CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

THE TERMINATION OF HARMONIC CURRENTS
IN A CLASS-C AMPLIFIER

A graduate project submitted in partial satisfaction of
the requirements for the degree of Master of Science in
Engineering

by

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<tr>
<td>$B_{V_{cbo}}$</td>
<td>transistor collector-to-base breakdown voltage</td>
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<tr>
<td>$f$</td>
<td>frequency</td>
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<td>$f_o$</td>
<td>center frequency</td>
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<td>$i_b$</td>
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<td>$i_c$</td>
<td>instantaneous collector current</td>
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<td>$i_{c,ave}$</td>
<td>average (dc) collector current</td>
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<td>$i_n$</td>
<td>instantaneous nth harmonic current</td>
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<td>$I_{bp}$</td>
<td>peak value of base current</td>
<td></td>
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<td>$I_{c,max}$</td>
<td>maximum instantaneous value of base current at any point in its cycle</td>
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<td>$I_{cp}$</td>
<td>peak value of collector current</td>
<td></td>
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<tr>
<td>$I_{c,max}$</td>
<td>maximum instantaneous value of collector current at any point in its cycle</td>
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<td>$I_{cm}$</td>
<td>transistor's maximum rated collector current</td>
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<td>$I_{can}$</td>
<td>Fourier coefficient of the nth harmonic of collector current</td>
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<td>$P_i$</td>
<td>input power</td>
<td></td>
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<td>$P_{i_{ac}}$</td>
<td>ac input power</td>
<td></td>
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<tr>
<td>$P_{i_{dc}}$</td>
<td>dc input power</td>
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<tr>
<td>$P_o$</td>
<td>output power</td>
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<tr>
<td>$P_{o_{ac}}$</td>
<td>ac output power</td>
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<tr>
<td>$P_{on}$</td>
<td>output power of the nth harmonic</td>
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<td>$R_i$</td>
<td>transistor input resistance</td>
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<td>$R_{l}$</td>
<td>transistor load resistance</td>
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<td>$R_{n}$</td>
<td>load resistance of the nth harmonic</td>
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\( t \)  
- time

\( v_b \)  
- instantaneous base voltage

\( v_c \)  
- instantaneous collector voltage

\( v_i \)  
- instantaneous input voltage

\( v_l \)  
- instantaneous load voltage

\( V_b \)  
- sum of threshold voltage and base bias voltage

\( V_{bb} \)  
- base bias voltage

\( V_{cc} \)  
- power supply voltage

\( V_{cp} \)  
- peak collector voltage

\( V_{cp,\text{max}} \)  
- maximum value of peak collector voltage

\( V_i \)  
- peak value of input ac voltage

\( V_{l,\text{ave}} \)  
- average load voltage

\( V_{\text{lan}} \)  
- Fourier coefficient of the nth harmonic of load voltage

\( V_{\text{sat}} \)  
- transistor collector saturation voltage

\( V_{\text{th}} \)  
- transistor base threshold voltage

\( Z_{\text{ln}} \)  
- load impedance for the nth harmonic

\( \beta \)  
- transistor current amplification factor

\( \eta_c \)  
- collector efficiency

\( \eta_{c,\text{max}} \)  
- maximum collector efficiency

\( \eta_{\text{cf}} \)  
- fundamental collector efficiency

\( \theta \)  
- transistor conduction angle

\( \omega \)  
- radian frequency

\( \omega_0 \)  
- center radian frequency
ABSTRACT

THE TERMINATION OF HARMONIC CURRENTS IN A CLASS-C AMPLIFIER

by

Stephen James Ferry

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The performance of a low-frequency class-C amplifier was analyzed. A simple nonsaturating model was assumed. Power output, power gain, and collector efficiency were found to increase as the power supply voltage was increased. Collector efficiency increased and power output decreased as the conduction angle decreased. Not all collector voltage waveforms are possible with the classic (textbook) collector current if passive harmonic impedances are assumed. In particular, the square-wave collector voltage waveform is not possible. When the collector current waveform was assumed to be a rectangular wave, much greater fundamental output power was possible if the correct harmonic impedances were used. Collector efficiency was also improved. The study showed that the collector current waveform should be analyzed before the collector load is selected. Then harmonic impedances may be selected for best performance.
Chapter 1

INTRODUCTION

The class-C amplifier is defined as an amplifier in which the output current flows for less than one-half the input voltage cycle [1]. Class-C amplifiers are generally used in high-power alternating current applications. The dc-to-ac efficiency of class-C amplifiers is greater than that of class-A or class-B amplifiers. This greater efficiency results in a lower dc current requirement for the amplifier and less power dissipation in the amplifier for a given ac output power.

The amplification process in a class-C amplifier is nonlinear. As a result of this nonlinear process, the input sinusoidal voltage is transformed to an output current which is rich in harmonics. How these harmonic currents are terminated will determine the waveform of the output voltage.

Most analyses of a class-C amplifier assume a high-Q parallel resonant circuit on the output of the amplifier [2]. This results in a zero-impedance load to the harmonic currents and thus no voltage is developed at the harmonic frequencies. Alternate harmonic loads have been considered for a class-B amplifier [3]. However, that particular analysis cannot be extended to a class-C amplifier. If it were, it would require that some of the harmonics be terminated in nonpassive loads.
Thus, there is a lack of discussion of class-C harmonic loading in the literature. The purpose of this paper is to study analytically the parameters which affect the performance of a class-C amplifier. The emphasis will be on harmonic loading effects, i.e., on the impedance presented to the amplifier at the various harmonic frequencies.

The following questions will be treated: What is the effect of power supply voltage and conduction angle on power gain, output power, and dc-to-ac efficiency? What are the coefficients of the harmonic components of output current in the classic (textbook) class-C amplifier? What are the limitations on output voltage waveform with a fixed output current waveform? What are the effects on dc-to-ac efficiency and output power of various output harmonic loads?

Certain assumptions will be made to limit the scope of the analysis. A low-frequency transistor model will be assumed. The transistor model is ideal. The relationships among the various transistor parameters (base voltage, base current, collector current) are not functions of either their magnitude or of frequency. The transistor is assumed not to go into saturation.

This paper will be limited to a theoretical analysis of the problem. Research in the literature will be used to verify parts of the theory.
Chapter 2

AN ANALYSIS OF THE CLASSIC CLASS-C AMPLIFIER

In this chapter, the transistor model and the classic class-C amplifier circuit are described. The equations for the base and collector current are derived. Then the equations for the ac input power, the ac output power, and the average current are found. Finally, the gain, output power, collector efficiency, and maximum collector current are derived as functions of power supply voltage and conduction angle.

TRANSISTOR MODEL

The common-emitter configuration will be analyzed. Figure 1 shows the assumed simple model for the low-frequency transistor [4].

\( V_{th} \) is the threshold voltage of the base-emitter junction. \( V_{th} \) must be exceeded before the base current \( i_b \) may flow. This representation leads to transistor input characteristics of Figure 2.

