CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

TARGET MOTION SIMULATION AND ANALYSIS

A thesis submitted in partial satisfaction of the requirements for the degree of Master of Science in Engineering

by

James Lowell Moore, Jr.

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The thesis of James Lowell Moore, Jr., is approved:

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ABSTRACT

TARGET MOTION SIMULATION AND ANALYSIS
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This paper presents the development of a technique for the evaluation of passive sonar Target Motion Analysis (TMA) methods. A computer model is developed which (1) simulates a variety of target and observer (own ship) engagement geometries and (2) generates noisy target relative bearing data from a zero mean Gaussian process, with no presmoothing for these geometries.

Also presented is the development of a TMA procedure which utilizes the Rosenbrock minimization technique. The FORTRAN subroutines developed to implement the Rosenbrock method are general in format. These subroutines are capable of solving any four variable unconstrained minimization problem and, by a simple extension of the program logic, can be used to solve any unconstrained minimization problem.
The computer simulation model is then used to compare the Rosenbrock method with a modified version of a well-known matrix inversion technique (CHURN). This comparison is made over a range of engagement geometries, ambient noise levels, and time intervals between bearing measurements (TBBM). The results indicate that the Rosenbrock method produces solutions as accurate as the modified version of CHURN, without the risk of generating singular or near singular matrices. However, the Rosenbrock method requires considerably more computer time and is more sensitive to TBBM.

The computer simulation model is demonstrated to be an effective tool for evaluating TMA techniques in the passive sonar environment. With a minimum of modification, the simulation model can be adapted to evaluate any passive sonar TMA technique.
1.0 INTRODUCTION

Techniques for detecting and locating objects in the ocean, by the sounds emitted by these objects, have been of interest for many years. While the modern age of sonar may be said to date back to the start of World War II, sonar has its origins deep in the past. For example, in 1490 Leonardo da Vinci wrote: "If you cause your ship to stop, and place the head of a long tube in the water and place the outer extremity to your ear, you will hear ships at a great distance from you." [4] From these simple beginnings sonar has developed into a useful tool for the ocean environment.

Historically, the main impetus for sonar development has come from its military applications. However, as the food and mineral resources on land are depleted, increased emphasis is being placed on the development of oceanological and industrial uses of sonar. Some of these recent applications are the fish finder, river flowmeter, acoustic navigation beacon, and acoustic ship docking systems. [1, 8, 22, 24]

Sonar systems are generally classified as either active or passive. Sonar systems are said to be active when sound is purposely generated by one of the system components. The sound waves generated by this component travel through the sea to the target, and are returned as
sonar echoes to a hydrophone, which converts sound into electricity. This electrical output is amplified and processed in various ways and is finally applied to a control or display device to accomplish the purpose for which the sonar set was intended. Active sonar systems are said to "echo-range" on their targets. Passive or listening sonar systems use sound radiated by the target. Here, only one-way transmission through the sea is involved, and the system centers around the hydrophone used to listen to the target sounds. [22]

This research explores the use of passive sonar systems to locate targets moving on the surface of the ocean. There are three basic kinds of underwater noise which emerge in the passive sonar process: radiated noise, self-noise, and ambient noise.

Radiated noise is the acoustic output of marine vehicles and can be utilized by passive sonar systems to locate and track these vehicles. Whenever a vehicle is propelled through the ocean, radiated noise is produced. The magnitude of this noise depends on many factors, but it can be classified as follows:

1. Hydroelastic noise is the "singing" produced by a propeller blade or a structural member as it moves through the water. This noise is not always present, and the tendency to produce this noise can be controlled by proper design.
2. Under certain conditions, pressures are reduced at points on the hull or the thrust producer to values less than that of the vapor pressure of water, and bubbles are formed. Cavitation noise is produced when these bubbles collapse as they move into areas of higher pressure. The tendency to produce cavitation is a function of vehicular speed and the ambient pressure. This tendency is not always present, and the minimum speed at which cavitation can occur for a given ambient pressure can be controlled to a certain extent by the design of the surfaces.

3. The nature of flow noise is not yet completely understood. It may be associated with turbulence; and, if it is, it can be controlled by design. However, there is evidence that flow noise is produced even when turbulence is absent; if this is true, there will always be flow noise when a vehicle is moving in the water.

4. Machinery noise may be generated by equipment used to propel the vehicle or to perform auxiliary functions within the vehicle. Any operating machinery, whether it is an engine used to generate power or a relay used to perform some electrical switching function, is a source of noise. Mechanical noise, therefore, is always present unless the vehicle is propelled by its own buoyancy or by an initial impulse and it contains no moving mechanical component. The magnitude of mechanical noise depends on the design of the moving mechanical components and the effectiveness with which they are acoustically isolated. [2, 24]
Self-noise is the radiated noise generated by the passive sonar system and its platform. Since self-noise originates with the passive sonar system and its platform, it is therefore subject to control by appropriate design and usage of the system and platform. [24] For example, one tactic often employed to minimize self-noise is described as sprint and coast. That is, the searcher sprints to a location and then coasts with engines and other equipment secured, listening for the target. [16]

Ambient noise is produced by a number of environmental factors. Weather conditions, geographic location, marine life, and seismic activity all impact the ambient noise level. Since these environmental factors are beyond control, research in this area has focused on developing a capability for predicting ambient noise characteristics. These studies suggest that, if the sample is clean, that is, does not contain obvious noise from nearby seismic activity, biological sources and the like, then the instant value statistics, such as amplitude distributions, are Gaussian. [3, 7, 12, 24]

Thus the passive sonar system provides relative bearing information for a radiated noise source corrupted by additive ambient noise. The obvious tactical advantage of using this information to locate and track a target, without risking detection by the target, has given impetus to the development of techniques for tracking targets using passive sonar. The methods developed to date deal primarily
with constant velocity targets. One of the most prominent of these methods is a collection of matrix inversion techniques which are loosely referred to as the CHURN techniques.

The original CHURN technique was developed by the U.S. Navy for the Fire Control System MK 113 Mod 2. [10] An unclassified version of this technique was presented by Bruckheim and O'Neil (B&O) [5] as a basis for comparison with a target motion analysis technique they had developed. The CHURN technique presented by B&O solves the target motion problem by minimizing the noise which is corrupting the relative bearing measurement. This minimization is accomplished by a least-squares procedure which sets the first partials of the sum of the squares of the error equal to zero. The resulting set of equations is then solved by matrix inversion. In addition, this method does not pre-smooth the raw bearings, nor does it impose any constraints on range, bearing, course, or speed.

Due to the nature of the target motion analysis problem, the matrix inversion technique used by CHURN is constantly running the risk of developing a singular or near singular matrix. It would therefore be useful if an alternative method to CHURN could be found, which does not use matrix inversion, to solve the target motion problem.

Thus, the major objectives of this research are to:

1. Develop a computer simulation model capable of evaluating target motion analysis techniques.
2. Develop an alternative method to the CHURN technique for solving the target motion problem.

3. Evaluate the effects of target maneuvers on both CHURN and the alternate target motion analysis method.
2.0 APPROACH

The development of the models required to achieve the objectives of this paper can be divided into three phases. The first phase is to develop a model to simulate the relative motion of two ships in the passive sonar environment. The second phase is to implement the modified CHURN technique to meet the objectives of this paper. The third and final phase is the development of the alternative to the CHURN technique.

2.1 Relative Motion Simulator

The initial task of this study was to develop a simulation model for the relative motion of two ships. It was also required that the model simulate erroneous relative bearing measurements between the two ships. This simulation model will be referred to as the Relative Target Motion Analysis (RTMA) model.

The development of the RTMA model involved the following steps:

1. Definition of the coordinate system.
2. Development of a method for representing target and observer (own ship) motion in the defined coordinate system.
3. Definition of the actual bearing from the relative motion of the two ships.
4. Development of the simulated bearing error term (noise).

2.1.1 Coordinate system

The coordinate system used for the RTMA model (see figure 2-1) is a north-east right-hand coordinate system whose axes are directed north and east from the origin. The north-directed axis is the positive \( y \) axis, the east-directed axis is the positive \( x \) axis. The north-east plane is horizontal and lies in the plane of the sea. [23] Horizontal heading angles are measured clockwise with respect to the \( y \)-axis. For example, a ship proceeding due east would have a heading angle of \( 90^\circ \).

2.1.2 Target and own-ship motion

To provide the greatest flexibility in the type of target and own-ship engagement geometries which can be simulated, the RTMA model allows changes in speed, and provides a mechanism to allow several types of target or own-ship motion. The ability to change either target or own-ship speed was accomplished by simply specifying the number of bearing measurements between a change in speed and the new speed. There are three types of motion currently defined by the RTMA model for either the target or own ship. These three motion types are linear, linear maneuvering (zig-zag), and nonlinear (sinusoidal). The equations used to implement the motion types are developed in the following paragraphs.
Figure 2-1. Coordinate system used for the Relative Target Motion Analysis (RTMA) Model
Linear motion is simulated by specifying a course heading, \( \theta \), and initial position \((X,Y)\). The velocity components and new positions of either the target or own ship are generated by using the following equations

\[
\begin{align*}
V_x &= V \sin \theta \quad (1) \\
V_y &= V \cos \theta \quad (2)
\end{align*}
\]

where

- \( V_x \) = x component of the velocity
- \( V_y \) = y component of the velocity
- \( V \) = velocity vector
- \( \theta \) = heading angle

and

\[
\begin{align*}
X_i &= X_{i-1} + V_x t_i \quad (3) \\
Y_i &= Y_{i-1} + V_y t_i \quad (4)
\end{align*}
\]

where

- \( X_i \) = x coordinate of position
- \( Y_i \) = y coordinate of position
- \( t_i \) = time period between relative bearing readings

