A DEVELOPMENTAL MATHEMATICS CURRICULUM
FOR KINDERGARTEN

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Arts in Elementary Education
by
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ABSTRACT

A DEVELOPMENTAL MATHEMATICS CURRICULUM FOR KINDERGARTEN

by

Marilyn Rehwald

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Mathematics programs throughout the United States have changed greatly since 1963 (the Cambridge Conference). Mathematics curriculum today includes more than just working through textbooks. The state of California has a recommended mathematics framework which suggests nine broad mathematics topics. It also suggests providing children with concrete experiences in mathematics.

The purpose of this study was to develop a kindergarten mathematics curriculum which meets the needs of the children in the Las Virgenes Unified School District. This curriculum was to reflect the district’s needs and follow the state guidelines.

A review of the related literature indicated a strong popularity for the “stage” theory of cognitive development. This theory suggests that each child passes through stages of intellectual development. The noted proponent of this theory is Jean Piaget. He, as well as others, were cited as authorities on the characteristics of children at each developmental stage. Literature was reviewed on performance objectives to support the writing of the curriculum.

The kindergarten mathematics curriculum was developed on the basis of this review. The curriculum was evaluated by a kindergarten manipulative mathematics
inventory, KMMI (included in the appendix). Statistics from this district-given test were compared to the standardized Wide Range Achievement Test — Mathematics, WRAT-Math.

The following conclusions were made from the study:

1. Kindergarten children are not all at the same developmental stage; curriculum should reflect children’s needs.

2. The results on table 5.2 comparing the KMMI and the WRAT-Math tests were significant at the 0.001 level.

3. Immature children score at the low end of the scale on both tests.

4. The results indicate the need for further study.

This study includes the mathematics curriculum, the evaluation instrument to test the curriculum, and a suggested record-keeping form. These materials have been copyrighted (June 1974).
CHAPTER I

INTRODUCTION

Until the launch of Sputnick in 1957, there had been very few changes in mathematics curriculum at the elementary and secondary levels. Now the needs of society have brought about a revolution in the field of mathematics. The National Defense Education Act termed mathematics a critical curriculum area in 1958 and provided federal financial assistance for strengthening instruction of mathematics at both the elementary and secondary levels.

New trends in mathematics curriculum have been emerging with this new interest. Educators from all over the United States met in 1963 at the Cambridge Conference and suggested mathematics curriculum changes which would: (1)

1. Delete certain aspects of classical drill.
2. Add problems illustrating new mathematical concepts.
3. Be based on a generally heuristic cognitive patterned approach to learning.

A statewide advisory committee on mathematics met in California in 1962 and again in 1967 to determine the direction of mathematics curriculum in the state of California. Recommendations of this committee, discussed in the First and Second Strands Reports (2), include a broad-based spiral curriculum which becomes more rigorous as understanding is increased.

At the time of the 1962 report, there was no textbook series containing the recommended mathematics content, and teachers did not have an adequate background for teaching this content. Since 1967, the state has provided both the textbooks containing the content and the inservice education providing teachers with the background for teaching mathematics in elementary school. (2)

THE PROBLEM

Tyler (31) states that curriculum needs to be developed to fill the “gaps” between what the child has already learned before entering school and what he needs
to learn. Emphasis must be placed on teaching the learner what he does not know as opposed to reteaching him something he does know. Tyler also noted that a curriculum needs to be developed focusing on the learner, on contemporary life (or community) and on reports from specialists in the field. The state framework (2) constitutes one report from specialists in the field. It outlines the curriculum content for mathematics in very general terms. The textbooks, also provided by the state, are more specific in content. According to Tyler's philosophy, then, it is necessary for each district (if it is fairly homogeneous) or each school to determine its specific curriculum content based on the needs of the learner and the community.

It is the writer's purpose to develop a mathematics curriculum for the kindergarten population in the Las Virgenes Unified School District. Based on district needs, the curriculum will be written in terms of performance objectives. The sequentially planned organization of this curriculum will follow the state framework and will encompass all of the mathematics strands. In addition, the writer proposes to develop an evaluation device for testing the effectiveness of the curriculum developed. Although a written test would be more economical timewise, the evaluation device will be a manipulative one whereby teachers test children individually. Children in kindergarten are at different stages of development, and individual evaluation of each child affords the teacher a better opportunity to assess the child and determine his level of thought and understanding. (8, 12) A guide for administering the individual test will be a part of this project to assure the consistency of teacher administration of the test within the school and within the district.

ORGANIZATION OF THE PROJECT

In chapter II will be found a review of the literature dealing with the learning theories related to mathematics and a review of the literature to substantiate the use of performance objectives in curriculum writing. Chapter III describes the method used to determine the curriculum. Chapter IV describes the curriculum developed specifically for the Las Virgenes Unified School District and the supportive materials for this curriculum. Chapter V discusses the testing instrument and its reliability as
compared to the Wide Range Achievement Test -- Mathematics (WRAT). Chapter VI will contain the summary, conclusions, and recommendations of this project.
CHAPTER II

REVIEW OF THE LITERATURE

LEARNING THEORIES

Hilda Taba (30), in her book on curriculum development, compared three major categories of learning theories: the theory of mental discipline, the behaviorist theory, and field theories. The theory of mental discipline assumes that the mind inherently contains all the attributes or faculties of intelligence, and it is the goal of education to bring these forth by the exercise of acquiring knowledge. The behaviorist theory assumes that man is a collection of responses to given stimuli from the environment. Field theories assume that learning is a cognitive process and that insight, intelligence, and organization are fundamental characteristics of human response. Unlike the theories of mental discipline and stimulus-response, field theories or cognitive theories of learning require an interaction among others for learning to occur.

The cognitive theories seem to be the most popular theory of learning in the education world today. The British Infant Schools stress a math lab or human interaction approach to learning. (18) The Strands Report (2) stresses that the best climate for learning is one which is pupil centered, activity oriented, and one which provides for interaction between children.

Piaget maintains that intellectual development is based on experience, of which physical activity and social interaction are the two major ingredients. (3) Duckworth maintains that cooperation with one's peers is important for clarifying ideas. (34) The remainder of this section reviews the literature dealing with cognitive learning theories which involve human interaction.