Assume that \( i_C = \beta i_b \) for all \( v_{c} \) and \( i_c \) (\( \beta \) is the current amplification factor, a constant). Also assume that the collector voltage cannot go below a value of \( V_{Sat} \). This assumption means that \( v_C \) goes just down to \( V_{Sat} \) but the transistor does not go into saturation [5]. An analysis of the transistor in saturation is beyond the scope of this work.
Figure 1

Low-Frequency Transistor Model
Figure 2
Transistor Input Characteristics
Another assumption is that the maximum value of $v_c$ does not exceed $(2V_{cc} - V_{sat})$. This assumption allows comparison of various collector voltage waveforms. This is a practical assumption since the collector voltage of most transistors is limited by the collector-to-base breakdown voltage, $BV_{cbo}$. In many cases, the circuit designer adjusts $V_{cc}$ so that $(2V_{cc} - V_{sat})$ is just below $BV_{cbo}$.

THE CLASSIC CLASS-C OPERATION

Figure 3 shows the classic configuration of a class-C transistor amplifier [6]. It is assumed that all reactive components are lossless. $C_c$ is a very low reactance coupling capacitor. It is used to prevent $L_1$ from shorting out $V_{bb}$. The tank circuits formed by $L_1, C_1$ and $L_2, C_2$ are resonant at the operating frequency; that is,

$$\omega_0^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}. \quad (1)$$

In addition, the input and output circuits are very high $Q$; thus,

$$X_{c1} = \frac{1}{\omega_0 C_1} = X_{L1} = \omega_0 L_1 \ll R_i, \quad (2)$$

$$X_{c2} = \frac{1}{\omega_0 C_2} = X_{L2} = \omega_0 L_2 \ll R_e. \quad (3)$$

The harmonic components of $v_b$ are nearly zero because of the very low reactance of $C_1$ at higher frequencies. The same situation applies in the collector circuit because of $C_2$. Thus, $v_b$ and $v_c$ are nearly pure sinusoids at the frequency $\omega_0$. 
\( v_i = V_i \cos \omega_0 t \)

**Figure 3**

Classic Class-C Amplifier Configuration
From Figure 3, it is seen that

\[ v_b = v_i - V_{bb} = V_i \cos \omega t - V_{bb}. \quad (4) \]

\( V_{bb} \) is an external power supply which may be varied to set the conduction angle of the amplifier. \( V_{bb} \) is in series with \( V_{th} \), and \( V_b \) can be defined as:

\[ V_b = V_{bb} + V_{th}. \quad (5) \]

When \( v_b \) reaches the value \( V_{th} \), the base begins to conduct. Thus, the conduction angle, \( \theta \), is defined by

\[ V_{th} = V_i \cos \frac{\theta}{2} - V_{bb} \]

\[ V_{th} + V_{bb} = V_b = V_i \cos \frac{\theta}{2} \]

\[ \theta = 2 \cos^{-1} \frac{V_b}{V_i}. \quad (6) \]

The base voltage waveform is shown in Figure 4a. The peak value of the base current is \( V_i/R_i \). The base current is given by

\[ i_b = \frac{V_i}{R_i} (\cos \omega t - \cos \frac{\theta}{2}) \quad |\omega t| < \frac{\theta}{2} \]

\[ i_b = 0 \quad \frac{\theta}{2} < |\omega t| < \pi. \quad (7) \]

By use of the assumption that \( i_c = \beta i_b \),

\[ i_c = \beta \frac{V_i}{R_i} (\cos \omega t - \cos \frac{\theta}{2}) = I_{cp} (\cos \omega t - \cos \frac{\theta}{2}) \]

\[ |\omega t| < \frac{\theta}{2} \]

\[ i_c = 0 \quad \frac{\theta}{2} < |\omega t| < \pi. \quad (8) \]
Figure 4a

Base Voltage Waveform in Classic Class-C Amplifier
The maximum value of the collector current, $I_{c,\text{max}}$, is

$$I_{c,\text{max}} = I_{cp}(1 - \cos \frac{\theta}{2}).$$

(9)

The base and collector current waveforms are shown in Figure 4b and Figure 4c, respectively.

The collector voltage must be a pure sinusoid at a frequency $\omega_0$ since no harmonic voltages can exist. The peak value of the sinusoid is $V_{cp}$. The average or dc collector voltage must be the power supply voltage, $V_{cc}$, since there is zero dc voltage drop across $L_2$. The collector voltage waveform is shown in Figure 4d. The equation is

$$v_c = V_{cc} - V_{cp} \cos \omega t.$$  

(10)

The figure shows that

$$V_{cp,\text{max}} = V_{cc} - V_{sat}.$$  

(11)

As Figure 3 shows,

$$v_c = V_{cc} - V_L.$$  

(12)

Thus,

$$v_c = V_{cp} \cos \omega t.$$  

(13)

The load voltage waveform is shown in Figure 4e. Note also that $v_{\text{ave}} = 0$.

POWER GAIN

Now the power gain of the amplifier will be calculated. The total input power is

$$P_i = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} v_b \times i_b \, dt$$
\[ i_b = \frac{(V_i - V_b)}{R_i} \]

\[ i_{b,\text{max}} = \frac{(V_i - V_b)}{R_i} \]

\[ I_{bp} = \frac{V_i}{R_i} \]

\[ I_{C,\text{max}} = \beta \left( \frac{V_i - V_b}{R_i} \right) \]

\[ I_{cp} = \beta \frac{V_i}{R_i} \]

**Figure 4b**
Base Current Waveform in Classic Class-C Amplifier

**Figure 4c**
Collector Current Waveform in Classic Class-C Amplifier
Figure 4d

Collector Voltage Waveform in Classic Class-C Amplifier

Figure 4e

Load Voltage Waveform in Classic Class-C Amplifier
\[
P_i = \frac{\omega}{2\pi} \int_{-\pi/2}^{\pi/2} (V_i \cos \omega t - V_{bb}) \frac{V_i}{R_i} (\cos \omega t - \cos \frac{\theta}{2}) dt
\]

\[
P_i = \frac{V_i^2}{4\pi R_i} [\theta - \sin \theta] - \frac{V_{bb}V_i}{\pi R_i} [\sin \frac{\theta}{2} + \frac{\theta}{2} \cos \frac{\theta}{2}].
\]

The second term is the dc term, so that

\[
P_{i_{dc}} = \frac{1}{4\pi} \frac{V_i^2}{R_i} [\theta - \sin \theta]. \tag{14}
\]

The output power may be calculated by use of

\[
P_o = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} V_{cp} \sin \omega t dt.
\]