Linear maneuvering (zig-zag) motion is a simple extension of linear motion. The velocity components and position of the vessel are calculated using the linear motion equations. The zig-zag motion is achieved by implementing these equations according to the following scheme. First, the number of course changes are specified as a function of the number of relative bearing readings between each course change. Second, the heading angle after each course change is defined by
\[ \theta_n = \theta \quad n = 1, 3, 5, \ldots, M \]  
\[ \theta_m = \theta + \frac{\pi}{2} \quad m = 2, 4, 6, \ldots, M \]

where \( \theta \) = initial specified own ship or target heading angle

\( M \) = number of course changes during simulation

Nonlinear (sinusoidal) motion is simulated using a sine wave function. The target or own-ship velocity and position components are approximated in the following manner. [11, 15]

Let

\( A \) = amplitude of the sine function

\( P \) = period of the sine function

\( S \) = speed of the vessel (constant)

\( K = \frac{2\pi}{P} \)

Then for the \( i^{th} \) iteration, we can define

\[ Y_i = A \sin K X_i \]

where

\( Y_i \) = \( y \) coordinate of the vessel at \( t_i \)

\( X_i \) = \( x \) coordinate of the vessel at \( t_i \)

The \( y \) component of the velocity is given in terms of the \( x \) component \((V_x)\) as

\[ V_{y_i} = (K A \cos K X_i) V_{x_i} \]  

The speed of the target is given by

\[ S^2 = V_{y_i}^2 + V_{x_i}^2 \]

Substituting equation (9) into equation (10) yields
\[ S^2 = V_{x_i}^2 + (K A \cos K X_i)^2 V_{x_i}^2 \]  
(11)

or

\[ S^2 = V_{x_i}^2 [1 + (K A \cos K X_i)^2] \]  
(12)

Solving equation (12) for \( V_{x_i} \) gives

\[ V_{x_i} = \frac{S}{[1 + (K A \cos K X_i)^2]^{1/2}} \]  
(13)

The x coordinate of the target can then be approximated for the \( i \)th iteration in terms of the previous value using

\[ X_i = X_{i-1} + V_{x_{i-1}} t_i \]  
(14)

Thus, by simply specifying an initial position and speed, equations (8), (9), (13) and (14) may be solved to provide target or own-ship velocity and position components.

2.1.3 Relative bearing angle between target and own ship

By using the appropriate equations, described in paragraph 2.1.2, for the type of motion being simulated, the target and own-ship position at any time \( t \) is known. Therefore, the relative bearing angle \( \beta \) (see figure 2-2) can be defined by

\[ \tan \beta_i = \frac{\Delta X_i}{\Delta Y_i} \]  
(15)

where \( \beta_i \) = bearing angle at time \( t_i \),
\( \Delta X_i \) = distance between own ship and target in X direction
\( \Delta Y_i \) = distance between own ship and target in Y direction
Figure 2-2. Target and own ship position at time $t_i$
Solving equation (15) for $\beta_i$ we have

$$\beta_i = \tan^{-1} \left( \frac{\Delta X_i}{\Delta Y_i} \right)$$

(16)

The actual observed relative bearing $B_i$ would be given by:

$$B_i = \beta_i + \epsilon$$

(17)

where $B_i =$ observed relative bearing at time $t$

$\epsilon =$ error or ambient noise level of the relative bearing measurement.

The term $\epsilon$ will be developed in the following paragraphs.

2.1.4 Development of the simulated error term (ambient noise)

The RTMA model is independent of the distribution of the error term; however, as indicated earlier, the work of Calderon, Green and Arase [7, 12, 3] indicated that ambient noise levels have the form of a Gaussian random variable.

The Gaussian or standard normal distribution is a continuous, symmetrical distribution, with the mean at the origin. The Gaussian distribution has been frequently applied in computer modeling and simulation to describe most measurement phenomena. For example, measurement error in angular or linear dimensions, scores on a test, heights or weights of men, women or children, all have been shown to follow a Gaussian distribution. [17, 21]

The Gaussian distribution derives its usefulness from the Central Limit Theorem. The Central Limit Theorem
states that the probability distribution of the sum of $M$
indeed and identically distributed random numbers
$X_i$ with respective means $\mu_i$ and variances $\sigma_i^2$, as $M$ becomes
very large, asymptotically approaches the Gaussian distri-
bution with a mean and variance given by

$$\mu = \sum_{i=1}^{M} \mu_i \quad (18)$$
$$\sigma^2 = \sum_{i=1}^{M} \sigma_i^2 \quad (19)$$

Therefore, the Central Limit Theorem allows the
use of a Gaussian distribution to represent overall measure-
ments on effects of independently distributed additive
errors regardless of the distribution of the measurements
of individual errors. [19]

A mathematical interpretation of the Central Limit
Theorem may be given. If $r_1, r_2, \ldots, r_N$ are uniformly dis-
tributed independent random variables (i.e., $\mu_i =$ constant
$= \theta$ and $\sigma_i^2 =$ constant $= \sigma^2$), then as $M$ becomes large, equa-
tions (18) and (19) are given by

$$\mu = \sum_{i=1}^{M} \mu_i = M \theta \quad (20)$$
$$\sigma^2 = \sum_{i=1}^{M} \sigma_i^2 = M \sigma^2 \quad (21)$$

and from the standard form of the Gaussian distribution

$$z = \frac{x - \mu}{\sigma} \quad (22)$$
Substituting the values of \( \mu \) and \( \sigma \) given by equations (20) and (21) into equation (22) yields

\[
Z = \frac{\sum_{i=1}^{M} r_i - M \theta}{\sqrt{M} \sigma}
\]  

Then from the definition of a standard Gaussian distribution equation (23) defines a standard Gaussian variate.

Thus, for a set of \( K \) uniformly distributed random variables defined over the interval \( 0 \leq r_i \leq 1 \), \( \theta \) and \( \sigma \) are given as follows:

\[
\theta = \frac{a + b}{2} = \frac{0 + 1}{2} = \frac{1}{2}
\]

\[
\sigma = \frac{b - a}{\sqrt{12}} = \frac{1 - 0}{\sqrt{12}} = \frac{1}{\sqrt{12}}
\]  

Substituting equations (24) and (25) into equation (23) yields

\[
Z = \frac{\sum_{i=1}^{K} r_i - K}{\sqrt{K/12}}
\]  

Substituting the value of \( Z \) given in equation (22) into equation (26) and solving for \( X \) we obtain

\[
X = \sigma_x \left( \frac{12}{K} \right)^{1/2} \left[ \sum_{i=1}^{K} r_i - \frac{K}{2} \right] + \mu_x
\]

where \( X \) is a Gaussian distributed random variable.

The literature \([19, 21]\) recommends that the smallest value of \( K \) used to generate \( X \) be \( K = 10 \). Therefore, by choosing \( K = 12 \), equation (27) simplifies to
\[ X = \sigma_X \left( \sum_{i=1}^{12} r_i - 6.0 \right) + \mu_X \]  \hspace{1cm} (28)

By defining

\[ \epsilon = \sigma \left( \sum_{i=1}^{12} r_i - 6.0 \right) \]  \hspace{1cm} (29)

then equation (28) has the same form as equation (17) and the error or ambient noise is defined by equation (29).

The RTMA model described in section 2.1 has been implemented as part of the Target Motion Simulation and Analysis Program (TMSAP) described in Appendix A. The RTMA portion of TMSAP simulates a specified target and own-ship engagement geometry for a specified number of time units. Using this information, TMSAP calculates at specified time intervals actual values of target and own-ship position, target velocity, and actual relative bearing. TMSAP then calculates the ambient noise term given by equation (29) and adds it to the actual relative bearing to obtain the observed bearing measurement.

2.2 Implementation of the CHURN Technique

The CHURN technique provides the initial position and velocity components of a target traveling in a straight line at a constant speed. Therefore, once the CHURN technique has converged on these target values, the target position and speed for linear motion at any time \( t_i \) is easily calculable. Since in this application (especially nonlinear motion) the current target position and speed,
rather than the initial conditions, are of primary interest, it would be advantageous if the CHURN technique could be modified to directly solve for the current values.

This objective can be accomplished by a simple reformulation of the basic CHURN technique. The reformulated CHURN technique, which will be referred to as the PREDIC technique, begins with equation (15) and proceeds in the following manner.

In equation (15), the relative bearing angle was defined at time $t_i$ as

$$\tan \beta_i = \frac{\Delta X_i}{\Delta Y_i}$$  \hspace{1cm} (30)

Since from figure 2-2

$$\Delta X_i = X_{T_i} - X_{S_i}$$  \hspace{1cm} (31)

$$\Delta Y_i = Y_{T_i} - Y_{S_i}$$  \hspace{1cm} (32)

where $X_{T_i} = X$ coordinate of target at time $t_i$

$X_{S_i} = X$ coordinate of own ship at time $t_i$

$Y_{T_i} = Y$ coordinate of target at time $t_i$

$Y_{S_i} = Y$ coordinate of own ship at time $t_i$

Substituting equations (31) and (32) into equation (30) yields

$$\tan \beta_i = \frac{X_{T_i} - X_{S_i}}{Y_{T_i} - Y_{S_i}}$$  \hspace{1cm} (33)

If there were no ambient noise (i.e., $\epsilon = 0$), equation (30) could be written
\[
\frac{\sin \beta_i}{\cos \beta_i} = \frac{X_{T_i} - X_{S_i}}{Y_{T_i} - Y_{S_i}}
\]  
(34)

By rearranging equation (34), a residual, \( R_{L_i} \), could be formed such that

\[
R_{L_i} = (X_{T_i} - X_{S_i}) \cos \beta_i - (Y_{T_i} - Y_{S_i}) \sin \beta_i
\]

\[
= 0
\]  
(35)

