ are corresponding sides. Name the other two sets of corresponding sides.
Use the figures above to answer the questions.

1. $\angle R^\circ =$
2. $\angle S^\circ =$
3. Is $\triangle$ GHI similar to $\triangle$ RST? Why?
4. Which angles corresponds to these angles: $\angle G$, $\angle S$, $\angle H$, and $\angle R$?
5. Which sides correspond to these sides: $\overline{GH}$, $\overline{ST}$, $\overline{HI}$, $\overline{GI}$, and $\overline{TK}$?
6. Do you think that the two triangles have the same shape?
7. Do you think that every triangle is similar to itself?
COMPARING SIMILAR TRIANGLES

You know that the segment joining two points A and B is represented by the symbol $\overline{AB}$. The length of that segment is represented by the symbol $AB$. Keep in mind that $\overline{AB}$ is a segment, but $AB$ is a number. $AB$ is also called the distance between the points A and B.

Length of $\overline{AB} = AB$

Use the figures below to answer the questions.

\[ AB = 1'' \]
\[ BC = \frac{3}{2}'' \]
\[ CA = 3/4'' \]
\[ DE = 2'' \]
\[ EF = 1'' \]
\[ FD = 1\frac{1}{4}'' \]

1. Compute the following ratios:
\[
\frac{AB}{DE} \quad \frac{BC}{EF} \quad \frac{CA}{FD}
\]
Compare your three answers.

2. Compute these three ratios:
\[
\frac{DE}{AB} \quad \frac{EF}{BC} \quad \frac{FD}{CA}
\]
Compare your three answers.

3. (a) Compute the perimeter, $AB + BC + CA$, of $\triangle ABC$.
(b) Compute the perimeter, $DE + EF + FD$, of $\triangle DEF$.
(c) Compute the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$.
(d) Compare your answer of 3(c) with your answer of 1.
4. Compute this ratio:

\[
\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF} \quad \text{(The ratio of the altitude of } \triangle ABC \text{ to the altitude of } \triangle DEF \text{ is 1 to 2.)}
\]
Suppose we are given any two similar triangles, \( \triangle ABC \) and \( \triangle DEF \). From the work from the preceding lesson, we know these four statements are true.

1. \[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}
\]

2. \[
\text{Perimeter of } \triangle ABC = \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}
\]

3. \[
\frac{CS}{FT} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}
\]

The preceding lesson did not show this, but the altitudes CS and FT (how "tall" the triangles are) could be determined by measuring.

4. \[
\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{CA^2}{FD^2} \quad (AB^2 \text{ means } AB \times AB)
\]

You should note that the areas are in the same ratio as the squares of the lengths of the corresponding sides. This is natural since area is measured in square units while length is not.

Use the diagram to answer questions 1-5.
1. Find the coordinates of point U for which \( \triangle STU \sim \triangle ABC \).

2. Find the coordinates of point Y for which \( \triangle WXY \sim \triangle ABC \).

3. Find the ratio of the lengths of the sides of \( \triangle GHI \) to the lengths of the corresponding sides of \( \triangle STU \).

4. Find the ratio of the lengths of the sides of \( \triangle STU \) to the lengths of the corresponding sides of \( \triangle WXY \).

5. Find the ratio of the area of \( \triangle ABC \) to the area of \( \triangle GHI \).

Use this diagram to answer questions 6-12.

6. Find the coordinates of a point C for which \( \triangle ABC \sim \triangle XYZ \).

7. Find the coordinates of a point F for which \( \triangle DEF \sim \triangle XYZ \).

8. Find the ratio of the lengths of the sides of \( \triangle DEF \) to the lengths of the corresponding sides of \( \triangle ABC \).

9. Find the coordinates of a point U for which \( \triangle UVW \sim \triangle XYZ \).
10. Find the ratio of the lengths of the sides of \(\triangle DEF\) to the lengths of the corresponding sides of \(\triangle UVW\).

11. Find the ratio of the perimeter of \(\triangle ABC\) to the perimeter of \(\triangle XYZ\).

12. Find the ratio of the area of \(\triangle UVW\) to the area of \(\triangle XYZ\).
RATIONS AND SEGMENTS

You can discover a useful principle by studying the diagram below.

The distance AL is half the distance LP. Therefore L divides the segment AP in the ratio 1:2 (AL:AP as 1:2). The distance MP is half the distance MB. Therefore M divides the segment PB in the ratio 1:2. Now, guess the ratio in which K divides AB.

Test your guess by comparing the two lengths AK and KB.

Find the coordinates of the described points on the segment AB in the diagram.

1. C, so that AC:CB = 1:1. 5. G, so that AG:GB = 5:1
Find the coordinates of the described points on segment $PQ$ in the diagram.

9. K, which is the midpoint of $PQ$.
10. L, so that $PL:LQ = 1:3$.
13. V, so that $PV:VQ = 3:5$.
15. S, so that $PS:PQ = 7:8$.
RATIO AND SIDES OF TRIANGLES

If you divide two sides of a triangle into the same ratio and then join the dividing points with a segment, you will discover some surprising facts.

ΔABC has the vertices (2,2), (8,5), and (11,-1). Make a copy of the diagram.

1. Find the coordinates of the point D on AB such that AD:DB = 1:2. Plot D.

2. Find the coordinates of the point E on AC such that AE:EC = 1:2. Plot E. Draw DE.

3. Is ΔADE similar to ΔABC? Explain your answer.

4. What is the ratio of BC to DE?

5. What are the coordinates of the midpoint of AC? of the midpoint of BC?

ΔPQR has vertices (-7,2), (1,-2), and (9,6). Make a copy of the diagram.
6. Find the coordinates of a point $S$ on $QR$ such that $QS:SR = 3:1$.

7. Find the coordinates of a point $T$ on $PR$ such that $PT:TR = 3:1$.

8. What is the ratio of the perimeter of $\triangle PQR$ to the perimeter of $\triangle TSR$?

9. What is the ratio of $TS$ to $PQ$?

10. Write the coordinates of a point $V$ on $QR$ such that $QR:VR = 3:5$.

11. Write the coordinates of a point $W$ on $PR$ such that $PW:WR = 3:5$.

12. What is the ratio of $QV$ to $QS$? of $VS$ to $QR$?
Notes to Teacher on Proportion

Consider the following problem: A basketball player scored 38 goals in 72 attempts during a game. If he continued at this rate, how many shots would he have to attempt to score 25 goals in the next game? The box statements (proportions) could be the following:

\[
\begin{array}{c|c}
\text{attempts} & 36 & n \\
\hline
\text{goals} & 38 & 25 \\
\end{array}
\quad \text{OR} \quad
\begin{array}{c|c}
\text{goals} & 38 & 25 \\
\hline
\text{attempts} & 72 & n \\
\end{array}
\]

The box statement

\[
\begin{array}{c|c}
38 & 72 \\
\hline
25 & n \\
\end{array}
\]

would yield the correct "n," but it cannot be as clearly related to the problem under consideration (as the other two box statements are). It becomes important during oral discussion that the teacher encourage, or insist, that students write labels as well as numerals when the box statement is established.
PROPORTIONS

Example: Mary Jo has a cookie recipe which calls for 1 1/3 cups of flour, 1/2 teaspoon of salt and other ingredients. The recipe makes 16 cookies.