The collector current, \(i_c\), is the sum of the currents through \(R_c\), \(L_2\), and \(C_2\). However, since \(L_2\) and \(C_2\) are lossless, all the power must be dissipated in \(R_c\). With the use of Equations (8) and (13),

\[
P_o = \frac{\omega}{2\pi} \int_{-\theta/2\omega}^{\theta/2\omega} V_{cp} \sin \omega t \times \frac{\beta V_i}{R_i} (\cos \omega t - \cos \frac{\theta}{2}) dt
\]

\[
P_o = \frac{\beta V_i}{R_i} \frac{V_{cp}}{4\pi} [\theta - \sin \theta]. \tag{16a}
\]

\[
P_o = \frac{I_{cp} V_{cp}}{4\pi} [\theta - \sin \theta]. \tag{16b}
\]

The dc term is zero. This is logical since

\(V_{lave} = 0\). The maximum power out occurs when \(V_{cp} = V_{cc} - V_{sat}\). Then

\[
P_{o,\text{max}} = \frac{I_{cp}(V_{cc} - V_{sat})}{4\pi} [\theta - \sin \theta]. \tag{16c}
\]
Now the power gain, $G_p$, is

$$G_p = \frac{P_{oac}}{P_{i ac}} = \frac{\frac{\beta V_i V_{cp}}{4\pi R_i} [\theta - \sin \theta]}{V_i} \frac{V_i^2}{4\pi R_i} [\theta - \sin \theta]$$

$$G_p = \frac{\beta V_{cp}}{V_i}. \quad (17a)$$

$G_p$ increases as $\beta$ and $V_{cp}$ increase, but $G_p$ decreases as $V_i$ increases. The gain must decrease as $V_i$ increases unless $V_{cp}$ is allowed to increase also. The maximum power gain is

$$G_{p,\text{max}} = \frac{\beta(V_{cc} - V_{\text{sat}})}{V_i}. \quad (17b)$$

This shows that the gain as well as the output power may be increased by increasing $V_{cc}$.

**COLLECTOR EFFICIENCY**

The collector efficiency, $\eta_c$, is defined as the ratio of ac output power to dc input power at the collector.

$$\eta_c = \frac{P_{oac}}{P_{i dc}}$$

$$P_{i dc} = i_{c,\text{ave}} \times V_{c,\text{ave}}$$

$$i_{c,\text{ave}} = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} i_c \, dt = \frac{\omega}{2\pi} \int_{-\theta/2\omega}^{\theta/2\omega} I_{cp} [\cos \omega t - \cos \frac{\theta}{2}] \, dt$$

$$i_{c,\text{ave}} = \frac{I_{cp}}{\pi} [\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2}] \quad (18)$$
\[ v_{c, \text{ave}} = V_{cc} \]

\[ P_{idc} = \frac{V_{cc}I_{cp}}{\pi} \left[ \sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2} \right] \]  
(19)

From Equations (16b) and (19),

\[ \eta_c = \frac{I_{cp}V_{cp}}{4\pi \frac{V_{cc}I_{cp}}{\pi} \left[ \sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2} \right]} \]

\[ \eta_c = \frac{V_{cp} \left[ \frac{\theta - \sin \theta}{\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2}} \right]}{4V_{cc}} \]  
(20a)

\[ \eta_{c, \text{max}} = \frac{V_{cc} - V_{sat}}{4V_{cc}} \left[ \frac{\theta - \sin \theta}{\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2}} \right] . \]  
(20b)

By application of l'Hôpital's rule to Equation (20b), it is found that

\[ \lim_{\theta \to 0} \eta_{c, \text{max}} = \frac{V_{cc} - V_{sat}}{V_{cc}} . \]

But since \( V_{cc} \gg V_{sat} \), in general,

\[ \lim_{\theta \to 0} \eta_{c, \text{max}} \approx 1 . \]

However, note from Equation (16b) that

\[ \lim_{\theta \to 0} P_{o, \text{ac}} = 0 . \]

Thus, as the conduction angle goes to zero, the collector efficiency approaches 100 percent, but the power out goes to zero.

Note from Equation (20b) that
Thus, as $V_{cc}$ is increased, the collector efficiency increases. However, since $V_{cc} \gg V_{sat}$, in general, the improvement in efficiency from increasing $V_{cc}$ is not great.

MAXIMUM COLLECTOR CURRENT

Since most transistors are limited to some maximum collector current, it may be necessary to calculate this parameter.

The equation for $I_{c,\text{max}}$ will now be derived. The output power

$$P_{oac} = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} V_{cc} I_{L} \, dt = \frac{1}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} V_{cc}^{2} \, R_{L} \, dt$$

$$= \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} V_{cc}^{2} \cos^{2} \omega t \, R_{L} \, dt$$

$$P_{oac} = \frac{V_{cc}^{2}}{2R_{L}} . \quad (21)$$

Comparing Equations (16b) and (21):

$$\frac{V_{cc}^{2}}{2R_{L}} = \frac{I_{cp} V_{cc}}{4\pi} \left[ \theta - \sin \theta \right]$$

$$I_{cp} = \frac{2 \pi V_{cc}}{R_{L} \left[ \theta - \sin \theta \right]} . \quad (22)$$

And use is made of Equation (9) to obtain

$$I_{c,\text{max}} = \frac{2 \pi V_{cc}(1 - \cos \theta)}{R_{L} \left[ \theta - \sin \theta \right]} \quad (23)$$
In many cases, the object of the design is to get the maximum output power from a given transistor. As noted above, there is a tradeoff between collector efficiency and output power, both of which are functions of the conduction angle, \( \theta \). \( \theta \) should be chosen for the minimum acceptable efficiency. \( V_{cc} \) should be chosen such that \( 2V_{cc} - V_{sat} < BV_{cbo} \). Then \( R_L \) should be calculated using Equation (25) with \( I_{c,max} = I_{cm} \), the maximum rated collector current for the transistor. The above procedure will result in the maximum output power for a given transistor at a given collector efficiency.
Chapter 3

FOURIER ANALYSIS OF THE COLLECTOR CURRENT WAVEFORM IN THE CLASSIC CLASS-C AMPLIFIER

EQUATIONS FOR FOURIER ANALYSIS

The following equations will be used in the analysis of the load voltage and collector current waveforms. A periodic function of time, \( f(t) \), may be represented in the frequency domain by [7]:

\[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)
\]

where

\[
a_n = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} f(t) \cos n\omega t \, dt
\]

\[
b_n = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} f(t) \sin n\omega t \, dt
\]

\[
a_0 = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} f(t) \, dt \quad \text{(the dc term)}
\]

The amplitude spectrum of the component frequencies is given by

\[
|c_n| = \frac{1}{2} \sqrt{a_n^2 + b_n^2} \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots
\]

When \( f(t) \) is an even function of \( t \), that is, when \( f(-t) = f(t) \), then

\[
a_n = \frac{2\omega}{\pi} \int_{0}^{\pi/\omega} f(t) \cos n\omega t \, dt \quad f(t) \text{ even}
\]

\[
b_n = 0
\]

