SIMULATIONS OF
MISSILE SYSTEMS

A thesis submitted in partial satisfaction of the requirements for the degree of Master of Science in Test and Evaluation

by

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May, 1976
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ABSTRACT

Simulations of Missile Systems is a thesis project report presented in two parts. Each part concerns a separate project and is complete in itself. The format for the report is that specified for submittal by the Pacific Missile Test Center, Point Mugu, which was the agency for whom the work was done.

In the first part, "Analysis of the Pod-Mounted Tracking Antenna--Cruise Missile Project," a dynamic analysis is performed on a radar antenna for the purpose of selecting drive motors, gear ratios, and damping factors. The second part, "Determination of Intruder Probability of Acquisition Grids," a low-altitude, anti-ship missile is computer modeled so that a statistical study could be done to identify the hazardous area around the target ship. This can then be used to study needed capabilities for range clearance whenever a missile is to be tested on the range.
Analysis of the Pod-Mounted
Tracking Antenna
(Cruise Missile Project)
SUMMARY

This report analyzes the design of the pod-mounted tracking antenna developed for the Cruise Missile Project. In particular, it analyzes the tracking rate in azimuth and elevation, and it selects parameters which determine the tracking rate—system damping and geartrain ratios.

The method was to mathematically model the dynamics of the antenna and solve the model through simulation on Tymeshare-CSMP (Continuous System Modeling Program).

The simulation showed the system to be underdamped with the damping factor equal to .27, and to be satisfactory in tracking (tracking rate of at least 20 degrees per second) for geartrain ratios between 400:1 and 1400:1. Error in position was less than 1 degree.

To achieve a damping factor of .75, a tachometer was modeled into the system on each axis. The tachometer generates a voltage proportional to the velocity and of opposite sense which is added to the respective error signal voltage. For the azimuth axis, the tachometer output is -.175 volts per degree per second. Similarly, the elevation tachometer output is -.075 volts per degree per second. After each damping voltage has been added to its respective error signal voltage, it is amplified by a factor of 25 and fed to the corresponding axis motor.
Finally, optimum performance is obtained when both azimuth and elevation geartrain ratios are 590.8:1. This is reduced by 4:1 for the gearmotor ratios.
INTRODUCTION

The subject of this report is the pod-mounted tracking antenna shown in figure 1. The antenna is to be used as instrumentation in support of the Cruise Missile Project.

The purpose of the work reported herein is to determine if the antenna, in its present design configuration, will meet prescribed criteria for tracking rate and position error and to provide the basis for design changes, if necessary, which will meet the above criteria. This work is authorized under job order 6N3G3A5.

The organization of the report is as follows:

1) Mathematical model.
2) Tymeshare-CSMP simulation.
3) Results.
4) Conclusions.
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MATHEMATICAL MODEL

The antenna in figure 1 can be modeled dynamically as

\[ \alpha_k = \frac{\Sigma T_k}{\Sigma J_k} \]

where the \( k \) subscript denotes either azimuth (\( k=1 \)) or elevation (\( k=2 \)).

The antenna is divided into its component links, figures 2 through 10, and each component is then analyzed separately. Depending on the geometry of the link and its motion, a particular link can be modeled by a moment of inertia (\( J \)) or by a torque (\( T \)).

**Moment Calculations.**

The components of motion in azimuth, hereafter referred to as \( \theta_1 \), are shown in figures 2 through 5. The link in figure 2 has motion in translation only and will be treated in the section on inertial torques.

Figures 3 and 4 illustrate components A and B in figure 1 which pivot about x. \( \theta_1 \) is measured clockwise about this pivot.

Figure 6 contains the calculations for the moment of inertia about x of link A (likewise link B since the two are identical). The result of 37.76 lbm-in\(^2\) is entered in table 1 of figure 11.

