DYNAMIC BEHAVIOR OF A MEMRISTIVE CIRCUIT MODEL FOR LONG-BASE P-N JUNCTION DIODES

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Engineering

by

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List of Symbols

A  junction area
C_{j}  junction capacitance
C_{d}  diffusion capacitance
D_{h}  hole diffusion constant
\varepsilon  dielectric permitivity of the semiconductor
H_{c}  the combinance parameter used in lumped models
H_{D}  the diffusance parameter used in lumped models
I_{s}  diode saturation current
J_{h}  hole current density
L_{h}  hole diffusion length
N_{A}  acceptor concentration in the p-type region
N_{D}  donor concentration in the n-type region
p^{'}  excess hole concentration
q  magnitude of electron charge
q_{h}  total excess stored hole charge
q_{m}  charge of memristance
S  the storance parameter used in the lumped models
S_{h}  surface recombination velocity in the p-type region
\psi_{0}  built-in voltage
\mu_{e}  electron mobility
\mu_{p}  hole mobility
\tau_{h}  hole recombination life time
V_{T}  thermal voltage
ABSTRACT

DYNAMIC BEHAVIOR OF A MEMRISTIVE CIRCUIT MODEL FOR LONG-BASE p-n JUNCTION DIODES

by

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A memristive diode model is a new simple lumped circuit model for junction diodes. This model which was postulated by Dr. Leon O. Chua [1] contains only four elements; namely, two controlled current sources, a nonlinear capacitor, and a memristor. The memristor is defined by $R_m(q_m) = V_m/i_m$ as a linear charge-controlled resistor which is the element of a two-terminal circuit. $R_m(q_m)$ is a function of the charge $q_m$ passing through its terminals [1]. In this paper, the model is operated in order to show it is able to simulate the diode’s transient behaviors under both reverse and forward
operating modes by using long-base diodes. From the result of Chapter 3, the model is found able to give the storage time and the fall time more accurately than other models which were used to model the junction diode under reverse transient operation. Under forward transient operation, the model also can simulate the diode's conductivity modulation characteristic.

The model is also compared with the one-lump model, two-lump model and two-capacitor model.

Some of the main ideas and derivations in this paper stay closely to the laboratory report which was presented by Dr. Leon O. Chua and Dr. Chong-Wei Tseng. (Department of Electrical Engineering and Computer Sciences and the Electronics Research Laboratory, University of California, Berkeley.)
Chapter 1

INTRODUCTION

If we want to simulate the long-base p-n junction diodes with a model, the capability of this model in simulating the dynamic behaviors of real junction diode during reverse and forward transient operations must be shown.

In this paper the model is compared with some models which were previously used to simulate the dynamic behaviors of p-n junction diodes. First, we choose two-capacitor model (2) which shows an ideal junction-law diode in parallel with a nonlinear junction capacitance $C_j(v_j)$ in Fig. 1(a). Second, we choose the one-lump model and the two-lump model (3) which consist of some unconventional elements such as combinance $H_e$, diffusance $H_d$, and storance $S$ as shown in Fig. 1(b)(c). Our objective is to show that the memristive diode model is the most accurate and simplest model to satisfy the essential features of the ideal diode with comparison to those above diodes models.
\[ i_j = I_s \left( \exp\left(\frac{v_j}{V_T}\right) - 1 \right) \]

\[ V_T = kT/q \]

\[ C_j(v_j) = \frac{K_a}{2(\phi_b - v_j)^{1/2}} \]

\[ \phi_b = \text{built-in voltage} \]

\[ C_d(v_j) = I_s \tau / V_T \exp(v_j/V_T) \]

\[ I_s = \text{saturation current} \]

\[ K_a \text{ and } \tau \text{ are diode parameters} \]

---

Fig. 1. The model of junction diode: (a) two-capacitor model.
Fig. 1. (continued) The models of junction diodes:

(b) one-lump model, (c) two-lump model.
Dynamic Behavior of Junction Diode Switching Transients

We consider the simple diode shown in Fig. 2, in which the switch is in position 1 for \( t < t_o \) and the diode current is in the steady state with \( i(t) = I_f \). At \( t = t_o \), the switch falls down to position 2, so \( i(t) = -I_f \). Therefore, when \( t < t_o \) the circuit is under forward transient operation and when \( t \geq t_o \) the circuit is under reverse transient operation.

![Fig. 2. Reverse transient behavior of junction diodes: simplified circuit for measuring diode reverse transient voltage and current.](image)
(a) Dynamic Behaviors During Reverse Transient

Before $t_0$ in Fig. 2., the voltage $V(t) = E_f$. Also, assume that $|E_2| > |E_f|$. When the switch is thrown from position 1 to 2, the reverse transient waveforms $V(t)$ and $i(t)$ are as shown in Fig. 3. At $t = t_0$, there is a small instantaneous drop in $V(t)$ from $E_f$ to $E_0$ but the instantaneous drop in $i(t)$ from $I_f$ to $-I_r = -E_2/R$ is much larger. The instantaneous change is a reversal of $\partial p'_n/\partial x$ at $x = 0$ form $-I_f/qAD_h$ to $I_r/qAD_h$. The reverse current across the junction consists of holes being withdrawn from the n-type region into the p-type region. Initially there can be little change in $p'_n(0)$, and hence little change in voltage. The diode remains with a forward-bias voltage, although the current has reversed. During the time interval $t = t_0 + \tau_s$, the current $i(t)$ remains essentially constant until the voltage waveform crosses the time axis. After the voltage crosses the time axis, the time interval $\tau_f$ specifies the time for the voltage to settle to 90% of its final steady value. During this same interval, $\tau_f$, the reverse current drops to 10% of its original value. The time interval $\tau_s$ is called the storage time and $\tau_f$ is called the fall time. The storage time and fall time represent two important figures of merit for switching diodes. For abrupt
junctions, the storage time and the transition time are
determined by the ratio $I_r/I_f$ and the minority-carrier
lifetime. The relationship between the normalized
storage time $\tau_s/\tau_h$ versus $I_r/I_f$ for long-base diodes
as predicted by the models discussed above and the ideal
junction diodes are shown in Fig. 4. The lowermost
curve is the exact solution of the diode diffusion
equation (3); the corresponding relationships as
predicted by the two-capacitor model, one-lump model
and the two-lump model have been given in (3) are
reproduced here (Fig. 4). Finally, the relationship
predicted by the memristive diode is also shown in
Fig. 4 for comparison purposes. Observe that the
relationships predicted by the two-capacitor model and
one-lump model differ significantly from the ideal one
(the lowermost curve in Fig. 4), while that predicted
by the memristive diode model to be presented in
Chapter 2 represents a much better approximation.