(26)
The amplitude spectrum then is

\[ |c_n| = \frac{1}{2} a_n \quad \text{f(t) even} \quad (27) \]

\[ n = 0, \pm 1, \pm 2, \pm 3, \ldots \]

**HARMONIC COMPONENTS OF COLLECTOR CURRENT**

The harmonic components of the collector current will be determined from Equations (8) and (26). Note that \( i_c \) is an even function of \( \omega t \).

\[ I_{can} = \frac{2\omega}{\pi} \int_{0}^{\pi/2} I_{cp} (\cos \omega t - \cos \frac{\theta}{2}) \cos n \omega t \, dt \]

\[ I_{can} = \frac{I_{cp}}{\pi} \left[ \frac{\sin(n+1)\theta}{n+1} + \frac{\sin(n-1)\theta}{n-1} - \frac{2 \cos \frac{\theta}{2} \sin \frac{n\theta}{2}}{n} \right] \quad (28a) \]

\[ I_{can} = \frac{I_{cp}}{\pi n (n^2 - 1)} \left[ \sin \frac{\theta}{2} \cos \frac{\theta}{2} - n \cos \frac{n\theta}{2} \sin \frac{n\theta}{2} \right] \quad (28b) \]

\[
\frac{I_{ca0}}{2} = i_{c, ave} = \frac{I_{cp}}{\pi} \left[ \sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2} \right]. \quad (29)
\]

(Note that Equation [29] agrees with Equation [18] for \( i_{c, ave} \).

\[ I_{cal} = \frac{I_{cp}}{2\pi} (\theta - \sin \theta) \quad \text{positive } 0 < \theta < \pi \quad (30a) \]

\[ |I_{cal}| = \frac{I_{cp}}{4\pi} (\theta - \sin \theta) \quad (30b) \]

\[ I_{ca2} = \frac{2I_{cp}}{3\pi} \sin^3 \frac{\theta}{2} \quad \text{positive } 0 < \theta < \pi \quad (30c) \]
The signs of fourth and fifth harmonics are negative for some values of \( \theta < \pi \). This means that these harmonic currents are out of phase with the fundamental. However, the total instantaneous collector current is always positive, i.e., flows out of the transistor. Positive collector current was assumed in the original model since the ideal diode forces the base current to be positive and the collector current is proportional to the base current.
Chapter 4

AN ANALYSIS OF SEVERAL LOAD VOLTAGE WAVEFORMS IN THE CLASS-C AMPLIFIER

HARMONIC COMPONENTS OF LOAD VOLTAGE

Now, various load voltage waveforms will be compared. The limits of \( v_c \) are \( \pm (V_{cc} - V_{sat}) \) because of the limits on \( v_c \). The harmonic components of each waveform will be determined. The object will be to maximize the fundamental component of \( v_L \). This will maximize the fundamental power out since the fundamental component of \( i_c \) is the same in all cases. It will also maximize the collector efficiency since \( i_{c,ave} \) and \( v_{c,ave} \) are the same in all cases. Note that the power out is given by

\[
P_{on} = 2 \times |V_{lan}| \times |I_{lan}|, \tag{31}
\]

since, in the amplitude spectrum, every harmonic component appears on both sides of the frequency origin. The load impedance at each harmonic frequency may be determined from

\[
Z_{lan} = \frac{V_{lan}}{I_{lan}}. \tag{32}
\]

SINUSOIDAL LOAD VOLTAGE

First, the classic case where \( v_c = (V_{cc} - V_{sat}) \cos \omega t \) will be analyzed. The waveform is shown in Figure 5.
Figure 5

Sinusoidal Load Voltage
\[
V_{\text{lan}} = \frac{2\omega}{\pi} \int_0^\pi (V_{\text{cc}} - V_{\text{sat}}) \cos \omega t \cos n\omega t \, dt
\]

\[
V_{\text{lan}} = \frac{(V_{\text{cc}} - V_{\text{sat}})}{\pi} \left[ \frac{\sin (n+1)\pi}{(n+1)} + \frac{\sin (n-1)\pi}{(n-1)} \right]
\]

\[
V_{\text{la0}} = 0.
\]

This result was expected since \( V_{\text{\text{ave}}} = 0 \).

\[
V_{\text{lan}} = (V_{\text{cc}} - V_{\text{sat}})
\]

\[
V_{\text{lan}} = 0 \quad n \neq 1.
\]

All the harmonic components other than the fundamental are zero. This was expected since the load voltage is a pure sinusoid at the fundamental frequency.

\[
|V_{\text{lan}}| = \frac{1}{2} (V_{\text{cc}} - V_{\text{sat}}).
\]  

(33)

Now, with the use of Equations (30a), (31), and (33):

\[
P_{\text{oac}} = 2 \times \frac{I_{\text{cp}}(\theta - \sin \theta)}{4\pi} \times \frac{1}{2} (V_{\text{cc}} - V_{\text{sat}})
\]

\[
P_{\text{oac}} = \frac{I_{\text{cp}}(V_{\text{cc}} - V_{\text{sat}})}{4\pi} (\theta - \sin \theta). 
\]  

(34)

It is seen that Equation (34) agrees with Equation (16c).

**CLIPPED SINUSOID LOAD VOLTAGE**

Now the clipped sinusoid load voltage waveform of Figure 6 will be considered.

\[
V_\lambda = (V_{\text{cc}} - V_{\text{sat}}) \quad |\omega t| \leq \phi
\]
Figure 6
Clipped Sinusoid Load Voltage

\[ \frac{V_{cc} - V_{sat}}{\cos \phi} \]

\[ V_{cc} - V_{sat} \]

\[ -\left(\frac{V_{cc} - V_{sat}}{\cos \phi}\right) \]
\[ V_\phi = \frac{(V_{cc} - V_{sat})}{\cos \phi} \cos \omega t \quad \phi \leq |\omega t| \leq \pi - \phi \]

\[ V_\phi = -(V_{cc} - V_{sat}) \quad |\omega t| > \pi - \phi \]

\( \phi \) is in no way related to the conduction angle of the amplifier, \( \theta \).