The antenna link in figure 5 has a complex moment about x since it varies as a function of its position in
elevation (designated as \( \theta_2 \)). Drawings c and d in figure 5 have moments of inertia about \( x \) of (c)

\[
\frac{1}{12} \, m(12^2 + 1^2) + \frac{1}{2} mD^2
\]

\[
m = 15
\]

\[
D = 1
\]

\[
\frac{1}{12} \, (15)(12^2 + 1^2) + 15 (1^2) = 196.25
\]

and (d)

\[
\frac{1}{12} \, m(12^2 + 12^2) + \frac{1}{2} mD^2
\]

\[
m = 15
\]

\[
D = 1
\]

\[
\frac{1}{12} \, (15)(12^2 + 12^2) + 15 (1^2) = 375.00
\]

Since the moment is a function of \( r^2 \) and \( r \) is a function of \( \cos \theta_2 \) and \( \sin \theta_2 \) for cases c and d respectively, the moment about \( x \) is approximately

\[
196.25 \cos^2 \theta_2 + 375.00 \sin^2 \theta_2
\]

Link E, figure 5, has a moment about \( y \) (corresponding to \( \theta_2 \)) of

\[
\frac{1}{12} \, m(12^2 + 1^2) + \frac{1}{2} mD^2
\]

\[
m = 15
\]

\[
D = 1
\]

\[
\frac{1}{12} \, (15)(12^2 + 1^2) + 15 (1^2) = 196.25
\]

The remaining links are analyzed in the following section.
**Inertial Torque Calculations.**

Link C in figure 2 can be modeled as an inertial torque in $\theta_1$. The mass center of C is located at pin $r$ which always moves tangent to the arc about pivot $x$. Then its inertial torque $(T_1)$ is

$$-m r \alpha_1 r = -m r^2 \alpha_1$$

$m = 1.562 \text{ lbm}$

$r = 7.63 \text{ in}$

$$- (1.562)(7.63^2) \alpha_1 = -90.89 \alpha_1$$

This value is entered for links C and D in table 1, figure 11.

Link F in figure 7 is similar; its inertial torque being $-1.40 \text{ lbm}$. The mass is 624 lbm and $r$ is 1.5 in.

Horizontal acceleration is the only external aceleration which directly affects $\theta_1$. The mass center of the entire mechanism is located on a line $y'-x'$ with pivots at A and B in figure 1. As $\theta_1$ increases, this line is displaced from the centerline of the mechanism by $r \sin \theta_1$, and the torque due to horizontal acceleration is

$$T_{gh} = -m r g_h (r \sin \theta_1)$$

$m = 26 \text{ lbm}$

$r = 1 \text{ in}$

$$-26 (1) g_h (1 \sin \theta_1) = -26 g_h \sin \theta_1$$
where $g_h$ is the external horizontal acceleration.

In $\theta_2$, link $F$ is modeled as in figure 8 and entered in table 2, figure 11. $T_{gy}$ is the inertial torque due to an external vertical acceleration.

Link $F$ is influenced in $\theta_2$ by both vertical and horizontal accelerations. Figure 9 shows this relationship.

Motor Torque Calculations.

The motor data is given in figures 12 and 13. This data was supplied by Globe (motor type LL, 24 volts, winding number -1). Figure 14 demonstrates how an expression for the motor torque may be derived. The voltage from a to b is $RI - E_g$ where $E_g$ is the generated back emf.

From figure 12,

$$T_m = K_t - b$$

where $K_t$ is the torque constant.

Knowing that $E_g$ varies as $K_n$ times $n$ where $K_n$ is the motor generator constant and $n$ is the rpm, then

$$V = RI - E_g = R(T + b)/K_t - K_n(n)$$

$$T_m = (K_n n + V)K_t/R - b$$

From figure 12,

$$K_t = 2.38$$

$$b = .529$$

Given when $V = 24$ and $n = 0$ (from figure 13),

$$T_m = 5.5$$
Solving for \( R \),

\[ R = 9.47 \]

Then given when \( n = 11300 \) and \( V = 24 \),

\[ T_m = 0 \]

\( K_n \) is solved for giving,

\[ K_n = -0.001938 \]

and

\[ T_m = (-0.000487 n + 0.251 V - 0.529) \]

To obtain \( n \) in terms of \( \theta \),

\[ n = (\theta (60)/2 )N_g \]

where \( N_g \) is the geartrain ratio.