Mary Jo wishes to make 32 cookies, or twice as many as the recipe makes. How much flour and salt should she use?

Solution: She must use 2 x 1 1/3 cups flour or 2 2/3 cups of flour. She must use 2 x 1/2 teaspoon of salt or 1 teaspoon of salt.

In the example the ratio of cookies to salt is 16 : 1 or 32 : 1 (32 cookies per 1 teaspoon of salt). The ratio of cookies to flour is 16 : 1 1/3 or 32 : 2 2/3.

The ratios in this example can be written as box statements.

<table>
<thead>
<tr>
<th>cookies</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>tsp. of salt</td>
<td>1/2</td>
<td>1</td>
</tr>
</tbody>
</table>

The ratio \( \frac{32}{2 \frac{2}{3}} \) can be simplified \( \frac{32}{2 \frac{2}{3}} = \frac{32 \times 3}{8/3 	imes 3} = \frac{96}{24} = 4 \). Thus we can write:

<table>
<thead>
<tr>
<th>cookies</th>
<th>16</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>cups of flour</td>
<td>1 1/2</td>
<td>1</td>
</tr>
</tbody>
</table>

A number sentence which states that two ratios are equal is called a proportion.
**PR 1**

Note the following:

<table>
<thead>
<tr>
<th>32</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12</th>
<th>$1\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{3}$</td>
</tr>
</tbody>
</table>

In any proportion, the "cross products" are equal:

$1 \times 16 = \frac{1}{2} \times 32$ and $1 \times 16 = 1 \frac{1}{3} \times 12$. The products are shown by the arrows.

Check the cross products to see if the box statements are proportions.

1. cupcakes | 24 | 32  
cups of sugar | $1\frac{1}{2}$ | 2

2. bolts | 45 | 84  
cost | $.80 | $1.40$

3. miles | 80 | 90  
hour | $1\frac{1}{3}$ | $1\frac{1}{2}$

4. hours | 44 | 30  
hour | $\frac{55}{3}$ | $\frac{45}{3}$

5. tiles | 950 | 300  
cost | $111$ | $54$

6. inches | 3 7/8 | 2 3/4  
feet | 31 | 22
<table>
<thead>
<tr>
<th></th>
<th>cu. in.</th>
<th></th>
<th></th>
<th>cans of soup</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>3.5</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1b.</td>
<td>1.05</td>
<td>3.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>cost</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.37</td>
<td>1.40</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>cu. feet</th>
<th></th>
<th></th>
<th>lb. of chlorine</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td>2</td>
<td>25</td>
<td></td>
<td>1½</td>
<td>2½</td>
<td></td>
</tr>
<tr>
<td>gallon</td>
<td>5</td>
<td>180</td>
<td></td>
<td>5000</td>
<td>9000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>yd.</th>
<th></th>
<th></th>
<th>doz.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11.</td>
<td>5</td>
<td>15</td>
<td></td>
<td>2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>ft.</td>
<td>15</td>
<td>25</td>
<td></td>
<td>1.78</td>
<td>5.34</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>12.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
HOW TO WRITE BOX STATEMENTS, HOW TO CHECK TO SEE IF A STATEMENT IS A PROPORTION.

Example: On a trip a family drove 400 miles in 10 hours. The next day they drove 8 hours, at the same rate, and went 320 miles.

Solution: First set up two ratios, each expressing miles/hour and put them into a box statement.

Sentence 1: Family drove 400 miles in 10 hours

\[
\begin{array}{c|c}
\text{miles} & 400 \\
\text{hours} & 10
\end{array}
\]

Sentence 2: They drove 8 hours, at the same rate, and went 320 miles.

\[
\begin{array}{c|c}
\text{miles} & 320 \\
\text{hours} & 8
\end{array}
\]

Check to see if statement is a proportion:

\[
\begin{array}{c|c|c}
\text{miles} & 400 & 320 \\
\text{hours} & 10 & 8
\end{array}
\]

\[
\begin{align*}
8 \times 400 &= 3200 \\
10 \times 3200 &= 3200
\end{align*}
\]

Yes, the statement is a proportion.

Write a box statement that represents the description.

Then check to see if the statement is a proportion.

1. Ellan got $2.10 for baby sitting 5½ hours. The following week she sat 3 hours and was paid, at the same rate, $1.20.

2. A salesman earned a commission of $75 for making sales of $1,500. The next month he made $5,000 worth of sales and earned, at the same rate, $250 commission.
3. A variety of wheat yields 18 pounds of white flour from 1 bushel of wheat. At that rate, 4½ bushels gives 81 pounds of white flour.


5. Sam studied 6 hours and got 82 on a test. The next week he studied 4 hours and got 69 on his test.

6. A LP record provided 18 minutes of music. Five such records provide 90 minutes of music.

7. One Friday a restaurant used 220 small cakes to serve 100 people. The following Friday they used 275 cakes to serve 125 people.

8. One year Joe Flack paid $950 income tax for an income of $5,200. The following year he paid $1,020 income tax for an income of $5,800.

9. A car used 8.4 gallons of gasoline to travel approximately 143 miles. The next day it used 10.2 gallons of gasoline to travel 173 miles.
SOLVING PROPORTIONS

Since the cross products in a proportion are equal, we need only three values in a proportion to find the fourth.

Example 1: Mary Jo's recipe calls for $1 \frac{1}{3}$ cups of flour for 16 cookies. How much flour does she need for making 60 cookies?

Solution:

<table>
<thead>
<tr>
<th>cups of flour</th>
<th>1 $\frac{1}{3}$</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>cookies</td>
<td>16</td>
<td>60</td>
</tr>
</tbody>
</table>

$16 \times c = 60 \times 1 \frac{1}{3}$
$c = \frac{80}{16} = 5$ cups

Example 2: A car travels 120 miles in 3 hours. At the same rate, how far will it travel in 5 hours?

Solution:

<table>
<thead>
<tr>
<th>hours</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>miles</td>
<td>120</td>
<td>m</td>
</tr>
</tbody>
</table>

$3 \times m = 120 \times 5$
$m = \frac{600}{3} = 200$ miles

Example 3: A car traveled 180 miles on 12 gallons of gasoline. What was the mileage per gallon?

Solution:

<table>
<thead>
<tr>
<th>miles</th>
<th>180</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>gallons</td>
<td>12</td>
<td>1</td>
</tr>
</tbody>
</table>

$12 \times m = 180 \times 1$
$m = \frac{180}{12} = 15$ miles per gallon

Solve the following by using box statements:

1. Joe Flack drove 75 miles in 1$\frac{1}{2}$ hours. How long would he take to drive 200 miles?
2. At the same rate as in Problem 1, how long would it take Joe to drive 320 miles?