The cognitive theorists of learning maintain that children progress through stages. Duckworth suggests that all children go through stages of learning development. (34) Frazier suggests that children go through these stages at different rates. (11) Hyde sees the sequence of these stages as continuous, overlapping, and irreversible, and that transition from one stage to the next is a gradual process, during
which “group structures are incomplete.” At each stage, however, a “child’s structures or mental operations exhibit certain characteristics.” (12:53)

STAGES OF COGNITIVE DEVELOPMENT

Piaget is often quoted by those psychologists who believe children go through distinct stages of development. Piaget is classified as a genetic epistemologist, and not a child psychologist. He is mainly concerned with the nature of knowledge and the process by which it is acquired. (8) His method for arriving at his stages of child development has been largely clinical. Piaget observed a child’s behavior in his surroundings; then he formulated a hypothesis based on many observations. On the basis of many years of observing and formulating hypotheses, Piaget has defined four stages of development in children. (8)

The first stage, sensorimotor, lasts from birth to roughly age two. It is during this stage that the infant moves from purely reflex actions in dealing with objects to discriminating between actions with objects. During this stage, the infant begins to coordinate his senses. However, his behavior is restricted to direct interaction with the environment. (12, 19)

The second stage, preoperational stage, begins at about the age of two and lasts until age seven. The child enters this stage when he is able to begin to manipulate the environment. The child is very egocentric during this stage. He can focus on only one aspect of a situation at a time. The child’s thinking lacks a proper sequence and direction. An event and explanation are not related. This is the stage at which words become symbols or representations of reality. (12, 14, 19)

The third or concrete operational stage begins at about age seven, lasting until age eleven. Children at this stage are beginning to use some logic as they perceive events. Things are as they are, not as they seem to be. A child in this stage of development can hold two relationships in his mind at a time. (12, 14, 19)

The fourth or formal operations period encompasses the ages of eleven to fifteen. The child has a capacity for abstract thought based on formal logic apart from experience and experiment. (12, 14, 19)
The succession of stages is consistent even though the chronological ages at which they appear may differ from child to child. Piaget suggests four factors which influence the gradual transition of a child from one stage to the next: maturation, experience, social transmission, and equilibration. Maturation refers to the nervous system. Experience refers to the experiences which a child has and also to his reactions to these experiences. Social transmission involves the structure of the child or his mental operations which allow him to understand the language being used. Equilibration is a self-regulation factor which is the fundamental factor in the learning process. (12) Equilibration is the coordination of the first three factors. (8) It is an active mental process which operates outside the individual's awareness and which is brought about by the child reflecting, in an autoregulatory sense, on his own activities. (8)

Stendler summarizes Piaget's equilibration factor and theory of cognitive development in this way:

"For Piaget, intelligence is not something that is qualitatively or quantitatively fixed at birth, but rather, is a form of adaptation characterized by equilibrium. Part of man's biological inheritance is a striving for equilibrium in mental processes as well as in other physiological processes. Twin processes are involved: assimilation and accommodation. The child assimilates information from the environment which may upset existing equilibrium, and then accommodates present structures to the new information so that equilibrium is restored." (27:336)

Stated in a different way, assimilation and accommodation need to be in a state of equilibration. Once equilibration is achieved, maturation and experience help to explain cognitive development. (26) This is not something which happens once with intellectual development as the outcome; it is a constant and on-going process. It is a child's constant striving for equilibrium through the process of assimilating and accommodating information that determines the child's stage of intellectual development.

PIAGET'S THEORY OF COGNITIVE DEVELOPMENT AND MATHEMATICS

In relating Piaget's theory of cognitive development to mathematics, two more ideas must be clarified: conservation and reversibility. Gruen defines conservation as
the realization that a particular quality of an object remains invariant in spite of changes in other irrelevant dimensions. (37) In other words, a child is able to realize that certain attributes of an object are constant even though there appears to be a change in the object or its attributes. (26) Reversibility is movement back to the original starting point. To possess the concept of reversibility, a child must be able to hold the beginning and ending points in a constant relationship. For example, given a ball of clay, a child will divide the clay into two balls which are the same size; then he will roll one ball into a sausage. When asked which has more clay, the ball or the sausage, many children cannot see that the two clay objects contain the same amount of clay in spite of the fact that clay has neither been added to nor taken away from the ball or the sausage. A child capable of verbalizing that both contain the same amount of clay is capable of reversibility and is at the level of conservation. Conservation is defined by Piaget as the “necessary condition for any mathematical understanding.” (22:4) Conservation is probably achieved by actually thinking of the inverse operation and realizing that by applying the inverse operation, one would arrive at the original operation. (53) It was stated earlier that a child is constantly going through the dual processes of assimilation and accommodation. As a child accommodates new information into his structure, he moves toward conservation. A child conserves quantity before number, mass before weight, and weight before volume. (8, 22)

It has been suggested that the concept of conservation is learned from direct observation or from social reinforcement. There are theoretical difficulties in attempting to explain conservation with either of these possibilities, as “experimental efforts to induce conservation through either of these possibilities have not proved successful.” (53:1057) Sowada (50) suggests that the process of conservation can be short-circuited by giving a child verbal formulas or a set of rules that he can commit to memory without assimilating the concept to his present organization or without accommodating his present organization to fit the new concept.

Cathart (33) has cited studies which were designed to train children at an earlier stage to recognize any or all of the identity, reversibility, or compensatory relationships necessary for conservation. The studies (Berlin, 1965; Pufall, 1967; Sigel,
Roeper and Hooper, 1966; Smeslund, 1961; Smith, 1968; Towler, 1967; Wallach and Spratt, 1964; and others) have indicated that training in Piaget-type tasks results in no significant change in inducing conservation at an earlier age. Investigators are now ignoring chronological age as the determining factor in establishing readiness for a task. They are looking instead at a child's mental age. (12)

Piaget maintains that "... it is a great mistake to suppose that a child acquires ... mathematical concepts just from teaching ... When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them (mathematical concepts) comes only with his mental growth." (21:406) One of the problems in mathematics teaching today is that too often the child is made to fit the mathematics rather than making the mathematics fit the child.

"... Children develop mathematical concepts by themselves, independently and spontaneously ..." (21:406) Mathematical understanding is based on experience. Children need to find their own answers. If they read about it (or are given the right answers), "... learning will be deformed as is all learning that is not the result of the subject's own activity." (34:317-319; 7) General ways of knowing must be actively constructed by the child through his interaction with the environment. (41) "Knowing is action on objects." (17) No one can make sense of mathematical symbols unless he has carried out actions with materials. A lack of experience with concrete materials leads to inadequate mental operations and the inadequate development of abstractions required for more advanced math. (17) Maier notes that "... each developmental aspect begins with concrete ordinary experiences or problems. Only after complete mastery of a concrete experience does development proceed toward the mastery of its corresponding abstraction." (16:156) The mathematical idea does not reside in the material itself, but in the abstraction of the actions undertaken with the material. And it is the mastery of the abstraction that Piaget refers to as conservation of an idea or concept. (14, 17)

Because a child's experimentation with or action on materials is necessary, Piaget maintains that teachers can only create situations in which a child can invent and discover structures for himself. (17) The basic environment must be created within which
learning can take place. Brearly (6) suggests that it is important for a teacher to intervene in a child's learning when appropriate to help him focus on the relevant mathematical elements.