1Actually the uppermost relationship shown in Fig. 4
is the solution of the equation $-I_r = dq_h'/dt + q_h'/\tau_h$,
where $q_h'$ is the charge of excess carriers stored in
base, $I_r$ is the magnitude of the initial reverse current.
But this equation is precisely the governing equation
for the two-capacitor model during reverse transient [2].
The fall time $\tau_f$ depends on how fast the residual stored charge can be discharged. The two-capacitor model has predicted fall times which are typically three order of magnitude smaller than actually observed [4]. Consequently, it is unsatisfactory for analyzing many high speed circuits, such as switching circuits. The lumped model could predict a much more accurate $\tau_f$ so long as a sufficient number of lumped sections are chosen. A typical number required for accurate prediction has been reported to be 20 sections [4]. Such model is often too complicated for simulating circuits containing many diodes. In contrast to this, it will be shown in Chapter 2 that the memristive diode model is capable of approximating an accurate $\tau_f$ by only adjusting one of the model parameters. Additionally, from the computer circuit simulation point of view, the memristive diode model is much more economical since it requires only four elements.
Fig. 3. Reverse transient behavior of junction diodes: the qualitative waveforms of diode voltage and current during reverse transient operation.
Two-capacitor model

Memristor diode model ($\alpha = 0.5$, $\beta = 1.5$)

One-lump model

Fig. 4. The relationships between the normalized storage time $\tau_s/\tau_h$ and the reverse-to-forward current ratio $I_r/I_f$ as predicted by different models for long-base junction diode.
Fig. 5. Forward transient behaviors of junction diodes: the simplified circuit for measuring diode forward transient voltage.

(b) Dynamic Behaviors During Forward Transient

Fig. 5 illustrates the forward transient measurement for a junction diode. When the diode is initially in equilibrium, $i_s(t)$, as shown in Fig. 6(a), is a step current at $t = 0$. The corresponding diode voltage waveform $v(t)$ for $t \geq 0$ is shown in Fig. 6(b). In Fig. 6(b) the voltage waveform increases monotonically until it reaches its steady state value when the applied current amplitude $I_f$ is very small ($I_f = I_3$). When the current is increased ($I_f = I_2$), an oscillation exists before the voltage $v(t)$ reaches the steady state value. If the applied current amplitude is sufficiently large ($I_f = I_1$), the voltage waveform exhibits an initial
overshoot and then decreases monotonically to its steady state value.

The phenomena mentioned above is called conductivity modulation of a junction diode. This behavior is affected by the stored charge as in Fig. 7 which shows the variation of the spatial distribution of $p_n$ at various intervals of time after $t = 0$.

Among the three models shown in Fig. 1, the two-lump model can simulate the forward transient behaviors more accurately than using the other two models. This is achieved through the introduction of a storance $S$ whose value is controlled by the junction current $i_j(t)$, which in turn is a function of the junction voltage $v_j(t)$. Observe that as the diffusance $H_d$ is charged or discharged, $S$ decreases or increases, thereby providing a mechanism for simulating the conductivity modulation (3). However, it is not a good model for simulating reverse transient behaviors as we have already indicated earlier. We will later show the memristive diode model is capable of predicting both forward and reverse transient behaviors. The memristor will be seen to play a crucial role in simulating the charge storage effect in the diode base.
Fig. 6. Forward transient behaviors of junction diodes:
(a) Applied current $i_s(t)$. (b) The voltage waveforms during forward transient operation corresponding to three different values of $I_f$. 

$I_1 > I_2 > I_3$
Fig. 7. The hole distribution in the n-type region of a diode during the transient following applications of a constant forward current.
Chapter 2

THE DEVICES OF MEMRISTIVE DIODE MODEL

From junction diode physics, we know there is a thin transition space charge layer\(^2\) at the diode junction, and a neutral region\(^3\) where the resistance depends on the locating carriers. There are two different ways the carriers will go as they flow through a diode. First, they can alter the amount of stored charge by flowing into the space charge layer. Second, the carriers can be stored or recombined in the neutral region after passing through the space charge layer. Therefore, the

\(^2\)The region straddling the metallurgical junction, which contains the electric field and the space charge, and across which is developed the contact potential, is usually referred to as the space charge layer.

\(^3\)The space charge layer is sandwiched between two regions in which the electrostatic potential is constant, and the electric field and charge density are zero. These two regions are called the neutral regions.
conductance in the neutral region will be changed corresponding to the variations of carrier concentration. Based on this theory, the model is set up as shown in Fig. 8.