\[ V_{\phi\text{an}} = \frac{2\omega}{\pi} \int_0^{\phi/\omega} (V_{cc} - V_{sat}) \cos n\omega t \, dt \]

\[ + \frac{2\omega}{\pi} \int_{\phi/\omega}^{\pi-\phi/\omega} \frac{V_{cc} - V_{sat}}{\cos \theta_1} \cos \omega t \cos n\omega t \, dt \]

\[ - \frac{2\omega}{\pi} \int_{\pi-\phi/\omega}^{\pi/\omega} (V_{cc} - V_{sat}) \cos n\omega t \, dt \]

\[ V_{\phi\text{an}} = \frac{2(V_{cc} - V_{sat})}{\pi} \frac{(1 - \cos n\pi) \sin n\phi}{n} \]

\[ - \frac{1}{2 \cos \phi} \left[ \frac{[1 + \cos(n+1)\pi] \sin(n+1)\phi}{n+1} \right] \]

\[ - \frac{\sin(n-1)\pi \cos(n-1)\phi}{n-1} \]

\[ + \frac{[1 + \cos(n-1)\pi] \sin(n-1)\phi}{n-1} \]

\[ V_{\phi\text{a0}} = 0 \quad \text{as expected since } v_{\phi,\text{ave}} = 0. \]

\[ V_{\phi\text{an}} = 0 \quad \text{for } n \text{ even.} \]

\[ V_{\phi\text{an}} = \frac{2(V_{cc} - V_{sat})}{\pi} \frac{2 \sin n\phi}{n} \]

\[ - \frac{1}{\cos \phi} \left[ \frac{\sin(n+1)\phi + \sin(n-1)\phi}{n+1} \right] \quad \text{n odd} \]
\[ V_{\text{val}} = \frac{2(V_{cc} - V_{sat})}{\pi \cos \phi} \left[ \frac{\pi}{2} - \phi + \frac{\sin 2\phi}{2} \right]. \quad (36) \]

Note that \( V_{\text{val}} = V_{cc} - V_{sat} \) when \( \phi = 0 \). Also, it can be shown that \( V_{\text{val}} \) is maximized when \( \phi = \pi/2 \). The value of \( V_{\text{val}} \) when \( \phi = \pi/2 \) may be found by application of l'Hôpital's rule:

\[ \lim_{\phi \to \pi/2} V_{\text{val}} = \frac{4(V_{cc} - V_{sat})}{\pi}. \quad (37) \]

When \( \phi = \pi/2 \), the waveform of \( v_\ell \) is a square wave.

**SQUARE-WAVE LOAD VOLTAGE**

The square-wave load voltage is shown in Figure 7.

\[ v_\ell = (V_{cc} - V_{sat}) \quad |\omega t| \leq \pi/2 \]

\[ v_\ell = -(V_{cc} - V_{sat}) \quad \pi/2 \leq |\omega t| \leq \pi. \]

The harmonic components will be found from

\[ V_{\ell n} = \frac{2\omega}{\pi} \int_{0}^{\pi/2\omega} (V_{cc} - V_{sat}) \cos n\omega t \, dt \]

\[ = \frac{2\omega}{\pi} \int_{\pi/2\omega}^{\pi/\omega} (V_{cc} - V_{sat}) \cos n\omega t \, dt \]

\[ V_{\ell n} = \frac{2}{n\pi}(V_{cc} - V_{sat})[2 \sin \frac{n\pi}{2} - \sin n\pi] \quad (38) \]

\[ \frac{V_{\ell a0}}{2} = v_{\ell, \text{ave}} = 0. \quad (39) \]

The average voltage is zero, as expected.

\[ V_{\text{val}} = \frac{4}{\pi}(V_{cc} - V_{sat}) \quad (40) \]
Figure 7
Square-Wave Load Voltage
\[ V_{k an} = 0 \quad n \text{ even} \quad (41) \]
\[ V_{k an} = -\frac{4}{n\pi}(V_{cc} - V_{sat}) \quad n = 3, 7, 11 \cdots \quad (42) \]
\[ V_{k an} = +\frac{4}{n\pi}(V_{cc} - V_{sat}) \quad n = 5, 9, 13 \cdots \quad (43) \]

Now the output power at the fundamental frequency will be determined.

\[ P_{o1} = 2 \times |V_{kal}| \times |I_{cal}| \]
\[ P_{o1} = 2 \times \frac{1}{2} \times \frac{4}{\pi}(V_{cc} - V_{sat}) \times \frac{I_{cp}(\theta - \sin \theta)}{4\pi} \]
\[ P_{o1} = \frac{(V_{cc} - V_{sat}) I_{cp}(\theta - \sin \theta)}{\pi^2}. \quad (44) \]

The fundamental output power given by Equation (44) is greater than the power out in the usual case \([\text{Equations (34)} \text{ and (16c)}]\) by a factor of \(4/\pi\).

Since the power out at the fundamental frequency is of primary concern, the fundamental collector efficiency, \(\eta_{cf}\), will be defined \(\eta_{cf} = P_{o1}/P_{idc}\). In the square-wave case,

\[ \eta_{cf} = \frac{\frac{1}{\pi^2}(V_{cc} - V_{sat}) I_{cp}(\theta - \sin \theta)}{\frac{1}{\pi} V_{cc} I_{cp}(\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2})} \]
\[ \eta_{cf} = \frac{\frac{1}{\pi} (V_{cc} - V_{sat}) \times (\theta - \sin \theta)}{V_{cc} \times (\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2})}. \quad (45) \]

The fundamental collector efficiency given by Equation (45) is greater than the collector efficiency in the classic
case, Equation (20b), by a factor of $4/\pi$. If it is assumed that $V_{\text{sat}} = 0$ and the limit as $\theta \to 0$ is taken, one finds that, in the square-wave case,

$$\lim_{\theta \to 0} \frac{\eta_{cf}}{V_{\text{sat}}} = \frac{4}{\pi}.$$ 

This reveals that the fundamental collector efficiency is greater than 100 percent. At first glance, this would appear to violate the law of conservation of power, since the amplifier is putting out more ac power than the dc power it receives from the power supply. This apparent anomaly can be explained by the following: Equation (42) shows that

$$V_{z_{a3}} = -\frac{4}{3\pi} (V_{cc} - V_{\text{sat}}).$$

Thus,

$$Z_{z_{a3}} = \frac{V_{z_{a3}}}{I_{c3}} = \frac{-4}{3\pi} \frac{(V_{cc} - V_{\text{sat}})}{I_{cp} \sin^3 \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$= -\frac{2(V_{cc} - V_{\text{sat}})}{I_{cp} \sin^3 \frac{\theta}{2} \cos \frac{\theta}{2}}. \quad (46)$$

Therefore, the required impedance at the third harmonic frequency is a negative resistance. This means that power must be delivered to the amplifier at the third harmonic frequency in order to have the square-wave load voltage wave form.
NEGATIVE RESISTANCE HARMONIC TERMINATION

A circuit which would deliver power to the amplifier at harmonic frequencies can be built. Such a circuit on the collector of a class-C amplifier is shown in Figure 8.

The collector sees a negative resistance at the third harmonic frequency. This negative resistance could be synthesized by use of an active element. The negative resistance would deliver power to the amplifier at the third harmonic frequency and thus the principle of conservation of power in the class-C amplifier would not be violated. The amplifier is delivering more fundamental power than the dc power it is receiving from the power supply. The difference in power comes from the third harmonic power delivered by the negative resistance.

It should be noted that the total system (amplifier plus negative resistance) is no greater than 100 percent efficient, since the negative resistance must also receive dc power to convert to the third harmonic power it delivers to the amplifier.