The role of language is crucial for aiding in clarifying, refining and extending a child's ideas, and it helps the child focus his thoughts on mathematical relationships. It is important for teachers to use a precise language to help the child move from perceptual thought to conceptual thought. Language and action help a child organize his experiences so he can generalize them to other situations. It is through language that the crystallization of relationships is made possible." (6:98) Teachers must provide the materials and language which are appropriate to the thinking level of the child and which provoke the quest for further clarification and extension of mathematical ideas. This should not be confused with rote verbalism which can lead to later failure in mathematics. (6) Discussing the role of language, Piaget states that the person spoken to selects the words spoken according to his own interests and distorts them in favor of previously formed concepts. He further notes that all children feel that they understand what is said and that they are understood by the listener. (24)

JUSTIFICATION FOR USE OF PERFORMANCE OBJECTIVES

Research has indicated that the teacher must create an environment in which children can work and make discoveries through the teacher's careful intervention at the appropriate time. (6) For this to happen, the teacher must have clearly in mind what the objectives of the mathematics program are and how to achieve them.

The idea of behavioral objectives became popular at the turn of the century with the advent of the scientific movement in education. (53) Within the last few years, the debate has been intense over the issue of stating curriculum in terms of measurable objectives. (59) Jackson (39) conducted a small study of 20 teachers to determine if "outstanding" teachers made use of objectives. "Outstanding" was determined and defined at the administrative level. His study indicated that these "outstanding" teachers felt hampered by the use of behavioral objectives.
Johnson, Tyler, and Bloom strongly support the use of goals in curriculum development. (40, 31, 5) Johnson (40) defines a goal as a statement of what the learner is to be like when he has successfully completed a learning experience. It is a description of a pattern of behavior which the learner will demonstrate after the learning has taken place. Bloom (5) stresses the need to clearly specify curriculum in terms of content and in terms of behavior expected of the learner. With curriculum clearly specified, the teacher is better able to select appropriate learning experiences in order to bring about the prescribed behavior change.

Merritt (42) disputes psychologists who insist that the use of behavioral objectives makes teaching less humanistic. He states that the current curriculum needs to be translated into more measurable terms, resulting in information which would improve instruction. Included in the objectives there needs to be the appropriate learning activities and provisions for evaluating the desired outcome.

If changes in behavior are to take place because of the interaction of a student with specific learning material, then the objectives need to be stated more clearly than if the behavior change is to result from interaction with the teacher. (31) Johnson (40) notes that too often the relative merits of a material or textbook are discussed apart from the goals that can be reached through their use.

Because mathematics is a sequential and scientific subject, it is an easy subject for writing objectives and developing evaluative devices. (35) Pinard and Laurentdeau (45) emphasize that such evaluation needs to be qualitative, not quantitative. The need is to study the wrong answers in order to make curriculum changes as opposed to studying the number of correct responses on a test. Evaluation of children in relation to the objectives, according to Tyler (31), is for the purpose of identifying the strengths and weaknesses of the program, not the child. In conclusion, Johnson supports the use of performance objectives in curriculum writing when he states that "only when tests measure performance in terms of goals..." does a teacher "... fully realize the child's achievement." (40:114)
CHAPTER II

DEVELOPMENT OF THE KINDERGARTEN
MATHEMATICS CURRICULUM

The development of the kindergarten mathematics curriculum for Las Virgenes closely followed Tyler's philosophy of curriculum development. As stated in chapter I, Tyler recommends that curriculum development be a threefold process: listen to experts in the field, look at the community, and look at the learner. (31) The experts in the field included the cognitive theorists (discussed in chapter II), the California Mathematics Advisory Board (Strands Report), and the publishers of the state-provided kindergarten textbooks.

The community is the Las Virgenes Unified School District. It is a suburban area located in the western part of Los Angeles County. Ninety-eight percent of the community is white, native English-speaking, upper middle class and professional in employment. The average home costs about $40,000. Approximately seventy percent of the adult population is college educated (statistics made available through the office of the District Assistant Superintendent). The community has set high expectations for its children as illustrated by favorable tax overrides in recent years. In 1972, members of the community were involved in an extensive goal-setting conference. A top-priority goal resulting from this conference was the understanding of mathematics.

Piaget (14) has suggested that mathematical concepts are irregular and patchy in less able and deprived communities. An investigation in Aden, South Arabia, illustrates the inverse of this theory. The Aden study involved 144 children: 72 male and 72 female. Of this number, 48 were European children whose fathers were involved in the military, the government or business; 48 were Arabs; 24 were Indians; 24 were Somalis. The purpose of the investigator's study was to determine if the children of Aden exhibited responses similar to those found by Piaget in the children in Geneva. The findings of the study were termed positive by the investigator. Furthermore, a quantitative comparison of test scores by community indicated that the European children scored "significantly higher" than the children from the other communities. (12:199)
investigator suggested that the background experiences of the European children caused the higher scores.

The Aden study has implications for the Las Virgenes Unified School District. The learner in Las Virgenes comes to school with an understanding of many mathematical concepts. The population of the community has provided many experiences for its children.

The kindergarten curriculum, written in terms of performance objectives, was developed after looking at the learner's needs, the community's priorities, and the learning theories expounded by the experts. The curriculum, originally written in 1972, has been revised three times. The latest revision was made after the author's manipulative mathematics test was given to 80 kindergarten children from eight classes in the district.
CHAPTER IV

THE KINDERGARTEN MATHEMATICS CURRICULUM

The kindergarten curriculum was developed as a part of a spiral curriculum for the elementary grades. Kindergarten children who have achieved mastery of the kindergarten objectives, based on the Kindergarten Manipulative Mathematics Inventory, are considered to have the mental age of a first-grade child, and need to be given mathematical experiences from the next level of the curriculum.

The curriculum guide contains the performance objectives and a list of suggested supportive materials which can be used to achieve the objectives. It emphasizes an activity approach to mathematics through the use of physical objects usually found in the kindergarten environment and through the use of other materials available to the kindergarten teacher. Pages from the current state-adopted kindergarten mathematics textbooks — Houghton Mifflin, Modern School Mathematics, Structure and Use (H.M. Text) and Singer Publishing Company, Sets and Numbers (S.P.C.) — have been correlated with the performance objectives.

The objectives are organized by Strand as suggested in the State Mathematics Framework.
PERFORMANCE OBJECTIVE

I. Numbers and Operation
   A. Number
      1. Given a verbal direction to count as high as he can, the child will correctly count by rote from 1 to 25 within 3 minutes with 80% accuracy.
      2. Given two identical sets of numerals (0-10), the child will correctly match the numerals within 5 minutes with 100% accuracy.
      3. Given verbally any number from 0-5, the child will be able to correctly write the numeral within 1 minute with 80% accuracy.

SUGGESTED ACTIVITIES

1. Objects
   - Playing cards
   - Dominoes
   - Number, shape cards (t. made)
   - Worksheets (District math file)

2. Counting activities in class:
   1. chairs
   2. children
   3. milk
   4. blocks
   5. calendar
   6. other

3. Classroom activities:
   1. the number of absentees
   2. the number of work stations
   3. my “age” (5 years old)
   4. the number for fingers on one hand, toes on one foot
   5. other
4. Given a pictorial representation of a set containing 0-10 elements, the child will be able to tell how many elements are contained in the set within 1 minute with 80% accuracy.

5. Given a set of objects and any 3 numeral cards (1-10), the child will construct sets containing that many elements within 3 minutes with 80% accuracy.