However, since \( \epsilon \neq 0 \), the observed bearing \( B_i \) is given by equation (17) as

\[
B_i = \beta_i + \epsilon
\]  
(36)

Substituting equation (36) into equation (35) gives

\[
R_{L_i} = (X_{T_i} - X_{S_i}) \cos B_i - (Y_{T_i} - Y_{S_i}) \sin B_i
\]

\[
\neq 0
\]  
(37)

Up to this point, the development of PREDIC has been identical to the CHURN technique. [5] The only real difference between the two methods centers around the value of the target position \((X_{T_i}, Y_{T_i})\) which is substituted into equation (37). The CHURN technique substitutes the following equations into equation (37)

\[
X_{T_i} = X_{T_0} + V_x t_i
\]  
(38)

\[
Y_{T_i} = Y_{T_0} + V_y t_i
\]  
(39)

where \( X_{T_0} = \) initial X coordinate of the target

\( Y_{T_0} = \) initial Y coordinate of the target

\( V_x = \) initial X component of velocity
\[ V_Y = \text{initial } Y \text{ component of velocity} \]
\[ t_i = \text{elapsed time} \]

The PREDIC technique, however, substitutes the following equations into equation (37)
\[ X_{T_i} = X_{T_{i+1}} - V_x \Delta t_i \quad (40) \]
\[ Y_{T_i} = Y_{T_{i+1}} - V_y \Delta t_i \quad (41) \]

Thus, the PREDIC residual is given by the equation
\[ R_{L_i} = (X_{T_{i+1}} - V_x \Delta t_i - X_{S_i}) \cos B_i \]
\[ - (Y_{T_{i+1}} - V_y \Delta t_i - Y_{S_i}) \sin B_i \quad (42) \]

\( R_{L_i} \) then is a direct measure of the ambient noise level because equation (42) would equal zero if \( \varepsilon = 0 \). If we choose a vector of unknowns \( \mathbf{V} \) defined by
\[
\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_x \\ X_{T_{i+1}} \\ V_y \\ Y_{T_{i+1}} \end{bmatrix} \quad (43)
\]

then the estimation problem is to choose a value of \( \mathbf{V} \) which minimizes the sum of the squares of \( R_{L_i} \). Thus, the problem becomes to find
\[
\sum_{i=0}^{m} R_{L_i}^2 = \min \quad (44)
\]

where \( m = n-1 \), \( n = \text{the number of relative bearing measurements} \). The solution to equation (44) is obtained by solving the following equation
\[ \sum_{i=0}^{m} R_i \frac{3 R_i}{3 V_j} = 0 \quad j = 1, 2, 3, 4 \]  \hspace{1cm} (45)

where \( V_j \) is defined in equation (43). Substituting equation (42) into equation (45) and rearranging provides a vector equation of the form

\[ \mathbf{V} = \mathbf{A}^{-1} \mathbf{B} \]  \hspace{1cm} (46)

where

\[ \mathbf{V} = \begin{bmatrix}
-[(m-1) t_1 \cos B_1 (X_{S_1} \cos B_1 - Y_{S_1} \sin B_1)] \\
\cos B_1 (X_{S_1} \cos B_1 - Y_{S_1} \sin B_1) \\
[(m-1) t_1 \sin B_1 (X_{S_1} \cos B_1 - Y_{S_1} \sin B_1)] \\
-\sin B_1 (X_{S_1} \cos B_1 - Y_{S_1} \sin B_1)
\end{bmatrix} \]

\[ \mathbf{X} = \begin{bmatrix}
[(\Delta t_1)^2 \cos^2 B_1] \\
\Delta t_1 \cos^2 B_1 \\
-(\Delta t_1)^2 \sin B_1 \cos B_1 \\
\Delta t_1 \sin B_1 \cos B_1
\end{bmatrix} \]

The solution of equation (46) yields the current values of target position and speed.

The PREDIC method described in this section has been implemented as a subroutine in the TMSAP computer program described in Appendix A. The TMSAP computer program simulates relative bearings corrupted by additive noise and own-ship position. The PREDIC subroutine uses this data to form the matrices \( \mathbf{A} \) and \( \mathbf{B} \) of equation (46).
A standard FORTRAN routine is then utilized to solve equation (46) for the current values of target position and velocity.

2.3 Development of an Alternative to the \textsc{CHURN} Technique

While \textsc{PREDIC} obtains a solution by minimizing the linearized residual given by equation (46), with only moderate requirements on computer speed and memory, a reduction in accuracy must be accepted.

This reduction in accuracy results from the fact that minimizing the residual given by equation (46) does not simultaneously minimize the actual error term. This may be illustrated for the \textsc{PREDIC} technique in the following manner.

If we begin again with the true identity given by equation (35), we have

\[
(XT_{i+1} - V_x \Delta t_i - XS_i) \cos \beta_i
- (YT_{i+1} - V_y \Delta t_i - YS_i) \sin \beta_i = 0
\]  

(47)

However, from equation (17)

\[
\beta_i = B_i - \epsilon_i
\]

(48)

For ease of notation, let

\[
A_i = XT_{i+1} - V_x \Delta t_i - XS_i
\]

(49)

\[
C_i = YT_{i+1} - V_y \Delta t_i - YS_i
\]

(50)
Substituting equations (48), (49) and (50) into equation (47) yields

\[ A_i \cos (B_i - \varepsilon_i) - C_i \sin (B_i - \varepsilon_i) = 0 \]  

(51)

Expanding equation (51) and using the following approximations

\[ \sin \varepsilon_i \approx \varepsilon_i \]
\[ \cos \varepsilon_i \approx 1, \text{ small } \varepsilon_i \]  

(52)

equation (51) becomes

\[ A_i (\cos B_i \cos \varepsilon_i + \sin B_i \sin \varepsilon_i) - C_i (\sin B_i \cos \varepsilon_i - \cos B_i \sin \varepsilon_i) = 0 \]

\[ A_i (\cos B_i + \varepsilon_i \sin B_i) - C_i (\sin B_i - \cos B_i \varepsilon_i) = 0 \]

\[ (A_i \cos B_i - C_i \sin B_i) + \varepsilon_i (A_i \sin B_i + C_i \cos B_i) = 0 \]

\[ \varepsilon_i = \frac{C_i \sin B_i - A_i \cos B_i}{C_i \cos B_i + A_i \sin B_i} \]  

(53)

Substituting the values of \( A_i \) and \( C_i \) given by equations (49 and 50) into equation (53) gives

\[ \varepsilon_i = \frac{(Y T_{i+1} - V_x \Delta t_i - YS_i) \sin B_i - (X T_{i+1} - V_x \Delta t_i - XS_i) \cos B_i}{(Y T_{i+1} - V_x \Delta t_i - YS_i) \cos B_i + (X T_{i+1} - V_x \Delta t_i - XS_i) \sin B_i} \]  

(54)

Notice that the numerator of equation (54) is the same as the residual given by equation (35). Since \( \varepsilon_i \) of equation
(54) is the quantity which really should be least-squares minimized, the PREDIC approach accounts only for the numerator without regard to the denominator. Thus, to determine the target's current position and speed, the following problem should be solved:

\[
\sum_{i=0}^{m} \varepsilon_i^2 = \min
\]

(55)

where \( \varepsilon_i \) is defined by equation (54).

Therefore, the alternative method should, if possible, be capable of solving both equation (44) and equation (55).

Since the minimization problem given by equation (55) is very difficult to solve by analytic methods, a numerical technique must be utilized to provide a solution. There are quite a number of numerical techniques capable of minimizing equations of the form given by equation (55). However, the objectives of this study restrict the candidates for selection to those techniques which meet the following criteria:

1. Due to the possibility that a maneuvering target will produce a singular matrix, the technique should not utilize matrix inversion.

2. The technique should not require the formulation of a gradient because of the difficulty of defining derivatives for equation (55).
3. The technique should be able to solve unconstrained minimization problems.

4. The technique should strike a balance between speed of convergence and computer programming complexity.

The procedure developed by H. H. Rosenbrock [20] has characteristics which satisfy all four of the selection criteria. The Rosenbrock method is a modification of the alternating variable method. The alternating variable method chooses each variable in turn, keeping all others constant, and the extremum is obtained by an appropriate single variable search method. While this method is very straightforward, it suffers from this simplicity by being very slow. [6]

The two modifications to the alternating variable method proposed by Rosenbrock produced one of the most robust methods available for unconstrained optimization when derivatives are not available. The first of the modifications is to avoid the single variable optimization for each direction in turn. Instead, a step of predetermined length is taken in each direction and these step lengths are modified after each calculation. For example, if a step is taken in the "X1-direction" and a better result is obtained, this is considered a "good" direction and the stepsize is increased (usually by a factor of 3) for the next exploration of the X1-direction. If a "worse" result is produced, then a shorter stepsize (usually one-half the previous stepsize) in the opposite direction is chosen for
the next search in the X1-direction. This procedure is followed as each of the variables are considered in turn. [6, 9, 14]

The second modification resulted from Rosenbrock recognizing that, if the search axes were reoriented along the most successful overall direction, the number of steps required to find the optimum would be reduced considerably. The question then became when and how to change the axes. The first question can only be answered by experience, and it has been found that the most useful criterion is to change axes when at least one "success" and one "failure" have been obtained in each of the N directions of an N variable problem. The question of how to change the axes has been resolved by using the Gram-Schmidt Orthogonalization Process. [6, 9, 14]

The first direction is taken to be the resultant of the steps taken with the previous set of directions. The remainder are calculated by the Gram-Schmidt Orthogonalization Process. This process produces a set of N orthonormal unit vectors for any set of N linearly independent vectors. The detailed proofs of independence, etc., can be found in Mirsky. [18]

To implement the Rosenbrock method for the four variable minimization problems given by equations (44) and (55), an arbitrary starting point and stopping criteria must be defined. The arbitrary starting point (Vx, XT, Vy, YT) is defined by the following scheme depending on the
The stopping rule employed for this application was to assume a solution has been reached when (1) the stepsize is equal to or less than 0.0001, or (2) the absolute difference between final function values in successive iterations is less than or equal to 0.001. These values for stepsize and the absolute difference were found by trial and error to be the best compromise between accuracy and the amount of computer time required for convergence.