Finally,

\[ T_m = (-0.004651 N_g \dot{\theta} + 0.251 V - 0.529) \]

and the output of the motor-geartrain combination is \( T_m \) times \( N_g \) times the gearbox efficiency (\( \eta_g \)).
The equations generated in the last chapter are now combined in figure 15.

So that $T_m$ will be in $\text{lbm-in}^2/\text{sec}^2$,

$$\text{(in-oz x lbf/16 in-oz x 12 in/ft)}$$

$T_m$ is multiplied by $12/16$.

Also, since $g_h$ and $g_v$ are in terms of $\text{ft/sec}^2$, these terms are multiplied by 12 to put everything in correct units.

Notice that $\dot{\theta}/\dot{\theta}$ is multiplied times $0.529$ in the expression for $T_m$. This is necessary since $0.529$ represents coulomb friction which must act in opposition to the motion of the motor.

Figure 16 is a description of the user-defined blocks for the CSMP program which solves for $\alpha_1$ and $\alpha_2$. Figure 17 is a block diagram of the model.

A description of the block diagram is as follows:

<table>
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<th>Block(s)</th>
<th>Function</th>
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<tr>
<td>21, 18</td>
<td>generates either a stationary or moving target</td>
</tr>
<tr>
<td>6</td>
<td>subtracts target and damping ($B_1$) from the position and amplifies the result</td>
</tr>
<tr>
<td>15</td>
<td>limits the maximum and minimum voltage from the amplifier</td>
</tr>
</tbody>
</table>
1, 3, 4 generates $\alpha_1$ given $N_g$, $N_g'$, and $g_h$

8 integrates to velocity

9 integrates to position

The blocks in the lower half of the figure perform a similar function for $\alpha_2$. 
RESULTS

Since the model is a second order system, the solution can contain both real and imaginary parts. If the solution contains imaginary parts, the response is sinusoidal and will probably require some damping to limit the amplitude of the succeeding peaks. It is generally held that for optimum response, the damping should be approximately .7 of critical.°

The first trial was run without any added external damping, and the system was found to have an inherent damping ratio (β) of .27 for θ₂. By increasing B₂ (block 7 of figure 17) to .075, β was increased to .8. Similarly, by increasing B₁ (block 6 of figure 17) to .175, β for θ₂ was increased to .65.

The geartrain ratio (NG) was 48.4, and the efficiency (Ne) was .459. These figures were entered as parameters in blocks 1 and 2 of figure 17.

In order to generalize the data for selection of the optimum geartrains, the velocity-time curves were obtained (figure 21). Figure 23 shows the maximum velocity

---

which could be obtained with the 494.4 geartrain ratio. A good approximation to the velocity curve is a triangular waveform which becomes trapezoidal when the velocity reaches the limiting velocity. This is indicated by the dotted line in figure 18. Then the time to move and lock on any \( \theta \) is related to \( \theta \) by

\[
\theta = \frac{1}{4} mt^2 - \left( \frac{1}{2} mt - V_{lim} \right)^2 / m
\]

where \( m \) is the slope.

The limiting velocity for any geartrain ratio can be computed from the motor torque equation and the gear reduction ratio. Since the voltage to the motor is a function of the damping, the equation for \( T_m \) becomes

\[
T_m = -0.004651 N_g \dot{\theta} + 0.251(1432.39)(1 - B\dot{\theta}) - 0.529
\]

Knowing the values of \( B \) from the previous simulation, it is possible to compute the maximum velocity for any \( N_g \) by solving the case where \( T_m \) equals zero. Since the mechanism also has some internal damping and since the amplifier output is constrained to plus or minus 22 volts, the actual limiting velocity is less than that given in the above equation. Figure 19 provides the data for computed maximum velocity relative to actual maximum velocity, or actual maximum velocity equals .5727 times computed maximum velocity for \( \dot{\theta}_1 \) and .6746 times computed maximum velocity for \( \dot{\theta}_2 \).
Maximum velocity is also limited in that motor speed cannot exceed 11300 rpm (figure 13). This means that $\dot{\theta}$ is less than or equal to $1183.33/N_g$. Therefore, the actual maximum velocity is the actual to computed velocity factor times the smaller of the two computed velocities.