6. Given any numeral (1-10), the child will say the numeral which precedes it and follows it within 2 minutes with 100% accuracy.

7. Given pictures of a series of three objects, the child will identify the ordinality of the objects: first, middle, last, within 2 minutes with 100% accuracy.
8. Given verbally a simple open sentence, a child will suggest what will make it true within 2 minutes with 100% accuracy.

B. Place Value

C. Operations: Addition

9. Given sets of objects, the child will join two sets together. He will state the number in set I, the number in set II, and the number in the new set formed within 2 minutes with 80% accuracy.

Operations: Subtraction

10. Given a set of objects, the child will separate it into two sets, stating the number in each new set formed within 2 minutes with 80% accuracy.

8. District math file
   Classroom experiences

9. Cuisenaire Rods
   Objects as counters, toys, etc.
   Cards
   Children
   Pencils, crayons, etc.
   Dominoes
   H.M. Text: pp. 85-91

10. Cuisenaire Rods
    Objects as counters, toys, etc.
    Cards
    Children
    Pencils, crayons, etc.
    Dominoes
    H.M. Text: pp. 92-96
D. Fractions

11. Given a set of 2, 4, 6 objects, the child will separate the set into two equal sets in 5 minutes and name one of the new sets formed as being \textit{one-half} of the original set with 100% accuracy.

II. Geometry

1. Given five patterns, the child will reproduce the patterns within 10 minutes with 80% accuracy.

2. Given five repeated patterns, the child will indicate verbally or on paper what should come next to continue the pattern within 10 minutes with 80% accuracy.

11. Objects as counters, toys, etc.
   - Paper folding
   - Crackers, other breakable foods
   - Cuisenaire Rods
   - Children
   
   \textit{H.M. Text}: pp. 28-29

1. Beads
   - Flannel board cutouts
   - Teacher-prepared cards
   - Blocks: kdg., A blocks, parquetry
   - Geo-boards

2. Beads
   - Flannel board cutouts
   - Teacher-prepared cards
   - Blocks: kdg., A blocks, parquetry
   - Cuisenaire Rods
   - Geo-boards
   
   \textit{H.M. Text}: pp. 24-27
3. Given five sets of two similar geometric shapes (curves), the child will tell which is bigger and which is smaller within 5 minutes with 80% accuracy.

4. Given circles, triangles, squares, and rectangles, the child will name these shapes (curves) within 2 minutes with 100% accuracy.

5. Given verbally the terms circle, triangle, rectangle, the child will make drawings of these shapes within 3 minutes with 100% accuracy (defined by the attribute of the shape: correct number of sides, etc.).

6. Given a simple closed curve, the child will place markers to indicate what the terms outside, inside, on are in relation to the given closed curve within 2 minutes with 100% accuracy.

3. Attribute blocks
   Cuisenaire Rods
   Flannel board cutouts
   Paper cutouts

4. Attribute blocks, parquetry blocks
   Flannel board cutouts
   Paper cutouts
   Shapes within environment
   H.M. Text: 15-21

5. Patterns for tracing
   Shapes
   S.P.C. Text: pp. 54-55

6. Playground markings, children
   Pictures of curves
   H.M. overhead transparencies (gr. 1)
   Geo-boards
   Boxes and objects
   H.M. Text: pp. 11-21
III. Measurement

1. Given a variety of containers, the child will experiment with volume and make verbal comparisons of two vessels using the terms *holds more, holds less* during a 20-minute work period with 100% accuracy.

2. Given the terms *day* and *hour*, within 1 minute the child will be able to say which is longer with 100% accuracy.

3. Given verbal statements related to time (dinner time), the child will tell what happens at that time within 2 minutes with 100% accuracy.

4. Given two objects and told to make length comparisons, the child will say which is longer and which is shorter within 1 minute with 100% accuracy.

5. Given two objects of the same length, placed so that the objects are not lined up with each other, the child will say that the objects are the same length within 1 minute with 100% accuracy.

1. Water play table
   Buckets of dry ingredients as sand, rice, flour
   Vessels for measurement

2. Classroom experiences
   Clocks

3. Clocks
   Pictures of activities

4. Nonstandard units of measure: pipe cleaners, ice cream sticks, string, straws, etc.
   Ruler, yardstick
   *H.M. Text:* pp. 3-4
   *S.P.C. Text:* pp. 26-31

5. Nonstandard units of measure: pipe cleaners, ice cream sticks, string, straws, etc.
   Ruler, yardstick
Given two objects and told to make weight comparisons, the child will say which weighs more and which weighs less within 1 minute with 100% accuracy.

IV. Applications

1. Given pictures related to the child’s environment, the child will name those pictures which are related to math in some way. He will tell how the picture is related to and/or tell the purpose of each math-related picture.

V. Statistics and Probability

1. Given a closed situation where the child watches the teacher place two colors of physical objects into a sack (say red and blue), the child will state a probable color of the object removed within 1 minute with 100% accuracy.

VI. Sets

1. Given three pictorial representations of sets containing 0-10 objects, the child will identify the empty set within 30 seconds with 100% accuracy.

Pan balance
Objects to weigh: rocks, feather, assorted boxes
Scales
H.M. Text: p. 6

1. Objects related to math: scales, thermometer, money, telephone, home address, license plate, calendar
Pictures of math-related objects
Have children cut out math-related pictures from magazine

1. Blocks: 2 colors
Marbles: 2 colors
Objects
Opaque sack

1. Plates and objects or counters
Picture cards
Children, hula hoops
H.M. Text: pp. 55-57
2. Given two sets of objects (0-10 in each), the child will show that the sets are equivalent by placing them in a one-to-one correspondence (moving the objects of set close to the objects of the other set) within 5 minutes with 100% accuracy.

3. Given two nonequivalent sets (0-10 in each), the child will say within 1 minute which set contains the larger number of objects and which set contains the lesser number of objects with 100% accuracy.

4. Given numerals or sets (1-5 in each set), the child will arrange them in order from fewest to most within 2 minutes with 80% accuracy.

2. Blocks
   Children
   Flannel board cutouts
   Classroom experience: milk/straws, children/milk, children/chairs
   H.M. Text: pp. 33-37

3. Blocks
   Children
   Flannel board cutouts
   Classroom experience: milk/straws, children/milk, children/chairs, tables/chairs
   H.M. Text: pp. 1-2, 36-41, 82

4. Clothes line and numeral cards or picture cards
   Picture cards
   Nesting boxes, lids
   Number cards
   Sequence, number puzzles
   Simple dot-to-dot pictures
   H.M. Text: pp. 60-62
   S.P.C. Text: pp. 74-75
VII. Functions and Graphs

1. Given a pictorial representation comparing two sets (two variables), the child will say one true statement about the graph in 1 minute with 100% accuracy.

2. Given a Cartesian grid (3 x 3) with pictures at intersecting points, the child will say the name of the picture at the given number pair within 1 minute with 75% accuracy.