3. Joe used 15.4 gallons of gasoline to drive 320 miles. How many miles per gallon was that?

4. A small car travels 28.4 miles per gallon of gasoline. How much will it use for a 650 miles trip?

5. A brand of antifreeze calls for 5 quarts of water for each 2 quarts of antifreeze. The cooling system of a certain car calls for 7 quarts of antifreeze. How much water must be put in the cooling system?

6. Joe Flack figured he spent $281 on his car for 4,250 miles of driving. He expects to drive 10,000 miles next year. How much money should he expect to spend?

7. If 30 miles per hour is equal to 44 feet per second, then 45 miles per hour is equal to how many per second?

8. It cost $17.60 to make a 220-mile trip. What is the cost in cents per mile?

9. An inner-city bus averages 35 miles per hour. How long will it take to make a 237-mile trip?

10. A car traveled 167 miles on 9.7 gallons of gasoline. How far can it travel on a full tank of 18 gallons?
Solve each of the following problems using proportions (box statements):

1. In making good coffee 2 level tablespoons of ground coffee are used for 1 measuring cup (8 oz.) of brewed coffee. A large automatic coffee-maker makes 24 measuring cups of coffee. How many level tablespoons are needed to make a full pot of coffee?

2. Claire has a recipe for fruit cup which yields 6 portions. The recipe calls for 3 cups of strawberries and 2/3 cup of confectioners' sugar, among other ingredients. Claire wishes to make 15 portions.
   a. How many cups of strawberries does she need?
   b. How much sugar does she need?

3. In a recipe for cheese puffs, an 8-ounce package of cheese is used for 32 servings. How much cheese would you need for 50 servings?

4. A governor's wife was told that a pound of sliced turkey could serve 4 people at a buffet dinner. How much sliced turkey should she order for 220 guests?

5. A jelly recipe calls for 3 quarts of raspberries and 7 1/2 cups of sugar. Mrs. Reed bought 10 quarts of raspberries for making jelly. How much sugar should she use?
6. A chicken-rice recipe calling for 3 cups of cooked rice and 1 1/2 cups of cooked, diced chicken serves 8 portions. How much of each (ingredient) would you use for 12 portions?

7. A recipe for scalloped potatoes calls for 2 quarts of sliced potatoes and 10 1/2 ounces of creamed soup to serve 8 people. How much of each ingredient is needed to serve 6 people?

8. A punch recipe calls for 3 quarts of pineapple juice, 8 lemons, 8 oranges, 6 quarts of ginger ale, and 2 cups of sugar. It serves 35. The student council plans to serve punch to 250 guests at a prom. How much of each ingredient will they need?

9. A restaurant manager told a hostess that he figures on 250 appetizers for 100 guests. How many should he order for 72 guests?
When you make a map of a territory or a drawing of an object, you represent actual distance by much smaller distances called drawing distances. At the bottom of a map or drawing, you show the scale ratio, which tells the ratio of drawing distances to actual distances.

Example: If the scale ratio is 1:12, find the drawing distance if the actual distance is 3'.

\[
\begin{align*}
\text{scale ratio} & : \frac{\text{drawing distance}}{\text{actual distance}} = \frac{1 \text{ in.}}{12 \text{ in.}} \quad (3' = 36") \\
\frac{\text{drawing distance}}{\text{actual distance}} & = \frac{1}{12} \cdot \frac{d}{36} \\
12 \times d & = 36 \\
d & = \frac{36}{12} = 3 \text{ inches}
\end{align*}
\]

If the scale ratio is 1:12, find the drawing distances that represent these actual distances.

1. 2'  
2. 5'  
3. 8'  
4. 6 yards
5. 4'  
6. 24'  
7. 14'  
8. 12'6"
PR 5

If the scale ratio is $\frac{1}{2}''$ to 1', what actual distances are represented by these drawing distances?

9. 2''  
10. 8''  
11. 3''  
12. 1\frac{1}{2}''

13. 1'  
14. 6''  
15. 3\frac{1}{2}''  
16. 1\frac{3}{4}''

The diagram below is not drawn to scale, but four pairs of actual room lengths are given. The scale is supposed to be $\frac{1}{2}''$ to 1'. Find the correct lengths of the items numbered 17-24 in the diagram.
UNIT MEASURES USING PROPORTIONS

Perhaps you have noticed that every proportion can be written as a number sentence involving \( n \) (or some other letter) as the unknown part of the proportion.

Consider the following problem: How much are 5 shirts at $2.95 each? It can be described by a proportion.

\[
\text{cost} \quad \frac{\$2.95}{1} \quad \frac{c}{5} \quad 1 \times c = 5 \times 2.95
\]

\[
c = \$14.75
\]

Solve the following by using proportions:

1. A package of grass seed has directions: "Use 1 pound per 250 square feet." The package contains 5 pounds of seed. How many square feet of lawn can be seeded with one package?

2. A concentrated insect spray comes in a 10-ounce can. The directions say, "Use 1 part in 15 parts of water." The can costs $1.85. Another spray comes ready to use, a 1-gallon jar costing $1.29. If they are equally effective, which is the better buy? (1 qt. = 32 oz.)

3. Last year Mr. Stemson harvested 450 bushels of potatoes from 9 acres of potato plants. This year he wishes to market 700 bushels. How many acres must he plant?

4. An evergreen shrub has grown 5" in 2 years. It is now 21" tall. At the same rate of growth, how tall will it be in 5 more years?
5. Three baseball players have the following batting records:

<table>
<thead>
<tr>
<th>Player</th>
<th>At Bat</th>
<th>Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norm</td>
<td>136</td>
<td>37</td>
</tr>
<tr>
<td>Charlie</td>
<td>223</td>
<td>46</td>
</tr>
<tr>
<td>Eddie</td>
<td>253</td>
<td>62</td>
</tr>
</tbody>
</table>

Find the ratio (with the denominator of 1) of the number of hits to the number of times at bat for each player and list them in order.

6. Three backs on a football team have the following record:

<table>
<thead>
<tr>
<th>Player</th>
<th>Times Carried</th>
<th>Yards Carried</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom</td>
<td>18</td>
<td>141</td>
</tr>
<tr>
<td>Jay</td>
<td>32</td>
<td>295</td>
</tr>
<tr>
<td>Spider</td>
<td>22</td>
<td>108</td>
</tr>
</tbody>
</table>

For each player, find the ratio: yards gained per carry. List them in order.

7. Three gallons of Paint A were used to paint 1,275 square feet of wall and ceiling. Four gallons of Paint B were used to paint 1,450 square feet of wall and ceiling. Which paint covered more square feet per gallon?

8. Mr. Sloan needs 1,050 pounds of fine sand to mix with 6 sacks of cement. He decides to use 20 sacks of cement. How much sand does he need?

