Mechanically automated steering for a four wheel mobile platform with differential drive kinematics and controls

A thesis submitted in partial fulfillment of the requirements
For the degree of Master of Science in Mechanical Engineering

By

Ara A. Mekhtarian

May, 2014
The thesis of Ara A. Mekhtarian is approved:

_________________________________________  _________________
Dr. Stewart Prince                                       Date

_________________________________________  __________________
Professor Aram G. Khachatourians                        Date

_________________________________________  __________________
Dr. C.T. Lin, Chair                                      Date

California State University, Northridge

ii
# Table of Contents

Signature Page ii  
List of Tables vi  
List of Figures vii  
Abstract xi  
Introduction 1  

I- Mobile Platform configuration  
  I.1- A consideration of number of wheels 3  
  I.2- Ackerman steering vs. differential drive 5  
  I.3- Omni-directional passive caster wheel 7  
  I.4- The proposed platform configuration 9  
  I.5- Drive systems for the proposed platform design 11  

II- Mechanical design of the wheel  
  II.1- An introduction to the multi-directional active wheel 12  
  II.2- Examples of Omni-directional active wheel designs 13  
  II.3- Four wheel steering vs. Omni-directional wheels 17  

III- Potential multi-directional active wheel designs 20  
  III.1- Sectioned linear slide wheel 21  
  III.2- Alternating dually wheel 23  
  III.3- Gimbal spherical wheel 26  
  III.4- Mechanically automated four-wheel-steering 30  
  III.5- The mechanical design matrix 35
IV. First-iteration of prototyping the proposed platform
   IV.1- Prototype motion and controls
   IV.2- Analysis of drive motor speeds and torques
   IV.3- Differential gear boxes
   IV.4- Machining custom designed parts
   IV.5- Welded dynamic steel space frame
   IV.6- Wheels and tires
   IV.7- Prototype component layout
   IV.8- Platform weight and battery run time analysis
   IV.9- Cost analysis of the prototype
   IV.10- Manual testing and analysis

V. Second iteration prototype design
   V.1- Manual testing of the second iteration prototype

VI. Third iteration prototype design
   VI.1- Steering pivot offset torsion spring force analysis

VII. The autonomous test plan
   VII.1- Kinematics of the proposed platform
   VII.2- Optimizing the steering pivot offset
   VII.3- Programming the Vex microcontroller
   VII.4- Calibration
   VII.5- Autonomous test results

VIII. Conclusion

References
<table>
<thead>
<tr>
<th>Appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix-A</td>
<td>86</td>
</tr>
<tr>
<td>Appendix-B</td>
<td>89</td>
</tr>
<tr>
<td>Appendix-C</td>
<td>89</td>
</tr>
<tr>
<td>Appendix-D</td>
<td>90</td>
</tr>
</tbody>
</table>
List of Tables

Table-1: Mechanical design comparison matrix 37
Table-2: Comparison of logic system microcontrollers 41
Table-3: Prototype weight analysis 52
Table-4: Cost analysis of the first iteration prototype 63
Table-5: Torque and gravity force component 65
Table-6: Results 80
Table-A3-1: Straight path calibration data 89
Table-A3-2: Turning radius calibration data 89
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comparison of three and four wheel platforms</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Ackerman steering and Differential Drive</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Caster wheel</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>Comparison of two differential drive platforms</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Examples of drive systems for the proposed platform design</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>Introducing the multidirectional active wheel</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>Swedish Omni-directional wheel</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>Airtrax forklift</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>Spherical Omni-directional wheel</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>Multi-material anisotropic friction wheel</td>
<td>15</td>
</tr>
<tr>
<td>11</td>
<td>Spherical Drive System (SDS)</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>Four wheel steering</td>
<td>17</td>
</tr>
<tr>
<td>13</td>
<td>Simple path example</td>
<td>18</td>
</tr>
<tr>
<td>14</td>
<td>Sectioned linear slide wheel components</td>
<td>21</td>
</tr>
<tr>
<td>15</td>
<td>Sectioned linear slide wheel Assembly</td>
<td>21</td>
</tr>
<tr>
<td>16</td>
<td>Wheel section realignment feature</td>
<td>22</td>
</tr>
<tr>
<td>17</td>
<td>Wheel section realignment simulation snapshots</td>
<td>22</td>
</tr>
</tbody>
</table>
Figure 18: Alternating dually wheel design  23
Figure 19: Simulation of the sliding mechanism  23
Figure 20: Alternating dually wheels  24
Figure 21: Gimbal spherical wheel  26
Figure 22: Gimbal spherical wheel simulator  27
Figure 23: Simulation of Gimbal wheel without any interference  28
Figure 24: Starting position of the Gimbal wheel  28
Figure 25: Interference between the drive shaft and the wheel  29
Figure 26: Four-wheel steering, wheel position configurations  30
Figure 27: Four wheel steering, Wheel design  31
Figure 28: Four-wheel steering, platform design  32
Figure 29: Four-wheel steering, functionality  33
Figure 30: Vex control system  42
Figure 31: Vex motor  43
Figure 32: Platform traveling up an incline  43
Figure 33: Differential gearbox application  45
Figure 34: Differential gear box modifications  46
Figure-35: Differential gear box assembly and Break-in run time 46

