CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

DIGITAL ENVIRONMENT

COMPUTER ANALYSIS

(DECA)

A graduate project submitted in partial satisfaction of the requirements for the degree of Master of Science in Engineering

by

Steven Ludwig Barrows

August, 1975
The graduate project of Steven Ludwig Barrows is approved:

______________________________

Committee Chairman

California State University, Northridge
August, 1975
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ABSTRACT

Digital Environment
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This report contains descriptions of two projects completed at the Pacific Missile Test Center, Point Mugu, California. The first deals with factors which affect the probability of identification of objects on Cathode Ray Tube (CRT) displays. The second was the establishment of the amplitude phase relationship for gyro head to body coupling in the SIDEWINDER missile.
Cathode ray tube (CRT) displays have become an integral part of cockpit hardware. They are used heavily in reconnaissance operations. A camera mounted on the underbody of the aircraft picks up visual images which are displayed on a CRT monitor. This allows aircraft personnel to view enemy weapons from the air. This system has great capabilities over simple viewing from a window since large magnifications are possible. The greatest advantage, though, is that the system enables the viewer to perceive "invisible" light since cameras are not as limited as the human eye and electronic processing produces a visible image on a CRT display. The range of the spectrum of major interest now is the infrared.

The process of viewing an image on a CRT display introduces new fields of investigation. The size of the display image, the resolution of the display image and the contrast of the image with its background are factors which influence the accuracy with which objects can be recognized. This project intends to make usable quantitative information available. The ability to identify an object on a screen from a range of objects was of special interest. That is, not merely the location of an object but also its proper identification was required.
Forty reports on the capabilities of CRT displays were investigated. These reports were the results of experiments done with untrained personnel. In these experiments the subjects were instructed to view a screen and identify objects. For example, one group of experiments involved the recognition of military targets. The subject was given a pictorial list of targets such as tanks, guns and supply trucks. He then viewed a CRT screen which had a terrain displayed on it. Within this terrain was one or more of the given figures. The subject was then scored on his ability to correctly identify the object.

Table one contains a listing of the independent variables used in the tests. Each of these variables was a factor which was believed to affect the probability that a subject would correctly identify an object displayed on a CRT screen. Display lines may be a term unfamiliar to the reader. A CRT display does not show a continuous picture. Actually the image is made up of a series of lines. A beam of electrons is swept across the face of the display (See Diagram one). In general the more lines involved the better is the quality of the picture.

Each report selected was searched for each of the variables listed in Table one. In every report some of these variables were constant. Often a variable was not even listed in a report. In that case the author of the
### Table One

**List of Independent Variables**

1. The number of different types of symbols displayed.

2. The type of symbol displayed (numeric, military, etc.).

3. The size of the symbol on the display screen.

4. The size of the display screen.

5. The number of lines on the display screen.

6. The number of lines in the symbol.
LINE PATTERN ON THE CATHODE RAY TUBE

Diagram One
report was contacted or an intelligent "guess" was made as
to the value of that variable in the report. Once the data were collected they were placed on a TYMSHARE data file.

The TYMSHARE data file allows extensive lists to be made up and stored on a main library disk. Once a data file is complete the TYMSHARE system has a large number of analysis routines which can be easily run to investigate relations between data vectors. Since a reduction of the data collected was the intent of this project, collection of the data on computer files was thought advantageous. Once this data file was complete a print out of it was produced (See Appendix A). The independent variables are listed as vectors. The last column contains the accuracy vector which was the one known dependent vector. Table two is included in Appendix A to explain the abbreviations of vector names used within the program.

Analysis

The analysis was done to establish a rating of the relative effectiveness of each independent variable in the prediction of the accuracy of recognition. In order to proceed with this it was necessary to remove a bias from the accuracy vector. Clearly, as the number of symbols decreases there exists a good chance of merely guessing the outcome. The probability of guessing an outcome is the inverse of the number of symbols. This
bias was removed by the use of a d' statistic. The value of the accuracy was taken as the mean of a biased population while the inverse of the number of symbols was taken as the mean of the bias. The means of the two distributions in terms of a standard deviation were subtracted. This yielded a d' statistic which was a good unbiased estimator of the accuracy. From these calculations a new vector was formed entitled ZSCORE and the analysis was continued.