In Fig. 8(a) the one-dimensional p-n junction diode is shown with an n-type region of width \( W_n \) and junction area \( A \). From the definition of long-base diode, we assume that \( W_n \) is much greater than the diffusion length \( L_h \). Also assume that the p-type region is much more heavily doped than the n-type region, so the diode current is nearly equal to the junction hole current. Two important conditions in this assumption are now emphasized again: \( N_A \gg N_D \) and \( W_n \gg L_h \).

In Fig. 8(b) \( i_1 \) is used to simulate the leakage of carriers flowing through the space charge layer \( C_j \) is used to show the effect of the transition layer, \( i_2 \) is used to denote the recombination of carriers, and \( R_m \) is used to simulate the conductance of the neutral regions.

From basic physical principles, these four elements are characterized and derived as follow:
Fig. 8. (a) A one-dimensional junction diode, (b) the memristive model for junction diode.
A. The nonlinear junction capacitance $C_j(v_j)$.

For simplicity we choose the standard expression\textsuperscript{4} for $C_j(v_j)$ as derived from the depletion approximation for a one-dimensional diode with an abrupt-step and single-side (i.e., $N_A \gg N_D$) junction [3]. This expression is derived as follows:

Using the depletion approximation, the magnitude of the charge in either half of the depletion layer of an abrupt junction is:

$$q_j = qAW_nN_D = qAW_PN_A$$  \hspace{1cm} (1)

\textsuperscript{4}The junction capacitance $C_j(v_j)$ is an expression of the dynamic changes of the stored charge in the space-charge layer as shown in Fig. 9.
Fig. 9. (a) Charge in the space-charge layer as function voltage.

(b) Distribution of space charge in the dipole layer of an abrupt p-n junction.
The widths of the p-type and n-type portions of the depletion layer are:

\[ W_p = WN_D/(N_A+N_D) \]  
\[ W_n = WN_A/(N_A+N_D) \]

where \( W \) is the total width of the space-charge layer and:

\[ W = \left( \frac{2\epsilon/q (\psi - V_j) (1/N_A+N_D)}{1/N_A+N_D} \right)^{1/2} \]

From Eqs. (1)(2b)(3)

\[ q_j = qAW(N_A/N_A+N_D)N_D \]

\[ = qA \left[ \frac{2\epsilon/q (\psi - V_j) (1/N_A+1/N_D)}{(N_A/N_A+N_D)N_D} \right]^{1/2} \]

\[ = A \left[ 2q \epsilon (\psi - V_j)/(N_A+1/N_D) \right]^{1/2} \]

The junction current associated with changes in \( q_j \) is:

\[ i_j = -dq_j/dt = -dq_j/dv \cdot (dv/dt) \]

the current \( i_j \) may then be described by a junction capacitance \( C_j(V_j) \), given by:

\[ C_j(V_j) = -dq_j/dv = -d/dv \left[ A \left( 2q \epsilon (\psi - V_j)/(N_A + 1/N_D) \right)^{1/2} \right] \]

\[ = A/2 \left[ 2q \epsilon / (\psi - V_j)(N_A + 1/N_D) \right]^{1/2} \]
\[ = A/2 \left( 2q e N_D / (\psi_o - v_j) \right)^{1/2} \quad N_A \gg N_D \]

\[ = K_a (\psi_o - v_j)^{1/2} / 2 \]

where \[ K_a = A (2q e N_D)^{1/2} \]

B. The memristor \( R_m(q_m) \)

The memristor is defined by \( R_m(q_m) = v_m / i_m \) as a linear charge-controlled resistor which is the element of a two-terminal circuit. \( R_m(q_m) \) is a function of the charge \( q_m \) passing through its terminals (1).

The current carrier density by holes and by electrons is equal to the product of their respective mobilities and average velocities. The current density, or the charge density per unit time transported by holes and electrons is:

\[ I_n = n q \mu_n E \quad (7a) \]

and

\[ I_p = p q \mu_p E \quad (7b) \]

whereby the total electrical current density \( I \) may be expressed as:

\[ I = q (n \mu_n + p \mu_p) E = \sigma E \quad (8) \]

For \( N_A \gg N_D \), the diode resistance is mainly
distributed by the base region where the conductance is much smaller than the p-type region. Under low injection condition where electrons can be considered as the majority carriers, the conductivity of the base region is:

$$\sigma(x) = q\left\{\mu_n n_o + \mu_p p_n(x)\right\} \quad (9)$$

where x is the distance from the junction to the base region, and $p_n(x)$ is the hole concentration at x. For a one-dimensional diode, the steady state diffusion equation (3) is:

$$D_h \frac{\partial^2 p_n(x)}{\partial x^2} = \frac{\partial p_n(x)}{\partial t} + \frac{[p_n(x) - p_{n_0}]}{\tau_h} \quad (10)$$

The excess hole concentration at x is defined as:

$$p'(x) = p_n(x) - p_{n_0}. \quad (11)$$

Eq. (10) can be rewritten as:

$$D_h \frac{\partial^2 p'_n(x)}{\partial x^2} = \frac{\partial p'_n(x)}{\partial t} + \frac{p'_n(x)}{\tau_h} \quad (12)$$

Under dc or static condition, $\frac{\partial p'_n(x)}{\partial t} = 0$, so Eq. (12) becomes:

$$D_h \frac{\partial^2 p'_n(x)}{\partial x^2} = \frac{p'_n(x)}{\tau_h}$$

$$\frac{\partial^2 p'_n(x)}{\partial x^2} = \frac{p'_n(x)}{D_h \tau_h}$$
\[ \frac{d^2 p_n^*(x)}{dx^2} - \frac{p_n^*(x)}{L_h^2} = 0 \tag{13} \]

where \( L_h^2 = D_h T_h \).

In order to solve Eq. (13) two boundary conditions are considered. The first is at \( x = 0 \) and the second is at \( x = W_n \).