The Rosenbrock method described in this section has been implemented as three subroutines in the TMSAP computer program outlined in Appendix A. The TMSAP computer program simulates the observed bearings corrupted by additive noise and the position of own ship. The first subroutine (FUNCT) uses this information to form the sum of the squares for the appropriate error residual. The second subroutine (ROSEN) begins at an arbitrary starting point and minimizes the error term generated by FUNCT. The third subroutine (GRAM) is used by ROSEN to produce the orthogonal unit vectors used during the minimization process.
3.0 RESULTS

Comparisons between the PREDIC and Rosenbrock techniques for the problem given by equation (44) were made by simulating several engagement geometries under various noise level conditions and time intervals between bearing measurements (TBBM). For all comparisons considered, own ship followed the modified linear (zig-zag) cycle given in figure 3-1, with magnitudes of the X and Y components of velocity each equal to two. The number of these cycles simulated for own ship was a function of the total time simulated for each geometry.

There are four target vector elements estimated by each technique. The convergence pattern for all four elements is approximately the same for each method. Therefore, only the absolute difference between the actual and numerical solution for the X component of target position was used to compare these methods.

Three different linear engagement geometries were considered. These are illustrated in figures 3-2 through 3-4, and may be referred to as closing target, crossing target, and opening target geometry, respectively. Four maneuvering target engagement geometries were evaluated and are depicted in figure 3-5.
Figure 3-1. Own ship trajectory for all geometries
Figure 3-2. Closing target geometry

Figure 3-3. Crossing target geometry
Figure 3-4. Opening target geometry

Figure 3-5. Maneuvering target geometry
3.1  **Sensitivity of the PREDIC and Rosenbrock Solutions to a Change in the Time Interval Between Bearing Measurements**

The sensitivity of the two solutions to the time between bearing measurements was evaluated by comparing linear target motion solutions for the crossing target geometry. Assumed values of TBBM's were 1.0, 5.0, 10.0, 25.0, and 50.0 time units, while all other variables were held constant. Time started at 1.0 and advanced using the appropriate time increment. One bearing measurement (including noise) was made at the end of each increment. The results of these sensitivity comparisons are presented in figures 3-6 through 3-10, respectively.

An examination of the data presented in figures 3-6 through 3-10 indicates that both methods are sensitive to an increase in TBBM. However, the Rosenbrock Method appears to be considerably more sensitive than PREDIC for intervals greater than 10.0 time units. A TBBM of 5.0 time units appears to be the best compromise between the amount of computer time utilized and the rate of solution convergence for both methods.

3.2  **Sensitivity of the PREDIC and Rosenbrock Solutions to a Change in Noise Level**

The solution sensitivity was compared for the opening, crossing, and closing target geometries. TBBM was held constant at 5.0 time units. Each geometry was simulated for each of four noise standard deviations (σx) of 0.10, 0.25, 0.50 and 1.0. The changes in the solution
Figure 3-6. Target motion solutions for a TBBM of 1.0 time units
Figure 3-7. Target motion solutions for a TBBM of 5.0 time units
Figure 3-8. Target motion solutions for a TBBM of 10.0 time units
Figure 3-9. Target motion solutions for a TBBM of 25.0 time units
Figure 3-10. Target motion solutions for a TBBM of 50.0 time units
characteristics caused by these variations in ambient noise level are presented in figures 3-11 through 3-16. Figures 3-11 through 3-13 are noise level effects on the PREDIC solution for the three geometries. Figures 3-14 through 3-16 depict noise level effects on the Rosenbrock solution.

The data presented in figures 3-11 through 3-16 shows that both methods experience approximately the same degradation in the rate of convergence for increasing noise levels. In addition, these results indicate that the rate of convergence is somewhat dependent on the engagement geometry. For both methods, the rate of convergence is slower for the opening and closing geometries compared to the crossing geometry. These differences become more marked as the noise level is increased.

3.3 The Effect of a Maneuvering Target on the Solutions Produced by the PREDIC and Rosenbrock Methods

The target was maneuvered at a constant speed in a sinusoidal crossing path (see figure 3-5, page 31) with amplitudes of 0.0, 1.0, 5.0, and 30.0 range units. The results of these target maneuvers on the PREDIC and Rosenbrock solutions are presented in figures 3-17 and 3-18, respectively.

An examination of this data shows that both methods experience declines in the rate of convergence for an increase in the amplitude of the maneuver. With the exception of the largest maneuvering amplitude, these curves are
very similar to those for the linear target geometries at increased noise levels.
Figure 3-11. Noise level effects on the PREDIC solution for the opening target geometry.
Figure 3-12. Noise level effects on the PREDIC solution for the crossing target geometry
Figure 3-13. Noise level effects on the PREDIC solution for the closing target geometry.
Figure 3-14. Noise level effects on the Rosenbrock solution for opening target geometry
Figure 3-15. Noise level effects on the Rosenbrock solution for crossing target geometry
Figure 3-16. Noise level effects on the Rosenbrock solution for closing target geometry.
Figure 3-17. The effect of target maneuvers on the PREDIC solution
Figure 3-18. The effect of target maneuvers on the Rosenbrock solution
4.0 DISCUSSION AND RECOMMENDATIONS

The results indicate that the PREDIC and Rosenbrock methods produce equivalent solutions to equation (44). However, the Rosenbrock method requires approximately 2.5 times as much computer time as PREDIC. Also, for all engagement geometries, noise levels, and TBBM's examined in this study, PREDIC did not produce a singular matrix. Therefore, even though the Rosenbrock method does not run the risk of producing a singular matrix, it does not seem justified to recommend its use over PREDIC at this time. It is recommended, however, that both methods be compared for additional engagement geometries, especially those with a maneuvering target.

A direct comparison of the results in section 3.0 with those produced by Bruckheim and O'Neill (B&O) [5] is not possible because B&O solve for initial rather than current target values. However, a qualitative comparison can be made of the magnitude of the errors. This comparison indicates that both PREDIC and Rosenbrock have significantly smaller errors in target position and velocity estimates as compared with those encountered by B&O. This difference is perplexing, especially in the case of PREDIC since it is a direct extension of the CHURN technique described by B&O. The fact that both PREDIC and Rosenbrock independently converge on the same solution would seem to indicate that
errors in the formulation and/or coding of both methods have not occurred. A possible explanation is that B&O may have implemented the simulation of the noise term differently than the method described in section 2.1.4. However, the magnitude of the differences seems to discount this explanation. To insure that this apparent discrepancy with the results of B&O is not just a random occurrence, this research could be extended by rerunning the engagement geometries using different random number seeds for the generation of the noise term.

In the actual ocean environment, it is not uncommon to encounter standard deviations of ambient noise with magnitudes of 0.3 to 0.5 degrees. The results of this study indicate that both PREDIC and Rosenbrock are only marginally effective at these noise levels. This reduction in performance may have been due to the fact that equation (44) uses a "linearized" residual. The residual formed by equation (55) may not be as sensitive to the noise level. An attempt to verify this was made by using the Rosenbrock method to solve equation (55). This proved unsuccessful due to the fact that the solution was very dependent on the starting point. Initial starting points very near the actual solution were required for the Rosenbrock method to converge. One method which might be employed to avoid this difficulty is to use solutions produced for equation (44) PREDIC as starting points for solving equation (55) using Rosenbrock. The feasibility of this recommendation could
be evaluated using the TMSAP computer program.

The results also indicate that changing the TBBM effectively changes the magnitude of the rate at which the observed bearings are changing. For example, at a TBBM of 1.0 time units, the change in observed bearing for successive readings is from 0.1 to 0.3 degrees, while for a TBBM of 50.0 time units, these changes range from 7.0 to 25.0 degrees. Similar rates of change in bearing angles would occur for targets at very long or very short range. The long range targets for a constant TBBM would produce rates of change in bearing angles similar to those produced by a TBBM of 1.0 time units in this study. For a short range target, the same TBBM would produce rates of change similar to the TBBM of 50.0 time units. This is illustrated in figure 4-1 for a stationary observer and two targets moving at the same speed.

By inspection of figure 4-1, it is obvious that

$$A_2 - A_1 > B_2 - B_1$$

Thus, the sensitivity of both methods, especially Rosenbrock, to changes in TBBM also implies a sensitivity to the range of the target. To find the magnitude of this dependence, if any, the range of the target could be varied for the various geometries while holding the TBBM constant.

The results presented indicate both methods are capable of solving the maneuvering target problem only for small amplitudes of the maneuver. The fact that the
Figure 4-1. Illustration of long and short range effects on the rate of change of the observed bearing.
decrease in solution accuracy is similar to that encountered for a change in the noise level indicates that the application of some type of smoothing may reduce the sensitivity of these models to target maneuvers. Development of a tracking model for a maneuvering target presents a formidable problem. An examination must be made of the basic assumptions used to solve the linear target motion problem, with a view to amending or extending them for the maneuvering target case. As these new techniques evolve, they can be evaluated using the TMSAP computer program developed as a part of this research.

Thus, the findings of this research can be summarized as follows:

1. For the range of conditions examined in this study, PREDIC is superior to the Rosenbrock method by virtue of its minimal use of computer time.

2. Both methods perform better than the CHURN technique studied by Bruckheim and O'Neill.

3. For real-world noise conditions, both techniques are only marginally effective.

4. The sensitivity of the solutions to target maneuvers indicates that these models are capable of tracking targets which have small maneuvering amplitudes.