The slopes in figure 18 are easily calculated, and since the gear ratio is a torque multiplier, which translates into an acceleration multiplier, the slope for any $N_g$ is only a multiple of the ratio of $N_g$ in question to $N_g$ from the figure times the slope just calculated.

Finally, figure 20 shows the means to calculate the time to move to and lock on position $\theta$. Figures 21 and 22 demonstrate the relative position response times for various gear ratios.
CONCLUSIONS

The need for added damping of the mechanism was demonstrated in the last section. It was found that \( B_1 \) should equal \( .175 \) and \( B_2 \) should equal \( .075 \). This means that a tachometer voltage output used for damping as in figure 23 would be attenuated to \( .175 \) volts per degree per second for \( \dot{\theta}_1 \) and \( .075 \) volts per degree per second for \( \dot{\theta}_2 \).

In selecting the gear ratios, figures 27 and 22 indicate that there is a range of values for \( N_g \) which would be acceptable. For optimum response over the range of operation, \( N_g \) should equal approximately 900 for both \( \theta_1 \) and \( \theta_2 \). Since there is a 4:1 reduction upon exit from the gearmotor (figure 23), the gearmotor ratios should be 225:1. The closest gearmotor ratio supplied by Globe is 147.7:1.

The above parameters run on the computer yield an error less than 1 degree and are capable of tracking at 20 degrees per second. Additionally, the simulation performed with no ill effects when subjected to plus or minus 3 g's vertical and 1 g horizontal.
Figure 1

Top View

Side View

Scale 5:1
Figure 2

Link C

Scale 5:1

Figure 2
\[ J_x = \frac{1}{12}(m)(b^2 + c^2) + mD^2 \]

\begin{align*}
\text{Case 1:} & \quad m = 1.513 \text{ lbm} \\
& \quad b = 15.125 \text{ in} \\
& \quad c = 0.5 \text{ in} \\
& \quad d = 1 \text{ in} \\
& \quad J_x = 30.39 \text{ lbm-in}^2 \\
\text{Case 2:} & \quad m = 0.098 \text{ lbm} \\
& \quad a = 0.25 \text{ in} \\
& \quad D = 6.08 \text{ in} \\
& \quad J_x = 3.63 \text{ lbm-in}^2 \\
\text{Case 3:} & \quad m = 2.019 \text{ lbm} \\
& \quad b = 2 \text{ in} \\
& \quad c = 2.5 \text{ in} \\
& \quad D = 1 \text{ in} \\
& \quad J_x = 3.74 \text{ lbm-in}^2 \\
\Sigma J_x &= 37.76 \text{ lbm-in}^2
\end{align*}

Figure 6
\[ T_{t1} = -m_1 r^2 \alpha_2 \]
\[ m_1 = 0.25 \text{ lbm} \]
\[ r = 5.84 \text{ in} \]
\[ T_{t1} = -8.65 \times 10^2 \text{ lbm-in}^2/\text{sec}^2 \]
\[ T_{t2} = -m_2 d^2 \alpha_2 \]
\[ m_2 = 0.370 \text{ lbm} \]
\[ d = 3.25 \text{ in} \]
\[ T_{t2} = -3.91 \times 10^2 \text{ lbm-in}^2/\text{sec}^2 \]
\[ T_t = -12.56 \times 10^2 \text{ lbm-in}^2/\text{sec}^2 \]

\[ T_{gh} = (-m_1 r \cos \theta - m_2 d \cos(\theta + 15)) g_h \]
\[ = (-1.48 \cos \theta - 1.20 \cos(\theta + 15)) g_h \]

\[ T_{gv} = (m_1 r \sin \theta + m_2 d \sin(\theta + 15)) g_v \]
\[ = (1.48 \sin \theta + 1.20 \sin(\theta + 15)) g_v \]

\[ \text{lbm-in-ft/sec}^2 \]