At \( x = 0 \)

\[ p_n^*(x) = p_n^*(0) = p_{no}(\exp(v_j/V_T) - 1) \tag{14} \]

where \( v_j \) is the applied voltage across the diode junction, and \( p_n^*(x) = p_n(x) - p_{no} \) is the excess hole concentration at \( x \) in the base region.

For junction diodes of finite size \( (5) \), \( p_n^*(x) \) is defined as:

\[ p_n(x) - p_{no} = A \cosh \frac{x-x_{o+}}{L_h} + B \sinh \frac{x-x_{o+}}{L_h} \tag{15a} \]

where \( x_{o+} \) is the thickness of the space charge layer, and if the layer is very thin, so \( x_{o+} \) approaches to zero.

Eq. (15a) becomes:

\[ p_n'(x) = p_n(x) - p_{no} \]

\[ = A \cosh(x/L_h) + B \sinh(x/L_h) \tag{15b} \]
If \( x = 0 \), Eq. (15b) becomes:

\[
P_n'(0) = A
\]  
(16)

At \( x = \overline{V}_n \):

\[
S_n p_n'(\overline{V}_n) = \frac{1}{q} J_h(\overline{V}_n) = -D_n \varphi p_n'(x)/\partial x\bigg|_{x = \overline{V}_n}
\]

(17)

where \( S_n \) is the surface recombination velocity (3),

and \( J_h(x) \) is the hole current density at \( x \).

The solution of Eq. (13) from Eqs. (14-17) is then:

\[
S_n p_n'(\overline{V}_n) = -D_n \varphi p_n'(x)/\partial x\bigg|_{x = \overline{V}_n}
\]

\[
S_n \left[ A \cosh(\overline{V}_n/L_n) + B \sinh(\overline{V}_n/L_n) \right] = -D_n \partial/\partial x \left[ A \cosh(x/L_n) + B \sinh(x/L_n) \right]\bigg|_{x = \overline{W}_n}
\]
\[ S_h(A \cosh(W_n/L_h) + B \sinh(W_n/L_h)) = -D_h/L_h[A \sinh(W_n/L_h) \]
\[ + B \cosh(W_n/L_h)] \]

\[ A(S_h \cosh(W_n/L_h) + D_h/L_h \sinh(W_n/L_h)) = \]
\[ -B(D_h/L_h \cosh(W_n/L_h) + S_h \sinh(W_n/L_h)) \]

\[ B = \frac{\left( S_h \cosh(W_n/L_h) + D_h/L_h \sinh(W_n/L_h) \right) p_n'(0)}{-\left( D_h/L_h \cosh(W_n/L_h) + S_h \sinh(W_n/L_h) \right)} \]

so

\[ p_n'(x) = p_n'(0) \cosh(x/L_h) - \]
\[ p_n'(0) \left[ \frac{S_h \cosh(W_n/L_h) + D_h/L_h \sinh(W_n/L_h)}{D_h/L_h \cosh(W_n/L_h) + S_h \sinh(W_n/L_h)} \right] \sinh(x/L_h) \]
\[ = p_n'(0) \left[ \frac{\cosh(x/L_h)}{\cosh(W_n/L_h) + D_h/S_h L_h \sinh(W_n/L_h)} \right] \sinh(x/L_h) \]

\[ \text{(18)} \]

If the recombination center can recombine all excess hole concentration, the \( S_h \) becomes very large, and for the long-base conditions,

\[ p_n'(x) = p_n'(0) \left[ \cosh(x/L_h) - \coth(W_n/L_h) \sinh(W_n/L_h) \right] \]
\[ = p_n'(0) \left[ \cosh(x/L_h) - \sinh(x/L_h) \right] \text{ } W_n \gg L_h \text{ (19)} \]

From the excess hole concentration, we can get the
excess minority charge \( q_h \) (which is stored in the neutral region),

\[
q_h = \int_0^{\mathcal{W}_h} A q_p'(x) dx = A q_p'(0) L_h
\]

(21)

Then

\[
p_n'(0) = q_h / A q L_h
\]

(22)

so Eq. (20) can be rewritten as:

\[
p_n'(x) = q_h / A q L_h \left( \cosh(x/L_h) - \sinh(x/L_h) \right)
\]

(23)

Assuming \( q_h = q_m \) (that will be justified later), then Eq. (23) becomes

\[
p_n'(x) = q_m / A q L_h \left( \cosh(x/L_h) - \sinh(x/L_h) \right)
\]

(24)

Substituting Eq. (24) into Eq. (11)

\[
p_n(x) = p_n'(x) + p_{no}
\]

\[
= p_{no} + q_m / A q L_h \left( \cosh(x/L_h) - \sinh(x/L_h) \right)
\]

(25)

Then conductivity of the memristor is found from Eq. (25)

\[
\sigma(x, q_m) = q_m n_{no} + q_m \left( p_{no} + q_m / A q L_h \left( \cosh(x/L_h) - \sinh(x/L_h) \right) \right)
\]

(26)

Finally the memristance is given by:

\[
R_m(q_m) = \int_0^{\mathcal{W}_h} dx / A \sigma(x, q_m)
\]

(27)
C. The controlled current source $i_2 = I_2(q_m)$

We define the recombination of carriers $i_2 = q_m/\tau_h$ for long-base diode at steady state. Then $dq/dt = 0$ represents no current flow through the memristor, and $i = i_2$ where $i$ is the diode current, again given by:

$$i = AJ_h(0) = -AqD_h \frac{\partial p_n^*(x)}{\partial x} \bigg|_{x=0} = \frac{AqD_h p_n^*(0)}{L_h}$$

(28)

where $\frac{\partial p_n^*(0)}{\partial x} = -p_n^*(0)/L_h$ from the derivation of Eq. (20).