9. Mr. Marz borrowed $500 and paid $22.50 in interest. Mr. Carr borrowed $200 and paid $9.00 in interest. Which man paid the lower interest rate?
Notes to Teacher on Percentage

Most teachers would have the students write $0.75 \times 200 = 150$ as the way to solve the problem: What is 75% of 200? In using the A, B, C method, the student need not worry about what parts of a problem would be multiplied (or whatever operation). The above problem would be worked by the A, B, C method like this: A is to find, B is 75, and C is 200.

$$\frac{A}{C} = \frac{B}{100} \quad \frac{A}{200} = \frac{75}{100} \quad 100 \times A = 75 \times 200 \quad A = \frac{15000}{100} = 150$$

The A, B, C method in this particular case is longer and does produce the same answer as compared to the "old" method. When students recognize this, the shorter method is not to be discouraged.

BUT, it may be pointed out that if the problem were "what is \% of 16?," the A, B, C method is superior to the "old" method. Fractions fit easily in the proportion $A/C = B/100$. $A = \frac{1}{6} \times \frac{16}{100}$

$$100 \times A = \frac{1}{6} \times 16$$

$$A = \frac{4}{100} = 1/25$$

Using the A, B, C method, the student does not need to change \% to a decimal--an operation that is generally difficult for the general mathematics student.

The following worksheets dealing with calculating percent are: PC 10 with practice in identifying A, B, and C; PC 11 with practice in finding B; PC 12 with practice in finding A; PC 13 with practice in finding C; and PC 14 with a mixture of problems like those that were in PC 11, PC 12, and PC 13.
Another way of stating ideas you have already studied is by a percent. What is a percent? A percent is a ratio. A way of making comparisons.

Let's first look at the meaning of a percent.

*per:* for each or for every (as in $2 per hour)

*cent:* a word root meaning hundred, as *century* (we describe 100 cents = 1 dollar)

*percent:* for each hundred

Translating sentences to a form involving ratios will help to make the meaning of a percent clearer. The symbol % means percent.

**Example 1:** We spend 45% of our taxes for defense.

**Translation:** We spend 45/100 of our taxes for defense, or for each $100 of our taxes, we spend $45 for defense.

**Example 2:** 8% of the students are absent.

**Translation:** 8/100 of the students are absent, or for each 100 students, 8 are absent.

Using ratios, translate each of the following sentences:

1. She saved 10% of her salary.
2. A state charges 5% sales tax.
3. This candy is 40% sugar.
4. A team won 62% of the games it played.
5. He receives 8% commission on all of his sales.
6. Of the passenger planes owned by a company, 30% have jet engines.

7. A governor receives 53% of the vote.

8. A quarterback completed 42% of his passes.

9. The purchase of an automobile required a 20% down payment.

10. Mary Jo answered correctly 87% of the questions on a test.
SHOWING PERCENTS ON A NUMBER LINE

We can show percents on a number line.

If we call the distance from A to B one unit, we can name point C as \( \frac{1}{4} \).

If we now take the same unit AB and divide it into 100 equal parts as shown,

\[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]

we can now name point C as \( \frac{25}{100} \). If we call the unit AB 100%, we can show the number line unit as

\[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]

and we can name point C as 25%. Thus \( \frac{1}{4} = \frac{25}{100} = 25\% \).

Name each point as a fractional number and as a percent.

1. \[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]
2. \[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]
3. \[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]
4. \[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]
5. \[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]
6. \[
\text{A} \quad \downarrow \quad \text{C} \quad \downarrow \quad \text{B}
\]
PC 2

7. \[ \downarrow \]

8. \[ \downarrow \]
USING PROPORTION TO FIND PERCENTS

Since a percent is a ratio, it may be calculated by using a proportion.

Example 1: Sam answered 24 questions correctly out of a total of 30. What percent did he answer correctly?

Solution:

<table>
<thead>
<tr>
<th>number correct</th>
<th>24</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>

\[ 30 \times x = 24 \times 100 \]

\[ r = 2400/30 = 80 \]

Sam answered 80% of the 30 questions correctly.

Example 2: In a school with an enrollment of 800 students, 40 were absent. What percent was absent?

Solution:

<table>
<thead>
<tr>
<th>number absent</th>
<th>40</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>800</td>
<td>100</td>
</tr>
</tbody>
</table>

\[ 800 \times x = 40 \times 100 \]

\[ r = 4000/800 = 5 \]

Therefore 5% of the students were absent.

Solve the following by using proportions:

1. A team played 80 games and won 40. What percent of the games did they win?
2. A family with an income of $100 per month spends $100 per month for housing. What percent of their income do they spend for housing?

3. A store charged $200 for a refrigerator and made a profit of $40. What was the store's percent of profit?

4. Sam applied for a job selling magazine subscriptions. He was offered 40 cents (commission) for every $2.50 worth of sales. What was his percent of commission?

5. Jane was receiving a salary of $50 per week. She was given a raise of $3.50 per week. What percent of her salary was the raise?

6. Sam had 45 correct answers on a test of 50 questions. What percent of the questions did he answer correctly?

7. Alice's father paid $175 down on a car priced at $875. What percent of the price was his down payment?

8. Alice's father paid $105 interest on the balance of $700 when he bought the car in Problem 7. What percent of the balance was the interest?

9. Alice saves $6 per week out of her wages of $40 per week. What percent of her wages did she save?

10. A club spent $82 from a treasury of $102.50. What percent of the treasury did they spend?
OTHER PROBLEMS WITH PERCENTS

Example 1: A businessman plans to spend 15% of his receipts for advertising. He expects receipts of $950 next week. How much does he expect to spend for advertising next week?

Solution:

\[
\begin{align*}
\text{for advertising} & \quad \frac{15}{100} = \frac{S}{950} \\
\text{total receipts} & \quad 100 \\
\end{align*}
\]

\[100 \times S = 15 \times 950 \]

\[S = \frac{15 \times 950}{100} = 142.50\]

Example 2: Two weeks ago the businessman in Example 1 spent $78 for advertising. This was 15% of his receipts. What was his receipts that week?

Solution:

\[
\begin{align*}
\text{for advertising} & \quad \frac{15}{100} = \frac{T}{78} \\
\text{total receipts} & \quad 100 \\
\end{align*}
\]

\[15 \times T = 78 \times 100 \]

\[T = \frac{78 \times 100}{15} = 520\]

His receipts were $520.

Solve the following problems by proportion.

1. About 20% of the students from Tilton High School go to college. If Tilton's enrollment is 1,380 students, about how many students will go to college?

2. In a certain newspaper 40% of the classified ads result in the sale of the articles advertised. One Sunday the paper had 1,960 ads. About how many sales resulted from these ads?

3. Mrs. Banks deposited $550 in a savings bank which pays 5% interest. How much interest did she receive for 1 year?
4. A club spent $280 in expenses for a dance. They spent $84 of it for refreshments. What percent did they spend for refreshments?

5. Joann received a 5% raise in pay which amounted to $4.00 per week. How much was she receiving before the increase?

6. A certain stock pays a dividend of $4.50 per share, which is 4% of the value of the share. How much is each share worth?

7. Mr. Lazar sold $1,800 worth of cameras for which he received a 3% commission. How much did he make (his commission in $'s)?

8. You are to take a test of 60 short questions. The teacher says that a score of 70% is the passing grade. How many right answers must you have to pass?

9. A man borrowed $300 from a finance company and paid back $372 by the time he made his last payment. What percent extra did he pay?