The circuit above is clearly not practical. Although the efficiency of the amplifier is increased, the total system efficiency is not. In fact, the system efficiency would be reduced, since there would be less than 100 percent dc-to-ac conversion in the negative resistance.
Figure 8

Negative Resistance Termination of Third Harmonic Current
CONCLUSION

In the classic class-C amplifier, the sign of the coefficient of the third harmonic component of collector current is positive. Thus, it is not possible to develop a square-wave collector voltage without going to exotic negative resistance techniques, since the square-wave collector voltage has a negative third-harmonic coefficient. If the square-wave collector voltage could be developed, it would lead to very high output power and collector efficiency.

In general, the collector voltage waveform is limited by the collector current waveform if passive loads are assumed. The sign of each coefficient of collector voltage must be the same as the sign of the corresponding coefficient of collector current. Otherwise, negative resistance terminations are required.
Chapter 5

LOADING OF HARMONIC CURRENTS IN
A GENERAL CLASS-C AMPLIFIER

The loading of harmonic currents in a general class-C amplifier will be considered. Two different collector current waveforms will be analyzed. The effect of the harmonic load on the performance of the circuit will be determined. In each case, it will be assumed that the collector current range is from 0 to $I_{cm}$ (the transistor's maximum rated collector current) and the collector voltage may swing from $V_{sat}$ to $2V_{cc} - V_{sat}$. It will also be assumed that the conduction angle $\theta = 5\pi/6 = 150^\circ$. Only the load of the first five harmonics will be considered. To consider a larger number of harmonics would unnecessarily complicate the analysis. Also, most collector voltage waveforms can be closely approximated by the first five harmonics. Thus, it is not necessary to deal with the higher harmonics.

The object of the design of the collector load is to maximize the value of the Fourier coefficient of the fundamental component of the load voltage relative to the peak value of the sum of all the harmonics (i.e., the load voltage waveform). This will maximize the fundamental output power since the fundamental component of collector current is assumed constant.
The circuit for maximizing the fundamental component of load voltage is shown in Figure 9. The three parallel resonant circuits are assumed to be very high Q.

The paths of each harmonic component of current are shown in the figure. Each harmonic current is shorted by reactive elements in the parallel networks where it is not resonant. Thus, each harmonic voltage appears only across its respective load resistor (fundamental across R₁, third across R₃, and fifth across R₅).

In Figure 9, there are load resistors for the odd harmonics only. The even harmonics see zero impedance. Thus, the even harmonic voltages are zero. The reason for this is that the even harmonic load voltages cause the peak value of load voltage to be higher, not lower, than the fundamental. This is undesirable. The odd harmonic load voltages cause the peak value of load voltage to be less than the fundamental (if the magnitudes of the odd harmonic components are chosen properly).

CASE A

In this case, the collector current waveform will be the classic class-C amplifier waveform. The load impedance will be changed. The Fourier coefficients of collector current are found from Equations (29) and (30).

\[
\begin{align*}
I_{ca1} &= 0.337 \ I_{cm} \tag{47a} \\
I_{ca2} &= 0.191 \ I_{cm} \tag{47b} \\
I_{ca3} &= 0.0495 \ I_{cm} \tag{47c}
\end{align*}
\]
$L_1$ and $C_1$ are resonant at $\omega_o$
$L_3$ and $C_3$ are resonant at $3\omega_o$
$L_5$ and $C_5$ are resonant at $5\omega_o$

Figure 9
Alternate Collector Load
As discussed above, the even harmonic impedances are zero. In this case, the third harmonic coefficient is positive, so it only can increase the peak of the load voltage waveform. Therefore, the third harmonic impedance is also made to be zero. Voltage is allowed to develop at the fifth harmonic since its coefficient is negative. The following waveform gives a peak value for the fundamental:

\[ V_L = (V_{cc} - V_{sat}) \times 1.05 \times (\cos \omega t - 0.06 \cos 5\omega t) . \quad (48) \]

The load voltage waveform with its components is shown in Figure 10.

\[ V_{\lambda 1} = 1.05 (V_{cc} - V_{sat}) \quad (49a) \]
\[ V_{\lambda 5} = -0.063(V_{cc} - V_{sat}) . \quad (49b) \]

The output power is:

\[ P_{o1} = 2 \times |V_{\lambda 1}| \times |I_{ca1}| = 2 \times \frac{1.05(V_{cc} - V_{sat})}{2} \times \frac{0.337 I_{cm}}{2} \]
\[ = 0.177(V_{cc} - V_{sat})I_{cm} \quad (50a) \]

\[ P_{o5} = 2 \times |V_{\lambda 5}| \times |I_{ca5}| = 2 \times \frac{0.063(V_{cc} - V_{sat})}{2} \times \frac{0.0122 I_{cm}}{2} \]
\[ = (3.84 \times 10^{-4})(V_{cc} - V_{sat})I_{cm} . \quad (50b) \]
Figure 10
Case A Load Voltage

\[ v_L = 1.05(V_{cc} - V_{sat})(\cos \omega t - 0.06 \cos 5\omega t) \]

\[ V_{cc} - V_{sat} \]

\[ 1.05(V_{cc} - V_{sat})\cos \omega t \]

\[ -0.063(V_{cc} - V_{sat}) \times \cos 5\omega t \]

\[ -(V_{cc} - V_{sat}) \]
The load resistances are:

\[ R_1 = \frac{V_{ja1}}{I_{ca1}} = \frac{1.05(V_{cc} - V_{sat})}{0.337 I_{cm}} = 3.12 \frac{V_{cc} - V_{sat}}{I_{cm}} \]  \hspace{1cm} (51a)

\[ R_5 = \frac{V_{ja5}}{I_{ca5}} = \frac{0.063(V_{cc} - V_{sat})}{0.0122 I_{cm}} = 5.16 \frac{V_{cc} - V_{sat}}{I_{cm}} \]  \hspace{1cm} (51b)

The fundamental efficiency is:

\[ \eta_{cf} = \frac{P_{o1}}{P_{idc}} = \frac{0.177(V_{cc} - V_{sat})I_{cm}}{0.200 V_{cc} I_{cm}} = 0.886 \frac{V_{cc} - V_{sat}}{V_{cc}} \]  \hspace{1cm} (52a)

The total efficiency is:

\[ \eta_c = \frac{P_{o1} + P_{o5}}{P_{idc}} = 0.888 \frac{V_{cc} - V_{sat}}{V_{cc}} \]  \hspace{1cm} (52b)

CASE B

Now consider the case where the collector current is rectangular, as shown in Figure 11. This current could have resulted from the base being driven by a rectangular wave. This could be done to improve the efficiency and output power of a power amplifier stage. Several papers in the literature have dealt with the subject of rectangular collector current in transistor amplifiers \[\{8,9,10\}\].