The method used to evaluate the data was a simple linear correlation. Correlation methods test an assumption that a dependent variable is a function of a number of independent variables. If the assumption is correct a correlation analysis will produce a number (correlation coefficient) which has a magnitude close to one. If the assumption was false the correlation coefficient will have a magnitude close to zero. Only simple correlation was used in this report. Simple correlation is performed on the assumption that the dependent variable is directly related to only one independent variable. This is scored the same way except that when the coefficient is positive, the dependent variable tends to increase as the independent variable increases, whereas, it will decrease if the coefficient is negative. Of course there is an infinite number of ways in which a dependent variable can be related to an independent variable.
Appendix B contains the results of a simple correlation. This is an output from a program available on the TYMSHARE system. This program was run using the data collected. The line of interest is the bottom line which lists the correlation coefficients associated with each independent variable and the dependent variable ZSCOR. These data are interpreted as explained; that is, CONT has a higher correlation than WLIN and, therefore, it is viewed as a greater determiner of the outcome. Thus, the variables have been quantitatively ranked as to significance. The data were very extensive so there is strong reason to assume that these results are reliable. Confirmation of this prediction would be a good basis for further experimentation.
The purpose of this project is twofold. The first area of concern is to perform laboratory measurements of seeker gyro transient precession rate in relation to gyro head angle. The second is to establish the amplitude phase relationship from the data derived for gyro head to body coupling in the SIDEWINDER missile.

The SIDEWINDER missile tracks a target by following the infrared light given out by its jet exhaust. The guidance section consists of a photocell that is sensitive to infrared light. A servo system keeps the missile body aligned with the gyro seeker by activating small jets which reorient the missile body (See Diagram two). The missile ignites when it is within a specified distance from the target.

**Laboratory**

The missile guidance unit was strapped to a rotating platter. The platter was placed on a bench along with a signal source. This approach allows tracking of a signal source at different angles (See Diagram three). The platter was placed in its "zero" position until the seeker was tracking the target. Then the platter was rotated. With the seeker still tracking, the source was shut off. By rotating both the platter and the guidance system body, all points of the target plane were reached.
SIDEWINDER TARGET TRACKING DESCRIPTION

FIRST THE SEEKER "SEES" THE TARGET

THEN THE BODY REALIGNS

Diagram Two
SIDEWINDER MISSILE TEST BENCH

Diagram Three
The missile guidance unit has two voltage outputs which when electronically processed produce orthogonal voltage references for the seeker position relative to the guidance unit body. In this way the seeker position can be easily monitored. Since the linearity of these voltages was suspect, calibration data were taken at intervals throughout the target plane. Reduction of these data showed the two signals to be approximately linear. A total of nine runs were made. Included is a graph of these data. The arrows indicate progression with time.

The next step was to attempt a curve fit on the data (See Appendix C). There was reason to believe from primary data analysis that the data approximately follow a spiral. A general form is given in Diagram four. Therefore, the spiral is a function of the four variables which are constraints of velocity, angle, and final position. Generated values for \( x(t), y(t) \) were calculated by the use of \( z \) transforms. Since there are four variables there will be sixteen partial differential equations (second order). Therefore matrix solution was used in evaluation of incremental adjustments to the variables.

The equations involved are described in Diagram five ("S" is explained in Appendix C). These equations have the form:

\[ AX = Y \]

Inversion creates the solution:

\[ X = A^{-1}Y \]
\[ x = \text{Azimuth Component of Head Angle (degrees)} \]
\[ y = \text{Elevation Component of Head Angle (degrees)} \]

Progression of data with time
Spiral Form

\[ \dot{\Delta A} = \lambda (\Delta A e^{j\phi}) \]

\[ \Delta A = \hat{A}(t) - \hat{A}_F \]

\[ \hat{A}_F = f(x_f, y_f) \]

Constraints:

\( \lambda \) velocity

\( \phi \) angle

\( x_f, y_f \) final position

Diagram Four
MATRIX FORM OF THE INITIAL EQUATIONS

\[
\begin{bmatrix}
\frac{\partial s^2}{\partial x^2} & \frac{\partial s^2}{\partial x \partial y} & \frac{\partial s^2}{\partial y^2} \\
\frac{\partial s^2}{\partial x \partial f} & \frac{\partial s^2}{\partial x^2} & \frac{\partial s^2}{\partial x \partial \phi} \\
\frac{\partial s^2}{\partial y \partial f} & \frac{\partial s^2}{\partial y \partial \phi} & \frac{\partial s^2}{\partial y^2}
\end{bmatrix}
\begin{bmatrix}
\Delta x_f \\
\Delta y_f \\
\Delta \lambda \\
\Delta \phi
\end{bmatrix}
= 
\begin{bmatrix}
\frac{2s}{2x_f} \\
\frac{2s}{2y_f} \\
\frac{2s}{2\lambda} \\
\frac{2s}{2\phi}
\end{bmatrix}
\]

Diagram Five
Diagram Six
which is the method employed by the computer program.

**Program**

Diagram six is a block diagram of the program. The function of each block is described as follows:

- **Subroutine**
  - **Track T**
  - Given a value of \( t \), searches a \( t(j) \) file for \( t(j) < t < t(j+1) \) then returns \( j \).

- **Value Generation**
  - Generates values of the function through interpolation for values of \( t \) lying between experimental data points.

- **Assignment of Constraints**
  - Sets up a file for the derivation of the partial derivatives.

- **Index Monitor**
  - Deletes computation of duplicate derivatives.

- **Compute**
  - Controls the calculation from mathematical models of \( S, x(t), y(t) \).

- **Subroutine**
  - **GYSSQD**
  - Controls the \( z \) transforms and calculates \( S \).

- **Form Matrix and Solve**
  - Performs the inversion of the partial derivative matrix.

- **Return**
  - Decision process for "Is \( S \) small enough"
Increment  Adds calculated increment to
Constraints  previous guess for a.

Output t, x(t),  Forms output file.
y(t), x_t, y_t

Given the experimental values and some guesses for the constraints this program calculates the "best fit" parameters for a spiral. The program and a sample output are included in Appendix D.

Output

The first line of output contains the values of the constraints which produced the lowest mean squared deviation along with a print out of that deviation. The first column of data is the reference time. The second and fourth columns are interpolated values of the normalized input values. The third and fifth columns are the computed values from the z transform using the constraints given above. Table three contains a listing of results.
### Table Three

**Tabulated Results**

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**Average:**

| 0.100  | -3.08| -3.50    | -2.28    |
Bibliography


Super FORTRAN, a Superset of I Level FORTRAN Four, (1970), TYPSHARE, Inc., California.

Statpac, (1972), TYPSHARE, Inc., California.
Appendix A

CRT Data Files
This is a complete listing of the data collected on CRT displays. Included is a table of the abbreviations used.
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<td>ACCU</td>
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Appendix B

Correlation

This is the result of a correlation performed on the collected data. This program was run on the TYMSHARE system.
PLEASE LOG IN: CAP25342:11;

PROJECT CODE:

TYMSHARE C11 1/17/75 13:20
-STATPAK

1>LOAD REDUC

2>REPLACE NSYM=LOG(NSYM)

3>RENAME NSYM AS LSYM

4>CORRELATION

CORRELATION MATRIX
LSYM ALPHA GEOM WLIN HMIN WMIN HLIN CONT ACCU ZSCDF

LSYM
1.0000

ALPHA
.9109 1.0000

GEOM
-.0933 -.4286 1.0000

WLIN
-.5326 -.6480 .4020 1.0000

HMIN
-.3212 -.1416 -.2450 .3634 1.0000

WMIN
-.2909 -.1592 .0169 .5119 .8258 1.0000

HLIN
.2896 .3231 -.1686 .0651 .2547 .1350 1.0000

CONT
.9197 .9832 -.3643 -.6494 -.1987 -.1832 .3324 1.0000

ACCU
.0979 .0597 .0611 .3616 .4485 .2632 .5482 .0491 1.0000

ZSCDF
.4733 .4007 .0299 .1069 .2855 .1437 .5778 .3966 .8944 1.0000
Appendix C

Theory of Curve Fitting

This is a complete discussion of the second derivative method of curve fitting.
Theory of Curve Fitting