From Eq. (21),

$$q_h^* = Aq p_n^*(0)L_h = AqD_h p_n^*(0) \tau_h / L_h$$

(29)

where $L_h^2 = D_h \tau_h$

Equations (28) and (29) together give

$$i = \frac{AqD_h p_n^*(0)}{L_h} = q_h^*/\tau_h$$

(30)

therefore

$$i = i_2 = q_h^*/\tau_h = q_m/\tau_h$$

(31)

That means at the steady state the excess hole charge $q_h^*$ is equal to the stored charge in the memristor
and the definition \( i_2 = q_m/\tau_h \) is set.

D. The controlled current source \( i_1 = I_1(i,i_j,V_j,q_m) \)

We define:

\[
I_1(i,i_j,V_j,q_m) = I_{1f}U(i) + I_{1r}U(-i) \tag{32}
\]

where

\[
U(i) = \begin{cases} 
1 & i > 0 \\
1/2 & i = 0 \\
0 & i < 0 
\end{cases} \tag{33}
\]

is a step function.

(1) Derivation of \( I_{1f} \)

The upper part of the circuit in Fig. 8(b) is equivalent to the model circuit with an ideal junction-law diode in parallel with a diode diffusion capacitance \( C_d(v_j) \). When \( i > 0 \), \( I_{1f} \) is given by:

\[
I_{1f} = \frac{q'^*}{\tau_h} + \frac{dq'^*}{dt} \tag{34}
\]

and from Eq. (29)

\[
q'^*_h = \frac{AqD_h p_n'(0)}{L_h} \tau_h
\]

\[
= \tau_h \frac{AqD_h}{L_h} p_{no} \left( \exp \left( \frac{v_j}{V_T} \right) - 1 \right)
\]

\[
= \tau_h I_s \left[ \exp \left( \frac{v_j}{V_T} \right) - 1 \right] \tag{35}
\]
\[ p_n' (0) = p_{no} \left\{ \exp \left( \frac{v_j}{V_T} \right) - 1 \right\}, \quad V_T = kT/q \quad (36a) \]

and

\[ I_s = \frac{AqD_h}{L_h} p_{no} \quad (36b) \]

\( I_s \) is the diode saturation current, \( V_T \) is the thermal voltage of an ideal diode. Eq. (34) can be rewritten as:

\[
I_{1f} = \frac{\tau_h I_s \left( \exp \left( \frac{v_j}{V_T} \right) - 1 \right)}{\tau_h} + \frac{\tau_h}{V_T} I_s \exp \left( \frac{v_j}{V_T} \right) \frac{dv_j}{dt}
\]

\[ = I_s \left( \exp \left( \frac{v_j}{V_T} \right) - 1 \right) + C_d(v_j) \frac{dv_j}{dt} \]

\[ = I_s \left( \exp \left( \frac{v_j}{V_T} \right) - 1 \right) + C_d(v_j) \left\{ \frac{i_j}{i(v_j)} \right\} \quad (37) \]

where

\[ C_d(v_j) = \frac{I_s \tau_h}{V_T} \exp \left( \frac{v_j}{V_T} \right) \quad (38) \]

and \( I_{1f} \) satisfies the static and dynamic characteristics of the diode.

(2) Derivation of \( I_{1r} \)

We define:

\[ I_{1r} = i - C_d(v_j) \max(\gamma_a, \gamma_b) \quad (39) \]

where

\[ \max(\gamma_a, \gamma_b) = \gamma_a \quad \text{whenever} \quad \gamma_a \geq \gamma_b \]

\[ = \gamma_b \quad \text{whenever} \quad \gamma_a < \gamma_b \]
and

\[ \gamma_a = \frac{i}{C_d(v_j)\{1 + \sigma \left[ \frac{|q_m| + I_S \tau_h}{(I_i + I_S) \tau_h} \right] U(-v_j) \}} \]  

(40)

\[ \gamma_b = -\max(\gamma_c, \gamma_d) \]  

(41)

\[ \gamma_c = \frac{I_S \{\exp(v_j/V_T)-1\} - i}{C_d(v_j) + C_j(v_j)} \]  

(42)

\[ \gamma_d = \frac{\left\{ \frac{V_T}{AqL_h} \right\} q_m}{p_{no} \left\{ 1 + \exp\left( \frac{V_i}{(1-0.25U(-v_j)V_T)} \right) \right\} \left\{ 0.5 \left[ \frac{|q_m| + I_S \tau_h}{(I_i + I_S) \tau_h} \right]^\beta \right\} } \]  

(43)

The two empirical parameters in Eqs. (40-43) can be selected in order to get a more accurate prediction for the fall time \( \tau_f \) and the storage time \( \tau_s \). Usually we pick \( \beta = 1.5 \), and \( 0.5 \leq \sigma \leq 10 \) [1] depending on the residual stored charge in the base region and the discharge rate of a chosen diode at \( t = t_o + \tau_s \).

Consider again the circuit shown in Fig. 2. The reverse transient behavior occurs after \( t = t_o \). The corresponding carrier concentration distribution \( p_n(x) \) at that time is shown in Fig. 10. At \( t = t_o + \tau_s \), the \( p_n(x) \) at \( x = 0 \) reaches the equilibrium value \( p_{no} \) which is shown by a dashed line. The slope of \( p_n(x) \) at \( x = 0 \)
Fig. 10. The distribution of the carrier concentration \( p_n(x) \) in the base region during reverse transient operation.

is proportional to the diode current \( I \). For a long-base diode, the relationship between \( \frac{T_5}{T_h} \) and \( \frac{I_r}{I_f} \) is given by (7):

\[
\text{erf} \left( \left( \frac{T_5}{T_h} \right)^{1/2} \right) = \frac{1}{1 + \frac{I_r}{I_f}}
\]

where \( \text{erf} \) = error function.