The Fourier coefficients of the collector current will be calculated by use of Equation (26). In this case,

\[ f(t) = I_{cm} \begin{cases} |\omega t| & \leq \theta/2 \\ 0 & \theta/2 < |\omega t| \leq \pi \end{cases} \]

The result is
Figure 11
Rectangular Collector Current
\[ I_{\text{can}} = \frac{\theta I_{\text{cm}}}{\pi} \sin \frac{n\theta}{2} \]  

\[ \frac{I_{\text{ca0}}}{2} = i_{\text{c,ave}} = \frac{I_{\text{cm}} \theta}{2\pi} \]  

\[ I_{\text{ca1}} = \frac{2I_{\text{cm}}}{\pi} \sin \frac{\theta}{2} \]  

\[ I_{\text{ca2}} = \frac{I_{\text{cm}}}{\pi} \sin \theta \]  

\[ I_{\text{ca3}} = \frac{2}{3} \frac{I_{\text{cm}}}{\pi} \sin \frac{3}{2} \theta \]  

\[ I_{\text{ca4}} = \frac{1}{2} \frac{I_{\text{cm}}}{\pi} \sin 2\theta \]  

\[ I_{\text{ca5}} = \frac{2}{5} \frac{I_{\text{cm}}}{\pi} \sin \frac{5}{2} \theta . \]  

With \( \theta = \frac{5}{6}\pi \), the following are the results:

\[ i_{\text{c,ave}} = 0.417 I_{\text{cm}} \]  

\[ I_{\text{ca1}} = 0.615 I_{\text{cm}} \]  

\[ I_{\text{ca3}} = -0.150 I_{\text{cm}} \]  

\[ I_{\text{ca5}} = 0.0330 I_{\text{cm}} . \]  

In Case B, the load will be assumed to be the same as the classic class-C amplifier. Thus, the resultant collector voltage will be a sinusoid at the fundamental frequency with no higher harmonic components.
The fundamental output power is:

\[ P_{o1} = 2 \cdot |V_{\text{al}}| \cdot |I_{\text{cal}}| = 2 \times \frac{V_{cc} - V_{sat}}{2} \times \frac{0.615 \cdot I_{cm}}{2} \]

\[ = 0.308(V_{cc} - V_{sat})I_{cm} \quad \text{(55)} \]

The dc input power is:

\[ P_{i_{dc}} = V_{c, ave} \times i_{c, ave} = V_{cc}(0.417 \cdot I_{cm}) \quad \text{(56)} \]

Then the collector efficiency is:

\[ \eta_{c} = \frac{0.308(V_{cc} - V_{sat})I_{cm}}{0.417 \cdot V_{cc} \cdot I_{cm}} = 0.737 \frac{(V_{cc} - V_{sat})}{V_{cc}} \quad \text{(57)} \]

The load resistance is:

\[ R_{l} = \frac{V_{\text{al}}}{I_{\text{cal}}} = \frac{V_{cc} - V_{sat}}{0.615 \cdot I_{cm}} = 1.63 \frac{V_{cc} - V_{sat}}{I_{cm}} \quad \text{(58)} \]

**CASE C**

For this case, a harmonic load will be selected to improve the performance for the rectangular collector current waveform. Again, the even harmonic impedances are zero. This time, the sign of the third harmonic coefficient of collector current is negative. The coefficients of each harmonic voltage are chosen to maximize the value of the fundamental component. This results in the following coefficients:

\[ V_{\text{la1}} = +1.2(V_{cc} - V_{sat}) \quad \text{(59a)} \]

\[ V_{\text{la3}} = -0.24(V_{cc} - V_{sat}) \quad \text{(59b)} \]

\[ V_{\text{la5}} = +0.042(V_{cc} - V_{sat}) \quad \text{(59c)} \]
The resulting load voltage is shown in Figure 12, along with its component harmonic voltages. Note that the fundamental component is larger than the peak of the composite voltage. The addition of the higher harmonic voltages has resulted in "leveling out" the composite voltage. Now the output power in each harmonic is

\[ P_{o1} = 2 \times |V_{z1}| \times |I_{cal}| = \frac{1.2(V_{cc} - V_{sat})}{2} \times \frac{0.615 \cdot I_{cm}}{2} \]

\[ = 0.369(V_{cc} - V_{sat})I_{cm} \] \hspace{1cm} (60a)

\[ P_{o3} = 2 \times \frac{0.24(V_{cc} - V_{sat})}{2} \times \frac{0.150 \cdot I_{cm}}{2} \]

\[ = 0.0180(V_{cc} - V_{sat})I_{cm} \] \hspace{1cm} (60b)

\[ P_{o5} = 2 \times \frac{0.042(V_{cc} - V_{sat})}{2} \times \frac{0.0330 \cdot I_{cm}}{2} \]

\[ = 6.93 \times 10^{-4}(V_{cc} - V_{sat})I_{cm} \] \hspace{1cm} (60c)

The total output power is

\[ P_{total} = 0.388(V_{cc} - V_{sat})I_{cm} \] \hspace{1cm} (60d)

The dc input power is given by Equation (56). It is the same as in the previous case, since the average collector voltage and current are the same as before.
\[ v_L = 1.2(V_{cc} - V_{sat})(\cos \omega t - \frac{1}{5} \cos 3\omega t + 0.035 \cos 5\omega t) \]

Figure 12
Case C Load Voltage
The fundamental collector efficiency is

\[ \eta_{cf} = \frac{0.369(V_{cc} - V_{sat})I_{cm}}{0.417 V_{cc} I_{cm}} = 0.885 \frac{V_{cc} - V_{sat}}{V_{cc}} \]  (61a)

The total collector efficiency is

\[ \eta_{c} = \frac{0.388(V_{cc} - V_{sat})I_{cm}}{0.417 V_{cc} I_{cm}} = 0.930 \frac{V_{cc} - V_{sat}}{V_{cc}} \]  (61b)

Now each load resistor will be calculated.

\[ R_1 = \frac{V_{l,al}}{I_{ca,1}} = \frac{1.2(V_{cc} - V_{sat})}{0.615 I_{cm}} = 1.95 \frac{V_{cc} - V_{sat}}{I_{cm}} \]

\[ R_3 = \frac{V_{l,a3}}{I_{ca,3}} = \frac{-0.24(V_{cc} - V_{sat})}{-0.150 I_{cm}} = 1.60 \frac{V_{cc} - V_{sat}}{I_{cm}} \]

\[ R_5 = \frac{V_{l,a5}}{I_{ca,5}} = \frac{0.042(V_{cc} - V_{sat})}{0.033 I_{cm}} = 1.27 \frac{V_{cc} - V_{sat}}{I_{cm}} \]

SUMMARY OF RESULTS

This section compares the performance in the classic class-C amplifier, Case A, Case B, and Case C. The results are tabulated in Table I.

The classic case and Case A have the same collector current waveform. Case A has a harmonic load which improves both the power out and collector efficiency. The output power in Case A is 0.2 db higher than the classic case and the efficiency is 4.4 percent higher.