10. The Fries family spends 35% of its income for food. The monthly income is $425. How much is spent for food each month?

11. A prize cow gave 42 pounds of milk which was 11% butterfat (cream). How much cream (in pounds) was there in the milk?

12. A 4-ounce sample of ground hamburger was tested and found to contain 1.3 ounces of fat. What was the percent of fat?
Solve the following problems by proportion.

1. Mr. Sloane said he made a 9% profit last year on an investment of $2,500. How much profit did he make?

2. A car was advertised for $1,355 with 20% down. How much was the down payment?

3. Ruth saved 60¢ one week, which was 15% of her babysitting money. How much had she earned?

4. Sue had 16 correct answers on a test, which was a score of 80%. How many questions were on the test if they all rated equally?

5. A Georgian team won 6 out of 9 league games. What percent of the league games did they win?

6. A charity campaign raised $24,500 out of a quota (goal) of $30,000. What percent did it raise?

7. Of $380,000 raised in a charity campaign, 5% went for office expenses. How much was spent for office expenses?

8. A concrete mixture contains 30% water (by weight). A truck delivers 3,400 pounds of concrete. How many pounds of water are in the concrete delivered?

9. An oil tank holds 280 gallons when full. The tank is 32% full. How many gallons of oil are there in the tank?

10. On a certain Tuesday 10 out of 44 T.V. shows shown after 6:00 p.m. were westerns. What percent of the shows were western?
COMPARISON, USING PERCENT

Example: Mrs. Walker invested $850 and made a profit of $51 in one year. Mr. Sampson invested $1,300 and made a profit of $65 in one year. Which investment resulted in the larger percent of profit?

Solution:

Mrs. Walker's profit \[
\frac{51}{850} = r \quad 850 \times r = 51 \times 100
\]
Mrs. Walker's investment \[850 \times r = 5,100 \quad r = \frac{5,100}{850} \quad r = 6\]

Mr. Sampson's profit \[
\frac{65}{1300} = r \quad 1300 \times r = 65 \times 100
\]
Mr. Sampson's investment \[1300 \times r = 6,500 \quad r = \frac{6500}{1300} \quad r = 5\]

It is now clear that Mrs. Walker made 1% more or $1 more for each $100 invested than Mr. Sampson made.

Solve. Round all answers to the nearest tenth of 1%.

1. One class completed 75 projects out of 110 started.

Another class completed 35 out of 40 started. Compare their percents of completion.

2. Here are the records of four teams:

<table>
<thead>
<tr>
<th>Team</th>
<th>Won</th>
<th>Lost</th>
<th>Played</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bobcats</td>
<td>22</td>
<td>18</td>
<td>40</td>
</tr>
<tr>
<td>Bears</td>
<td>23</td>
<td>19</td>
<td>42</td>
</tr>
<tr>
<td>Comets</td>
<td>19</td>
<td>18</td>
<td>37</td>
</tr>
<tr>
<td>Owls</td>
<td>21</td>
<td>19</td>
<td>40</td>
</tr>
</tbody>
</table>

For each team, find the percent of wins and list them from best to poorest.
3. The James family saves $12 per month out of an income of $400. The Burns family saves $18 per month out of an income of $500. Which family has the higher rate of savings and by how much?

4. Al made 13 hits in 30 times at bat. Reno made 15 hits in 34 times at bat. Compare their batting averages (percents).

5. Julie answered 36 questions correctly on a test of 48 questions, and June answered 32 correctly on a test of 40. Who had the higher percent score and by how much?

6. Mr. Harris made a one-year car loan for $800 and paid $90 in carrying charges. How much higher was the rate Mr. Hunter paid?

7. One company charged $1.25 delivery charge for a parcel worth $30. Another company charged $2.25 for a parcel worth $50. Which charged the higher rate? How much higher was it?

8. A quarterback completed 31 forward passes out of 70 attempts. Another quarterback completed 26 passes out of 55 attempts. Who had the better completion rate? How much better was it?

9. One market ground 18 pounds of fat and 70 pounds of lean meat into a batch of hamburger. Another market ground 28 pounds of fat and 95 pounds of lean meat into a batch of hamburger. Compare the percentages of fat in each.

10. You own two kinds of stock. One stock, valued at $90 per share, paid you a dividend of $3.30 last year. The other, valued at $45 per share, paid you a dividend of $2.00. Which stock paid the higher dividend rate (percent)? How much higher was it?
FRACTION OF A PERCENT

These questions confuse many people. Let's examine them carefully.

What does \( \frac{1}{2} \% \) mean? What does 2.5\% mean?

What does .1\% mean? What does 1\% mean?

In each of the examples which follow, a set of 100 squares will represent 100\% of 1.

\[
\frac{1}{2} \% = \frac{1}{2} \text{ of } 1\% \\
1\% = .01 \\
\frac{1}{2} \text{ of } 1\% = \frac{1}{2} \times .01 = .005
\]

\[
.1\% = .1 \text{ of } 1\% \\
1\% = .01 \\
.1\% \text{ of } 1\% = .1 \times .01 = .001
\]

\[
2.5\% = 2.5 \text{ of } 1\% \\
1\% = .01 \\
2.5 \text{ of } 1\% = 2.5 \times .01 = .025
\]
Change each of the following to decimal form. Then use a set of 100 squares, as in the examples given, to diagram each percent.

1. \(1\frac{3}{4}\%\)  
2. \(3/10\%\)  
3. \(2\frac{1}{10}\%\)  
4. \(2\frac{3}{4}\%\)  
5. \(4\frac{3}{4}\%\)

6. \(1\frac{4}{10}\%\)  
7. \(1\frac{3}{4}\%\)  
8. \(3\frac{1}{4}\%\)  
9. \(2\frac{7}{10}\%\)  
10. \(1\frac{3}{4}\%\)

Calculate the following:

11. \(1\frac{3}{4}\%\) of $500  
12. \(3/10\%\) of $150  
13. \(2\frac{1}{10}\%\) of $3,500  
14. \(2\frac{3}{4}\%\) of $1,280  
15. \(4\frac{3}{4}\%\) of $8,500

16. \(1\frac{4}{10}\%\) of 2,300  
17. \(1\frac{3}{4}\%\) of 2,300  
18. \(3\frac{1}{4}\%\) of $680  
19. \(2\frac{7}{10}\%\) of $150  
20. \(1\frac{3}{4}\%\) of $660
PERCENTS GREATER THAN 100%

Just as fractions may be greater than 1, such as 7/4, some problem's solutions give ratios greater than 1.

This means that percents figured will also be greater than 1 or greater than 100%.

Example: A gift shop sold $800 worth of gifts in November and $2,000 worth of gifts in December. The December sales were what percent of the November sales?

\[
\frac{\text{December sales}}{\text{November sales}} = \frac{2000}{100} = \frac{100 \times 2000}{800} = 250
\]

The December sales were 250% of the November sales.

Solve. Round answers to the nearest one tenth (of 1%).