Figure-36: SolidWorks model and actual machined pivot blocks 47

Figure-37: SolidWorks model of the dynamic frame 48

Figure-38: Ball bearing wheel pivot joints 49

Figure-39: Soft foam rubber tires vs. Vex tires 50

Figure-40: Component layout 51

Figure-41: Free-body diagram of front right wheels 55

Figure-42: Pivot point is placed to be offset along the driving axle 56

Figure-43: Pivot shift mechanism 57

Figure-44: Platform right turn simulation 58

Figure-45: Platform straight path simulation 59

Figure-46: FM transmitter 61

Figure-47: Manual Test course 61

Figure-48: 3rd iteration wheel design 63

Figure-49: 3rd iteration Platform assembly and rendering 64

Figure-50: Tilting angle of vertical link 65

Figure-48: Local and global reference frames 67
Figure-49: Turning radius 69

Figure-50: Ackerman steering kinematics 70

Figure-51: steering pivot offset range 72

Figure-52: RobotC Algorithm example 74

Figure-53: Vex PIC programming hardware 74
Abstract

Mechanically automated steering for a four wheel mobile platform with differential drive kinematics and controls

By

Ara A. Mekhtarian

Master of Science in Mechanical Engineering

In comparison to the carlike Ackerman steering, differential-drive mobility is known to improve the mechanical maneuverability of an autonomous ground vehicle, to simplify the control system and optimize its efficiency. However, most two-wheel differential drive platforms suffer from unbalanced weight distribution, and underperform due to a lack of wheel traction. A pioneer solution is the proposed mobile platform, with mechanically automated steering driven by four powered wheels, which improves the maneuverability of the Ackerman wheel mobile platform configuration, but yet maintains the simple kinematics and controls of a differential drive system. The mechanical design of each wheel eliminates the need for an external control system to mechanically steer or rotate the orientation of each of the four wheels.

The scope of this thesis includes the following: the mechanical design and the kinematic analysis, machining and assembly of a scaled model prototype, and test cases for both manual and autonomous control modes of the proposed mobile platform. Finally, obtained results from the autonomous tests are presented along with recommendations for future improvements of the mechanical design of the proposed mobile platform.
Introduction

The goal of this thesis project is to conceive and design an ideal mobile platform, in terms of maneuverability, motion control simplicity, energy efficiency, and platform stability. The project begins with research and comparisons among different mobile platform configurations, and identifies the platform that delivers the ultimate performance, but yet has the simplest motion control system. The term mobile platform is in reference to any type of a ground vehicle or robot, varying from a small powered wheel chair platform to a larger passenger transportation vehicle. Within the chosen mobile platform configuration, a mechanical design improvement to the wheel design is introduced, which makes the mobile platform more maneuverable and energy efficient, and at the same time simpler to control. A comparison is finally made to similar wheel designs, to show why the suggested design is preferable over the other designs.

The mechanical design portion of this project outlines three wheel designs that achieve the same performance objective through different methods of locomotion, and demonstrates motion simulations of the functionality of each wheel design. The three wheel designs are compared and graded using a mechanical design matrix, in order to indentify the ultimate wheel design for further development and testing. Finally a kinematic analysis of the motion of the overall platform using the chosen wheel design is performed, in order to develop the motion control equation that maps the speed of each wheel to the motion path of the mobile platform, knowing the geometry of the platform and the wheels.

A physical prototype of the proposed mobile platform is assembled, not only to test if the suggested concept physically works, but to also test how accurately controllable the platform is both manually and autonomously. The mechanical parts composing the platform of the
The prototype consists of both off-the-shelf and custom fabricated parts. The microcontroller that controls the motion of the wheels is planned to be programmed to control the motion of the platform autonomously, or through a radio receiver that receives signals from a manually controlled radio transmitter.

The testing portion of the project includes two different test plans. The manual control test is performed to show that any user can fully control the prototype platform through a simple test course. The open-loop controlled pre-planned path test, on the other hand, is carried out to show how predictable the position of the mobile platform can be at any given time by pre-programming the mobile platform to travel from one known location to another, and measuring the results.
I- Mobile Platform Configuration:

I.1- A consideration of the number of wheels:

In order to determine the optimal number of wheels, for an ideal mobile platform, a comparison is made between two potential candidate configurations. The first platform consists of three wheels, which is the minimum number of surface contact points required for a mobile platform to be stable. The second platform consists of four wheels in the configuration shown in figure-1. Both platforms have Omni-directional wheels which allow the wheels to travel in one direction, and to slip in another direction.

Two main criteria are considered in comparing the mobile platforms, Omni-directional mobility and platform stability at different speeds. From the perspective of Omni-directional mobility, three wheels are sufficient to achieve three degrees of freedom in motion needed in the inertial reference frame, or travel in the x and y directions and rotation about the z-axis. From this viewpoint, the four wheel platform appears to have a redundancy by having an additional wheel. However, from the perspective of platform stability, adding the fourth wheel offers significantly higher stability than the three wheel platform. Comparing the angles between any two wheel axle, \( \alpha \), for each design, not only is the angle for the four wheel design 25\% smaller than that of the three wheel platform, but the four wheel platform offers support from all four corners of a typically rectangular platform, achieving optimal stability in the vertical direction.
It is concluded that, even though the four wheel mobile platform may appear to have redundancy from the Omni-directional mobility standpoint, it is the platform of choice since the fourth wheel offers the required platform static stability.
I.2- Ackerman steering vs. differential drive

During the previous analysis of mobile platform mobility and stability, it has been established that four wheels are necessary. The next paragraphs describe an analysis of platform maneuverability, in which two four-wheel platform designs are compared: the first with an Ackerman steering mechanism similar to a typical passenger transportation vehicle, and the second equipped with a differential drive where the speeds of the two drive wheels control both the speed and the direction of the platform. Both platforms have two degrees of freedom in the inertial reference frame.