5. The TMSAP computer program developed for this study is an effective tool for use in evaluating Target Motion Analysis (TMA) techniques. It is capable of simulating maneuvering target geometries (including zig-zag and
sinusoidal maneuvers), as well as any linear engagement geometry. Its modular design allows complete flexibility in adapting the program for the evaluation of any TMA technique. In addition, the subroutines developed to implement the Rosenbrock method were designed to be general and, with a simple extension of the program logic, may be used to minimize any unconstrained function.
5.0 REFERENCES


APPENDIX A

The Target Motion Simulation and Analysis Program (TMSAP) is written in FORTRAN IV and is compatible with the CDC 3170 computer system.

The TMSAP program consists of a main program, eight subroutines and one FIND Library routine. Table A-1 provides a brief description of the main program and the subroutines. All of the subroutines, with the exception of PREDIC and FUNCT, were designed to be independent and are not limited to this application. For example, subroutines ROSEN and GRAM can be used to solve any four variable non-linear minimization problem by the Rosenbrock method.

Program input is by card reader in sets of three data cards for each case. The program is structured to allow any number of cases to be run consecutively, as shown in figure A-1. Table A-2 describes the format of each input card. Table A-3 defines each input variable.

TMSAP output is by line printer and consists of the case description, a table describing the target and own ship relative motion simulation, a table of the linear solution produced by PREDIC, and a table presenting the Rosenbrock Method solution produced by ROSEN. The TMSAP output variables are defined in table A-4. An example of the output formats is presented in Appendix B as part of a sample problem.
Figure A-2 is a high level flow diagram for TMSAP. Following figure A-2 is a complete listing of TMSAP, with the exception of the FIND Library subroutine SIMQ. A description of SIMQ may be found in [13].
Table A-1. Description of the Target Motion Simulation and Analysis Program (TMSAP) Main Program and Subroutines

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A360</td>
<td>This function transforms any angle into an angle between 0 and 360 degrees</td>
</tr>
<tr>
<td>ERROR</td>
<td>Calculates the error (noise) term as a Gaussian random variable</td>
</tr>
<tr>
<td>FUNCT</td>
<td>Determines the current value of the function being minimized by the Rosenbrock Method</td>
</tr>
<tr>
<td>GRAM</td>
<td>Constructs a set of mutually orthogonal unit vectors in four dimensions using the Gram-Schmidt orthogonal procedure</td>
</tr>
<tr>
<td>HEADS</td>
<td>Calculates own ship position and velocity as a function of time for the own ship motion simulated, i.e., linear, zig-zag, or sinusoidal</td>
</tr>
<tr>
<td>HEADT</td>
<td>Determines target position and velocity as a function of time for the target motion simulated, i.e., linear, zig-zag, or sinusoidal</td>
</tr>
<tr>
<td>MAIN</td>
<td>Provides TMSAP input and output and controls the subroutine operations</td>
</tr>
<tr>
<td>PREDIC</td>
<td>Calculates the &quot;linearized&quot; solution to the nonlinear target motion problem using a modified version of the CHURN model</td>
</tr>
<tr>
<td>ROSEN</td>
<td>Determines the minimum of a four variable nonlinear function by means of the Rosenbrock Method</td>
</tr>
<tr>
<td>SIMQ</td>
<td>A FIND Library function which solves a matrix equation of the form $C = A^{-1}B$, where $A$, $B$ and $C$ are matrices. See reference [13] for a complete description.</td>
</tr>
</tbody>
</table>
Figure A-1. Data deck setup for the Target Motion Simulation and Analysis Program (TMSAP)
Table A-2. Input Card Format Description

<table>
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<tr>
<th>Variable Name</th>
<th>Card Column Number</th>
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<th>Values</th>
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<td>20A4</td>
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<td></td>
<td>YT</td>
<td>9-16</td>
<td>F8.2</td>
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<td></td>
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<td>NDT</td>
<td>22-24</td>
<td>I3</td>
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<td>25-27</td>
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<td>MOTT</td>
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<tr>
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<td>VS(1)</td>
<td>9-12</td>
<td>F4.0</td>
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<tr>
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<td>NUMS(1)</td>
<td>13-16</td>
<td>I4</td>
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<td></td>
<td>NUMT(2)</td>
<td>21-24</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>VS(2)</td>
<td>25-28</td>
<td>F4.0</td>
</tr>
<tr>
<td></td>
<td>NUMS(2)</td>
<td>29-32</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>VT(3)</td>
<td>33-36</td>
<td>F4.0</td>
</tr>
<tr>
<td></td>
<td>NUMT(3)</td>
<td>37-40</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>VS(3)</td>
<td>41-44</td>
<td>F4.0</td>
</tr>
<tr>
<td></td>
<td>NUMS(3)</td>
<td>45-48</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>VT(4)</td>
<td>49-52</td>
<td>F4.0</td>
</tr>
</tbody>
</table>
Table A-2. Input Card Format Description (continued)

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Card Column Number</th>
<th>Format</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Card 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NUMT(4)</td>
<td>53-56</td>
<td>I4</td>
<td>$0 \leq \text{NUMT} \leq 9999$</td>
</tr>
<tr>
<td>VS(4)</td>
<td>57-60</td>
<td>F4.0</td>
<td>$0. \leq \text{VS} \leq 999$</td>
</tr>
<tr>
<td>NUMS(4)</td>
<td>61-64</td>
<td>I4</td>
<td>$0 \leq \text{NUMS} \leq 9999$</td>
</tr>
<tr>
<td>VT(5)</td>
<td>65-68</td>
<td>F4.0</td>
<td>$0. \leq \text{VT} \leq 999$</td>
</tr>
<tr>
<td>NUMT(5)</td>
<td>69-72</td>
<td>I4</td>
<td>$0 \leq \text{NUMT} \leq 9999$</td>
</tr>
<tr>
<td>VS(5)</td>
<td>73-76</td>
<td>F4.0</td>
<td>$0. \leq \text{VS} \leq 999$</td>
</tr>
<tr>
<td>NUMS(5)</td>
<td>77-80</td>
<td>I4</td>
<td>$0 \leq \text{NUMS} \leq 9999$</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Definition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BYS</td>
<td>Initial value of own ship course (deg)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BYT</td>
<td>Initial value of target course (deg)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DT</td>
<td>Time interval between relative bearing measurements (sec)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| MOTS           | Type of own ship motion defined by its value:  
|                | 1 = Straight line  
|                | 2 = Zig-zag  
|                | 3 = Sinusoidal |
| MOTT           | Type of target motion defined by its value:  
|                | 1 = Straight line  
|                | 2 = Zig-zag  
<p>|                | 3 = Sinusoidal |
| NDS            | Number of time intervals between a change in own ship course |
| NDT            | Number of time intervals between a change in target course |
| NUMS           | Number of time intervals between a change in own ship speed |
| NUMT           | Number of time intervals between a change in target speed |
| SD             | Standard deviation of noise |
| TITLE          | Case identifier |
| TMAX           | Total time simulated |
| VS             | Own ship speed |
| VT             | Target speed |
| XS             | Initial X-coordinate of own ship position |
| XT             | Initial X-coordinate of target position |</p>
<table>
<thead>
<tr>
<th>Program Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>YS</td>
<td>Initial Y-coordinate of own ship position</td>
</tr>
<tr>
<td>YT</td>
<td>Initial Y-coordinate of target position</td>
</tr>
<tr>
<td>ZS</td>
<td>Amplitude of nonlinear own ship motion</td>
</tr>
<tr>
<td>ZT</td>
<td>Amplitude of nonlinear target motion</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>----------------</td>
<td>------------</td>
</tr>
<tr>
<td>BEAR1</td>
<td>Actual value of the relative bearing for the simulated target and own ship geometries</td>
</tr>
<tr>
<td>BYS</td>
<td>Initial value of own ship course</td>
</tr>
<tr>
<td>BYT</td>
<td>Initial value of target course</td>
</tr>
<tr>
<td>DEL</td>
<td>Absolute difference between the actual target position and velocity and the values calculated by the Rosenbrock Method</td>
</tr>
<tr>
<td>DT</td>
<td>Time interval between relative bearing measurements</td>
</tr>
<tr>
<td>DVELX</td>
<td>Absolute difference between actual target x-component of velocity and the value calculated by PREDIC</td>
</tr>
<tr>
<td>DVELY</td>
<td>Absolute difference between actual target y-component of velocity and the value calculated by PREDIC</td>
</tr>
<tr>
<td>DXT</td>
<td>Absolute difference between actual target x-component of position and the value calculated by PREDIC</td>
</tr>
<tr>
<td>DYT</td>
<td>Absolute difference between actual target y-component of position and the value calculated by PREDIC</td>
</tr>
<tr>
<td>NDS</td>
<td>Number of time intervals between a change in own ship course</td>
</tr>
<tr>
<td>NDT</td>
<td>Number of time intervals between a change in target course</td>
</tr>
<tr>
<td>OA</td>
<td>Current value of noise generated by ERROR</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>----------------</td>
<td>------------</td>
</tr>
<tr>
<td>OBY</td>
<td>Observed relative bearing between own ship and target</td>
</tr>
<tr>
<td>PVX</td>
<td>X-component of target velocity computed by PREDIC</td>
</tr>
<tr>
<td>PVY</td>
<td>Y-component of target velocity computed by PREDIC</td>
</tr>
<tr>
<td>PXT</td>
<td>X-component of target position computed by PREDIC</td>
</tr>
<tr>
<td>PYT</td>
<td>Y-component of target position computed by PREDIC</td>
</tr>
<tr>
<td>RVX</td>
<td>X-component of target velocity computed by ROSEN</td>
</tr>
<tr>
<td>RVY</td>
<td>Y-component of target velocity computed by ROSEN</td>
</tr>
<tr>
<td>RXT</td>
<td>X-component of target position computed by ROSEN</td>
</tr>
<tr>
<td>RYT</td>
<td>Y-component of target position computed by ROSEN</td>
</tr>
<tr>
<td>SD</td>
<td>Standard deviation of noise</td>
</tr>
<tr>
<td>T</td>
<td>Time of current simulation step</td>
</tr>
<tr>
<td>TITLE</td>
<td>Case identifier</td>
</tr>
<tr>
<td>TMAX</td>
<td>Total time simulated</td>
</tr>
<tr>
<td>VELX</td>
<td>Current value of the x-component of the simulated actual target velocity</td>
</tr>
<tr>
<td>VELY</td>
<td>Current value of the y-component of the simulated actual target velocity</td>
</tr>
<tr>
<td>VS</td>
<td>Own ship speed</td>
</tr>
<tr>
<td>VT</td>
<td>Target speed</td>
</tr>
<tr>
<td>XS</td>
<td>Current value of the simulated x-coordinate of own ship position</td>
</tr>
<tr>
<td>XT</td>
<td>Current value of the simulated x-coordinate of target position</td>
</tr>
</tbody>
</table>
Table A-4. Definitions of TMSAP Output Variables (continued)