1. Center High School had an enrollment of 1,300 students last year. This year they have an enrollment of 1,650 students. This year's enrollment is what percent of last year's enrollment?

2. Joe Davis earned $50 per week last year. This year he entered a skilled trade and is earning $110 per week. His present wage is what percent of last year's wage?

3. In 1948 a certain make of automobile sold for $1,300. Today the average price of that make is $2,200. Today's price is what percent of the 1948 price?

4. In 1960 Fayetteville had a population of 5,000. Today Fayetteville has a population of 11,500. Today's population is what percent of the 1960 population?
5. In problem 4, the 1960 population is what percent of today's population?

6. Mr. Sams bought a T.V. set for $180 in cash. Mr. Simpson bought the same kind of set on an installment plan and paid a total of $240. Mr. Simpson paid what percent of the cash price of $180?

7. Joel took a reading test and scored 180 words per minute. He received special instruction in how to read, and his rate improved to 420 words per minute. His improved reading rate was what percent of his old rate?
COMPARING AREAS

Consider the two rectangles below.

The area of rectangle A is what percent of the area of rectangle B?

We can see at a glance that the area of rectangle A is greater than the area of rectangle B. Thus, the percent is greater than 100%. Rectangle B was specially chosen. The diagram below now shows rectangle B as a set of 100 squares.

Placing rectangle B over rectangle A, we see

\[ \text{Rectangle A} = \text{Rectangle B} + \text{Rectangle OPQR} \]
Now placing rectangle B over rectangle A to the right of line segment OP, we see

Notice that rectangle OPQR contains 20 squares. Since there are 100 squares to the left of line segment OP, we see that

area of rectangle = 120% of the area of rectangle B.

On tracing paper, trace square B. For each of the following state:
its area is what percent of the area of square B.

1. 

2. 
A: B, C's OF PERCENT

What is left under the title of percent are the three types of percent exercises. These three types are like the following:

(1) What is 14% of 78?, (2) 31 is what percent of 78?, and (3) 17 is 61% of what?

These three types are in the general form: A is B% of C.

This general form has this proportion:

\[
\frac{A}{C} = \frac{B}{100}
\]

How does one know what is A, what is B, and what is C?

B is always the numeral in front of the percent sign (%).

C is always the numeral after the word of.

A is always the remaining part.

Example: 70% of 13 is what?

Solution: B is 70, C is 13, and you are to find A.

\[
\frac{A}{100} \times \frac{70}{13} = \frac{13 \times 70}{910} = \frac{910}{100} = 9.1
\]

Identify A, B, C and set up the proportion. DO NOT SOLVE.

1. 26 is 20% of what?
2. 50% of 16 is what?
3. What is 3% of 82?
4. Find 120% of 16.
5. 18 is what percent of 90?
6. 7% of 85 equals what?
7. What % of 48 is 18?
8. 2.5 is 40% of what?
9. .03496 is what % of .4?
10. 17% of what is 72?
PROPORTION TO FIND PERCENT

Example: 60 is what % of 160?

\[
\begin{align*}
A &= 60 \\
C &= 165 \\
\text{find } B
\end{align*}
\]

\[
\frac{A}{C} = \frac{B}{100}
\Rightarrow \frac{60}{165} = \frac{B}{100}
\Rightarrow 165 \times B = 100 \times 60
\Rightarrow B = \frac{6000}{165} = 37.\overline{5}
\]

60 is 37\(\frac{1}{2}\)% of 160.

Solve. Round each to nearest tenth (of 1%).

1. 8 is what % of 16?
2. 30 is what % of 120?
3. 27 is what % of 84.5?
4. 91.5 is what % of 782?
5. 7.1 is what % of 622.88?
6. What % of 59 is 48?
7. What % of 8 is 20?
8. 27 is what % of 36?
9. 231 is what % of 575?
10. 420 is what % of 180?
PC 12

PROPORTION IN PERCENT

Example: What is 12\(\frac{1}{2}\)% of 80.

Solution: A to find  \(B = 12\frac{1}{2} \times \frac{25}{2}\)  \(C = 80\)

\[
\frac{A}{C} = \frac{B}{100} \quad \frac{A}{80} = \frac{25}{2} \quad 100 \times A = 80 \times 25/2
\]

\[
A = 1000/100 = 10
\]

10 is 12\(\frac{1}{2}\)% of 80.

Remember the above example could be stated as: 12\(\frac{1}{2}\)% of 80 is what.

Calculate the following:

1. 50% of 80.
2. 33 1/3% of 9.
3. 70% of 70.
4. 75% of 40 is what?
5. 66 2/3% of 12.
6. What is 50% of 17?
7. 306% of \$412 is what?
8. 8.4% of 412.
9. 3% of \$318.20.
10. 6.6% of \$259.11.
Example: 26 is 20% of what.

Solution: \( A = 26 \quad B = 20 \quad C \) is find

\[
\begin{align*}
\frac{A}{C} &= \frac{B}{100} \quad \frac{26}{C} = \frac{20}{100} \\
100 \times 26 &= 20 \times C \\
2,600 &= 20 \times C \\
2600/20 &= C \\
C &= 130
\end{align*}
\]

26 is 20% of 130.

Calculate

1. 52 is 25% of what?
2. 36 is 6% of what?
3. 77 is 20% of what?
4. 45% of what is 72?
5. $66.30 is 22.2% of what?
6. $84.70 is 54% of what?
7. 1,260.72 is 305% of what?
8. 122% of what is $315.98?
9. 2\frac{3}{4}\% of what is 32?
10. 0.5 is 800% of what?
Calculate the following:

1. What is 40% of 2.5?
2. 17 is 850% of what?
3. 0.27 is 9% of what?
4. 4.7 is what% of 117.5?
5. What is 3.2% of 156.25?
6. 25.2 is what percent of 420?
7. What number is 17.5% of 40?
8. 0.03496 is what percent of 0.437?
9. 1.926 is 21.4% of what?
10. What is .25% of 4000?
PC 15

GEOMETRICAL INTERPRETATIONS

In the sketch, the circle touches the sides of the square.

Solve.

1. If the square is 10" on a side, the circumference of the circle is what percent of the perimeter of the square? (Round to nearest tenth)

2. The area of the circle is about what percent of the area of the square? (Use the dimensions given in problem 1)

3. Suppose the square in problems 1 and 2 measured 12". Answer the same two questions about the perimeters and area.

4. Answer problem 1 and 2 for a square with side 25".
PERCENT OF INCREASE AND DECREASE

Example: Butter rose in price from 60¢ per pound to 72¢ per pound. What was the percent of increase?

Solution:

\[
\begin{array}{c|c}
\text{increase} & r \\
\hline
\text{price} & 100
\end{array}
\]

Wait a minute! Our information seems to be incomplete. We have 2 prices, but we need an increase and a price.

The information is here, but we have to find it. The increase is 72 - 60 or 12 cents, and we want to find the percent of increase over the old price. Our solution should read:

\[
\begin{array}{c|c}
\text{increase} & 12 \\
\hline
\text{price} & 60
\end{array}
\]

\[
\frac{12}{60} = \frac{r}{100}
\]

\[
60 \times r = 12 \times 100
\]

\[
r = \frac{1200}{60}
\]

\[
r = 20
\]

The increase was 20%.

