CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

STATISTICAL VALIDATION OF A HYBRID COMPUTER SIMULATION

A graduate project submitted in partial satisfaction of the requirements for the degree of Master of Science in Engineering

by

Henry Morgan Cook, III

May, 1975
The graduate project of Henry Morgan Cook III is approved.

California State University, Northridge

May, 1975
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>vi</td>
</tr>
<tr>
<td>Chapter 1 - Description of SLICE</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 2 - Simulation Validation</td>
<td>4</td>
</tr>
<tr>
<td>Chapter 3 - Statistical Analysis</td>
<td>10</td>
</tr>
<tr>
<td>Chapter 4 - Conclusion</td>
<td>19</td>
</tr>
<tr>
<td>Bibliography</td>
<td>21</td>
</tr>
<tr>
<td>Appendix A - Computer Program Listing and Output, Statistical Tests</td>
<td>22</td>
</tr>
<tr>
<td>Appendix B - Computer Listing, Paper Tape Reader</td>
<td>35</td>
</tr>
<tr>
<td>List of Tables</td>
<td>iv</td>
</tr>
<tr>
<td>List of Figures</td>
<td>v</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1. Missile Velocity Component u For Two Simulations

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Missile Velocity Component u For Two Simulations</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Block Diagram of Missile Simulation</td>
<td>3</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Kolmogorov-Smirnov Test Using Data</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>From Table 1</td>
<td></td>
</tr>
<tr>
<td>Figure 3</td>
<td>Median and Quantile of <em>X</em></td>
<td>16</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Statistical Illustration Test Using Data</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>From Table 1</td>
<td></td>
</tr>
</tbody>
</table>
ABSTRACT

STATISTICAL VALIDATION OF A HYBRID COMPUTER SIMULATION

by

Henry Morgan Cook III

Master of Science in Engineering

May, 1975

In this report a hybrid computer missile simulation, developed at the Naval Missile Center, Point Mugu, California, is presented and described. Validation of computer simulations is then discussed in general. Two nonparametric statistics are presented as a means of quantitatively comparing two sets of data to determine if they are from the same probability distribution. By using these statistics one may determine the validity of a model. Computer listings are given for calculating the statistical tests.
CHAPTER 1

DESCRIPTION OF SLICE

The Simulation Laboratory for Infrared Countermeasures Evaluations (SLICE) is a recently developed laboratory tool for the investigation of jamming techniques on infrared-guided air-to-air missiles. This tool consists of two analog computers, a PACE and a BECKMAN, which are programmed to simulate the dynamic responses of a family of similar infrared (IR) homing missiles: SIDEWINDER, ATOLL, etc. To ensure greater validity of the model, and for greater ease in programming, some portions of the missile hardware, such as the signal processor, are included in the simulation. (See Figure 1 for a block diagram of the simulation.)

The target generator is modeled by electronic equipment consisting mainly of pulse generators. The target generator feeds the signal processor (missile hardware) which computes the direction to the target. The signal processor is followed by the fin servo model (simulated on the BECKMAN computer) which computes the amount of canard deflection to steer the missile toward the target. The airframe model (programmed on the BECKMAN with a table lookup program in the SIGMA 2) calculates the aerodynamic loads on the missile as a function of missile velocity, altitude, rates and canard deflection. The kinematics and transformations section (PACE computer) then computes the position of the missile and target with respect to a fixed reference system. This section is
driven by the target position module (PACE) and by the head gyro section (PACE). To close the loop, the target generator section is driven by the kinematics and transformations section to provide the proper signal to the signal processor.

In using this simulation, one may easily "dial in" a set of initial launch conditions for the missile under consideration, and with some precision, model the flight path the missile will follow on its course toward the target. The initial conditions that may be specified prior to missile launch include missile and target position, altitude, rates, and attitude, as well as the target's maneuvers, if any.
CHAPTER 2

SIMULATION VALIDATION

The main criticism of most simulations is that they do not accurately portray the system they try to model. The solution to this problem lies in performing a validation study of the simulation. This is what was attempted.

All simulations of physical systems are representations of real life and usually contain simplifications and are thus not exact replicas of what they intend to model. It is the aim of the simulator to model the system of interest in as realistic a manner as possible. A model is said to be valid if it accurately simulates the real life phenomena in question. The degree of validity that a model possesses is often a subjective property that is not always easy to determine.

Another important property of a model is its worth, or value. The value of one model is enhanced if it can explain or predict the actions of reality better than another. A model can have a high value in its ability to explain real life, but have a low validity if the model is not founded on scientific fact. An example might be a model that predicts the incidence of automobile accidents by the phase of the moon. This model might be quite accurate in its predictions and thus have high value, but in the absence of a clear relationship between the two phenomena, the model must have low validity. A model can also have low value and high validity. An example of such a case might be a set of
insolvable differential equations which exactly represent the real life situation. It should be understood that such cases as these are extreme, and that a good model should be high in both value and validity.