Figure 8
$$T_{gh} = mr \sin \theta \; g_h$$

$m = 15 \text{ lbm}$

$r = 1 \text{ in}$

$$T_{gh} = 15 \sin \theta \; g_h \text{ lbm-in-ft/sec}^2$$

$$T_{sv} = mr \cos \theta \; g_v$$

$$T_{sv} = 15 \cos \theta \; g_v \text{ lbm-in-ft/sec}^2$$

Figure 9
\[ T_i = -ma_r r \]

\[
\begin{align*}
\text{m} &= 1.562 \text{ lbm} \\
\text{r} &= 7.628 \text{ in} \\
\alpha &= \text{r} \\
T_i &= -90.89 \alpha \text{ lbm-in}^2/\text{sec}^2
\end{align*}
\]

\[ T_i = -mr^2\alpha \]

\[
\begin{align*}
\text{m} &= .624 \text{ lbm} \\
\text{r} &= 1.5 \text{ in} \\
T_i &= -1.40 \alpha \text{ lbm-in}^2/\text{sec}^2
\end{align*}
\]

Figure 10
Table 1

<table>
<thead>
<tr>
<th>Link</th>
<th>$J$</th>
<th>$T_i$</th>
<th>$T_{gv}$</th>
<th>$T_{gh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>37.76 lbm-in²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>37.76</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>-90.89 lbm-in²/sec²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>-90.89 lbm-in²/sec²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>196.25 cos²θ₂ + 375 sin²θ₂</td>
<td>-26 gₜh sinθ₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td>-1.4α</td>
<td>km-lbm-in-ft/sec²</td>
<td></td>
</tr>
</tbody>
</table>

$\Sigma J = 75.52 + 196.25 \cos^2 \theta_2 + 375 \sin^2 \theta_2$ lbm-in²

$\Sigma T_i = -183.18 \alpha$ lbm-in²/sec²

$\Sigma T_{gv} = 0$

$\Sigma T_{gh} = -26 gₜh \sin \theta_1$ lbm-in-ft/sec²

$T_m = (-0.06201 N_g \ddot{\theta}_2 + 0.251 V - 0.529 \ddot{\theta}_2 / \ddot{\theta}_1) N_g$/N_in-oz

Table 2

<table>
<thead>
<tr>
<th>Link</th>
<th>$J$</th>
<th>$T_i$</th>
<th>$T_{gv}$</th>
<th>$T_{gh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>196.25 lbm-in²</td>
<td>15 gₜv cosθ₂</td>
<td>15 gₜh sinθ₂</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>-12.56 lbm-in²/sec²</td>
<td>1.48 sinθ₂ + 1.20 sin(θ₂+15) gₜv</td>
<td>1.48 cosθ₂ + 1.20 sin(θ₂+15) gₜv</td>
<td></td>
</tr>
</tbody>
</table>

$J = 196.25$ lbm-in²

$T_i = -12.56 \alpha$ lbm-in²/sec²

$T_{gv} = (15 \cos \theta_2 + 1.48 \sin \theta_2 + 1.20 \sin(\theta_2+15)) g_v$ lbm-in-ft/sec²

$T_{gh} = (-1.48 \cos \theta_2 + 15 \sin \theta_2 - 1.2 \sin(\theta_2+15)) g_h$

$T_m = (-0.06201 N_g \ddot{\theta}_2 + 0.251 V - 0.529 \ddot{\theta}_2 / \ddot{\theta}_1) N_g$/N_in-oz

Figure 11
Figure 12
<table>
<thead>
<tr>
<th>Nominal No load Speed</th>
<th>Rated Torque</th>
<th>Nominal Break-away Torque</th>
<th>No Load Current</th>
<th>Nominal Current At Rated Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>11,300 rpm</td>
<td>1 oz-in</td>
<td>5.5 oz-in</td>
<td>.230 amps</td>
<td>.650 amps</td>
</tr>
</tbody>
</table>