The degree of maneuverability of a mobile platform is theoretically defined as the sum of the degrees of mobility and steerability ($\delta_M = \delta_m + \delta_s$). In the case of the Ackerman steering platform: $\delta_m = 1$, $\delta_s = 1$, and therefore $\delta_M = 2$. For the the differential drive platform $\delta_m = 2$, $\delta_s = 0$, and therefore $\delta_M = 2$. Another way to gauge the maneuverability of a vehicle is by measuring the physical turning radius of the platform. For the Ackerman steering for example, the smallest turning radius that the platform can achieve is $w/2$ ($w =$ width of platform), and that’s when one of the rear wheels experiences no translation but only rotation at its point of contact, and the rear axle rotates around this point. The differential drive platform however, can make a zero radius turn, where the drive wheels are rotated at the same speed but in opposite directions, and the platform rotates about its center vertical axis without traveling. It follows that, even though both the Ackerman steering and the
differential drive platforms have the same theoretical two degrees of maneuverability, the differential drive platform is physically more maneuverable since it’s capable of achieving pure rotation.
I.3- Omni-directional passive caster wheel:

The four-wheel differential drive mobile platform displayed in Figure-2 has two driving wheels (left and right), and two Omni-directional caster wheels (front and rear). Though caster wheels have many advantages, they also present restrictions to the platform under certain conditions, and have limitations in their applications of use.

The caster wheels are what allow the differential drive mobile platform in Figure-2 to make a zero radius turn. One of the best features of a caster wheel is the fact that it is a passive Omni-directional wheel, which means that the direction-of-travel of the wheel is dictated by the mobile platform, and theoretically, caster wheels don’t present any restrictions to the platform. However, in real-world conditions, a caster wheel does present restrictions.

A diagram of a caster wheel is shown in Figure-3. As long as the contact surface of the wheel is horizontal, as shown to the right in the figure, the normal force has no horizontal components, and the caster wheel functions as it theoretically should. However, when the contact surface is not horizontal as shown to the left, the normal force has the horizontal component Nx, which will partially exhaust the force F exerted by the platform to dictated the direction of the wheel. When the caster wheel resists turning in the direction desired by the mobile platform, it presents restrictions to the platform, both during sharp turns and on straight paths. As a result, a caster wheel is unstable at high speeds since the horizontal components of its normal forces, which continuously change directions with the terrain surface, causes the passive swivel of the wheel to oscillate about its
vertical axis at high frequencies. When the platform is travelling at high speeds, the oscillation of the caster wheels result in violent shaking forces of the platform, and distort the vehicle’s straight path motion. Furthermore, all passive wheels, including caster wheels, generally add friction drag to the drive train of a mobile platform.
I.4- The proposed platform configuration:

An ideal mobile platform would take advantage of the maneuverability and the simplicity of the kinematic and control systems of a differential drive platform configuration, yet maintain a stable four-wheel configuration, without using any passive caster wheels. Though such a platform would only have two degrees of freedom in the inertial reference frame, this platform design may be preferable for both automatically and manually controlled mobility for the following reasons.

Human beings have an almost 180-degree forward facing horizontal field of view, but our color perception and visual concentration ability is limited to the center portion of this field of view. Electronic sensors that perceive the environment of an autonomous mobile platform, such as laser range finders and cameras have similar field of view limitations, which are usually limited to the front of the vehicle. When information about the environment on both sides of the vehicle is not always available, translation of the mobile platform from one side to another does not prove to be very useful. Adding sensors to increase the perception angle can easily overwhelm the onboard data processor, due to the large amount of data transmitted by the environment perception sensors. A mobile platform that can travel in one direction, and that can perform pure rotation, as suggested, has sufficient maneuverability to travel to any desired destination through any desired path and achieve any desired final orientation position. Even though a platform with three degrees of mobility may achieve the same goal more efficiently, it requires that its path is clear in doing so, which it may or may not be capable of detecting.

Two four-wheeled differential drive mobile platforms are shown in Figure-4, and the following paragraph compares the two designs, and highlights the reasons why the proposed mobile platform, Design-2 is the platform of choice.
The first design (Design-1) has two active wheels, which provide a moderate amount of traction, but it also relies on some of its platform weight to be supported by two caster wheels, which cause friction drag and instability of the platform at high speeds. The simplicity of its drive and control systems makes Design-1 attractive. Design-2, however, has four active wheels, which provide superb traction, and uses no passive wheels for platform support, minimizing friction drag. At the same time, Design-2 maintains the same simple differential drive system as Design-1, where each motor controls the two front and rear wheels on each side of the platform. Finally, Design-2 is much more stable during sharp high speed turns. The configuration of the two wheels on each side of the platform, achieves premium platform support on each corner of the platform.
I.5- Drive systems for the proposed platform design

The simplest and most reliable mechanical method to drive all four wheels, is to have a separate drive motor for each wheel. So long that two motors on either side of the platform can be controlled to rotate at the same speed and direction, differential drive control of the mobile platform can be achieved. The two wheels on each side can also be mechanically synchronized to rotate at the same speed through one drive motor. There are different drive methods that can be utilized so that each motor of the proposed platform design drives two wheels synchronically.

Two examples of drive systems are shown in Figure-5. The first drive system (top) consists of four individual right angle gear reducers at each wheel, driven by the motors via drive shafts. The second drive method (bottom) consists of only two inline gear speed reducers that are mated to the motors, and that drive the wheels using timing belts and gear pulleys.

Figure-5: Examples of drive systems for the proposed platform design. www.Apexdynamics.com
II- Mechanical Design of wheel

II.1- An introduction to the Multi-directional wheel:

The proposed mobile platform, shown in figure-6, is very stable when traveling in the direction of travel of its wheels (y-direction, represented by the red vector). However the platform’s fixed wheels are restricted to any translation in the direction perpendicular to the direction of travel (x-direction). In order for the proposed platform to be capable of going through pure rotation, all four of its wheels need to travel along the marked circular path, or at a 45-degree angle from their actual travel paths (represented by the yellow vector). For the platform to travel in any direction within the range of two degrees of freedom relative to the inertial reference frame, the span of the travel directions of each of its wheels will vary from a straight path to a 45-degree deviation, in opposite directions (shown by the shaded areas in Figure-6, Top).