Solve.

1. A man bought a car priced at $2,500. When he finished his payments, he had paid $3,000 for the car. What percent extra (increase) did he pay?

2. A company had 800 employees one year and 600 the next year. What was the percent of decrease in the number of employees?
3. A power lawn mower costing the storekeeper $62 was sold for $87.50. What was the percent of increase in price? (This is called the profit.)

4. A basketball player averaged 16 points per game one year and 22.5 points per game the next year. What was the percent of increase in points?

5. A quart of milk cost 10¢ in 1939. If a quart of milk costs 28¢ today, what is the percent of increase in price?

6. A man earning $105 per week received a $5 per week raise. What was the percent of increase in his pay?

7. A union group earning $1.65 per hour was offered a raise to $1.73 per hour. What percent of increase was that?

8. A suit costing $60 was sold for $72.50. What was the percent of profit?

9. A man borrowed $300 from a finance company and paid back $372 by the time he made the last payment. What percent extra did he pay?

10. A storekeeper bought a typewriter for $90 and sold it for $78. What was the percent of loss?
KEY

RT 1
1. a. 4 to 1  b. 1 to 4  c. 4 to 5  d. 1 to 5
2. a. 11 to 5  b. 11 to 16  c. 5 to 16
3. 30 to 1 or 30 lb. per ft.
4. a. 1 to 1  b. 2 to 1  c. 2 to 1
5. a. 1 to 1  b. 3 to 4  c. 1 to 2
6. a. 9 to 50  b. 9 to 41
7. 1200 to 48 = 25 to 1
8. 1/4 to 16 = 9 to 1
9. 3:5 or 3/5
10. a. 9 to 4  b. 4 to 9  c. 4 to 1  d. 1 to 4
e. 1 to 15  f. reciprocals

RT 2
1. 5:1  2. 1:30  3. 1:20  4. 5:1  5. 50:3
6. 1:13  7. 7:5  8. 6:28

RT 3
1. 1:3  2. 1:36  3. 1:12  4. 1:16  5. 1:32,000
15. 5:5280 or 1:1056  16. 1:1056  17. 1:1056
18. 8 ft., 4 ft.  19. 15 in., 21 in.  20. 6 oz., 10 oz.
21. 3 sq. ft., 6 sq. ft.
KEY.

RT 4
1. 80  2. 60  3. Yes, G = R, I = S and H = T
4. R, I, T, G  5. \( \overline{RT}, \overline{IH}, \overline{TS}, \overline{GI}, \overline{KG}, \overline{HG} \)  6. yes  7. yes

RT 5
1. \( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \) They are -  2. 2/1, 2/1, 2/1 They are +
3. a. 2\( \frac{1}{2} \)"  b. 4\( \frac{1}{2} \)"  c. \( \frac{1}{2} \)  d. same

RT 6
1. U = (15, 7) or (15, -17)  2. Y = (9, -5) or (33, -5)
3. 3:2  4. 1:1  5. 1:4  6. C = (-4, 3) or (20, 3)
7. F = (5, 8) or (5, -10)  8. 3:4  9. U = (-10, 0) or (-10, 10)
10. 3:5  11. 2:1  12. 25:4

RT 7
1. (7, 4)  2. (5, 3)  3. (9, 5)  4. (3, 2)  5. (11, 6)
6. (7, 4)  7. (5, 3)  8. (4, 2\( \frac{1}{2} \))  9. (3, 1)
10. (-1, -1)  11. (7, 3)  12. (-3, -2)  13. (1, 0)
14. (5, 2)  15. (9, 4)  16. (1, 0)

RT 8
1. (4, 3)  2. (5, 1)  3. yes  4. 2:1  5. (6\( \frac{1}{2} \), \( \frac{1}{2} \))
(9\( \frac{1}{2} \), 2)  6. (7, 4)  7. (5, 5)  8. 4:1  9. 1:4
10. (4, 1)  11. (-1, 3\( \frac{1}{2} \))  12. 1:2 and 3:8

PR 1
KEY

PR 2

PR 3
1. 4 hours.  2. 6.4 hours.  3. 20.77 or 20.8 miles/gal.
4. 22.88 or 22.9 gal.  5. 17.5 qts.  6. $661.77
7. 66 ft. per second  8. $0.08  9. 6.77 or 6.8 hr.
10. 309.88 or 309.9 miles

PR 4
1. 48  2. a. $\frac{7}{2}$ b. 1 2/3  3. 12 1/2  4. 55 lbs.
5. 25 cups  6. 4 1/2 cups rice and 2 1/2 cups chicken
7. 1 1/2 qts. potatoes, 7 7/8 oz. soup  8. 21 3/7 qt. of pineapple juice, 57 1/7 lemons, 42 6/7 qt. of ginger ale, 14 2/7 cups of sugar  9. 180

PR 5
1. 2"  2. 5"  3. 8"  4. $\frac{1}{2}$ yd.  5. 4"  6. 2"
7. 14"  8. 12 1/2"  9. 4"  10. 16"  11. 6"  12. 3"
13. 24"  14. 12"  15. 7"  16. 30"  17. 6 3/4"
18. 6 7/8"  19. 4 3/4" x 3 3/4"  20. 3"  21. 2 1/8"
22. 2 1/2" x 3"  23. 2 1/4" x 1 3/4"  24. 2 1/3" x 3 3/4"
KEY

PR 6
1. 1,250 sq. ft.  2. conc. 1.1¢ per oz., ready to use 1¢ per oz.  3. 14 acres  4. 33.5"  5. 1st Norm (.272), 2nd Eddie (.245), 3rd Charlie (.206)  6. 1st Jay (9.22), 2nd Tom (7.83), 3rd Spider (4.91)  7. Paint A 425, Paint B 362½  8. 3,500 lb.  9. Neither, both paid $4.50 per $100

PC 1
1. She save $10 out of every $100 of her salary.  2. State charges $5 tax out of each $100 sales.  3. 40/100 of the candy is sugar.  4. A team won 62/100 of the games it played.  5. For each $100 of sales, he receives $8.  6. 30/100 of the passenger planes have jet engines.  7. A governor received 53 votes out of every 100 votes.  8. A quarterback completed 42/100 of his passes.  9. The purchase of an automobile required a down payment of 20/100 of the price.  10. Mary Jo answered correctly 87/100 of the test questions.