Case B has a rectangular collector current waveform. Although Case B has the same collector load as the classic case, the output power is much higher. This is because the rectangular waveform has a higher fundamental component than
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</tr>
<tr>
<td>Collector circuit</td>
<td>Fig. 3</td>
<td>Fig. 9</td>
<td>Fig. 3</td>
<td>Fig. 9</td>
</tr>
<tr>
<td>Load voltage</td>
<td>Fig. 13c</td>
<td>Fig. 12</td>
<td>Fig. 13c</td>
<td>Fig. 12</td>
</tr>
<tr>
<td>Fundamental output power [% ( I_{cm} (V_{cc} - V_{sat}) )]</td>
<td>16.9</td>
<td>17.7</td>
<td>30.8</td>
<td>36.9</td>
</tr>
<tr>
<td>Fundamental efficiency [% ( (V_{cc} - V_{sat})/V_{cc} )]</td>
<td>84.4</td>
<td>88.6</td>
<td>73.7</td>
<td>88.5</td>
</tr>
<tr>
<td>Total efficiency [% ( (V_{cc} - V_{sat})/V_{cc} )]</td>
<td>84.4</td>
<td>88.8</td>
<td>73.7</td>
<td>93.0</td>
</tr>
<tr>
<td>Harmonic output power [% fundamental]</td>
<td>0</td>
<td>0.22</td>
<td>0</td>
<td>5.1</td>
</tr>
<tr>
<td>Fundamental load resistance [( x (V_{cc} - V_{sat})/I_{cm} )]</td>
<td>2.97</td>
<td>3.12</td>
<td>1.63</td>
<td>1.95</td>
</tr>
<tr>
<td>Third harmonic load resistance [( x (V_{cc} - V_{sat})/I_{cm} )]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.60</td>
</tr>
<tr>
<td>Fifth harmonic load resistance [( x (V_{cc} - V_{sat})/I_{cm} )]</td>
<td>0</td>
<td>5.16</td>
<td>0</td>
<td>1.27</td>
</tr>
</tbody>
</table>

In each case, the limits on \( i_c \) were 0 to \( I_{cm} \).
In each case, the limits on \( v_c \) were \( V_{sat} \) to \( 2V_{cc} - V_{sat} \).
In each case, the conduction angle was \( 5/6 \pi = 150^\circ \).

Case A - Classic class-C amplifier collector current with collector load for improved performance.
Case B - Rectangular collector current with classic class-C amplifier load.
Case C - Rectangular collector current with collector load for improved performance.
Figure 13a

Collector Current, Classic Case and Case A
Figure 13b

Collector Voltage, Classic Case and Case B

Figure 13c

Load Voltage, Classic Case and Case B
the classic current waveform. The efficiency is lower in Case B than in the classic case because the average current is higher in Case B. Comparison of these two cases points out that the nature of the collector current waveform is important in deciding the collector load.

Case C has the best performance of all cases. It has a rectangular collector current waveform with a harmonic load which maximizes the fundamental component of the waveform relative to the peak of the waveform. The output power in Case C is 3.4 db greater than the classic case. In addition, the collector efficiency is considerably better, with only 7 percent of the input dc power being dissipated in the transistor. Note that there was significant power generated at the harmonic frequencies in Case C. However, none of this power was dissipated in the fundamental load resistor, and thus there would be no problem in filtering these harmonics.

FREQUENCY LIMITATIONS TO THEORY

The theory discussed above is limited to a low-frequency model. There are two factors which restrict the theory at high frequencies. These factors are parasitic reactances and the reduction in transistor gain at high frequencies.

At high frequencies, the parasitic inductance of the collector lead will present a high series reactance to the higher harmonics. This inductance, in conjunction with the
parasitic shunt package capacitance, will tend to minimize the effect of the higher harmonics on the transistor performance [11]. The combination produces a low-pass effect which greatly reduces the magnitude of high-order harmonic voltages on the output of the transistor. Thus, no matter what network is put on the output of the transistor, no significant voltage can be developed at the higher order harmonic frequencies.

The second limitation to the application of the theory at high frequencies is the roll-off of the transistor's gain ($\beta$) [12]. When $\beta$ is reduced at the higher harmonic frequencies, the higher harmonic components of collector current are significantly reduced. Since these currents are not available in the output, no voltage may be developed at the higher harmonic frequencies.

Not having the higher order components of collector current in the output also has an adverse effect on collector efficiency. Consider Figure 14, which shows the classic class-C collector current represented by the first five harmonics. The collector current is not zero for $|\omega t| > \theta/2$. Figure 13b shows that the collector voltage is greatest for $|\omega t| > \pi/2 > \theta/2$. Thus, this situation results in greater dissipation in the transistor when the higher harmonics are missing since

$$P_{\text{diss}} = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} v_c i_c \, dt.$$ 

Cases where the collector current flows for $|\omega t| > \theta$ have been observed in the literature [13,14].
Figure 14

Classic Class-C Collector Current
Represented by First Five Harmonics
Chapter 6

CONCLUSIONS

The following generalizations may be made on the performance of the classic class-C amplifier as a function of power supply voltage and conduction angle. The power gain and output power are directly proportional to the power supply voltage. Thus they may both be increased by increasing the supply voltage. The collector efficiency also increases as supply voltage increases, but the increase in efficiency is very small. The collector efficiency increases and the output power decreases as the conduction angle decreases. The power gain is not a function of conduction angle since both the input power and output power are proportional to the same function of conduction angle.

An analysis of the harmonic components of collector current in the classic class-C amplifier shows that the signs of the fundamental, second, and third harmonic components are positive (and thus in phase) for any conduction angle between zero and \( \pi \). The signs of the fourth and fifth harmonics are negative for some conduction angles between zero and \( \pi \). However, the total instantaneous collector current is always positive.

The following observations apply to the harmonic terminations of collector current in the classic class-C
amplifier. Not all collector voltage waveforms are possible with the classic class-C amplifier collector current if harmonic load impedances are assumed to be passive. In particular, if the collector voltage waveform is to be a square-wave, a negative resistance must be supplied at the third harmonic. This means that power must be delivered to the transistor at the third harmonic frequency. In general, with passive loads, the only possible collector voltage waveforms are those for which the signs of the Fourier voltage coefficients are the same as the signs of the corresponding Fourier current coefficients.

The study of the performance of an amplifier with various collector current waveforms and harmonic loads yielded the following results. The output power and efficiency of a class-C amplifier may be improved if the harmonics are terminated properly. The collector voltage waveform should be such that the fundamental component is greater than the peak of the waveform. This results in higher output power and efficiency. The rectangular collector current waveform yields much larger output power and efficiency than the classic case if the harmonics are terminated properly. In particular, the output power is 3.4 db greater and the efficiency is 7.6 percent greater for a conduction angle of 150°. The application of the harmonic loading theory is limited to frequencies where the higher harmonics are not attenuated by parasitic reactances or by the roll-off of the transistor gain.
REFERENCES


2. Ibid., p. 487.


5. Ibid., p. 43.


13. Harrison, op. cit.