A curve of one constant has the form:

\[ x = y = a(t) \]

The intent is to find a value of "a" such that the computed values of \( x, y \) agree as closely as possible to the experimental values. To accomplish this a sum is formed (see diagram seven). Clearly smaller values of "S" represent closer fits and \( S/(\text{number of values minus one}) \) equals the experimental variance "s^2". The objective is to minimize "S". In this example the constraint "a" must be chosen. The technique is to alter the value of "a" until a "best fit" is found. A simple computational model for deriving "a" is shown in diagram eight.

Consider "S" as a function of "a", \( S(a) \) then \( S(a) \) vs. "a" is some graph whose absolute minimum of \( S(a) \) occurs at the desired value of "a" (see diagram seven, bottom). To arrive at good guesses at this constraint consider the graph of the partial derivative of \( S(a) \) with respect to "a". Evaluate this partial at some initial guess for "a". A line with slope equal to the second partial derivative of \( S(a) \) can be evaluated at this guess (see diagram eight, bottom). This line can be used as an approximation to the curve and the value of the constraint "a" when this line intersects the horizontal axis is the new guess for "a".
CURVE FITTING

\[ S = \sum (x_t - x(t))^2 + (y_t - y(t))^2 \]

WHERE \( x_t, y_t \) ARE EXPERIMENTAL VALUES

\( x(t), y(t) \) ARE COMPUTED VALUES

Diagram Seven
COMPUTATIONAL METHOD

\[ a \in \{-\infty, \infty\} \]

- \[ x(t) = a(t) \]
- \[ y(t) = a(t) \]
- **COMPUTE**
  - \[ S \]
- **S = 0?**
  - **NO**
  - **YES**
- **DONE**

**Diagram Eight**
The reasoning behind this technique is that "S" is at its minimum value when its partial derivative with respect to "a" equals zero. The following is an explanation as to how the previously discussed method arrives at this value of "a". Consider the general equation for a line:

\[ y = mx + b \]

A transformation of axis yields the equation:

\[ y = m(x - a_0) \]

Assuming that the line drawn in diagram eight, bottom is a close approximation to the actual curve, it must intersect the horizontal axis at the desired value of "a". Thus a new, better guess has been arrived at (see diagram nine).
**COMPUTATIONAL RESULTS**

\[ y = m(x - a_0) \Rightarrow \frac{\partial s}{\partial a_g} = \frac{2s}{(\partial a_g)^2} (a_g - a_0) \]

**LET** \(-\Delta a = a_g - a_0\)

**THEN** \(a_g + \Delta a = a_0\)

**Thus, the second derivative indicates that** \(a_0 = a_g + \Delta a\) **is a new best guess.**

Diagram Nine
Appendix D

SIDEWINDER Program

This program performs a spiral curve fit using a set of input data.
$\text{IF TC.GYROXY}$

- \text{DIMENSION} M(5)
- \text{DIMENSION} N(10)
- \text{DIMENSION} TINT(50), XIN(50), YIN(50), TINT(501), XINT(501), YINT(501),
- \text{XC}(501), YC(501), F(3, 3, 3, 3), ALAMDA(3), APHI(3), AXF(3), AYF(3), P1F(4),
- 2P2F(4, 4), A(4, 4)

10 \text{READ}((5, 1), IT, NTIN, NMAT, TINMAX, XLAMO, PHI0, XFO, YFO, OLAG, DPHI, DXF, DYO)

1 \text{FORMAT}(315, 9FS.2)

2 \text{FORMAT}(1SF5.2)

70 \text{FORMAT}(1H0,1SF7.3)

88 \text{FORMAT}(1H11)

\text{C. \ : \ \ ; \ ; \INTERPOLATE \ AND \ INDEX \ ENTRY \ DATA}

H(1)=1.0E+17

S=1.0E+15

TINT(1)=TINT(1)