<table>
<thead>
<tr>
<th>Program Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>YS</td>
<td>Current value of the simulated y-coordinate of own ship position</td>
</tr>
<tr>
<td>YT</td>
<td>Current value of the simulated y-coordinate of target position</td>
</tr>
</tbody>
</table>
Input: Simulation Variables (see table A-2)

Output: Simulation Variables

SET: Flags and convert degrees to radians

CALL HEADT

CALL HEADS

Compute: Initial Actual Bearing

Figure A-2. TMSAP flow diagram
Compute: Observed Bearing

CALL A360

Output: Initial positions, velocities, and bearings

DO20 I=1,N

CALL HEADT

Figure A-2. TMSAP flow diagram (continued)
Figure A-2. TMSAP flow diagram (continued)
Figure A-2. TMSAP flow diagram (continued)
Initialize:
Non-time dependent variables

Has PREDIC been called 5 times?
Yes
Calculate:
Time dependent variables

CALL SIMQ

Has a singular matrix been formed?
Yes
Output:
Warning

Increment:
Non-time dependent terms

RETURN

Figure A-2. TMSAP flow diagram (continued)
Define: Initial search directions and step size

Initialize: Function value and set flags

Find one success and one failure in each search direction

JUMP = B

Is step size too small?

I > 4

Solution Converged?

Define: Most successful direction

Figure A-2. TMSAP flow diagram (continued)
Figure A-2. TMSAP flow diagram (continued)
Listing of TMSAP Computer Program

```plaintext
LN 0701 COMMON/1/ TITLE(26), OBEY(50), PVX(500), VY(500), VX(500), Y(500),
LN 0702 OIL(4), LC(201), R(120)
LN 0703 COMMON/2/ I(1)
LN 0704 COMMON/L2/ I(1), PVX, PVY, PYT
LN 0705 COMMON/L3/ I(1), PVX, PVY, PYT
LN 0706 COMMON/L7/ X(500), Y(500)
LN 0707 COMMON/L9/ I(1)
LN 0708 COMMON/L10/ X(500), Y(500), VX(500), VY(500), VT(10), NUM1(13), ZT
LN 0709 COMMON/L15/ VT(10), I(1), XX, YY, NOT, II, XT
LN 0710 COMMON/L19/ VX(10), NUM1(10), ZT
LN 0711 COMMON/L21/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0712 COMMON/L22/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0713 COMMON/L23/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0714 COMMON/L24/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0715 COMMON/L25/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0716 COMMON/L26/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0717 COMMON/L27/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0718 COMMON/L28/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0719 COMMON/L29/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0720 COMMON/L30/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0721 COMMON/L31/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0722 COMMON/L32/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0723 COMMON/L33/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0724 COMMON/L34/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0725 COMMON/L35/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0726 COMMON/L36/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0727 COMMON/L37/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0728 COMMON/L38/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0729 COMMON/L39/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0730 COMMON/L40/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0731 COMMON/L41/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0732 COMMON/L42/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0733 COMMON/L43/ I(1), OX, OY, Z, XNX, YNOS, JJ, XS
LN 0734 COMM .....
```

VELOCITY, AND RELATIVE BEARING

DO 10 I=2,N

10 N=N+1

CALCULATE CURRENT ACTUAL TARGET POSITION AND VELOCITY

CALL HEAD

CALL CURRENT ACTUAL OWN SHIP POSITION AND VELOCITY

CALCULATE ACTUAL AND OBSERVED VALUE OF RELATIVE BEARING

DO 50 KOUNT = 0

50 WRITE(61,105) (TITLE(I),I=1,20)

OUTPUT CURRENT ITERATION VALUES OF TARGET AND OWN SHIP POSITION, VELOCITY, AND RELATIVE BEARING

COMPUTE LINEARIZED SOLUTION TO SIMULATED TARGET MOTION ANALYSIS

DO 60 T=1,N

60 WRITE(61,113) (TITLE(J),J=1,20),SO
ANSI FORTRAN(2-3)/MASTER

USASI FORTRAN DIAGNOSTIC RESULTS FOR FTN.MAIN

NO ERRORS

THE FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

LS2    LS3    LS7    LTI
SUBROUTINE HEAD

THIS SUBROUTINE CALCULATES TARGET POSITION AND VELOCITY AS A FUNCTION OF TIME IN THE TARGET MOTION SIMULATED.

COMMON S, VS(5), (XS(10), VS(5)), VT(10), NUMT(5), A
COMMON NP, STR, JIN, NKK, A, QT, TI, KI

10 IN=1
20 GO TO 30, 35, 40

30 IF(IN-1) 10, 10, 15
10 DX=IN*37
DY=0.3*37
VT(I)=DX*VT(I)
VY(I)=DV*VT(I)
GO TO 100

15 VX(I)=DX*VT(I)
VY(I)=DY*VT(I)
X(I)=X(I)+10*VT(I)
Y(I)=Y(I)+10*VT(I)

40 IF(IN=1) 20, 25, 20

20 IF(IN=1) 20, 25, 20

30 IF(IN=1) 20, 25, 20

40 IF(IN=1) 20, 25, 20

50 IF(IN=1) 20, 25, 20

60 IF(IN=1) 20, 25, 20

70 IF(NK=1) 60, 70, 65

65 NK=0
SI=EV
BI=DX
DI=1
GO TO 55

75 IF(NK=1) 60, 70, 65

60 IF(NK=1) 60, 70, 65

80 IF(NK=1) 60, 70, 65

70 IF(NK=1) 60, 70, 65

90 IF(NK=1) 60, 70, 65

80 IF(NK=1) 60, 70, 65

90 IF(NK=1) 60, 70, 65

100 RETURN END
ANSI FORTRAN(2.2)/MASTER

USASI FORTRAN DIAGNOSTIC RESULTS FOR HEADT

NO ERRORS

THE FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

LTI
SUBROUTINE HEADS

THIS SUBROUTINE CALCULATES OWN SHIP POSITION AND VELOCITY AS A FUNCTION OF TIME
FOR THE OWN SHIP MOTION SIMULATED

COMMON/LST/X(I,J),Y(J,K)
COMMON/VT1/VS(I,J),VMS(I,J,K)
COMMON/VT2/VT1,IS,MKX,M,KX,KJ
COMMON/VT4/DT

IF(NUST(J,J)-M) 20,25,20
20 JJ=JJ+1
25 GO TO 10,25,40,110

LINEAR OWN SHIP MOTION

30 IF(IS-1) 10,10,15
10 OX=SI(J,JT)
OY=CO(J,JT)
IS=IS+1
GO TO 100

15 SX=OY•VS(JJ)
SY=OY•VS(JJ)
Y(J)=Y(J)+1•OY+SY
GO TO 106

ZIG-ZAG OWN SHIP MOTION

25 IF(IS-1) 25,45,50
20 OX=SI(J,JT)
OY=CO(J,JT)
IS=2
GO TO 100

50 KS=KS+1
GO TO 55

55 KS=KS+1
GO TO 15

SINE WAVE MOTION

45 IF(IS-1) 75,75,80
75 PI=3.14156256
C1=2•PI/5
Y1=Y(J)
TRAN
IS=2

80 SX=Y(JJ)/IS+1•X(J)*C1+2*Y(J)*C1*TX
SY=CO(J,1)*TX
TX=X(M)
Y(J)=Y(J)+1•Y1•SIN(TX)

100 RETURN
END

USASI FORTRAN DIAGNOSTIC RESULTS FOR HEADS

NO ERRORS
ANSI FORTRAN(2.2)/MASTER

THE FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

L57 LTI
SUBROUTINE ERROR (S,W)

This subroutine calculates the error (noise) term as a Gaussian random variable.