We know from Eqs. (29) and (35) that the excess carrier concentration \( p_n'(x) \) at \( x = 0 \) under steady state condition is related to the hole excess charge \( q_h' \) or the stored charge \( q_m \) by Eq. (22). Let us rewrite Eq. (22) using \( \bar{p}_n'(0) \) and \( \bar{q}_m \) to denote the steady state excess carrier concentration and memristor charge, respectively, as
follows: \[ \bar{p}_n^*(0) = \frac{q_m(t)}{AqL_h} \] (45)

Let us now define a quantity \( \bar{p}_n^*(0,t) \) as follows:

\[ \bar{p}_n^*(0,t) = \frac{q_m(t)}{AqL_h} \] (46)

Observe that Eq.(46) reduces to Eq.(45) under steady state conditions. Next, we postulate the following constraint on the rate of change of the excess carrier concentration \( p_n^*(x) \) at \( x = 0 \):

\[ \frac{dp_n^*(0)}{dt} = -\frac{\bar{p}_n^*(0,t)}{0.5 \tau_h \left( \frac{|q_m(t)| + I_s \tau_h}{|i(t)| + I_s \tau_h} \right)^\beta} \] (47)

To justify the reason for introducing this constraint, we observe that if \( \beta = 1.5 \), and if the initial reverse current \(-i(t_0) = I_r\) is equal to \( q_m(t_0) / \tau_h = I_r \), then since \( i(t) \approx i(t_0) \) for \( t \leq t_0 + \tau_s \), we have

\[ \frac{dp_n^*(0)}{dt} = -\frac{\bar{p}_n^*(0,t)}{0.5 \tau_h \frac{|q_m(t)| + I_s \tau_h}{|i(t)| + I_s \tau_h}} = -\frac{q_m(t)}{0.5 \tau_h} \]

\[ = \text{constant} \] (49)

The term \( I_s \tau_h \) and \( I_s \) in Eq.(47) are very small constants which are introduced to avoid computer overflow problem when \( q_m(t) = 0 \), or \( i(t) = 0 \).
Equation (49) implies that if we switch a diode with a reverse current whose magnitude $I_r$ is equal to the forward steady state current $I_f$, then $p_n'(0)$ will become zero at $t = t + 0.5 \tau_h$. This observation is consistent with that predicted by Eq. (44); namely, if $I_r \approx I_f$, then $\tau_s \approx 0.5 \tau_h$, i.e., $v_j(\tau_s) = v_j(0.5 \tau_h) = 0$.

Since
$$p_n'(0) = p_{no}(\exp(v_j/V_T) - 1)$$

$p_n'(0)$ also becomes zero at $t = t_o + 0.5 \tau_h$. Thus we see Eq. (47) indeed represents a "qualitatively" reasonable constraint. It follows from Eqs. (47) and (49) that

$$-\frac{dv_j}{dt} = \frac{V_T p_n'(0, t)}{p_{no} \exp(v_j/V_T) 0.5 \tau_h \left[ \frac{|q_m(t)| + I_s \tau_h}{(|q_m(t)| + I_s \tau_h) \tau_h} \right] \beta}$$

If we use only Eq. (51) to calculate $v_j(t)$ and $\tau_s$ during reverse transient operation, two problems that were postulated by Dr. Chua (1) are immediately arised:

(i) It predicts too long a storage time $\tau_s$ when $I_r \ll I_f$.

(ii) When $t > t_o + \tau_s$, $v_j(t) < 0$ and $\exp(v_j/V_T) \approx 0$.

Hence, $|dv_j/dt|$ becomes exceedingly large, therefore resulting in "too short" a fall time $\tau_f$, let alone the computer overflow problem that invariably arises (1). To overcome these problems, we first modify Eq. (51)
as follows:  

\[
\frac{dv_j}{dt} = \frac{V_{TP_n}'(0, t)}{\tilde{p}_{h_0} \left\{1 + \exp \left(\frac{v_j}{[1 - 0.25v_jU(-v_j)] V_T} \right) \right\} \left\{0.5 \left[\frac{[\tau \cdot (t) + I_s(t)]}{[\tau \cdot (t) + I_s(t)]} \right] \right\}}
\]

(52)

Observe that Eq. (52) is equivalent to Eq. (51) when \(v_j \gg V_T\). However, when \(v_j < 0\) there is no overflow problem since

\[
\exp \left(\frac{v_j}{[1 - 0.25v_jU(-v_j)] V_T} \right) \rightarrow \exp(-4/V_T), \text{ as } v_j \rightarrow -\infty
\]

Upon substituting Eq. (46) into Eq. (52) and dropping the argument \(t\), we obtain \(Y_d\) as defined by Eq. (43). Let us further postulate the two expressions \(Y_a\) and \(Y_c\) as defined in Eqs. (40) and (42). Note that from Eq. (42),

\[
Y_c[C_d(v_j) + C_j(v_j)] = I_s \left[\exp \left(\frac{v_j}{V_T} \right) - 1\right] - i. \text{ Hence, } Y_c\text{ is equal to } -dv_j/dt\text{ of the two-capacitor model.}
\]

Also observe that when \(v_j > 0\), and \(i < 0\). Eq. (40) becomes

\[
Y_a = \frac{i}{C_j(v_j)}
\]

(53)

which is the most negative among \(Y_a\), \(-Y_c\), and \(-Y_d\) since

---

6 This modification will differ significantly from Eq. (51) only when \(v_j < 0\). However, in this case the discrepancy is immaterial because, as will be shown later, when \(v_j < 0\), the model will automatically replace Eq. (51) with a
realistic expression; namely, \( dv_j/dt = \gamma_a \), where \( \gamma_a \) is defined in Eq. (40).