PC 2
1. 3/5; 60%  2. 5/10 or ½; 50%  3. 3/8; 37½%  4. 6/10 or 3/5; 60%  5. 6/12 or ½; 50%  6. 5/12; 41 2/3%  7. 13/20; 65%  8. 1/3; 33 1/3%

PC 3
1. 50%  2. 25%  3. 20%  4. 16%  5. 7%  6. 90%  7. 20%  8. 15%  9. 9.15%  10. 80%
<table>
<thead>
<tr>
<th>KEY</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC 4</td>
</tr>
<tr>
<td>1. 276</td>
</tr>
<tr>
<td>6. $112.50</td>
</tr>
<tr>
<td>PC 5</td>
</tr>
<tr>
<td>1. $225</td>
</tr>
<tr>
<td>6. 81 2/3%</td>
</tr>
<tr>
<td>10. 22.7%</td>
</tr>
<tr>
<td>PC 6</td>
</tr>
<tr>
<td>1. 1st class 68.2%, 2nd class 87.5%</td>
</tr>
<tr>
<td>3. Burns, 0.6%</td>
</tr>
<tr>
<td>6. 18.4%</td>
</tr>
<tr>
<td>9. 1st 20.5%, 2nd 22.8%, sec. 2.3% more fat</td>
</tr>
<tr>
<td>PC 7</td>
</tr>
<tr>
<td>1. .015</td>
</tr>
<tr>
<td>6. .014</td>
</tr>
<tr>
<td>PC 8</td>
</tr>
<tr>
<td>1. 126.9%</td>
</tr>
<tr>
<td>6. 133.3%</td>
</tr>
<tr>
<td>PC 9</td>
</tr>
<tr>
<td>1. 80%</td>
</tr>
<tr>
<td>6. 150%</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
</tr>
<tr>
<td>7.</td>
</tr>
<tr>
<td>8.</td>
</tr>
<tr>
<td>9.</td>
</tr>
<tr>
<td>10.</td>
</tr>
</tbody>
</table>

**PC 11**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50%</td>
<td>25%</td>
<td>31.9%</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>81.4%</td>
<td>75%</td>
<td>40.2%</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>250%</td>
<td>90%</td>
<td>9.5%</td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>233.3%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PC 12**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1. | 40 | 3 | 49 | 30 | 8 | 6.8%
| 2. | | | | | |
| 3. | | | | | |
| 4. | | | | | |
| 5. | | | | | |
| 6. | | | | | |
| 7. | $1,260.72 | 8. | $34.61 | 9. | $9.55 | 10. | $17.10 |

**PC 13**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>208</td>
<td>2.</td>
<td>600</td>
<td>3.</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>43.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>47.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>59.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>61.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>66.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>67.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>68.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>69.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>72.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>73.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>74.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>76.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>78.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>79.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>81.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>82.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>83.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>84.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>86.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>88.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>89.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>91.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>92.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>93.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>94.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>96.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>97.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>98.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>101.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>102.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### KEY

**PC 14**
1. 1  
2. 2  
3. 3  
4. 4  
5. 5  
6. 6  
7. 7  
8. 8  
9. 9  
10. 10

**PC 15**
1. 78.5%  
2. 78.5%  
3. 78.5%  
4. 78.5%, 78.5%

**PC 16**
1. 20%  
2. 25%  
3. 41.1%  
4. 40.6%  
5. 180%  
6. 4.8%  
7. 4.8%  
8. 20.8%  
9. 13.3%
EVALUATION QUESTIONS

The following are possible test questions. Answers appear after each question in parenthesis.

Ratio

Express as a ratio of the first measurement to the second measurement (in lowest terms).

1. 5 feet, 1 mile (1:1056)
2. 4 ounces, ½ pound (1:2)
3. 9 feet, 2 yards (3:2)
4. If the scale ratio is 1:10, what would be the drawing distance of a pole 20 feet? (2"
5. If the scale ratio is 1:5, what would be the actual distance represented by 3 inches on a drawing? (15)

6. Match the ratio in the right column with the proper statement in the left column.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>two yards to one foot</td>
<td>(K)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>one inch to two yards</td>
<td>(I)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>three pounds to two ounces</td>
<td>(A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>two gallons to one quart</td>
<td>(C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>one inch to one mile</td>
<td>(H)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>1 8/36 yards to 1 foot</td>
<td>(J)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>1 sq. in. to 1 sq. ft.</td>
<td>(L)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>1 dozen to 1 gross</td>
<td>(E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>1 in. to 1 cm.</td>
<td>(D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>1 cubic ft. to 1 cubic in.</td>
<td>(E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
EVALUATION QUESTIONS

11. 1 mile to 4 yards  (F)  K. 6:1
12. 1/3 ft. to 3 yd.  (G)  L. 1:144

7. Car A cruises at 70 m.p.h. Car B cruises at 55 m.p.h. What is the ratio of the speed of car A to car B? (14/11)

8. Simplify these ratios
   a. $7\frac{1}{2}$ to 2  (29/4)  b. $0.02$ per $1.75$  (2/175)

9. One car used 25.4 gallons of gasoline to travel 340 miles. Another car used 20.4 gallons to travel 304 miles. Compare their gas mileage. (1st car 13.4 miles/gal., 2nd car 14.9 miles/gal.)

10. For each player, find the ratio of runs batted in to the number of times at bat and list them in order.

<table>
<thead>
<tr>
<th>Player</th>
<th>at bat</th>
<th>runs batted in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hank</td>
<td>318</td>
<td>44  (.138)</td>
</tr>
<tr>
<td>Charlie</td>
<td>262</td>
<td>42  (.164)</td>
</tr>
<tr>
<td>Lou</td>
<td>164</td>
<td>25  (.152)</td>
</tr>
</tbody>
</table>

Proportion

1. The directions on a package of lawn fertilizer state, "Use 3 pounds per 100 square feet." How much should you buy to fertilize 1,925 square feet of lawn? (57.75 lb.)

2. The county tax rate in a county in Michigan was $5.12 per $1,000 of property value per year. What was the tax on a home valued at $6,800? ($34.82)
EVALUATION QUESTIONS

3. At a social a club used 8 gallons of punch to serve 40 members. They are planning a large affair and have sold 150 tickets. At the same rate, how many gallons of punch do they need?

4. A certain brand of cement calls for 38 gallons of water to 6.3 sacks of cement. How much water is used for each sack of cement? (6.03 gal.)

5. Is the following a true proportion? Twelve acres of farmland yielded 216 bushels of wheat. At the same rate, eighteen acres yielded 324 bushels of wheat. (yes)

6. A car travels 22.4 miles per gallon of gasoline. How far will it travel on a tank containing 16 gallons? (358.4 miles)

Percent

1. You must pass 65% on a spelling test of 120 words in order to pass it. How many words must you get correct to pass? (78)

2. A particular make of car motor used to require 12 hours of labor for an overhaul. It was redesigned and now requires only 9 hours of labor for an overhaul. The new time is what percent of the old time? (78)

3. Andrew spelled 35 words correctly from a list of 50 words. What was his score in percent? (70)

4. A team won 32 games out of 42. What percent did they win? (76.2)

5. A merchant decided to decrease the price of an $89.50 power mower by 20%. What was the sale price? ($89.50 - $17.90 = $71.60)
6. Last year Jim Eliot earned $70 per week. This year he earns $82 per week. What is the percent of increase in his pay? (17.1)

7. Center High School had 80 students absent out of 690 enrolled. Centerville High School had 45 students absent out of 540 enrolled. Which had the higher rate of absence? (Center 11.6%, Centerville 8.3%)