VIII. Logical Thinking

1. Given a task, the child will verbally demonstrate his knowledge of the terms all, some, none with 100% accuracy within 2 minutes.

IX. Problem Solving

1. Given 5 problems involving familiar processes from any strand and one-half hour, the child will tell the solution for 4 of the problems, using manipulative materials as necessary.

1. Classroom experiences with two variable graphs
   - Blocks
   - Graph paper
   - District math file

2. Floor marked with masking tape and children
   - Teacher-prepared charts

1. Attribute blocks
   - Colored objects
   - Parquetry blocks
   - Colored cubes
   - Mosaics
   - Bucket
   - District math file

1. Classroom activities
CHAPTER V

THE DATA

The author's Kindergarten Manipulative Mathematics Inventory (KMMI) was used with a random selection of kindergarten children to evaluate the performance objectives for the Las Virgenes Unified School District. The revised test and testing guide are found in the appendix.

The kindergarten teachers in Las Virgenes were given the kit of manipulative materials and a guide for the administration of the test. The teachers received verbal instructions for record keeping at a kindergarten staff development meeting. The test, without the items relating to the strands of Application and Problem Solving, was administered in May 1973 to 80 children.

In the spring of 1973, the total kindergarten population of the Las Virgenes Unified School District was given another mathematics test, the Wide Range Achievement Test, Mathematics (WRAT-Math). The district used the oral part of level one and the first four written problems (level one). The WRAT-Math is a standardized test.

COMPARISON OF THE TESTS

A simple comparison of the WRAT-Math and KMMI shows many similarities. For example, the same administration procedure is used for both tests. The teacher administers each test on a one-to-one basis. Teacher variability affecting a child's total raw score should be minimal because each testing guide provides the teacher with the precise vocabulary to use in administering the test. Both tests are content oriented and test mathematics achievement. Neither test demands much pencil/paperwork from the child, although the WRAT-Math requires the child to write answers to four written problems dealing with addition and subtraction (operations).

An analysis of content shows that the KMMI contains items similar to the WRAT-Math with one exception: inequalities. Two items on the WRAT-Math deal with inequalities. Understanding of the terminology "more than/less than" is tested in the
area of measurement on the KMMI. The WRAT-Math deals with addition and subtraction in the algorithm form; the KMMI utilizes a performance approach whereby the child manipulates objects to illustrate the concept of addition and subtraction, then verbalizes his understanding of the processes. The WRAT-Math provides data in the areas of Number and Operation and Problem Solving. The KMMI utilizes a problem-solving approach to test items in these mathematical strands: Number and Operation, Geometry, Measurement, Statistics and Probability, Sets, Functions and Graphs, and Logical Thinking.

ANALYSIS OF DATA

For the purposes of this study, the population was 80 kindergarten children from eight classes in the Las Virgenes Unified School District. These children were given both the KMMI and the WRAT-Math tests.

An analysis of the data indicates that more than fifty percent of the scores are above the mean on both tests: 58% of the scores are above the mean on the WRAT-Math; 83% of the scores are above the mean on the KMMI. The greater percentage of scores falling above the mean on the KMMI can be explained by the fact that the WRAT-Math was designed to be used in grades kindergarten through 12, while the KMMI was developed specifically for kindergarten.

Table 5.1
Comparison of Mean Scores, WRAT-Math and KMMI

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Total Raw Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>WRAT-Math</td>
<td>18.72</td>
<td>24</td>
</tr>
<tr>
<td>KMMI</td>
<td>35.27</td>
<td>38</td>
</tr>
</tbody>
</table>

N = 80
A Pearson product moment correlation coefficient was performed to determine the significance of the similarities apparent on both tests.

Table 5.2

Pearson Product Moment Correlation Coefficient, WRAT-Math and KMMI

<table>
<thead>
<tr>
<th>WRAT-Math</th>
<th>N = 80</th>
<th>r = 0.76</th>
<th>(p = &lt;0.001)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KMMI</td>
<td>N = 80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of the findings indicate a correlation of 0.76 which is significant beyond a 0.001 level. It appears that children who score high on the WRAT-Math will tend to have high scores on the KMMI and, conversely, children who score low on the WRAT-Math will also tend to score low on the KMMI.

A scattergram illustrates the correlation graphically:

```
   23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
  24  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
  23  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
  22  |   |   |   |   |   |   | 1  | 1  | 6  |   |   |   |   |   |   |   |
  21  |   |   |   |   |   |   | 2  | 2  | 2  |   |   |   |   |   |   |   |
  20  |   |   |   |   |   |   | 1  | 1  | 2  | 1  | 5  | 2  |   |   |   |   |
  19  |   |   |   |   |   |   |   |   |   |   |   |   | 5  | 4  | 2  |   |
  18  |   |   |   |   |   | 1  | 1  | 2  | 3  | 1  |   |   |   |   |   |   |
  17  |   |   |   |   | 1  | 3  | 2  |   |   |   |   |   |   |   |   | 1  |
  16  |   |   |   | 2  | 1  |   |   |   |   |   |   |   |   |   |   |   |
  15  |   |   | 1  | 1  | 1  | 1  | 1  |   |   |   |   |   |   |   |   |   |
  14  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
  13  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
  12  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
  11  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
  10  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
   9  | 1  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
```
The scattergram shows that three children attained the highest scores on both the WRAT-Math and the KMMI; one child received the lowest score on both tests. If dotted lines are drawn to divide the scattergram into quadrants, the high degree of correlation can be seen in the opposing quadrants: upper right and lower left. Of N = 80, 69 scores (or 86%) fall into these areas of high correlation.

The findings on table 5.2 indicate that the KMMI and WRAT-Math appear to measure the same content and that the KMMI is comparable to the WRAT-Math for testing mathematics achievement. However, because the KMMI evaluates this achievement in all of the nine strands of the mathematics curriculum as opposed to two strands that are sampled on the WRAT-Math, the KMMI can be used to obtain a more accurate diagnosis of a child’s achievement when that student is functioning at a kindergarten level.

The use of the KMMI is not without some drawbacks. The time factor in administering the test is much greater for the KMMI (35 minutes as opposed to 5 minutes). The KMMI requires more clerical work from the teacher than does the WRAT-Math. And also, the large number of manipulative pieces can make the administration of the KMMI cumbersome.

Since teaching is warranted only to the extent that it facilitates learning, an analysis of the KMMI data clearly indicates that the kindergarten children in the Las Virgenes School District are achieving the performance objectives.
CHAPTER VI
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study was the development of a kindergarten mathematics curriculum for the Las Virgenes Unified School District. The proposed curriculum was to meet district needs, focus on the needs of the learner, and include all nine mathematical strands as suggested by the California State Department of Education.

The district's requirements were twofold: the curriculum must be written in performance objectives, and it must be part of a spiral curriculum.