\( C_j(v_j) \) is very small (typically in the order of 10\(^{-11}\) farad). Hence, the function \( \max(\gamma_a, \gamma_b) \) must give \( \gamma_b \) when \( v_j > 0 \) and \( i < 0 \), where \( \gamma_b \) is as defined in Eq. (41); namely, \( \gamma_b = -\max(\gamma_c, \gamma_d) \). Observe that when \( v_j > 0 \), the function \( \max(\gamma_a, \gamma_b) \) actually selects between the greater of two possible values of \( |dv_j/dt| \); namely that predicted by \( \gamma_c \) which coincides with the two-capacitor model prediction, or that predicted by Eq. (43) which is based on our postulate of Eq. (47). Consequently, the relationship between the normalized storage time \( \tau_s/\tau_h \) and the reverse-to-forward current ratio \( I_r/I_f \) as predicted by the memristive diode model could approach the ideal relationship shown in Fig. 4 by an optimum choice of the model parameter \( \beta \). On the other hand, when \( v_j < 0 \) and \( i < 0 \), Eq. (43) gives (except when \( i \) has decayed to a very small value) too large a value for \( dv_j/dt \) because the exponential term in the denominator approaches zero when \( v_j < 0 \). Moreover, since

\[ C_d(v_j) = \frac{I_s \tau_h}{V_T} \exp(v_j/V_T) \approx 0 \]

and

\[ I_s \left( \exp(v_j/V_T) - 1 \right) \approx -I_s \]
when \( v_j < -V_T < 0 \), Eq. (42) can be approximated by

\[
Y_c \approx -\frac{\mathbf{i}}{C_j(v_j)} \quad (54)
\]

But since \( C_j \) is very small, Eq. (54) gives too fast a decaying rate for \( v_j(t) \) and \( i(t) \). Physically, it is the residual stored charge in the base (which can still support a rather large value of \( i(t) \) ) which prevents \( i(t) \) from decaying too fast after \( t = t + \tau_c \). To account for this effect, the function \( Y_a(\cdot) \) as defined in Eq. (40) is postulated.

When \( v_j < 0 \), Eq. (40) reduces to:

\[
Y_a = \frac{\dot{\mathbf{i}}}{C_j(v_j) \left\{ 1 + \alpha \left[ \frac{|I_m| + I_s \tau_h}{( |\mathbf{i}| + I_s) \tau_h} \right] \right\}} \quad (55)
\]

Observe that for sufficiently large value of \( \alpha \), \( |Y_a| \) will be sufficiently small such that the function \( \max(Y_a, Y_b) = Y_a \) when \( v_j < 0 \) and \( i < 0 \). For a particular diode the value of the empirical parameter \( \alpha \) is chosen so that the fall time \( \tau_f \) approximates the measured value as accurately as possible.

The preceding derivations show our model is indeed capable of mimicking the essential qualitative diode behaviors under both reverse and forward transients.
We now turn to some specific examples so that the quantitative behavior can be evaluated.
Consider a silicon diode with the following parameters:

\[
\begin{align*}
N_D &= 10^{15} \text{cm}^{-3} & T_h &= 10^{-7} \text{sec} \\
\mu_n &= 1350 \text{cm}^2/\text{v-sec} & W_n &= 10L_h \\
\mu_p &= 480 \text{cm}^2/\text{v-sec} & T &= 290^\circ \text{K} \\
I_s &= 0.5 \times 10^{-9} \text{ma} & \psi_o &= 0.9 \text{ v} \\
V_T &= 25 \text{mv} & n_i^2 &= 2 \times 10^{20} \text{cm}^{-6}
\end{align*}
\]

From these parameters, we obtain

\[
D_h = \mu_p V_T = 12 \text{cm}^2/\text{sec}
\]

\[
L_h = \sqrt{\frac{D_h T_h}{T}} = 1.09 \times 10^{-3} \text{cm}
\]

\[
n_{no} = N_D = 10^{15} \text{cm}^{-3}
\]

\[
p_{no} = n_i^2 (290^\circ \text{K})/n_{no} = 2 \times 10^5 \text{cm}^{-3}
\]

and

\[
i = I_s \left( \exp \left( \frac{V_j}{V_T} \right) - 1 \right) = \frac{A q D_h}{L_h} \ p_n^*(0)
\]

\[
= \frac{A q D_h}{L_h} \ p_{no} \left( \exp \left( \frac{V_j}{V_T} \right) - 1 \right)
\]

(56)
then

\[ I_s = \frac{A q D_n}{L_n} p_{no} \]
\[ = \frac{I_s L_n}{q D_n p_{no}} = 1.5 \times 10^{-1} \text{cm}^{-2} \quad (57) \]

A. During Reverse Transient

Let us consider that the diode in Fig. 2 has the above value. When \( t = t_0 = 0 \) the switch drops down to the position 2 to form a reverse bias. But before then, the forward current \( I_f = 10 \text{ma} \) and the voltage \( E_2 = 10 \text{volts} \). If we choose the parameter \( \alpha = 0.5 \) and \( \beta = 1.5 \), the results \( t/\tau_n \) versus \( i(t) \) and \( v(t) \) are shown in Fig. 11. There is a small instantaneous voltage drop for each curve in the voltage waveform when \( S \) is switched. The reason is that the reverse current at \( t = 0 \) is equal to the current flow through the memristor. Before \( t = \tau_3 \), the voltage waveform remains flat and the current is given by \( i(t) = (E_2 + V(t))/R \). This phenomenon can be explained by Eq. (49) where we have

\[ \frac{dp_n'(0)}{dt} = \text{constant} \]

That tells us the excess carrier concentration at \( x = 0 \) before \( t = \tau_3 \) decreases linearly with time \( t \) as the junction voltage decreases very slowly by the result of

\[ p_n'(0) = p_{no} \left( \exp \left( \frac{V_j}{V_T} \right) - 1 \right) \]

\[ V_j = V_T \ln \left[ 1 + \frac{p_n'(0)}{p_{no}} \right] \quad (58) \]
When $t > \tau_s$, $v_j(t) < 0$, then $dv_j/dt = \gamma \alpha$. The current follows Eq.(57) and decays approaching zero where the voltage drop is neglected.