DO 50 I=1,12
    Y=FANDHMO(1)

50    A=A+Y
    V=(1.6D0)*S
    RETURN
END

USASI FORTRAN diagnostic results for ERROR

No errors
FUNCTION A360(ANGLE)

THIS FUNCTION TRANSFORMS ANY ANGLE INTO AN ANGLE BETWEEN 0 AND 360 DEGREES

5 IF(ANGLE < 0, 0, 10)
10 IF(360. - ANGLE) < 11.11, 40
11 ANGLE = ANGLE - 360.
15 GO TO 10
30 ANGLE = ANGLE + 360.

USASI FORTRAN DIAGNOSTIC RESULTS FOR A350

NO ERRORS
SUBROUTINE PREDIC(T0,T1,P1,P2,D1,D2)

THIS SUBROUTINE CALCULATES A LINEARIZED SOLUTION TO THE NON-LINEAR TARGET MOTION PROBLEM

DIMENSION A(2),B(2),C(2),D(2)

COMMON/COS/,SIN(),SANG (),CANG (),SANG1 (),CANG1 (),SANG2 (),CANG2 ()

COMMON/XY/,XV(),XYV (),PT ,PVT,PTT

INITIALIZE TIME AND NON-TIME DEPENDENT VARIABLES

M=1

IF(ENT3=1) 5C,63,50

60 TNP=S,0,0

40 A(1)=D,0

50 B(1)=D,0

60 COS=(COS(D))

70 SIN=(_SIN(D))

80 COS2=COS**2

90 SANGJ1=COS(D)

100 SANGJ1=SANG(J1)

110 CANGJ1=CANG(J1)

120 T=0,0

130 IF(ENT3=5) 70,75,75

70 GO TO 32

75 A11=1,0

80 A11=1,0

90 A21=1,0

C21=0,0

100 T=0,0

210 A11=0,0

310 A11=0,0

410 K=1

CALCULATE TIME DEPENDENT TERMS OF TARGET MOTION MATRIX

DO 22 L=1,K

22 X=2

A11=1,0

B11=0,0

C11=0,0

A21=0,0

B21=0,0

C21=0,0

CONTINUE

DO 25 L=1,K

25 CONTINUE

IF(JIJ=1) 10,15,15

DO 10 J=1,14

10 GO TO 200

DO 15 K=1,3

15 GO TO 200

DO 30 K=1,3

30 GO TO 200
LN 0077  INVERT TARGET MOTION MATRIX
LN 0078  AND CALCULATE SOLUTION MATRIX
LN 0079
LN 0080  35 CALL SIMQIC(D,M,KS)
LN 0081
LN 0082  CHECK FOR SINGULAR MATRIX
LN 0083
LN 0084  IF(KS-1) 40,45,40
LN 0085
LN 0086  45 WRITE(61,105)
LN 0087
LN 0088  105 FOR(A'IX,37HPR:012 HAS PRODUCED A SINGULAR MATRIX)
LN 0089
LN 0090
LN 0091  GO TO 55
LN 0092
LN 0093  STORE SOLUTION MATRIX
LN 0094
LN 0095  40 PVX=0(1)
LN 0096  PVT=0(2)
LN 0097  PVY=0(3)
LN 0098
LN 0099
LN 0100  INCREMENT NON-TIME DEPENDENT
LN 0101  TERMS OF TARGET MOTION MATRIX
LN 0102
LN 0103  200 A(6)=A(6)*COS2
LN 0104  TEMP=TEMP+GOS3*SIN3
LN 0105  A(9)=TEMP
LN 0106  A(11)=A(11)
LN 0107  A(15)=A(15)*SIN32
LN 0108  TEMP=TEMP+GOS3*SIN3
LN 0109  B(2)=B(2)*COS2+TERM2
LN 0110  B(4)=B(4)*TERM2
LN 0111  B(8)=B(8)*TERM2
LN 0112  B(12)=B(12)*TERM2
LN 0113  J=J+1
LN 0114
LN 0115  55 RETURN
LN 0116
LN 0117  END
LN 0118
LN 0119
LN 0120
LN 0121

USASI FORTRAN DIAGNOSTIC RESULTS FOR PREDIC
NO ERRORS

THE FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE
LS2 LS7
SUBROUTINE ROSETIP

This Subroutine Calculates the Minimum of a Four Varia
Non-Linear Function by Plans

COMMON// B(K), S(I), IND, DEL(K), DEL(K), C1(K), D1(K), D3(1)

COMMON// A(I)

INITIAL SEARCH DIRECTIONS ARE DEFINED AS THE COORDINATE AXES

DEFINE STEP SIZE

DEFINE FUNCTION VALUES AND SET FLA\n
JUMP=1

FINO ONE SUCCESS AND ONE FAILURE IN EACH SEARCH DIRE\n
JUMP=2

FINO ONE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=3

FINO TWO SUCCESSES AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=4

FINO TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=5

FINO TWO SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=6

FINO THREE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=7

FINO THREE FAILURES IN EACH SEARCH DIREC\n
JUMP=8

FINO THREE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=9

FINO FOUR SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=10

FINO FOUR FAILURES IN EACH SEARCH DIREC\n
JUMP=11

FINO FOUR SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=12

FINO FIVE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=13

FINO FIVE FAILURES IN EACH SEARCH DIREC\n
JUMP=14

FINO FIVE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=15

FINO SIX SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=16

FINO SIX FAILURES IN EACH SEARCH DIREC\n
JUMP=17

FINO SIX SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=18

FINO SEVEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=19

FINO SEVEN FAILURES IN EACH SEARCH DIREC\n
JUMP=20

FINO SEVEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=21

FINO EIGHT SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=22

FINO EIGHT FAILURES IN EACH SEARCH DIREC\n
JUMP=23

FINO EIGHT SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=24

FINO NINE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=25

FINO NINE FAILURES IN EACH SEARCH DIREC\n
JUMP=26

FINO NINE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=27

FINO TEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=28

FINO TEN FAILURES IN EACH SEARCH DIREC\n
JUMP=29

FINO TEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=30

FINO ELEVEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=31

FINO ELEVEN FAILURES IN EACH SEARCH DIREC\n
JUMP=32

FINO ELEVEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=33

FINO TWELVE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=34

FINO TWELVE FAILURES IN EACH SEARCH DIREC\n
JUMP=35

FINO TWELVE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=36

FINO THIRTEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=37

FINO THIRTEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=38

FINO THIRTEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=39

FINO FOURTEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=40

FINO FOURTEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=41

FINO FOURTEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=42

FINO FIFTEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=43

FINO FIFTEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=44

FINO FIFTEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=45

FINO SIXTEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=46

FINO SIXTEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=47

FINO SIXTEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=48

FINO SEVENTEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=49

FINO SEVENTEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=50

FINO SEVENTEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=51

FINO EIGHTEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=52

FINO EIGHTEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=53

FINO EIGHTEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=54

FINO NINETEEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=55

FINO NINETEEN FAILURES IN EACH SEARCH DIREC\n
JUMP=56

FINO NINETEEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=57

FINO TWENTY SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=58

FINO TWENTY FAILURES IN EACH SEARCH DIREC\n
JUMP=59

FINO TWENTY SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=60

FINO TWENTY-ONE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=61

FINO TWENTY-ONE FAILURES IN EACH SEARCH DIREC\n
JUMP=62

FINO TWENTY-ONE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=63

FINO TWENTY-TWO SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=64

FINO TWENTY-TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=65

FINO TWENTY-TWO SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=66

FINO TWENTY-THREE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=67

FINO TWENTY-THREE FAILURES IN EACH SEARCH DIREC\n
JUMP=68

FINO TWENTY-THREE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=69

FINO TWENTY-FOUR SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=70

FINO TWENTY-FOUR FAILURES IN EACH SEARCH DIREC\n
JUMP=71

FINO TWENTY-FOUR SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=72

FINO TWENTY-FIVE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=73

FINO TWENTY-FIVE FAILURES IN EACH SEARCH DIREC\n
JUMP=74

FINO TWENTY-FIVE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=75

FINO TWENTY-SIX SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=76

FINO TWENTY-SIX FAILURES IN EACH SEARCH DIREC\n
JUMP=77

FINO TWENTY-SIX SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=78

FINO TWENTY-SEVEN SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=79

FINO TWENTY-SEVEN FAILURES IN EACH SEARCH DIREC\n
JUMP=80

FINO TWENTY-SEVEN SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
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FINO TWENTY-EIGHT SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=82

FINO TWENTY-EIGHT FAILURES IN EACH SEARCH DIREC\n
JUMP=83

FINO TWENTY-EIGHT SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=84

FINO TWENTY-NINE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=85

FINO TWENTY-NINE FAILURES IN EACH SEARCH DIREC\n
JUMP=86

FINO TWENTY-NINE SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=87

FINO THIRTY SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=88

FINO THIRTY FAILURES IN EACH SEARCH DIREC\n
JUMP=89

FINO THIRTY SUCCESS AND TWO FAILURES IN EACH SEARCH DIREC\n
JUMP=90

FINO THIRTY-ONE SUCCESS AND ONE FAILURE IN EACH SEARCH DIREC\n
JUMP=91

FINO THIRTY-ONE FAILURES IN EACH SEARCH DIREC\n
JUMP=92

FINO THIRTY-ONE SUCCESS AND TWO FAILURE...
USASI FORTRAN DIAGNOSTIC RESULTS FOR ROSEN

NO ERRORS

UNREFERENCED STATEMENT LABELS

00290
SUBROUTINE GRAM(S)

THIS SUBROUTINE CONSTRUCTS A SET OF
ORTHOGONAL UNIT VECTORS IN
FOUR DIMENSIONS USING THE
GRAM-SCHMIDT ORTHOGONAL PROCEDURE

DIMENSION S(4,4),IND(4),BETA(4,4),ALPHA(4)

TEST FOR ZEROS IN BEST SEARCH DIRECTION

DEFINE LINEARLY INDEPENDENT VECTORS

C000
GO TO 84
S(3,J,K)=1.0
GO TO 98
S(1,J,K)=1.0
CONTINUE

NO?ALIZE VECORS AND GENERATE NEW SEARCH DIRECTIONS

IF 15 J=1,4
AS=AS+5(S(1,J))**2
DO 143 J=1,4

TEMP=J

DO 145 J=1,4

BETA(1,J)=S(1,J)/AS

DO 145 J=1,4

ALPHA(J)=S(2,J)-TEMP*BETA(1,J)

DO 145 J=1,4

AS=AS+(ALPHA(J))**2

DO 150 J=1,4

BETA(2,J)=ALPHA(J)/AS

DO 150 J=1,4

TEMP1=TEMP2*BETA(1,J)*S(3,J)