As was mentioned before, it is often difficult to measure the validity and value of a model without relying on subjective judgement. The problem confronting the researcher then, is to devise a method by which tests may be made on the simulation in order to reduce to a minimum any reliance on his subjective judgement. This is where a good validation study comes into play.

A validation study of an analog or hybrid computer simulation can be broken down into three phases: (1) the static test phase, (2) the open loop phase, and (3) the closed loop phase.

The static test phase is the first step in a validation study. Often, a simulation that can be characterised as having a high validity, will be based on a system of differential equations that describe the real problem. On an analog computer, these equations can be solved by connecting together such basic components as integrators, summers, and potentiometers. A static check consists of solving the differential equations for a given set of initial conditions and seeing if the results match the output of the analog computer for the same set of conditions.

The second phase of a validation study can be termed the open loop phase. During this portion of the study, various subsystems of the simulation are tested in the open loop mode. In other words, each subsystem is tested as a separate system with no interaction from other parts of the simulation. For instance, for a missile simulation, one
might look at the frequency response of such assemblies as the head gyro, airframe, and fin servo model. Magnitude versus frequency (Bode) plots may then be made of these subsystems and compared with similar data obtained from real missile hardware. If all the subsystems are tested and show themselves to be accurate models of the real hardware, then one can be more confident of the success of his entire simulation.

The third and final phase of validation is the closed loop testing. During this phase, the simulation is allowed to operate as it would under normal conditions with interactions between all the subsystems being allowed. In the case of a missile simulation, after the missile model is launched, it locks onto a target, and proceeds to guide toward the target. The simulation is judged to be valid if it can accurately reproduce the actions of the system it is attempting to model. Again, in the case of a missile simulation, there is generally data available from the firing range on the missile in question. The missile simulation can then be "flown" using the same initial conditions as were employed on the flight range. If the simulation closely models the actions of the real missile, then the researcher can have some degree of confidence in his simulation.

VALIDATION OF SLICE

The validation of SLICE is currently in progress. This air-to-air missile simulation has already passed the static test phase and is now ready to finish the open loop phase.

The static test phase was performed by Mr. Royce Townsend of the Pacific Missile Test Center, who is currently in charge of the SLICE
program. This phase of the testing was accomplished by writing a digital (FORTRAN) program that computed the voltages at the outputs of all amplifiers, resolvers, and multipliers. By comparing these values with those actually present on the analog machine it was possible to detect mistakes in programming of the simulator. By eliminating these wiring and component problems at the onset, it was possible to proceed with the validation study with a minimum of lost time.

It was the responsibility of my colleague, Rodolfo Ayala, and me to perform the open loop testing portion of the validation study. The open loop testing was divided into two phases. During the first phase Bode plots of the head gyro subassembly were made. This later proved to be of little use since we were not able to obtain any data on the real missile head gyro. Since no data could be found on any other subassemblies (with the exception of the airframe) we decided to abandon this phase. The second phase of the open loop testing was the investigation of the airframe. To analyze the characteristics of the airframe, the decision was made to perform a series of "drop tests". To perform a drop test, one gives the missile simulation a set of initial conditions, launches the missile and allows the model to "fly" without tracking on a target. This is a generally accepted method of testing the airframe and since two other simulations of a type similar to ours were in existence, ample data was available.

These other simulations, one from the Naval Missile Center\(^1\) (NMC) and the other from the Naval Research Laboratory (NRL), Wash-

\(^1\)The Naval Missile Center has since been reorganized and is now the Pacific Missile Test Center (PAMTC).
ington, D.C., were used in validating our missile simulation. These two simulations are models of the SIDEWINDER Missile and are completely digital in nature, in contrast to our hybrid model.

My task was to develop a statistical procedure by which we could analytically compare the results of our simulation with these other two simulations. However, before any statistical tests could be employed, the problem of acquiring the capability of gathering data in the proper form from our simulation had to be overcome.

When we first became acquainted with SLICE, the only method of outputting data was in the form of strip recordings. Since a large amount of data needed to be gathered, strip charts would have proved to have been very cumbersome and time consuming. With this in mind, a solution was sought for our problem. Luckily, our minicomputer, the SIGMA 2, which was being used for part of our simulation, had the capability of sampling six different parameters for each run for a total of 6.3 seconds. Programs were written\(^2\) for our SIGMA 2 to accomplish this mission. The output of the minicomputer was in the form of an eight channel paper tape. Another program was written\(^3\) to have this paper tape read into a UNIVAC 1230 computer and stored in memory. A FORTRAN program was then written which would have these stored values punched out on a deck of data cards.