The literature was reviewed with the needs of the learner in mind. Piaget's four stages of cognitive development provided the main impetus for the performance curriculum, one that is activity oriented. The kindergarten child is at the level Piaget refers to as "preoperational"; he is generally classified as a nonconserver. Because a child at this level learns best through experimentation and manipulation of objects (3, 6), the curriculum emphasizes a multimedia approach for the teaching of mathematics. The KMMI, designed to test the child's achievement of the objectives, is a performance test.

CONCLUSIONS

The KMMI was compared to a standardized test, the WRAT-Math. With a significant correlation between the two tests, it can be concluded that the KMMI is as reliable an instrument as the WRAT-Math for measuring kindergarten achievement.

RECOMMENDATIONS

In the past, Las Virgenes has utilized the WRAT-Math to obtain data for a long-range research project on ascertaining the values of a Transitional First-Grade Program. Otherwise known as T-1, the class is composed of children who "are not yet ready" for first grade as determined by a battery of tests. Looking at the scattergram, the five lowest scores on the WRAT-Math are 9, 9, 11, 12, and 13. The corresponding
KMMI scores are 23, 29, 31, 32, and 30, respectively. Of the five children attaining these scores, four are in the T-1 class; the fifth child was recommended for placement in the class. The author recommends that the KMMI be used as a determinant in selecting T-1 candidates.

Secondly, the author recommends that an item analysis be done to determine if there exists a correlation between specific items on the KMMI and T-1 placement. The author suggests that low scores in the areas of measurement, statistics and probability, sets (objectives 2 and 3), logical thinking and problem solving might be indicative of the need for placement in Transitional first grade.

When the KMMI was first administered, the problem-solving test items were not used and therefore are not part of the correlation of the KMMI and WRAT-Math. Items used for testing Problem Solving on the KMMI are derived from six of the nine strands. The author suggests that this area, Problem Solving, be correlated with the WRAT-Math. If the correlation is significant, then the author makes a final recommendation that the KMMI without the Problem Solving section be used as a diagnostic tool throughout the kindergarten year and that the Problem Solving items be used at the end of the year to evaluate achievement.
BIBLIOGRAPHY


PERIODICALS


PUBLISHED PAPERS


KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Directions: Mathematics

This test is designed to be given on a one-to-one basis: one teacher or trained aide will work with one child at a time. Results should be recorded on the accompanying record-keeping device.

The KMMI is constructed to be both a diagnostic and an evaluation tool. A wise teacher avoids assuming that a child knows something until she has tested that child in a variety of ways. The record-keeping device in the strand of Number and Operation has therefore been elaborated to allow the teacher to diagnose and record what the child knows. Final evaluation can be written in a different color to differentiate it from the diagnosis.

Materials for the test are included in the testing kit. The two-dimensional materials are labeled by objective number on the back. The three-dimensional materials are labeled insofar as possible and are found in the box. In addition to the testing kit, the evaluator should have available for each child tested a piece of paper and a writing tool (crayon or pencil).

Although this is not a timed test, 25 to 35 minutes allows ample time for completion. The instructor may choose to use parts of the test at the conclusion of a unit, or the instructor may choose to give the entire test at the end of the year to test achievement and retention.

The teacher's guide provides the evaluator with the exact wording for each test item. The evaluator is encouraged to deviate as little as possible from this working. Any major deviation in wording used by the evaluator to evoke a response should be noted, as this will provide the instructor with more information about the child.
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics
Nontimed Manipulative Test
one to one

I. NUMBER AND OPERATION

A. NUMBER

1. No material
   1. “How high can you count?”

2. Number cards: 3, 5, 8; 2 sets
   2. Place the number cards in random order in front of the child. Hand the child the identical set of cards: “Can you find the numbers that are the same as these and match them together?”

3. Pencil/crayon Paper

4. Picture cards: 5, 7, 8
   4. Show child one card at a time. Each time say: “Without counting, how many dots do you see?” If child must count to respond, mark method of counting on record sheet (point to dots, movement of head, whisper, eye movement).

5. Teddy bear counters
   Number cards: 3, 6, 8
   5. Show child number cards one at a time. “Make a set of this many bears (point to number card).” Response. “How many bears are in this set? Prove it by counting.” Record any difficulty child has counting.

6. Number card: 4
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics

7. Ordinality card: (boys in a line)
   Show card to child: “Here is a picture of three boys. They are going to buy some ice cream... here (indicate).” “Point to the boy who is first in line to buy ice cream.” (response) “Point to the last boy in line to buy ice cream.” (response) “Point to the middle boy in line to buy ice cream.”
   Extend to verbal level. Record V on record sheet if child is at verbal level.
   Point to last boy: “Where in line is this boy?” (response: last, third)
   Point to first boy: “Where in line is this boy?” (response: first)
   Point to middle boy: “Where in line is this boy?” (response: middle, second)

8. No materials
   Use example first: “Somebody is sitting on the floor. Make this sentence true by telling me the name of one person who is sitting on the floor.”
   Math test item: “We have some doors in this room. What number will make this sentence true?”
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics

B. ADDITION
9. Set board: Addition
   7 teddy bear counters
   9. Place teddy bear counters on table. Place set board in front of child so large set is at bottom. "Put these teddy bear counters in these two sets (indicate sets at top with hand)." "How many are in here (point to first set) and how many are in here (point to second set)?" "If you joined these two sets into this big set (indicate with hand), how many would you have in all?" "Do it and tell me." "Can you tell me about what you did?"

   For children who cannot verbalize process:
   "How many teddy bears did you put here (indicate) and how many teddy bears did you put here (indicate)?" "When you moved them down here, how many do you have in all?" (record prompting on sheet)

C. SUBTRACTION
10. Set board: Subtraction
    5 teddy bear counters
    10. Place 5 teddy bear counters in large set.
        "How many teddy bears are in this set?" (response) "Take some teddy bears away. How many did you take away?" (response) "How many are left?" (response) "Can you tell me about what you did?"

    For children who cannot verbalize process:
    "How many did you start with, take away child’s response, leaves how many?" (record prompting on sheet)

D. FRACTIONS
11. 8 plastic counters
    11. "Here are some plastic counters. Put the counters into two piles so each pile has the same amount of counters." (response) "Does each pile have the same amount?" "How do you know?" (record what child says)
Teacher Instructions — Mathematics

II. GEOMETRY

1-2. Pattern card 3
   Flannel pieces
   Flannel board

1-2. Hand child pattern card one at a time.
   Have flannel pieces available to child.
   Give child flannel board. “Copy this pattern on your flannel board. Now read the pattern to me. If you were to continue this pattern, what would come next?” Repeat same directions for rest of pattern cards.

3. Geometric curve cards:

3. Show child similar geometric curve cards one at a time. “Here are two shapes or closed curves. Point to the shape which is bigger.” (response) “Now point to the one which is smaller.”

   Card 2: “Point to the bigger shape.”
   (response) “If this shape (point to child’s response) is bigger, tell me what size this one is.”

   Card 3: “Point to the smaller shape.”
   (response) “If this shape (point to child’s response) is smaller, tell me what size this one is.”

   Cards 4 & 5: Point to one shape: “This one is (pause for child’s response)?” Point to other shape: “This one is (pause for child’s response)?”