XINT(1)=XINT(1)

YINT(1)=YINT(1)

J=1

DO 3 I=1, NMAX

T1=TINT(I)*FLOAT(I)*DT

TINT(I+1)=T1

CALL TRACK(I, T1, NTIN, J)

DTDEL=(T1-TINT(J))/(TINT(J+1)-TINT(J))

XINT(I+1)=XINT(J)+((XINT(J+1)-XINT(J))*DTDEL

YINT(I+1)=YINT(J)+((YINT(J+1)-YINT(J))*DTDEL

3 \text{CONTINUE}

DO 6 I=1, IT

IF(S.GT.H(1)) GO TO 228

H(1)=S

H(2)=XLAMO

H(3)=PHI0

H(4)=XFO

H(5)=YFO

228 \text{CONTINUE}

AYF(1)=YFO-DYF

AYF(2)=YFO

AYF(3)=YFO+DYF

AXF(1)=XFO-DXF

AXF(2)=XFO

AXF(3)=XFO+DXF

ALAMDA(1)=XLAMO-OI AM

ALAMDA(2)=XLAMO

ALAMDA(3)=XLAMO+OIA M

APHI(1)=PHI0-OPHI

APHI(1)=PHI0

APHI(3)=PHI0+OPHI

\text{C. \ \ THIS \ IS \ PROGRAM \ SHUTTLE}

DO 7 L=1,3

YF=AYF(L)

DO 4 K=1,3

YF=AYF(K)

7 \text{CONTINUE}

DO 4 J=1,3

4 \text{CONTINUE}

29
C INDEX CHECK TO AVOID UNNECESSARY COMPUTATIONS

DO 4, 1 = 1, 3

XLA0 = XLA0A \[J\]

6 IF((IJKL.EQ.4).OR.(IJKL.EQ.8)) GO TO 5

IF((IJKL.EQ.12).OR.(IJKL.EQ.16)) GO TO 107

GO TO 4

5 IF (J.EQ.K) GO TO 107

GO TO 4

108 CONTINUE

CALL GYSSQE (LDT, XLA0, PHI, XF, YF, XINT, YINT, XC, YC, S, NMAX)

F(I; J, K, L) = S

71 FORMAT (E12, 3)

4 CONTINUE

NIMAX = NMAX + 1

DLH2 = 2 * CLAM

DPH2 = 2 * DPHI

DXYF2 = 2 * DXYF

PIF(1) = (F3, 2, 2) - F1, 2, 2) / DLH2

PIF(2) = 0

PIF(4) = (F2, 2, 3) - (F2, 2, 1) / DXYF2

PIF(3) = 0

P2F(1, 4) = (F3, 2, 3) + (F1, 2, 1) - (F3, 2, 1) + (F1, 2, 3) / (DLH2 * DXYF2)

P2F(4, 1) = P2F(1, 4)

P2F(1, 1) = (F3, 2, 2) + (F1, 2, 2) - 2 * (F2, 2, 2) / (DLH2 * DXYF2)

P2F(4, 1) = (F2, 2, 3) + (F2, 2, 1) - 2 * (F2, 2, 2) / (DXYF2)

DEL0 = P2F(1, 1) * P2F(4, 1) * P2F(4, 1) * DEL0

212 FORMAT (1H, E15.3)

A(1, 1) = P2F(1, 4) / DEL0

A(1, 4) = P2F(1, 4) / DEL0

A(4, 1) = A(1, 4)

A(4, 4) = P2F(1, 1) / DEL0

A(4, 2) = 0

A(2, 4) = 0

A(1, 3) = 0

A(4, 3) = 0

DEL0Y = (A(4, 1) * P1F(1) + A(4, 2) * P1F(2) + A(4, 3) * P1F(3) + A(4, 4) * P1F(4))

DEL0X = (A(1, 1) * P1F(1) + A(1, 2) * P1F(2) + A(1, 3) * P1F(3) + A(1, 4) * P1F(4))

YFO = YF + DEL0Y

XLA10 = XLA0 + DEL0X

IF (S, CT, H(1)) GO TO 229

H(1) = S

H(2) = XLA10

H(5) = YFO

229 CONTINUE

AYF(2) = YFO

ALAMDA(2) = XLA10

DO 204 L = 1, 3

YF = AYF(1)

204 L = 1, 3
None
EVALUATION

BARROWS

GARY Y.