With this, the problem of acquiring data from our simulation was solved. The task of selecting and implementing a number of statistical

\(^2\) by Dave Whittington of PMTC.

\(^3\) by Jim Waite and Audrey Barnett of PMTC.
tests with which to compare our simulation with those digital models from NRL and NMC was then approached.
CHAPTER 3

STATISTICAL ANALYSIS

Two statistical tests were chosen to be used in validating the hybrid model. The reason for choosing two tests is that each test measures only some of the attributes of the simulation data. Thus, by using a number of tests, the researcher can be more confident of the results of his validation study.

Several criteria were used in selecting the statistical tests for use in the validation study. Tests were desired that would be nonparametric in nature; that is, they should not depend on the underlying statistical distribution of the data. Secondly, tests that could be easily programmed on the computer were preferred. And lastly, at least one test was needed that could give information to the researcher as to the nature of the differences between any two sets of simulation data.

The first statistical test that came to mind was the chi-square test. This is a statistical nonparametric test that can be used in testing to see if two sets of data are from the same general distribution. This test is based on the following heuristic considerations. If we use the SIGMA 2 minicomputer to sample a parameter during a simulated missile firing, we will have a vector of random variables, $0_1, \ldots, 0_n$, for $i = 1$ through $n$. Each $0_i$ will be a value of a missile parameter (such as velocity, altitude, etc.) for each sampling
instance. We may then obtain a similar vector of random variables, $E_i$, for $i = 1$ through $n$, from the NRL or NNC simulation using the same initial launch conditions and sampling period. If we then assume a hypothesis that the SLICE simulation is valid, it would follow that each vector $O_i$ should come from the same probability distribution as the corresponding vector $E_i$. Then $\sum_{i=1}^{n} (O_i - E_i)$ would appear to measure the acceptability of the hypothesis. Since a positive difference means no more nor less than a negative one, perhaps $\sum_{i=1}^{n} (O_i - E_i)^2$ would be a better measure. If we then weigh each difference by the factor $\frac{1}{E_i}$, it can be shown (Cramer 1946) that the statistic $\sum_{i=1}^{n} \left( \frac{O_i - E_i}{E_i} \right)^2$ is approximately chi-square with $(n-1)$ degrees of freedom.

However, one basic difficulty with this statistic is that no more than 20% of the $E_i$ should be less than 5.0 and none should be less than 1.0 (Conover 1971). To get around this difficulty, one can combine adjacent values for $E_i$ so that these conditions are met. However, as the sample size is decreased the statistic loses its power. In view of the fact that some of the parameters that were to be studied had quite low values, the chi-square statistic was discarded.

The Kolmogorov-Smirnov statistic was then considered. This test is of the same nature as the chi-square; that is, it is a nonparametric test for comparing the distributions of two sets of data. It is the general feeling (Conover 1971) that this test is to be preferred over the chi-square, especially for small sample sizes. This is because the Kolmogorov-Smirnov test is exact, even for small samples, while the chi-square test assumes that the number of observations is large
enough so that the chi-square distribution provides a good approximation as the distribution of the test statistic.

The second statistical test that was chosen was the statistical illustration test. This test is graphical in nature and provides the researcher with a good indication of how two sets of data might differ.

The computer programs used to implement these two tests were adapted from those written for an earlier validation study of the PHOENIX Missile at Pt. Mugu (Archer and Flueger 1970). Listings of these programs appear in Appendix A.

THE KOLMOGOROV-SMIRNOV TEST

The Kolmogorov-Smirnov statistic is a general nonparametric test employed when comparing two sets of data. This method is completely general; that is, the computed statistic is distribution free and may be used to test any two sets of data. For our case, one set of data is from the SLICE simulation, the second set being from either the NNC or NRL simulation. This test would then help determine if the two sets of data are from the same general population (Smirnov 1948, Feller 1948, Archer 1969).

For our two-sample problem there are two sets of data: one set of \( n \) points from one source and one set of \( m \) points from another source. The Kolmogorov-Smirnov statistic is defined as

\[
D_{mn} = \left( \max_{x} \right) |F_m(x) - G_n(x)|
\]

where

\[
F_m(x) = \frac{1}{m} \left\{ \text{number of points from one sample which are less than or equal to } x \right\}
\]
and \( G_n = \frac{1}{n} \left\{ \text{number of points from the other sample which are less than or equal to } x \right\} \)

and the \( \left( \max_x \right) \) is taken over all points \( x \) which are greater than or equal to the minimum of both sets of data and which are less than or equal to the maximum of both sets of data. So, if \( F_m(x) \) and \( G_n(x) \) were to be plotted, then \( D_{mn} \) would be the maximum vertical distance between the two distributions.