Figure 13
\[ \Sigma T = \alpha \Sigma J \]

\[ \alpha_1 = \frac{\Sigma T}{\Sigma J} = ((0.006201 N_g \dot{\theta}_1 + 0.251 V - 0.529 \dot{\theta}_1/|\dot{\theta}_1|)N_g N_g \]

\[ = ((0.004651 N_g \dot{\theta}_1 + 51 V - 0.529 \dot{\theta}_1/|\dot{\theta}_1|)N_g N_g \]

\[ - 312 \sin \theta_1 g_h - 183.18 \alpha)/(75.52 + 196.25 \cos^2 \theta_2 + 375 \sin^2 \theta_2) \]

\[ \theta_1 \]

\[ \alpha_2 = ((0.004651 N_g \dot{\theta}_1 + 188.25 V - 39675 \dot{\theta}_2/|\dot{\theta}_2|)N_g N_g \]

\[ + (180 \cos \theta_2 + 17.76 \sin \theta_2 + 14.4 \sin(\theta_2 + 15))g_v \]

\[ - (17.76 \cos \theta_2 - 180 \sin \theta_2 + 14.4 \sin(\theta_2 + 15))g_h - 12.56 \alpha)/196.25 \]

\[ \theta_2 \]

Figure 15
Description of User-Defined Blocks

Function S10(E1,E2,E3,P1,P2,P3)
TDOT=E1
THA=1.8825*V-.004651*TDOT/Abs(TDOT+.000001)
TRQ=(THA-TMB)*P1*P2

FUNCTION S11(E1,E2,E3,P1,P2,P3)
THEA1=E1
TDOT=E2
TI=31.2*SIN(THEA1)*P1+183.18*TDOT
TRQ=P3
SMT=TRQ-TI

FUNCTION S12(E1,E2,E3,P1,P2,P3)
THEA2=E1
SMT=E2
SKJ=75.52+196.25*COS(THEA2)**2+375*SIN(THEA2)**2
TDOT=SMT/SKJ

FUNCTION S13(E1,E2,E3,P1,P2,P3)
THEA2=E1
THA=180*COS(THEA2)+17.76*SIN(THEA2)+14.4*SIN(THEA2+.2618)
GRAVH=17.76*COS(THEA2)+180*SIN(THEA2)+14.4*SIN(THEA2+.2618)
SMT=TRQ+GRAV*P2-GRAVH*P1
TDOT=(SMT-12.56*TDOT)/196.25

FUNCTION S14(E1,E2,E3,P1,P2,P3)
ERR=E1
POS=E2
TDOTR=E3
VCHNG=(ERR-POS)-(TDOTR*P1)
VOUT=14.32.39*VCHNG

Motor Torque

Sum of Torques for θ1

Angular Acceleration for θ1

Angular Acceleration for θ2

Error Signal Output Following 25V/V amp. Stage

Figure 16
Figure 17
Figure 19
Given: slope, time, and limiting velocity such that slope is mirror image after $t=1/2T$

Find: area under curve

$$A_{t1} = \frac{1}{2} t \left( \frac{1}{2} mt \right) = \frac{1}{4} mt^2$$

$$H_{t2} = \frac{1}{2} mt - V_{lim}$$

$$B_{t2} = 2 \left( H_{t2} / m \right)$$

$$A_{t2} = \frac{1}{2} H_{t2} B_{t2} = \left( \frac{1}{2} mt - V_{lim} \right) \left( \frac{1}{2} mt - V_{lim} \right) / m$$

$$= \left( \frac{1}{2} mt - V_{lim} \right)^2 / m$$

$$A = A_{t1} - A_{t2} = \frac{1}{4} mt^2 - \left( \frac{1}{2} mt - V_{lim} \right)^2 / m$$

Figure 20
Figure 21
Figure 22
Globe Type LL 24V Winding Dash no. -

Motor $\theta_1$

Motor Gear Box $\theta_1$

Motor Gear Box $\theta_2$

Amp

Error Sig.