SOURCE STATEMENT

IF N(S) =

17. 3.91, MSEDEV=1P714.7.

WRITE(6,1)

8 FORMAT(100,75H TIME(SECONDS) ANGLE(INPUT) ANGLE(LSTSQ) ANGLE

LY(INPUT) ANGLE(LSTSQ)/)

WRITE(6,2) (TINT(I),XINT(I),XC(I),YINT(I),YC(I),I=1,NMAX,10)

9 FORMAT(1H,1P5E15.7)

11 CONTINUE

GO TO 10

RETURN

ERROR MESSAGE NUMBER 1

END
SUBFTC_TRACKX
SUBROUTINE TRACKT(X,Y,I,IMAX,II)
DIMENSION X(IMAX)
IF(X(I-1))1,2,3
1=I-1
IF(I.LT.1) GO TO 4
IF(X(I-1))1,2,3
3=I+1
IF(I.LT.IMAX) GO TO 2
IF(X(I-1))5,2,3
5=I-1
RETURN
4=I+1
RETURN
2=MIN(D,I,IMAX-1)
6 RETURN
END
SIBFTC SIGMOY
SUBROUTINE GYSQ32(DT, X, Y, XF, YF, XINT, YINT, NMAX)
DIMENSION XINT(NMAX), YINT(NMAX)
DIMENSION X(NMAX), Y(NMAX)

SINPHI = SIN(PHI)
COSPHI = COS(PHI)
IF(ABS(SINPHI).LE.1.E-04) SINPHI = SIGN(1.E-04, SINPHI)

OMEGA = XAMCA * SINPHI
ZETA = XAMCA * COSPHI / OMEGA

X1 = XINT(1)
Y1 = YINT(1)

XFXOMG = ZETOMG * X1 + OMEGA * Y1
YFXOMG = ZETOMG * Y1 - OMEGA * X1

CALL GYR2T(DT, ZETA, OMEGA, A0, A1, B01, B02, B11, B12)

B1 = B01 + B11
B2 = B02 + B12

XG(1) = XINT(1)
YG(1) = YINT(1)

S = 0.0
DO 1 N = 1, NMAX
   X = A1 * X1 + B1 * XFXOMG - OMEXF
   Y = A1 * Y1 + B1 * YFXOMG + OMEGYF
   X0 = X1
   X1 = X
   Y0 = Y1
   YY(N+1) = Y
   YX(N+1) = X
   S = S + (X - XINT(N+1))**2 + (Y - YINT(N+1))**2
1 CONTINUE
RETURN
END
$10FCC ZIGYRC...
SUBROUTINE GYRTZ(T, ZETANU, OMEGXT, AO, A1, A01, B02, B12)
COMPLEX Z1, Z1STAR, C1, C1STAR, C2, C2STAR
CEGXT = OMEGXT
Z1 = CEKX(-ZETANU*OMEGXT, -OMEGXT)
Z1STAR = CONJG(Z1)
B2 = 1./ (1. + ZETANU*ZETANU)
B1 = ZETANU + B2
B2VR2 = 0.5*B2
510VR2 = 0.5*91
C1 = CMPLX(-910VR2, 320VR2)
C1STAR = CONJG(C1)
C2 = CMPLX(-B2VR2, B10VR2)
C2STAR = CONJG(C2)
AO = REAL(Z1*Z1STAR)
A1 = REAL(Z1*Z1STAR)
C1ZC1Z = REAL(C1*Z1STAR+C1STAR*Z1)
C2ZC2Z = REAL(C2*Z1STAR+C2STAR*Z1)
B01 = 31*AO+C1ZC1Z
B02 = 32*AO+C2ZC2Z
P11 = 31*AL-C1ZC1Z
B12 = 92*B2*AL-C2ZC2Z
RETURN
END
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