   If child is unable to verbalize bigger and smaller on cards 2-5, use directions for card 1 for entire test. Record that child is at nonverbal level: N-V.

4. Geometric curves:

4. Show child one curve card at a time: “What do we call this geometric shape?” “Do you know its name?” Repeat for all four shapes.
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics

5. Paper
Writing tool: pencil/crayon

Accuracy level of this item depends on attribute of shape and not the perfection of shape: A triangle has three sides, a rectangle has four sides, a circle has no sides.

6. Closed curve
Red teddy bear
Yellow teddy bear
Blue teddy bear

6. Place closed curve in front of child. Place teddy bears next to card. “Let’s pretend this (point to closed curve with hand) is a fence. Pick up the red teddy bear and put him inside the fence.” (response) “Pick up the yellow teddy bear and place him outside the fence.” (response) “Pick up the blue teddy bear and place him on the fence.” (response) If the child has difficulty picking up the correct color, guide him to the right teddy bear.

Extend to verbal level and record V on record sheet if child can verbalize terms inside, outside, on. Point to one teddy bear at a time: “Where is this teddy bear?”
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics

III. MEASUREMENT

1. Large glass
   Small glass
   Place the two glasses (same color) in front of child. “Which of these glasses would hold more?” (response) “Which of these glasses would hold less?” (response)
   If child appears confused or if child gives the wrong answer, allow him to prove answer to examiner with water: “Can you show me by filling one glass and pouring it into the other?” Do not clue child as to which glass to fill. If child is unable to make a response after filling glass with water, no score.

2. No material
   “Which is longer: one day or one hour?”
   If child is confused, clue: “Mother told you that you could spend one hour at Disneyland or you could spend one day at Disneyland. Which would you choose?” (response) “Would you be at Disneyland longer if you stayed one day, or would you be there longer if you stayed one hour?”

3. No material
   “What do you do at rest time in the class?”
   Acceptable responses: any behavior which is related to rest time in class. If there is no rest time, choose some time (snack time, play time) and evaluate response in terms of behavior related to that time.
4. Long stick  
   Short stick  

   4. Place two sticks in front of child. “Point to the longer stick.” (response) “Point to the shorter stick.” (response) Extend to verbal level and record V on record form. “What size is this stick?” (point to one stick; repeat for second stick) Appropriate responses: Big, bigger, longer, biggest, longest, little, small, littler, smaller, littlest, smallest.

5. Two sticks of equal length;  
   or two pencils; or two Cuisenaire rods

   5. Place the two sticks of equal length in front of child. “Which one is longer or are they both the same length?” (response) “How do you know?” (response) Move one stick so it extends beyond the other about one inch. “And now?” (response) “How do you know?”

6. Two similar-sized rocks,  
   weight difference of 6 oz.

   6. Place two rocks in front of child. “Here are two rocks. One of them weighs more than the other one. Pick them up.” (pause) “Show me the one that is heavier (weighs more).” (response) “Show me the one that is lighter (weighs less).” (response) Extend to verbal level and record V on record-keeping form. “Tell me about the weight of this rock.” (choose one rock; allow child to hold rock) Acceptable responses: weighs more, weighs less, heavier, lighter, heavy, light. “Now tell me about this rock.”
IV. APPLICATIONS

1. Chart of pictures

1. "Can you find the telephone on this page?" (response) "What do we call these symbols written on the telephone?" (response: numbers, numerals) "Other things on this page also have numbers or numerals written on them. Point to each picture and tell me if the picture would have numbers on it. Tell me where on this thing you would find the numbers." (Be sure child points to all pictures. Record answers on record-keeping device.)
V. STATISTICS AND PROBABILITY

1. 3 red blocks
   2 blue blocks
   opaque sack

1. Place all of the blocks on the table in front of the child. Show child the sack. “Pick up the blocks and place them in this sack. Now mix up the blocks by shaking the sack. (pause) If I reach into the sack and take out one block, what color do you suppose it might be?” (acceptable response: red, blue) “Why do you think it might be (child’s response)?” (response) “Could it be a different color?” (response) “Why?” (response) “Could it be purple?” (acceptable response: no) “Why not?” (response)

This test item should clue the teacher to the child’s logic level of thinking.
VI. SETS

1. Set card: 1, 0, 2

1. Place picture card in front of child. "Here is a picture of three sets. Point to the empty set."

2. Flannel board
   Felt pieces:
   4 birds, 6 trees

2a. Place 4 birds on a flannel board. Place the 6 trees in front of child. "Put some trees on the flannel board so that each bird has a tree. (pause) Remove the extra unused trees. "Are there more birds, more trees, or are they just the same?" (pause) "How do you know?" Proof may be counting, touching the objects at the same time, drawing an imaginary line, or by moving objects. If the child counts, continue, "What can you do so that I can really see that there is the same number?" (pause) If no response: "Is there something you can move to prove that there are the same number of birds and trees?" If there is still no response, drop it!

b. If child places trees in a one-to-one correspondence with birds, continue: "Watch me closely." Spread out trees by moving first and last tree. "Now are there more birds, more trees, or are they just the same? How do you know?" (record response)

c. "Watch closely." Move trees very close to each other, making length of line of trees shorter than birds. "Now are there more birds, more trees, or are they just the same? How do you know?" (record response)
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics

3. Flannel board
   Felt pieces:
   6 hearts, 5 bunnies

3. Place 6 hearts and 5 bunnies on flannel board so length of each line is same. “Are there more hearts, more bunnies, or are they just the same? How do you know?” (record response) “Can you prove it?” (pause) Spread out bunny line. “Are there more bunnies, or more hearts? How do you know?” (record response)

4. Number cards: 1-5
   (optional: dot configuration cards 1-5)

4. Give child numeral cards 2-5 in random order. Place card 1 down to left. “Place these cards down on the table (move hand to right) so the cards go from least to most.” If child cannot do this with number cards, use dot configuration cards and repeat same directions.
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Teacher Instructions — Mathematics

VII. FUNCTIONS AND GRAPHS

1. Graph

   1. Show child graph. "Look at this graph. It shows something about a birthday party. What does the graph tell you about the birthday party?" (Possible responses: 4 girls came, 3 boys came, 7 children came, more girls came, fewer boys came)

2. Coordinate graph

   2. Show child coordinate graph. "What picture is at 2, 3? 2, 3?" Extend to verbal level: "What number pair tells where the sun is?"
VIII. LOGICAL THINKING

1. 3 red blocks
   2 blue blocks

1. Place blocks in front of child.
   “Are all of these blocks red?”
   (possible response: no)
   “Some of the blocks are what color?”
   (possible response: red, blue)
   “How many blocks are green?”
   (possible response: none, zero)
IX. PROBLEM SOLVING

1. Story pictures (6) 1. “Here is a picture of (name object). Can you tell me a number story about the picture?” Use story pictures in sequence for addition and subtraction until child cannot verbalize story.