In using this test, we first assume a null hypothesis of no difference between the two distributions. Then for a given \( n, m \) and significance level (normally 5\%), one may find the critical value for \( D_{mn} \) from a set of standard statistical tables. If the computed value of \( D_{mn} \) is less than the critical value (denoted by \( D_{mn}^* \)), then the null hypothesis is accepted and we conclude that the two sets of data come from the same general population.

If the hypothesis of no difference between the two sets of data is accepted at the 5\% level, then the strongest statement that can be made is that it cannot be determined at the 5\% significance level that the two sets of data are different. It should be noted that, if the hypothesis is assumed to be correct, by chance alone once in 20 trials the computed value of our statistic will exceed the critical value at the 5\% level. Therefore, in using the 5\% level of significance we will be in error on the average once in 20 trials. This chance for error is one of the primary reasons for using a number of different tests in assessing the validity of the simulation.

An example of the Kolmogorov-Smirnov statistic is illustrated in Figure 2 using the data from Table 1. The computed value of \( D_{mn} \) is
Figure 2. Kolmogorov-Smirnov Test Using Data From Table 1.
Altitude = 1000 feet
Velocity = 0.8 Mach
Fin Deflection = 5 degrees

Initial Conditions

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>NRL Data (ft/sec)</th>
<th>SLICE Data (ft/sec)</th>
<th>Time (sec)</th>
<th>NRL Data (ft/sec)</th>
<th>SLICE Data (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>890</td>
<td>890</td>
<td>2.5</td>
<td>2205</td>
<td>2270</td>
</tr>
<tr>
<td>0.1</td>
<td>959</td>
<td>960</td>
<td>2.6</td>
<td>2158</td>
<td>2220</td>
</tr>
<tr>
<td>0.2</td>
<td>1029</td>
<td>1030</td>
<td>2.7</td>
<td>2116</td>
<td>2180</td>
</tr>
<tr>
<td>0.3</td>
<td>1098</td>
<td>1100</td>
<td>2.8</td>
<td>2076</td>
<td>2140</td>
</tr>
<tr>
<td>0.4</td>
<td>1167</td>
<td>1170</td>
<td>2.9</td>
<td>2036</td>
<td>2100</td>
</tr>
<tr>
<td>0.5</td>
<td>1234</td>
<td>1240</td>
<td>3.0</td>
<td>1997</td>
<td>2065</td>
</tr>
<tr>
<td>0.6</td>
<td>1301</td>
<td>1310</td>
<td>3.1</td>
<td>1959</td>
<td>2030</td>
</tr>
<tr>
<td>0.7</td>
<td>1368</td>
<td>1380</td>
<td>3.2</td>
<td>1923</td>
<td>1995</td>
</tr>
<tr>
<td>0.8</td>
<td>1434</td>
<td>1440</td>
<td>3.3</td>
<td>1888</td>
<td>1955</td>
</tr>
<tr>
<td>0.9</td>
<td>1499</td>
<td>1510</td>
<td>3.4</td>
<td>1853</td>
<td>1930</td>
</tr>
<tr>
<td>1.0</td>
<td>1563</td>
<td>1580</td>
<td>3.5</td>
<td>1820</td>
<td>1890</td>
</tr>
<tr>
<td>1.1</td>
<td>1627</td>
<td>1640</td>
<td>3.6</td>
<td>1788</td>
<td>1860</td>
</tr>
<tr>
<td>1.2</td>
<td>1690</td>
<td>1705</td>
<td>3.7</td>
<td>1756</td>
<td>1830</td>
</tr>
<tr>
<td>1.3</td>
<td>1752</td>
<td>1770</td>
<td>3.8</td>
<td>1726</td>
<td>1800</td>
</tr>
<tr>
<td>1.4</td>
<td>1814</td>
<td>1835</td>
<td>3.9</td>
<td>1696</td>
<td>1770</td>
</tr>
<tr>
<td>1.5</td>
<td>1874</td>
<td>1900</td>
<td>4.0</td>
<td>1668</td>
<td>1745</td>
</tr>
<tr>
<td>1.6</td>
<td>1935</td>
<td>1965</td>
<td>4.1</td>
<td>1640</td>
<td>1720</td>
</tr>
<tr>
<td>1.7</td>
<td>1994</td>
<td>2025</td>
<td>4.2</td>
<td>1613</td>
<td>1680</td>
</tr>
<tr>
<td>1.8</td>
<td>2053</td>
<td>2085</td>
<td>4.3</td>
<td>1586</td>
<td>1665</td>
</tr>
<tr>
<td>1.9</td>
<td>2110</td>
<td>2150</td>
<td>4.4</td>
<td>1561</td>
<td>1630</td>
</tr>
<tr>
<td>2.0</td>
<td>2167</td>
<td>2210</td>
<td>4.5</td>
<td>1538</td>
<td>1616</td>
</tr>
<tr>
<td>2.1</td>
<td>2214</td>
<td>2260</td>
<td>4.6</td>
<td>1512</td>
<td>1590</td>
</tr>
<tr>
<td>2.2</td>
<td>2241</td>
<td>2290</td>
<td>4.7</td>
<td>1488</td>
<td>1565</td>
</tr>
<tr>
<td>2.3</td>
<td>2248</td>
<td>2300</td>
<td>4.8</td>
<td>1465</td>
<td>1535</td>
</tr>
<tr>
<td>2.4</td>
<td>2236</td>
<td>2290</td>
<td>4.9</td>
<td>1443</td>
<td>1520</td>
</tr>
</tbody>
</table>