B. During Forward Transient

Consider Fig. 5, where the circuit is the same as shown for reverse transient measurement. Apply different current steps to the circuit, the voltage waveforms are shown in Fig. 12 as describe in Chapter 1.

When $i > 0$

$$I_1 = I_s \{ \exp(v_j/V_T) - 1 \} + C_d(v_j) \frac{dv_j}{dt}$$

And Fig. 8 in Chapter becomes Fig. 13. In that case, when $I_f$ is small capacitors $C_j$ and $C_d$ will charge slowly and the voltage of the memristor is negligible. Then the lowermost curve which increases monotonically to the steady state will be obtained in Fig. 12. On the other hand when the applied current $I_f$ is very large, the junction voltage $v_j(t)$ and capacitors $C_j$ and $C_d$ will then rise and charge quickly. At that instant, $q_m(t = 0) = 0$ corresponding to $R_m(0)$ is very high, and $I_f$ flowing through the memristor produces a high voltage. Therefore, the initial voltage jump at $t = 0$ is $V_m = R_m(0)I_f$. Then the uppermost of the voltage waveform in Fig. 12
Fig. 11(a). The reverse transient voltage response of a junction diode in the circuit shown in Fig. 2: the voltage waveforms corresponding to different values of $R$: $200\Omega$, $300\Omega$, $400\Omega$, $500\Omega$. 
Fig. 11(b). The reverse transient current response of a junction diode in the circuit shown in Fig. 2. The current waveforms corresponding to different values of $R$: $200\Omega$, $300\Omega$, $400\Omega$, $500\Omega$. 
Fig. 12. The forward transient voltage response of the diode in Fig. 5.
Fig. 13. The equivalent circuit of the memristive diode model which applies when the diode current $i > 0$.

$$r_j = \left( \frac{di_j}{dv_j} \right)^{-1} = \frac{1}{\frac{d}{dv_j} \left[ I_s \left\{ \exp \left( \frac{v_i}{\frac{V_T}{V_T}} \right) - 1 \right\} \right]}$$

$$c_d = \frac{\tau_h I_s}{V_T} \exp \left( \frac{V_j}{V_T} \right)$$

$$i_2(q_m) = q_m / \tau_h$$

$$R_m = \frac{1}{A} \int_{-\infty}^{\infty} \frac{dx}{(x, q_m)}$$

Fig. 14. The superposition of $V_j(t)$ and $V_m(t)$ to form the forward transient waveform $V(t)$: (a) high input current case (b) intermediate input current case (c) low input current case.
is shown. As the time increases, the voltage \( V_m(t) = i_m(t) R_m(q_m(t)) \) will decrease while the \( q_m(t) \) increases accordingly to decrease the resistance \( R_m(q_m) \). When \( i_2(q_m) \) increases while \( i_m(t) \) decreases is related to the equation \( i_m(t) = I_f - i_2(q_m) \). Then \( V_j \) increases quickly will cause the \( V_m \) decrease rapidly to zero.

Fig. 14(a)(b)(c) show the superposition of \( V_j(t) \) and \( V_m(t) \) where \( V(t) = V_j(t) + V_m(t) \) when three different currents are applied. Those results illustrate the Fig. 12 which is operated with large, intermediate and small magnitude of the current steps \( I_f \).
Chapter 4

CONCLUSION

In this paper, a memristive diode model is proved to be able to simulate a long-base ($\tau = \tau_h$) p-n junction diode's dynamic behaviors under both reverse and forward transient operations.

From the model testing, we find the memristor $R_m(q_m)$ is capable of simulating the conductivity modulation phenomenon during forward transient operation and the initial small voltage drop during reverse transient operation.

During reverse transient operation, several of the models such as two-capacitor model, one-lump model and two-lump model are chosen to compare with the idealized diode diffusion equation by using their normalized storage time $\tau_s/\tau_h$ vs $I_x/I_f$ relationship. Among those the memristive diode model under long-base condition gives the best approximation. This model is specified by 13 physical diode parameters $\{\psi_0, \xi, N_D, \mu_n, \mu_p, n_{no}, p_{no}, A, W_n, D_h, \tau_h, V_T, I_S\}$ and two empirical parameters $\{\alpha, \beta\}$. But it is much easier than other models to
predict the fall time \( \tau_f \) by adjusting only one empirical parameter. Although we choose sufficiently large numbers of lumped-model sections, the lumped-model is able to improve the accuracy of its transient behaviors \( [3] \), obviously we find the memristive diode model is more economical since it employs only four elements. This model also can simulate the reverse transient decay behaviors.

During forward transient operation, the model is found able to simulate the junction diode's conductivity modulation behaviors corresponding to three different values of current amplitude \( I_f \). When \( I_f \) is small, the model's voltage waveform is shown monotonically increasing other than the waveform decreasing monotonically while \( I_f \) is large. There is an oscillatory voltage waveform when \( I_f \) is under intermediate value.

In contrast to this, the memristive diode model is shown capable of predicting both forward and reverse transient behaviors where the memristor is used in simulating the charge storage effect in the diode base.
REFERENCES