Since each wheel is already active in the direction of travel, one way to achieve a deviation of a 45-degree angle from its direction of travel is by allowing a passive translation of the wheel in the direction normal to the direction of travel. Such a multi-directional active wheel, also known as an Omni-directional wheel, minimizes wheel friction of the proposed platform during sharp turns, and improves the overall efficiency of the vehicle.
II.2- Examples of existing Omni-directional active wheel designs:

Before beginning the design of an active multi-directional wheel, it’s important to understand how other existing wheel designs can achieve travel in multiple directions. The objective of this research is to explore the variety of designs of multi-directional active wheels that are either in use today, or are experimental prototypes. The following paragraphs highlight the concepts behind the following four different wheel designs:

1) The Swedish Omni-directional wheel
2) The spherical Omni directional wheel
3) The multi-material friction wheel
4) The Spherical Drive System (SDS)

The Swedish Omni-directional or Mecanum wheel, as pictured in figure-7, was invented by Bengt Ilon in 1973 when he worked for the Swedish company Mecanum AB. The wheel consists of an active center hub with passive rollers mounted at a 45-degree angle relative to the travel direction, all around the outer circumference of the wheel. The impacted result from each Mecanum wheel to the overall platform is a traction force at 45-degrees (represented by a red vector). When all four wheels are configured as shown in Figure-7, the platform is capable of three degrees of freedom in the inertial reference frame.

It’s also important to note that as a result of the direction of the traction force of the Swedish Omni-directional wheel, the magnitude of the travel force in a straight path (represented
by the green vector in Figure-7) is approximately seven tenth the magnitude of the actual traction force, which means that approximately 30 percent of the energy that goes into the wheel is lost to rolling during a straight path motion of the platform. A Mecanum wheel also presents challenges on uneven and unpaved terrain, since its ability to travel over obstacles in the passive direction is limited by the small diameter of the rollers. The main advantage of using Mecanum wheels on a platform is the side-to-side motion path. One practical use of this feature is demonstrated by the Airtrax forklift (pictured in Figure-8), with which very long payloads can be lifted and translated in any direction without the need to rotate them.

The second example is the spherical Omni-directional wheel design, which was developed by students at the Kaneko Higashimori Laboratory, at Osaka University in Japan in 2008. Though the kinematics and the control of the spherical wheel are identical to the Mecanum wheel, their mechanical designs are quite different, as can be seen in Figure-9. The spherical wheel consists of two semi-spherical passive halves, which are active in a direction perpendicular to the passive axis through a center drive shaft. When the drive shaft of each wheel is mounted at a 45-degree angle on the platform, as
shown in Figure-9, the platform achieves the same three degrees of freedom relative to the inertial reference frame as the platform with Swedish wheels.

The Multi-material anisotropic friction wheel, an interesting example of a multi-directional wheel design, was recently developed by a group of engineers for a project funded in part by the U. S. Army research laboratory. The outer circumference of the wheel pictured in Figure-10 is populated with nodes made from two different materials. The center section of each node is made from a high friction material to provide the necessary traction in the travel direction of the wheel. On both sides of the node a low friction material is casted to provide slip in the lateral direction. The nodes are designed to bend when the wheel is translated laterally, allowing the low friction material to make contact with the ground, as shown at the bottom of the figure.

The design advantage of the multi-material friction wheel is the fact that it does not have any moving mechanical parts, such as rollers. Even though the wheel was tested on a scale prototype model, it is not practical for full size vehicle applications. The low friction material may perform well on smooth indoor surfaces while supporting a light weight platform, but will not be able to provide sufficient wear resistance for harsh outdoor paved surfaces while supporting the weight of a much heavier vehicle.

The last example of a multi-directional wheel design comes from the Charles W. Davidson College of Engineering at San Jose State University, where students are working on
the Spherical Drive System (SDS) pictured in figure-11. The basic idea behind the drive and control systems of the spherical wheel are the same as the three Omni-directional wheel platform drive and controls. The SDS concept uses three active Omni-directional wheels to control the travel direction of the sphere wheel through direct friction drive. The team of students plans to use two spherical wheels on a platform resembling a motorcycle, which has three degrees of freedom relative to the inertial reference frame.

One disadvantage of SDS is the fact that the drive system relies on the friction between the Omni-directional wheels and the outer surface of the sphere to create a travel force in the desired direction. In harsh outdoor environments, dust and moisture collected on the outer surface of the sphere will reduce the friction coefficient significantly. The friction is also partially dependent on the weight of the rider. Another disadvantage of spherically shaped wheels is their bulky shape, or the ratio (1:1) of their width to diameter, which is much larger than most conventional wheels.
II.3- Four-wheel steering vs. Omni-directional wheels:

A second way to allow a wheel to travel in more than one direction, or to deviate from its fixed travel path is by steering the wheel, or rotating it about its vertical axis, in the desired direction of travel. The proposed mobile platform can be mechanically equipped with four additional motors to control the travel direction of each wheel by rotating it about the vertical axis. An example of a mobile platform with four-wheel steering control is shown in Figure-12. When all four wheels are rotated to a 45-degree angle as shown, the platform is capable of going through a pure rotational motion.

Some differences between a platform with a four-wheel steering, and one with multi-directional wheels are as follows. In addition to the drive motors, the four-wheel steering platform requires four more controllable motors, to control the rotational position of each wheel, which are not required for the Omni-directional wheel platform. A four-wheel steering design can be associated with a control system that’s significantly more complex than that of the Omni-directional wheel platform since it has more controllable degrees of freedom. The steering mechanism also adds a significant amount of undesired weight to the platform, and is less energy efficient than a multi-directional wheel.