Table 1. Missile Velocity Component u for Two Simulations
shown in Appendix A, page 34. In this case, \( D_{mn} = 0.1 \) and is less than the critical value (\( r^*_{mn} = 0.272 \)) at the 5% level, and the null hypothesis of no difference between the two sets of data is therefore accepted.

**THE STATISTICAL ILLUSTRATION TEST**

The statistical illustration test (Wilk and Gnanadesikan 1968, Archer and Plueger 1970) is a graphical analysis technique used to determine if any similarity exists between the probability distribution functions of two simulation parameters. The points on the graph (see Figure 4) consist of ordered pairs of quantile values. We define, for each real number \( p \) such that \( 0 < p < 1 \), the \( p^{th} \) quantile of the random variable \( X \) (or its probability distribution, \( F(x) \)), as any real number \( \xi_p \) satisfying the inequalities

\[
P([X < \xi_p]) \leq p \quad \text{and} \quad P([X > \xi_p]) \leq 1-p.
\]

In words, \( \xi_p \) is a number on the x axis (see Figure 3) that "divides" the probability distribution into two parts with the amount \( p \) below and \( 1-p \) above. So, the median, \( m \), would be equal to \( \xi_{0.5} \) and divides the distribution in half.

![Figure 3. Median and quantile of X.](image)
The graphs for the statistical illustration test are constructed so that quantile points from identical probability distributions will appear on a straight line along the diagonal of the graph. Points appearing parallel to, but off the diagonal indicate that the probability distributions of the two simulation parameters are the same except for a location parameter. That is, if \( F(x) \) and \( G(x) \) represent the two distribution functions then

\[
F(x) = G(x-\alpha)
\]

for some real number \( \alpha \). Another possibility is that the quantile points will appear on a straight line, but not parallel to the diagonal. This would indicate that the distributions are the same except for a difference in a scale parameter, i.e.,

\[
F(x) = G(x/\rho)
\]

for some real number \( \rho \), or that the distributions are the same except for a change of scale and location, that is,

\[
F(x) = G\left(\frac{x-\alpha}{\rho}\right)
\]

for some real numbers \( \alpha \) and \( \rho \).

An example of this method is shown in Figure 4, again using the data from Table 1. In this case, the points closely follow the diagonal. Thus, we can safely conclude that the two sets of data come from the same distribution.
Figure 4: Statistical Illustration Test Using Data From Table 1.
CHAPTER 4

CONCLUSION

Simulation validation offers many challenges to the researcher. Depending on his needs and the nature of the simulation, the validation procedure may vary widely. In all too many cases, only a cursory validation is performed. For simple simulations this may be all that is necessary. However, for those simulations that are more involved, such a validation procedure is clearly inadequate.

Deciding on the type and degree of validation study is not always an easy task. Should one use his intuition and subjective judgement? When does it become worth the effort to devise a validation procedure that can fully measure the merits of a simulation? There are no absolute answers to these questions; the researcher must weigh his needs in relation to the amount of time and money he is willing to expend in a validation study. A good validation will not in itself guarantee a good simulation, but it can be of use in determining its shortcomings.

The simulation that was presented in this report was quite involved. A proper validation of such a system would have to be conducted in different phases to allow the researcher to test in turn each portion of the simulation. The validation techniques that were presented were in support of the validation of the missile airframe. The method that was chosen involved comparing the SLICE airframe with
the airframe simulations produced at NRL and NMC in connection with their proven missile simulations. The completion of this phase of the validation study would allow the simulation to be tested in a closed loop configuration with some confidence of success. If the simulation completes the closed loop validation phase, then the researcher can be confident that the missile simulation will perform adequately.

The two statistical tests that were presented as tools in validating the airframe are quite general in nature and may be used to test the equality of probability distributions for any two sets of data. As such, they lend themselves to widespread use in many fields where such information is needed. The manner in which they complement each other is particularly useful in obtaining a feel for the differences in two sets of data. The Kolmogorov-Smirnov test alone gives only an accept-reject type of answer. Combined with the statistical illustration test, the differences can be visualized and so ideas as to the method of correcting any discrepancy may become readily apparent.