2. Flannel pieces 2. “Use some of these flannel pieces to construct a pattern.” (response) “If you were to continue the pattern, what would come next?” (response)

3. 5 pennies
   Candy chart 3. “Here are some pennies. Pretend you can spend all of these pennies at the candy store. (show chart) “Let’s say you will buy this lollipop.” (point to it) “What else will you buy to spend all of your pennies?”

4. Plasticine 4. Give child ball of plasticine. “Divide this into two balls so each ball has the same amount.” (response) Show child the two balls. “Do they both have the same amount?” (response: wait for a yes response) “Put this one over here.” (point) “Roll this one into a sausage or snake.” (response) “Now, do the ball and the sausage have the same amount of plasticine in them, or has one more?” (response) “Why?” (response) “Roll this one into a ball again.” (response) “Have the balls the same amount of plasticine in them, or has one more?” (response)

5. Set cards (2) 5. Show child one set card at a time. “What do you see on this card?” (response) “Why do they all belong in this set?” (If no response, clue: What’s the same about all of these?) Repeat for card 2.
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

Mathematics Record-Keeping Form

I. NUMBER AND OPERATION

1. Rote Counting 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 (circle number child counts correctly to, / means number is left out)

2. Matches numerals Yes ____ No ____

3. Write numbers 1 2 3 4 5 6 7 8 9 10 (/ = miss)

4. Tells how many elements are in a set 1 2 3 4 5 6 7 8 9 10 (/ = miss)

5. Constructs a set of 1 2 3 4 5 6 7 8 9 10 (/ = miss)
   a. Recognizes 0 1 2 3 4 5 6 7 8 9 10

6. Before/after 1 2 3 4 5 6 7 8 9 10

7. Ordinality First NV ____ V ____ Middle NV ____ V ____ Last NV ____ V ____

8. Open sentences Yes ____ No ____

Operations

1. Makes 2 sets ____ Joins sets ____ Verbalizes operation __________________________

2. Separates sets ____ Verbalizes operation __________________________

3. Fractions: 2 equal piles ____ Names new subset as 1/2 __________________________

II. GEOMETRY

1. Patterns 1. reproduction Yes ____ No ____
   2. what comes next 1 ____ 2 ____ 3 ____ 4 ____ 5 ____

3. Size Big: NV ____ V ____ Little: NV ____ V ____

4. Shape NV ____ V ____ NV ____ V ____ NV ____ V ____ NV ____ V ____

5. Reproduce shape O circle △ triangle □ rectangle (show direction)

6. Inside NV ____ V ____ Outside NV ____ V ____ On NV ____ V ____
III. MEASUREMENT

1. Volume comparison  Yes ___  No ___
2. Day/hour (longer)  Yes ___  No ___
3. Time: rest ___ dinner ___ snack ___ bed ___ work ___ play ___
4. Length comparison  Yes ___  No ___
5. Length comparison (comments) _______________________________________
6. Weight comparison  Yes ___  No ___

IV. APPLICATIONS

1. Telephone _________________________________________________________
2. Truck ____________________________________________________________
3. Ruler ____________________________________________________________
4. Bat/ball __________________________________________________________
5. Money __________________________________________________________
6. House __________________________________________________________
7. Measuring cup _____________________________________________________
8. Clock __________________________________________________________
9. Scale __________________________________________________________
10. Cake __________________________________________________________
11. Spoon __________________________________________________________
12. Calendar _________________________________________________________

V. PROBABILITY AND STATISTICS

Probable outcome stated possible color ___  stated impossible color ___
Comments __________________________________________________________________
VI. SETS

1. Recognizes empty set

2a. Equivalence: Make E set

   Method of knowing

   b. (move 1 set) more ___ less ___ same ___ why

   c. (move 1 set) more ___ less ___ same ___ why

3a. Nonequivalence more ___ less ___ same ___ method

   b. (move 1 set) more ___ less ___ same ___ why

4. Ordering sets pictures _______ numerals _______

   Comments ________________________________________

VII. FUNCTIONS AND GRAPHS

1. Interpreting graph: _______________________________________

2. Coordinate graph: ____ names pictures at ordered pair

   ____ names ordered pair

   Comments _______________________________________

VIII. LOGIC

Understands terminology: all _____ some _____ none _____

Comments ________________________________________
IX. PROBLEM SOLVING

1a. Tells addition number story: 3 ___ 5 ___ 8 ___

b. Tells subtraction number story: 1 ___ 4 ___ 0 ___

2. Makes own pattern
   Tells what comes next

3. Spends all of pennies

4. Divides plasticine into 2 balls: ___ (more) ___ ball ___ sausage
   ___ same   Why
   Return sausage to ball ____ : more ____ same ____

5. Attributes of set: 1
   2

KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND 1: NUMBER AND OPERATION

OBJECTIVE 2

OBJECTIVE 4

OBJECTIVE 5

OBJECTIVE 6

OBJECTIVE 7
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND I: NUMBER AND OPERATION

OBJECTIVE 9: ADDITION

OBJECTIVE 10: SUBTRACTION
OBJECTIVES 1 and 2
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND II: GEOMETRY

OBJECTIVE 3

OBJECTIVE 4

OBJECTIVE 6
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

STRAND IV: APPLICATION
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND VI: SETS

OBJECTIVE 1

OBJECTIVE 4

OBJECTIVE 4 (optional)
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND VII: FUNCTIONS AND GRAPHS

OBJECTIVE 1

A Birthday Party

OBJECTIVE 2
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND IX: PROBLEM SOLVING

OBJECTIVE 1

OBJECTIVE 1

OBJECTIVE 1
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY
STRAND IX: PROBLEM SOLVING

OBJECTIVE 1

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OBJECTIVE 1

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OBJECTIVE 1
KINDERGARTEN MANIPULATIVE MATHEMATICS INVENTORY

List of Manipulative (3D) Materials

Strand I: Number and Operation

Objective 5: Teddy bear counters (10)
Objective 9: Teddy bear counters (7)
Objective 10: Teddy bear counters (5)
Objective 11: 8 plastic counters or chips

Strand II: Geometry

Objective 1: Flannel board, flannel pieces
Objective 2: Flannel board, flannel pieces
Objective 6: Teddy bear counters (1 red, 1 yellow, 1 blue)

Strand IV: Measurement

Objective 1: Large glass, small glass
Objective 4: 2 sticks – 1 long, 1 short
Objective 5: 2 sticks – same length
Objective 6: 2 rocks – same size, different weight

Strand V: Statistics and Probability

Objective 1: Opaque sack
  3 red blocks, 2 blue blocks

Strand VI: Sets

Objective 2: Flannel board, flannel pieces (4 birds, 6 trees)
Objective 3: Flannel board, flannel pieces (6 hearts, 5 bunnies)

Strand VIII: Logical Thinking

Objective 1: 3 red blocks, 2 blue blocks

Strand IX: Problem Solving

Objective 2: Flannel board, flannel pieces
Objective 3: 5 pennies
Objective 4: Plasticine