Another difference between the two platforms is the fact that a steering mechanism requires more time to respond to quick direction changes, and as a result, the four-wheel steering platform has a discontinuous motion, and a delayed reaction time in comparison to the platform with Omni-directional wheels. The following example highlights how a platform with four-
wheel steering has a slower response time, and a less continuous motion path than a platform with multi-directional wheels. For this example, a simple path is chosen for two mobile platforms to travel through, where one platform has four wheel steering controls, and the second has multi-directional active wheels. The vehicles both start at the position shown in Figure-13, and need to reach the highlighted goal by traveling within the dashed boundary lines. The platform with multi-directional wheels will travel in the x-direction until the 90-degree turn is reached, go through a counter-clockwise rotation, and then travel in the y-direction to reach its goal.

The motion progress of the multi-directional wheel platform can be seen to be continuous in Plot-1, since during any time interval the vehicle is either translating in the x or y-directions or going through pure rotational motion. The motion progress plot for the platform with four-wheel steering control however, is not continuous as shown in Plot-2. During the time intervals: 1-2 and 3-4 the vehicle does not experience any type of motion. When this vehicle reaches the 90-degree turn, it has to come to a complete stop before rotating the travel direction of its wheels (time
interval 1-2) to be oriented in the direction shown in Figure-12, and before going through a rotation. In addition, once the vehicle is finished turning, it again has to stay motionless while it re-orient its wheels (time interval 3-4), before it can travel in the y-direction. Even if time intervals 1-2 and 3-4 can be minimized by designing a very fast rotating four-wheel steering control system, the vehicle will still need to come to a complete stop before the wheels can be rotated, forcing the vehicle’s motion progress to be discontinuous.

The advantages of Omni-directional wheels have been discussed, but it’s also important to note the disadvantages as well. A downside to using Omni-directional wheels is the fact that they make the mobile platform inefficient when travelling in a straight path and over long distances. Since the travel direction of the wheel is at a 45 degree angle relative to the straight travel path of the mobile platform, some energy is dissipated in the unnecessary passive motion within each wheel. Another disadvantage of the Omni-directional wheel is its limitation of travel over uneven surfaces due the small diameter of the passive rollers. If energy efficiency over long distances of travel, and mobility over uneven surfaces were the only factors to consider, the four-wheel steering design is advantageous.
III- Potential multi-directional active wheel designs:

The objective of the mechanical design part of this thesis project is to design, and analyze the motion of new wheel design concepts and compare them to the multi-directional active wheel designs currently in use. A total of four (4) ideas have traveled from the drawing board to three-dimensional modeling and animation using Solidworks to demonstrate the functionality of each idea. The first two designs have passive degrees of freedom, and can be used as Omni-directional wheels to achieve three degrees of freedom, but the last two designs limit the mobile platform in only achieving two degrees of mobility in the inertial reference frame. Following is a list of the wheel designs, where each design is referenced by its design highlights:

1) Sectioned wheel design
2) Alternating dually wheel
3) The Gimbal spherical wheel
4) Mechanically automated four-wheel-steering design
III.1- The sectioned linear slide wheel design:

As the focus shifts to the development of a multi-directional active wheel design, the first wheel design example is introduced. The sectioned-linear slide wheel consists of the two main components pictured in Figure-14, the individual sections that form the outer circumference of the wheel, and a center hub that all the sections slide into. The purpose of having each section of the wheel slide relative to the center hub (or drive axle) is to add a second and passive degree of freedom between the wheel and the travel surface. The orientation angle of each slide is created at 22.5-degrees from the direction normal to the direction of the wheel travel by, to restrict the motion of the platform from side-to-side. Different views of the assembly of the sectioned linear slide wheel are shown in Figure-15. A roller is attached through a center pivot pin on an extrusion on the back side of each wheel section as shown in the top view, and is used to realign each section to its neutral position, as explained in the next paragraphs.
On the frame of the sectioned linear slide platform, where the drive shaft of each wheel pivots, a funneling feature is added to realign each section to its center position on every revolution of the wheel. The realignment motion is created by the rotational motion of the wheel. Regardless of the position of each section as it ends its contact with the travel surface, after making almost a full revolution to reach the point of contact with the travel surface again, its position is centered by the contact between the rollers and the conical sides of the funneling feature of the frame, as shown in Figure-16. The funnel feature on the frame only covers the top 270 out of 360-degrees to allow each section the freedom to slide as it gets close to making contact with the travel surface. Three snapshot photos of the realignment process of an offset wheel section are pictured in Figure-17. The pictures are still frames of the Solidworks motion simulation that demonstrates the wheel section realignment process.
III.2- The alternating dually wheel design:

In comparison to the sectioned linear slide design, where the outer circumference of the wheel is made from individual sliding sections, the alternating dually wheel has an outer circumference that’s designed as a single part which slides at an angle normal to the direction of travel. Similar to the concept of the sectioned linear slide wheel, the purpose of the slide is to give the alternating dually wheel a second degree of freedom that’s passive. The sliding mechanism design of the alternating dually wheel, pictured in Figure-18, has three spokes each consisting of a five-bar link mechanism (as shown in the Top section view of Figure-18) that provide a spring loaded slide for the outer part of the wheel in both directions using a single spring. The combination of all three spoke mechanisms is what keeps the outer wheel concentric to the drive shaft during the entire spring loaded sliding process. A Solidworks motion simulation is created to demonstrate the sliding motion of the alternating wheel design, and is shown in still frame snapshots in Figure-19.
The obvious problem of the sliding motion of the outer circumference of the wheel is the fact that its sliding range in both directions is limited by the range of motion in the five-bar mechanisms. To solve this problem, and in an effort to create a continuous sliding motion of the wheel, two parallel sliding wheels are used as pictured in the top view of Figure-20, and their contact with the travel surface is alternated (thereby the name alternating dually). During the time that each wheel is not making contact with the travel surface, its slide springs are given the chance to release their potential energy, and move the wheel back to its centered position. The contact between each wheel and the travel surface is controlled by the lifting mechanism pictured in the front section view of Figure-20. The mechanism consists of a cam shaft, and a rocker arm with a follower roller. The cam shaft is designed to bring one wheel in contact with the travel surface before beginning to lift the other wheel. This process can be seen by the blue and black outlines in the cam shaft section view of figure-20, which are the contour shapes of the cam for each wheel. The entire system is driven by the center axle, which houses a center set of gears that drive the cam shaft, and two sets of gears on each side that drive the active rotation of both wheels. Due to the design configuration of the gears, the cam shaft rotates eight times faster than the wheel, which results in the wheel lifting
and dropping eight times per revolution. The motion of the alternating dually wheel is simulated in a Solidworks motion study to show its functionality.
III.3- The Gimbal spherical wheel concept:

Moving away from the idea of a linear slide to provide the passive degree of freedom to the wheel, the concept of a spherical wheel with a Gimbal drive mechanism is explored. The front and top views in Figure-21 highlight the shape and form of the partially spherical wheel, and its Gimbal drive shaft mechanism. Looking at the top view in Figure-21, it should be clear that the wheel cannot travel in a direction normal to the direction of active travel due to the partially spherical shape of the wheel, and because of the mechanical interference between the wheel and the drive shaft. However, the purpose of this exploration is to determine if the Gimbal spherical wheel can travel at a 45-degree angle from the direction of active travel, highlighted by the red and yellow vectors in the top view, and which is required for a platform to be capable of making a zero radius turn. The purpose of the Gimbal drive mechanism is to provide the necessary flexibility to the drive shaft when the wheel is traveling at an angle.

To test the concept of the Gimbal spherical wheel, a simulator is designed to provide the partially spherical wheel two different directions of travel relative to the vehicle’s frame. When a contact is created between the outer circumference of the wheel and the gray colored roller in Figure-22, while the pink roller is allowed to float on the wheel surface, the wheel will travel in the direction of active travel (represented by the green vector in the top view). When the pink roller is allowed to make contact with the outer surface of the wheel, and the gray colored roller
is allowed to float, the wheel will travel at the
deviated angle of the pink roller (represented by the
blue vector in the top view). The first test was
conducted with the required deviation angle of 45-
degrees, but the simulation failed due to
interference between the drive shaft and the wheel,
just as it was anticipated.

In conclusion, when the drive shaft of the
Gimbal spherical wheel is mounted at an angle
normal to the direction of active travel, the wheel is
not capable of traveling at the large deviated angle
of 45-degrees from a straight path. But, what if the
drive shaft is mounted at an angle of 22.5 degrees
from the direction normal to the direction of active
travel? Then, the Gimbal spherical wheel would
only need to deviate 22.5 degrees during a zero-
radius turn, but the wheel also needs to deviate
22.5-degrees from the direction of active travel in the opposite direction, during a straight path
motion.

During the next test, and as pictured in Figure-22, the Gimbal spherical wheel and the
simulator are designed for a 22.5-degree angle of deviated travel. The first test is conducted with
the starting position of the wheel placed in an orientation where the side surface of the wheel is
normal to the driving surface as pictured in the top and isometric views of Figure-22. When the
wheel is rotated with the surface contact of the pink roller, simulating a travel at an angle of 22.5-degree, though the wheel gets very close to the shaft as pictured in the still frames of the Solidworks motion study of Figure-23, the two never actually make contact. This simulation proves that the concept of the Gimbal spherical wheel works successfully under this specific condition.

However, the starting position of the Gimbal spherical wheel cannot always be guaranteed to be similar to that in Figure-22, where the side surface of the wheel is normal to the driving surface, when the wheel is travelling at the 22.5-degree angle. In the next simulation, the starting position of the wheel is set randomly, where the side surface of the wheel is not normal to the driving surface, as pictured in the top view of Figure-24. When the wheel is rotated, again with the contact surface of the pink roller, a mechanical interference occurs between the drive shaft and the wheel, and is pictured in a still frame of the Solidworks motion study in Figure-25 on the next page.

It follows that the Gimbal spherical wheel does not function properly under all the given conditions since the concept presents mechanical interference problems between its parts. The
failure of the concept to prove that a successful travel of the wheel in all the desired directions is possible limits the mobility and maneuverability of the overall platform.

Figure-25: Interference between the drive shaft and the wheel
III.4- The mechanically automated four-wheel-steering design:

It has previously been discussed in this report that four wheel steering is more efficient over long distances of travel, but the addition of motors and controls to rotate each wheel in the desired direction add weight, energy consumption and financial cost, in addition to complicating the control system. The reasoning behind the idea of automating the four-wheel steering mechanically is to eliminate the need for rotational position control of each wheel every time the vehicle needs to change its direction of travel. Such a design will be efficient both during long distance straight path travel and during turning, since no energy is dissipated into driving passive rollers or steering the wheels. In addition, the mechanically automated steering design will also reduce the weight and financial costs of the four-wheel steering platform since it does not require the additional motors to control the orientation of the wheels. Finally, such a design will utilize a simple differential drive control system, without the need for an additional control system for the steering.