Thus, it is the author's belief that these two statistical tests may be employed to good advantage in situations which require a knowledge of the differences between two sets of data. With tools such as these, the researcher can be more confident of the results of his validation study. Although the author was not able to complete the validation of the airframe simulation, he is confident the techniques presented here will ensure the successful completion of the study.
BIBLIOGRAPHY


APPENDIX A

COMPUTER PROGRAM LISTING AND OUTPUT

STATISTICAL TESTS
REAL KSX,KSX1,KSX2,KSX3,KSX4,KSX5,KSY,KSY1,KST,KST1,KST2,KST3,KST4,KST5
COMCMN/CX/CXI(9)
COMCMN/CY/CIY(9),CY2(5)
COMCMN/KSX/KSX1(9),KSX2(5),KSX3(5),KSX4(5),KSX5(5)
COMCMN/KSY/KSY1(9)
COMCMN/CT/CT1(8),CT2(8),CT3(8),CT4(8),CT5(8)
COMCMN/KST/KST1(9),KST2(8),KST3(8),KST4(8),KST5(8)
DIMENSION A(50),B(50),C1100),Z(50)
DATA h,m/50,50/
READ(5,5G10X1,GSY1,GSY2,KSX1,KSX2,KSX3,KSX4,KSX5,KSY1)
50 FORMAT(8(5A6,11,9A6)
READ(5,6C7,CT1,CT2,CT3,CT4,CT5,KST1,KST2,KST3,KST4,KST5)
60 FORMAT(9(8A6,11,9A6)
READ(5,1C0)A,B
100 FORMAT(10F5.2)
INT=100
CALL KCLSM(A,N,B,M,C,INT,0,6,X50,X55,X5S,XDM, IT,KST3,KSX3,KSY1)
CC 20 I=1,50
20 I(I)=FLCAT(I)*0.02
CALL PLCT(A,N,B,M,Z,2C,1,CT2,QX1,CY1)
WRITE(4,2CC)X50,X55,X5S,XDM
WRITE(6,2CC)X90=,F8.6/20X,5fX5S=,F8.6/20X,5fX5D
1M=,F8.6)
IF(XCM,LT,X95) GC TC 21
IF(XCM,LT,X95) GO TO 22
IF(XCM,LT,X5D) GO TO 33
31 WRITE(4,6O1)
GC TC 45
32 WRITE(4,6C2)
GC TC 45
33 WRITE(4,6C3)
GC TC 45
601 FORMAT(///,20X,60XDM IS LESS THAN X5S, THEREFORE ACCEPT H0 AT 1 PERCENT LEVEL)
602 FORMAT(///,20X,60XDM IS LESS THAN X5S, THEREFORE ACCEPT H0 AT 5 PERCENT LEVEL)
603 FORMAT(///,20X,61XDM IS LESS THAN X5C, THEREFORE ACCEPT H0 AT 10 PERCENT LEVEL)
45 STCP
END
SUBROUTINE KCMSM(A,J,B,K,C,1HT,IP1,IP2,X50,X55,X59,XCMH,IT,T,X,Y)
DATA 750,795,799 /1.2238476,1.3580987,1.6276236/
XCMH=CMH(A,J,B,K,C,J*K)
XX=(FLOAT(C+K)/FLOAT(J*K))**0.5
X5C=Z50*XX
X55=Z95*XX
X59=Z95*XX
IF(IT.EQ.-1) RETURN
CALL CRDER(A,J)
CALL CRDER(B,K)
CALL GRAPH(A,J,B,K,1HT,IP1,IP2,IT,T,X,Y)
RETURN
END
$RFTC STEST2

SUBROUTINE PLOT(R,M,S,H,Z,L,IT,T TG,XTG,YTG)

DIMENSION R(M),S(N),L(N)

IF(L(1).GT.50).GO TO 30

CALL CRDER(R,M)

CALL CRER(S,N)

XL=-5555559.

IF(L(1).LT.50).GO TO 30

CALL CRDER(R,M)

CALL CRER(S,N)

XL=-5555559.