DO 160 J=1,4

ALPHA(J)=S(3,J)-TEMP1*BETA(1,J)-TEMP2*BETA(2,J)

DO 170 J=1,4

AS=AS+(ALPHA(J))**2

DO 175 J=1,4

BET(1,J)=ALPHA(J)/AS

DO 175 J=1,4

TEMP2=TEMP1*BETA(2,J)*S(3,J)

DO 180 J=1,4

ALPHA(J)=S(3,J)-TEMP1*BETA(2,J)-TEMP2*BETA(3,J)

DO 190 J=1,4

AS=AS+(ALPHA(J))**2

DO 195 J=1,4

BETA(3,J)=ALPHA(J)/AS

DO 200 J=1,4

TEMP3=TEMP1*BETA(3,J)*S(4,J)

DO 210 J=1,4

ALPHA(J)=S(4,J)-TEMP1*BETA(3,J)-TEMP2*BETA(4,J)

DO 220 J=1,4

AS=AS+(ALPHA(J))**2

DO 225 J=1,4

BETA(4,J)=ALPHA(J)/AS

DEFINE NEW SEARCH DIRECTIONS

IF 20 J=1,4

RETURN
SUBROUTINE FUNCT (A, IP, SUMS)

THIS SUBROUTINE CALCULATES THE
VALUE OF THE FUNCTION MINIMIZED BY THE RONSENROCK METHOD.

DIMENSION A(4)
COMMON/CL1/, C1(500), C2(500)
COMMON/CT1/, X1(500), Y1(500)
COMMON/C11/, DT

N1=3

DO 100 I=1,N

N1=I

TN=I-1.

SUMS=0.

DO 10 I=1,N

N1=I

T1=TN(I)-A(I)*(TN+1)*YS(M)

T2=TN(I)-A(I)*TN+1*X(M)

T3=TN(I)-SSM1-T2*C(M)

T4=T3+TMP*SEP

SUMS=SUMS+T4

100 CONTINUE

RETURN

END

USASI FORTRAN DIAGNOSTIC RESULTS FOR FUNCT

NO ERRORS

THE FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STOPAGE

MAP

OBJ, LGO
This appendix presents a description of the TMSAP computer program outputs. This is achieved by executing a sample problem, which may also be used to test subsequent versions of TMSAP.

The sample problem target and own ship geometry are illustrated in figure B-1. The geometry illustrated in figure B-1 is referred to as "crossing" geometry.

The input data case set up for the sample problem is presented in table B-1. The data card formats follow those outlined in Appendix A.

The output of the TMSAP program is four tables of line printer data. The actual number of pages of line printer paper required to output these tables is a function of the particular case being studied. For the sample problem, the TMSAP consists of four sheets of line printer data.

The first sheet of the sample problem output is shown in figure B-2. This sheet is a reprint of the pertinent input data for immediate verification of the input and for record purposes for subsequent review.

The second sheet of the sample problem output is presented in figure B-3. This table lists the output of the simulation of target and own ship motion. The first column lists the clock time that has elapsed during the simulation beginning with time equals 1.0. The second and third
columns list the coordinates of the actual target position at the time indicated in column 1. The fourth and fifth columns list the X and Y components of target velocity. Columns 6 and 7 list the coordinates of own ship. Column 8 is the actual relative bearing between the target and own ship. The ninth column is the magnitude of the ambient noise present during the measurement of relative bearing. Column 10 is the observed relative bearing between the target and own ship corrupted by ambient noise.

The third sheet of the sample problem output lists the linear or PREDIC solution to the target motion problem. Column 1 is a list of the elapsed clock time. Columns 2 through 5 are the actual values of target position and velocity components. Columns 6 through 9 are the linear or PREDIC solution for the actual values of target position and velocity components. Columns 10 through 13 list the absolute difference between the values of actual target position and velocity and those calculated by the PREDIC technique. Note should also be taken that the PREDIC technique does not attempt a solution until the fifth relative bearing measurement. This was done to minimize the possibility of the formation of a singular matrix.

The fourth sheet output for the sample problem is a list of the solution of the target motion problem by use of the Rosenbrock method. Column 1 is a list of the elapsed clock time. Columns 2 through 5 are a list of the actual target position and velocity components. Columns 6
through 9 are a list of the target values calculated by the Rosenbrock method. Columns 10 through 13 list the absolute difference between the values of actual target position and velocity and those calculated by the Rosenbrock method.
Figure B-1. Sample problem target and own ship geometry.
<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Card Column Number</th>
<th>Input Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>1-80</td>
<td>SAMPLE PROBLEM</td>
</tr>
<tr>
<td>XT</td>
<td>1-8</td>
<td>-500.</td>
</tr>
<tr>
<td>YT</td>
<td>9-16</td>
<td>500.</td>
</tr>
<tr>
<td>DT</td>
<td>17-21</td>
<td>5.0</td>
</tr>
<tr>
<td>NDT</td>
<td>22-24</td>
<td>100</td>
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<td>26-27</td>
<td>10</td>
</tr>
<tr>
<td>XS</td>
<td>28-35</td>
<td>0.</td>
</tr>
<tr>
<td>YS</td>
<td>36-43</td>
<td>0.</td>
</tr>
<tr>
<td>BYT</td>
<td>44-47</td>
<td>90.</td>
</tr>
<tr>
<td>BYS</td>
<td>48-51</td>
<td>45.</td>
</tr>
<tr>
<td>SD</td>
<td>52-55</td>
<td>0.1</td>
</tr>
<tr>
<td>MOTT</td>
<td>57</td>
<td>1</td>
</tr>
<tr>
<td>MOTS</td>
<td>59</td>
<td>2</td>
</tr>
<tr>
<td>ZS</td>
<td>62-63</td>
<td>0.</td>
</tr>
<tr>
<td>ZT</td>
<td>66-67</td>
<td>0.</td>
</tr>
<tr>
<td>Tmax</td>
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<td>200.</td>
</tr>
<tr>
<td>VT</td>
<td>1-4</td>
<td>1.0</td>
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<tr>
<td>NUMT</td>
<td>6-8</td>
<td>500</td>
</tr>
<tr>
<td>VS</td>
<td>9-12</td>
<td>2.83</td>
</tr>
<tr>
<td>NUMS</td>
<td>14-16</td>
<td>500</td>
</tr>
</tbody>
</table>
SAMPLE PROBLEM

INITIAL VALUES USED FOR THIS SIMULATION
TARGET POSITION X = -500.00 Y = 500.00
OWN SHIP POSITION X = 0.00 Y = 0.00
TIME BETWEEN READINGS = 0.05 SEC
NUMBER OF READINGS BETWEEN A CHANGE IN TARGET COURSE = 100
NUMBER OF READINGS BETWEEN A CHANGE IN OWN COURSE = 10
TARGET HEADING = 90.
OWN SHIP HEADING = 45.
STD DEV OF NOISE = 0.100
TARGET VELOCITY = 1. OWN SHIP VELOCITY = 3.
TOTAL TIME SIMULATED = 200.

Figure B-2. Sample problem output (page 1)
<table>
<thead>
<tr>
<th>TIME</th>
<th>TARGET POSITION</th>
<th>OWN SHIP POSITION</th>
<th>ACTUAL BEARING</th>
<th>OBS. BEARING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
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<td>350.00</td>
<td>1.000</td>
<td>0.000</td>
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<tr>
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<td>510.00</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
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<td>1.000</td>
<td>0.000</td>
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<td>1.000</td>
<td>0.000</td>
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<td>1.000</td>
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<td>1.000</td>
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<td>1.000</td>
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<td>0.000</td>
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<tr>
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<td>1.000</td>
<td>0.000</td>
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<td>0.000</td>
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<td>1.000</td>
<td>0.000</td>
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<td>0.000</td>
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<td>0.000</td>
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<tr>
<td>166.</td>
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<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>171.</td>
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<td>0.000</td>
</tr>
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<td>186.</td>
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<td>1.000</td>
<td>0.000</td>
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<tr>
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<td>1.000</td>
<td>0.000</td>
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<tr>
<td>201.</td>
<td>-1649.33</td>
<td>350.00</td>
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<td>0.000</td>
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</table>

Figure B-3. Sample problem output (page 2)
<table>
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<tr>
<th>TIME</th>
<th>ACTUAL TARGET VALUES</th>
<th>LINEAR SOLUTION FOR TARGET VALUES</th>
<th>ABS. DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>POSITION X Y</td>
<td>POSITION X Y</td>
<td>VELOCITY X Y</td>
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<td>0.500E+03 0.600E+03</td>
<td>0.500E+03 0.600E+03</td>
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</tr>
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</tr>
<tr>
<td>3.0</td>
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<td>0.000E+00 0.000E+00</td>
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<td>0.000E+00 0.000E+00</td>
</tr>
<tr>
<td>5.0</td>
<td>0.700E+03 0.800E+03</td>
<td>0.700E+03 0.800E+03</td>
<td>0.000E+00 0.000E+00</td>
</tr>
<tr>
<td>6.0</td>
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<td>0.750E+03 0.850E+03</td>
<td>0.000E+00 0.000E+00</td>
</tr>
<tr>
<td>7.0</td>
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<td>0.800E+03 0.900E+03</td>
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</tr>
<tr>
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<td>0.900E+03 1.000E+03</td>
<td>0.000E+00 0.000E+00</td>
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</table>

Figure B-4. Sample problem output (page 3)
### Sample Problem

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<th>POSITION</th>
<th>VELOCITY</th>
<th>POSITION</th>
<th>VELOCITY</th>
<th>ABS. DIFFERENCE</th>
</tr>
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</tr>
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*Figure B-5. Sample problem output (page 4)*