In order to better design a mechanism to automate the steering of each wheel, it’s important to understand how the functionality and the control of the four-wheel steering platform are similar to that of the differential drive platform. The three conditions shown in Figure-26 (a,b and c) closely approximate most of the conditions that require steering for the four-wheel platform to operate using differential drive kinematics and controls. The concept is made easier to see by the blue and the green rectangles in Figures-26a, b and c, along with the wheels at the end of each rectangle, which represent a separate differential drive vehicle. It

![Figure-26]
follows that the four-wheel-steering platform can be closely approximated by two identical differential drive platforms that appear to be in tandem when traveling in a straight path or turning slightly (Figure-26a,b), and crossing one another when the platform is experiencing pure rotation (Figure-26c).

It is also important to understand the direction and the range of steering rotation of each wheel under the different mobility conditions. An important concept to note is the fact that the steering for the front and rear wheels on each side of the platform always rotate in opposite directions, and therefore can be synchronized to always rotate together. However, the wheels on the right and left side of the platform cannot be mechanically linked since in Figure-26b they are steered in the same direction, while in Figure-26c they are rotated in opposite directions. The last thing to note from Figure-26 is that the maximum turning range for steering each wheel is 45 degrees in each direction in order for the platform to be capable of going through a pure rotational motion, and to be able to turn in both directions as shown in Figure-26b.

The initial design in automating the steering mechanism includes the use of two closely mounted wheels for each corner of the platform, where each pair of wheels is driven by a separate motor through a differential drive gear box, similar to the solid rear axle of a rear wheel drive car (Figure-27, bottom view). The wheel and drive train assembly are mounted to the chassis through a passive pivot point centered on the drive axle, as shown in the isometric view in Figure-27. The reasoning
behind choosing the two wheel design with a pivot at the center of the two wheels is to allow the wheel steering at each corner of the platform to be controllable in both rotational directions. The concept behind using a differential gear box is to create a difference in travel between the two wheels, while maintaining the same amount of torque in each wheel during turning. This difference in travel between the two wheels becomes the driving force behind the automated steering. The direction each pair of wheels rotates is dictated by whichever wheel is less restricted to travel. The restriction in travel is mainly due to the difference in speeds or rotational directions of the right and left set of wheels of the platform.

The overall platform design is highlighted in Figure-28, where the chassis of the platform is shown in a red color, and the wheel pivots in all four corners are designed to be equidistant from front to rear and side to side, so that each pivot will need to rotate 45 degrees during pure rotation. The front and rear pivots on each side of the platform are linked through two four-bar-link mechanisms. The rotation of the front and rear wheels on each side of the platform is reversed through a set of gears, shown in a light blue color, and act as one of the links, along with the steering links that connect the gears to the pivots, which completes each of the four-bar-link mechanisms. Since we have already learned that the front and rear wheels on each side of the platform always need
to steer in opposite directions, the linking will guarantee that this motion takes place, and may also dampen over oscillation of the passive steering of each independent wheel.

The method by which the mechanically automated four-wheel-steering platform is predicted to operate can be shown in Figure-29, where the left two wheels are rotated to travel in the upward direction while the right two wheels are rotated to travel in the downward direction. The upper left corner is predicted to react according to a 45 degree restriction at the pivot point created by the platform, and shown by a white dashed line in the upper close-up picture. The platform at this pivot point is basically restricted to traveling along the dashed line. The wheel on the left side of the pivot is predicted to be less restricted in traveling where the wheel on the right side is more restricted. If the wheel on the left side of the pivot travels forward, while the wheel on the right remains static, the pivot is predicted to turn clockwise, as desired. The functionality of the four-wheel-steering concept is difficult to simulate in SolidWorks, but can be tested with a scale model. If for example the steering rotation direction of each wheel is not controllable in the

Figure-29: Four-wheel steering, functionality
desired manner, the location of the platform at any given time will be unpredictable, or a situation may occur where the platform will be completely immobile.
III.5- The design matrix:

In order to determine which wheel design should be pursued for further development a grading system is designed to compare all the wheel designs in a single mechanical design matrix. The grading of each design is based on the nine design criteria listed below, and a weight factor to gauge the significance of each criterion. The grade of a wheel design for each criterion can be one (1) for a poor design, two (2) for a fair design, and three (3) for a good design.

1. **Energy Efficiency** of a wheel design is determined by how little energy is dissipated into slip within the wheel as the wheel travels in different directions.

2. The grade for the **Mechanical functionality** of the design is based on how well the design performs under different conditions and environments. The more problems that can be identified with a design, the lower the grade of the design in this criterion.

3. The physical **Space requirement** of the wheel is measured as the ratio of the width to the diameter of the wheel. The larger the space required for the width of the wheel, the lower the grade for the wheel design in this criterion.

4. The physical **Maneuverability** of the platform, which is gauged by the minimal turning radius that a mobile platform using the specified wheel design can perform. The smaller the turning radius of the platform, the higher the grade of the wheel design is for this criterion.

5. **Quick response time** is determined by how fast a mobile platform, using the specified wheel design, can physically respond to a change in direction. The quicker the response time, the higher the grade for this criterion.
6. The grade for **Control system complexity** is higher when the number of motors and actuators that control the mobile platform are kept to a minimum, and therefore simplifying the control system.

7. The **Design for Manufacturability** (DFM) criterion is a measure of the complexity of the fabrication processes of the individual parts of the wheel assembly. The higher the complexity of the part features, the lower the grade for this criterion.

8. The grade for **Design for Assembly** (DFA) criterion is determined by the number of parts and fasteners that the final assembly of the wheel design requires. The simpler the assembly, the higher the grade for this criterion.

9. The **User comfort and convenience** criterion grade is determined by flaws in the mechanical design of the specified wheel, which may potentially result in vibrations or shaking forces in the mobile platform, in turn causing discomfort or inconvenience to the end user.

The weight factor for each criterion can be one (1) for a criterion that is less significant to the overall design, two (2) for a criterion that is moderately significant and three (3) for the criteria that are emphasized the most throughout the project. Following is a list of the weights for all nine criteria.