10 CONTINUE

W=Z(I)

FX(I)=FNTNLV(M,W,R)

FY(I)=FNTNLV(N,W,S)

XL=MAX1(XL,FX(I),FY(I))

YT=MAX1(YT,FX(I),FY(I))

IF(NF.EQ.0) NF=3

CALL CRFRPLT(FX,FY,L,1,1,44,NF,50.C,50.0,2,XL,YT,2,XL,YT,TTG,

1 X,1,YTG)

RETURN

30 WRITE(L,100)

100 FORMAT(1HO,55HNO MORE THAN 50 QUANTILE POINTS ARE ALLOWED, YOU WAT

ITED,15)

RETURN

END
SUBROUTINE ORDER(A,M)
DIMENSION A(M)
K=M-1
GO TO 20 I=1,K
X=A(I)
KK=I+1
DO 10 J=KK,M
IF(X.LE.A(J)) GO TO 10
TEMP=A(J)
A(J)=X
X=TEMP
10 CONTINUE
A(I)=X
20 CONTINUE
RETURN
END
$IBFTC STEST4
FUNCTION FS(M,X,A)
DIMENSION A(M)
RKK=0.0
DO 10 I=1,M
10 IF(A(I),LE,X) RKK=RKK+1.0
FS=RKK/FLOCAT(M)
RETURN
END
FUNCTION FTNINV(W,Y,A)
DIMENSION A(M)
IF(Y.GE.FS(M,A(I),A)) GO TO 10
FTNINV=A(I)
RETURN
10 IF(Y.LE.1.0) GO TO 20
FTNINV=A(M)
RETURN
20 DC 20 I=2,M
IF(Y.LE.FS(M,A(I),A)) GO TO 40
30 CONTINUE
40 X1=A(I-1)
X2=A(I)
Y1=FS(W,X1,A)
Y2=FS(W,X2,A)
FTNINV=(Y-Y1)*(X2-X1)/(Y2-Y1)+X1
RETURN
END
$10$ FTC $S T E S T E$

FUNCTION $D M N ( A , N , B , M , C , I )$

DIMENSION $A ( N ) , B ( M ) , C ( I )$

$K I = K$

$M A = M + N$

DC 10 I = 1 , N

10 C ( I ) = A ( I )

DC 20 I = 1 , M

$K I = K I + 1$

20 C ( K I ) = B ( I )

CALL ORDER ( C , MN )

$D W N = 0$

$M N = M N - 1$

DC 30 I = 1 , M N N

$X = C ( I ) * ( C ( I ) + 1 ) - C ( I ) / 2 . 0$

$Y 1 = F S ( X , A )$

$Y 2 = F S ( X , B )$

30 IF ( ABS ( Y 1 - Y 2 ) . G T . $D W N )$ $D W N = A B S ( Y 1 - Y 2 )$

RETURN

END
SUBROUTINE GRAPH(AA,J,BB,K,INTER,IP1,IP2,IT,TTC,XTC,XTG,YTG)

DIMENSION AA(J),BB(K)

IF(INTER.EQ.0) THEN
    NF=1
    IF1=INTER
    APIN=APIN1(AA(J),BB(K))
    AMAX=AMAX1(AA(J),BB(K))
    YYYL=0.0
    YYYU=1.0
END IF

CALL GRFPLT(AA,AA,0,1,44,NF,50.0,50.0,2,APIN,AMAX,2,YYYL,
            YYYU,TTC,XTC,XTG,YTG)

AINC=APIN1(AMAX-APIN)/FLCAT(INTER)

X1=APIN

Y1=FS(J,APIN,AA)

DLN1=X1

IF(Y1.EQ.0.0) GO TO 10

IZERC=NYV(Y1)

IX1=XXV(X1)

IX2=NYV(IX1)

CALL LINEV(IX1,IZERC,IX1,IX2)

CALL LINEV(IX1,IZERC,IX1,IX2)

CALL PCINTV(IX1,Y1,IP1)

10 DC 40 I=1,INTER

DUP1=DLM1+AINC

DLP2=FSJ(J,DUM1,AA)

IF(DU12.EQ.Y1) GO TO 40

IX1=XXV(X1)

IX2=XXV(DLN1)

Y1=NYV(Y1)

Y2=NYV(IX1)

CALL PCINTV(IX1,IX1,IP)

CALL PCINTV(IX1,IX1,IP)

IF(Y1.EQ.0.0) GO TO 30

CALL LINEV(IX1,IX1,IX1,IX1)

CALL LINEV(IX1,IX1,IX1,IX1)

CALL PCINTV(X1,Y1,IP2)

30 CC 60 I=1,INTER

40 CONTINUE

X1=APIN

Y1=FS(K,APIN,BB)

DUM1=X1

IF(Y1.EQ.0.0) GC TO 50

IX1=XXV(X1)

IZERC=NYV(C.0)

IX2=NYV(IX1)

CALL LINEV(IX1,IZERO,IX1,IX2)

CALL LINEV(IX1,IZERO,IX1,IX2)

CALL PCINTV(X1,Y1,IP2)