Inertial Elements

Globes (same as above)

Figure 23
Determination of Intruder Probability of Acquisition Grids
In this report, a low-altitude cruise, anti-ship missile is modeled on computer. The model describes the missile's search characteristics which can be altered depending on the launch mode. The search is autonomous and active radar. There are two basic search patterns employed by the missile: 1) only a small area surrounding the target is searched; range and bearing to the target are known. 2) an expanded pattern along the flight path of the missile is searched; only bearing to the target is known. In the two modes of launch, there are three launch platforms which may be used, aircraft, ship, or submarine.

The launch platform used does not significantly alter the search pattern; however, the launch mode makes a considerable difference. Therefore, the two modes will be considered separately.

For launch mode 1) an intruder on the range, possibly an undetected fishing boat, has been systematically placed around the target. Figure 7 shows the results which are contour lines of the probability that the intruder will be hit by the missile. For launch mode 2) the intruder has been systematically placed along the flight path of the missile. Figure 8 shows the results of these launches.
INTRODUCTION

The purpose of this report is to determine a probability grid which describes the probability of an anti-ship missile acquiring an intruder (i.e. fishing boat) given that the intruder is on the range and is undetected by range surveillance. The missile's search procedure will be modeled on a computer Monte Carlo style to generate the necessary data.

Unfortunately, the search procedure of the missile is classified and cannot be included in this report. However, a rudimentary explanation of it will be given. The report concerns itself instead with the analysis of the random elements which cause the missile's performance to be probabilistic. The Monte Carlo data can then be tabulated and contour lines constructed to put the data in graphical form.
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The model of the missile search procedure consists basically of six parts as shown in figure 2. This diagram is the same for both launch modes 1) and 2) as described in the summary. However, the outputs and the operation of the blocks will differ.

Upon launch the missile undergoes a heading correction to intercept the target position. This permits the missile to be launched at any angle, within limits, off the azimuth angle to the target. It then flies its corrected heading to the target and activates its radar seeker at a programmed position downrange which is a function of the range, the launch mode, and to some extent the launch platform (see summary). The actual position that the seeker turns on and the position at which the seeker is programmed to turn on may not be the same. Factors which cause this discrepancy are wind, downrange position uncertainty—uncertainties caused by the absence of complete uniformity in the missiles' propulsion systems, and sensor bearing error which is the error in the launch platform sensors to detect the exact azimuth angle to the target. The wind was modeled as a normal random variable of downrange and crossrange components. Each component would have a mean of 0 knots and a 1 sigma deviation of 10 knots. The two other factors were modeled similarly with the mean and 1 sigma deviation supplied by the contractor.
The seeker search pattern now opens up in an attempt to acquire the target. In correlation with figure 3, the search pattern is specified by a minimum and maximum range, a crossrange dimension (LCR), and a downrange dimension (LDR). This pattern will expand as the missile flies downrange. To determine if the missile has acquired a target, the tests shown in figure 4 are made. If either the target or the intruder is in the shaded area, it is considered for acquisition. Both objects have their own radar profile which is modeled by a single sweep probability of acquisition curve which is a function of the search range. A generated random number is compared with the acquisition probability of the seeker to determine if an acquisition has been made on that sweep. The missile position is then incremented for each radar sweep. In figure 4,

\[ \begin{align*}
Y_5 &= \text{missile crossrange position} \\
X_5 &= \text{missile downrange position} \\
X &= \text{downrange position of the target or intruder} \\
P_8 &= \text{seeker search angle} \\
F_3 &= \text{heading correction angle} \\
B_3 &= \text{sensor bearing error}
\end{align*} \]

In launch mode 1), the grid for positioning the intruder is that shown in figure 1. The target was located at several range distances and the intruder located at every grid position each time the target was moved.
In launch mode 2), the intruder was positioned according to the grid in figure 5.
RESULTS