50 CC 60 I=1,INTER
DUM1=CUM1+AINC
DUM2=FS(K,DUM1,IN)
IF(Y1.GE.DUM2) GO TO 80
IX1=NXV(X1)
IX3=NXV(CUM1)
IY1=AYV(Y1)
IY2=AYV(CUM2)
CALL PCINTV(CUM1,CUM2,IP2)
CALL PCINTV(CUM1,Y1,IP2)
IF(Y1.EQ.0.0) GC TG 70
CALL LINEV(IX1,IY1,IX2,IY1)
CALL LINEV(IX1,IY1,IX2,IY1)
70 CALL LINEV(IX3,IY1,IX3,IY2)
CALL LINEV(IX3,IY1,IX3,IY2)
Y1=DUM1
Y1=CUM2
60 CNTINUE
RETURN
END
SUBROUTINE GRPFLT(X,Y,APT,ND,NV,NC,NF,DCX,DCY,XLI,XUI,NYC, 1 YLI,YUI,T,A,G)
DIMENSION X(2),Y(2),T(8),O(5),AIS
DATA DELTA/1.0E-07/
EXTERNAL TABLV
CALL CSIZV(3,3)
CALL RITSTV(18,26,TABLV)
NP=ABS(APT)
XL=XLI
YL=YLI
YU=YUI
NC=10,130,10,NF
10 M=NC+1
NC=11,13,NXC
11 XU=X(I)
XL=X(I)
NC=12 I=M,NP,NC
XL=XMIN(XL,X(I))
12 XU=XMAX(XU,X(I))
13 NC=15,21,NYO
14 YU=Y(I)
YL=Y(I)
NC=20 I=M,NP,NC
YL=YMIN(YL,Y(I))
20 YU=YMAX(YU,Y(I))
21 XU=X*(1.+SIGN(DELTA,XL))
XL=X*(1.-SIGN(DELTA,XU))
YU=Y*(1.+SIGN(DELTA,YL))
YL=Y*(1.-SIGN(DELTA,YU))
IF = XMAX (XU,XU*(1.+SIGN (OCC1,XL)))
IF = YMIN (YL,YL*(1.+SIGN (OCC1,YL)))
21 IF = X*(1.+SIGN (OCC1,XL))
30 CALL CDXYV(XL,XU,DX,NI,IX,NX,DCX,IERRX)
CALL CDXYV(YL,YU,DY,NI,IY,NY,DCY,IERRY)
30 X=SIGN(X,1)
XL=SIGN(XL,1)
Y=SIGN(Y,1)
YL=SIGN(YL,1)
XU=X*(1.+SIGN(DELTA,XU))
XU=X*(1.-SIGN(DELTA,XL))
YU=Y*(1.+SIGN(DELTA,YU))
YU=Y*(1.-SIGN(DELTA,YL))
Y=SIGN(Y,1)
X=SIGN(X,1)
X=SIGN(X,1)
X=SIGN(X,1)
CALL CXDYV1,XL1,XUX=ICUP,IXP,NX,DCX,IERRX)
YU=Y*(1.+SIGN(DELTA,YU))
XDM IS LESS THAN X9, THEREFORE ACCEPT I/O AT 1 PERCENT LEVEL
APPENDIX B

COMPUTER PROGRAM LISTING

PAPER TAPE READER
CONTROL T
MAIN PROGRAM SLICE
DIMENSION A(64,6), K(64,6), SF(6)
READ (5,100) (SF(I), I=1,6), NUM
100 FORMAT (6F10.5,12)
N=6

C *** THIS SYCO SUBPROGRAM READS AN 8-CHANNEL BINARY-CODED PAPER TAPE
C *** IMMEDIATELY PROCEED THE 384 WORDS (768 FRAMES) OF DATA ON THE
C *** PAPER TAPE.
C *** THIS PROGRAM WAS WRITTEN BY AUDREY BARNETT AND JIM WAITE.
C *** CODE 0322.
DO 222 L=1,10
CALL TAPES(K+N)
DO 10 J=1,6
DO 10 I=1,54
10 A(I,J)=FLOAT(K(I,J))/SF(J)
WRITE(6,1)) ((A(I,J),J=1,6),I=1,64).
WRITE(7,1))((A(I,J),J=1,6),I=1,64).
1 FORMAT((F13.3)
222 CONTINUE
STOP.

ARRAY DATA
A
K
SF.
DP AND CP ARRAY DATA
SUBROUTINES
READU
TAPES
FLOAT
WRITE
CONSTANTS
DP AND CP CONSTANTS
EQUIV. ALLOC.
VARIABLES
SLICE
I
NUM
N
M
L
DP AND CP VARIABLES
ARRAY ALLOC.
A
K
SF
TEMP

IS-LAST ADDRESS RELATIVE TO GIVEN BASE. THE PROGRAM OCCUPIES 01730 LOCATIONS, COMMON AREAS NOT INCL.