The total number of launches per grid space in mode 1) was 72, and mode 2) it was 36. To determine when this number was sufficient for each launch and not use up more computer time than necessary, a curve fitting technique was employed. The general shape of the probability contours was assumed to be elliptical. Then a least squares curve fit to an ellipse could be made from an ordinary least squares routine to fit a second order polynomial by recognizing the following:

\[ y = a + bx + cx^2 \]

second order polynomial

\[ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \]

ellipse with center at \( h, k \)

and major axis parallel to the \( x \) axis

but since the major axis is coincident with the \( x \) axis, \( k = 0 \) and,

\[ y^2 = A + Bx + Cx^2 \]

where the solution for \( y^2 \) is in the form of a second order polynomial.

Then wherever \( y \) would ordinarily be entered into the least squares routine, enter \( y^2 \), and the resulting equation for \( y \) will be

\[ y = \left( A + Bx + Cx^2 \right)^{1/2} \]

This procedure was used in determining the four probability contours in the upper diagram of figure 6 about half way through the Monte Carlo simulation in both mode 1) and 2). It can be seen that one of the contours crosses over others in the diagram. As long as this happens, the
data is considered to be insufficient since if the lines were truly lines of constant probability it would be impossible for any of them to cross. The Monte Carlo was stopped when a good profile was obtained as in the lower diagram of figure 6.

Although the data was considered good, there were still small irregularities which needed to be smoothed or filtered in order to connect the probability points with continuous contour lines. To filter the data,

\[ Pa(i,j) = 0.7 \cdot Pa(i,j) + 0.064 \cdot (Pa(i,j-1) + Pa(i-1,j) + Pa(i,j+1) + Pa(i+1,j)) + 0.011 \cdot (Pa(i-1,j-1) + Pa(i-1,j+1) + Pa(i+1,j-1) + Pa(i+1,j+1)) \]

where \((i,j)\) denote row and column grid designations, and \(Pa\) is the probability associated with that grid space. The magnitude of the three coefficients 0.7, 0.064, and 0.011 were assigned for three reasons:

a) to insure that at each iteration the original value did not change too much

b) to weight immediately adjacent grid spaces more than diagonally adjacent grid spaces

c) to insure that the filtering would not have the overall effect of raising or lowering the probabilities

Five iterations of filtering were considered to be sufficient, after which the contour lines of constant probability in figure 7 could be constructed. Figure 7 is the probability map for mode 1) launches, and figure 8 is the result of the same procedure for mode 2) launches.
CONCLUSIONS

The results of this study should not be construed to indicate the hazard pattern for the missile if the range to the target is known. For that case these results would be quite inaccurate. However, to be used in a study external to this report, these results are quite useful. In the external study the problem is to determine the probability of hitting an undetected intruder somewhere on the range for all missile firings and all types of missiles during a certain time period. What has been indicated here is given that the intruder is somewhere on the range it describes the probability that it will be hit during a launch of this particular missile. Since over a period of time, the launch when the intruder appears on the range cannot be known, the launch range in this study must appear as a variable.
$Y$ - crossrange coordinate
$G5$ - off axis launch angle
$R_S$ - range to target at launch
$X$ - downrange coordinate

Figure 1
Figure 2
RS_MIN = R* - LDR

z4 ≤ RS_MIN ≤ a5

RS_MAX = R* + LDR

b5 ≤ RS_MAX ≤ c5

DRS = RS_MAX - RS_MIN

d5 ≤ DRS ≤ e5

PSI = ATAN(LCR/(R* + CDR)) - f5

g5 < PSI ≤ h5

Seeker
Search Limits

Figure 3
$Y_5 + (X_5 - X) \cdot \tan(P_8 - (F_3 - B_3))$

$Y_5 - (X_5 - X) \cdot \tan(P_8 + (F_3 - B_3))$

**Figure 4**
Figure 5.
Intruder Probability of Acquisition Grid

Figure 7
Intruder Probability of Acquisition Grid