1. Energy efficiency: 2
2. Mechanical functionality: 3
3. Space requirement: 1
4. Physical maneuverability: 3
5. Quick response time: 3
6. Control system complexity: 3
7. DFM (Designed for Manufacturability): 2
8. DFA (Designed for Assembly): 2
9. User comfort and convenience: 2

There are four criteria that are given the highest significance weight factor. The wheel design must mechanically function well, and the mobile platform that uses the wheel design must meet the three objectives of this project: be physically maneuverable, have quick response time and have a simple control system. The lowest significance weight factor is assigned to the space requirement criterion since most the multi directional wheel designs require almost the same amount of space. The DFM and DFA criterion are assigned moderate significance weights, and so is the user comfort criterion since a large portion of autonomous mobile platforms are unmanned vehicles.

A total of three (3) wheel designs are considered for comparison: the sectioned linear slide, the alternating dually, and the mechanically automated four-wheel-steering designs. The Gimbal spherical wheel design is not considered for further development since it has been shown by simulation that the concept will not function properly under all conditions. The final mechanical design comparison matrix is shown in table-1 below. The total grade for each wheel design is the sum of the individual products of the grade for each criteria and the weight of the criterion.

<table>
<thead>
<tr>
<th></th>
<th>Sectioned linear slide</th>
<th>Alternating dually</th>
<th>Automated four-wheel-steering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Efficiency</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Functionality</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Space</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Maneuverability</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Quick response</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>controls complexity</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>DFM</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>DFA</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>User comfort</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total Grade</strong></td>
<td><strong>43</strong></td>
<td><strong>46</strong></td>
<td><strong>48</strong></td>
</tr>
</tbody>
</table>
Since the sectioned linear slide and the alternating dually wheel designs achieve multi-directional travel through a passive degree of freedom, they are given lower grades of 2 for energy efficiency in comparison to the automated four-wheel steering design, which is given a 3 for good energy efficiency. For the mechanical functionality criterion, each the sectioned linear slide and the automated four wheel-steering designs may have problems that occur only under certain conditions, and may be resolved by further development of each of the wheel designs. For example, the linear slide design may present binding in the slides from harsh terrains like grass or mud. The functionality of the automated four-wheel-steering design is difficult to simulated electronically. It follows that both of these designs are only given a fair grade of (2) for this criterion. The alternating dually wheel design, on the other hand, functions mechanically well in the Solidworks motion simulations, and will cope well with harsh conditions, and is therefore given a good grade of three (3) for functionality.

For the space requirement design criterion, the automated four-wheel-steering design has a width to diameter ratio of less than one when steered at a maximum of 45 degrees in each direction, and is given a fair grade of two (2) for this criterion. The sectioned linear slide and the alternating dually wheel designs both have ratios of over one (1.2 and 1.5 respectively), and therefore are given poor grades of (1) for this criterion.

All three wheel designs are capable of turning in a zero radius and deserve good design grades of three (3) for the maneuverability criterion. The sectioned linear slide and the alternating dually wheel designs are very responsive to quick changes of direction and receive good grades of three (3) for the quick response criteria. However, since the automated four-wheel-steering design may require some time for its wheels to react in steering, it receives a fair design grade of two (2) for the quick response criterion. Since all three wheel designs share the
same differential drive control system, they all have a good grade of three (3) for the controls complexity criterion.

The sectioned linear slide wheel and the automated four-wheel steering designs don’t have too many complex parts to manufacture since some of their parts may be available off-the-shelf, and they have obtained fair grades of two (2) for the design for manufacturing criterion. Most of the parts of the alternating dually design need to be custom made and therefore the design is given a poor grade of (1) for the DFM criterion. The sectioned linear slide and the alternating dually wheel designs have the most number of parts in each of their assemblies (22 and 47 respectively), and are given poor design for assembly grades of one (1). The automated four-wheel-steering design, however, has only thirteen parts in its assembly and is given a fair grade of two (2) for the DFA criterion.

Finally for the user comfort criteria, the sectioned linear slide and the alternating dually wheel designs are predicted to potentially result in large shaking forces due to the fast movement of the wheel components, and are given poor grades of one (1) in this criteria. The automated four-wheel-steering design may be a friendlier design for the user comfort criterion, but may still result in some jerking in the platform during direction changes, and it’s given a fair grade of two (2). The automated four-wheel-steering design achieves the highest total grade of 48 points, and the sectioned linear slide design the lowest total grade of 43 points. In conclusion the automated four-wheel-steering concept is chosen as the candidate to be further developed. The following chapters of this report will expand and explain the multiple iterations of prototyping and testing of the concept of the automated four-wheel steering design.
IV- First-iteration of prototyping the proposed platform

The objective of fabricating and assembling a physical prototype is, first to demonstrate that the concept of the automated four-wheel-steering functions properly in real life conditions, and second to measure the accuracy of how well the mobile platform is controllable. Because of the uncertainty in the overall functionality of the concept and the design, it is a good engineering practice to start with a small scale model of a real life prototype of the platform, to test the concept in a shorter time frame, and to reduce the overall financial impact of the project. In order for a scale prototype model to be as close of a representation of the full scale model as possible, it’s important for the scale size of the prototype not to be too small to misrepresent the functionality of the concept. It’s also important for the scale model to be fabricated with close tolerances and to include all the mechanical features that the full scale model has. The scale size of the prototype platform for this project is chosen to be 1/3 the size of a powered wheelchair platform for the prototype to represent the wheelchair as closely as possible.

For the first test, a user will need to control the prototype platform manually, to show that the prototype functions under all the mobility conditions. The second test, however, requires that the prototype runs autonomously, and without any intervention by the user. To run both tests with the same prototype, the motion control system not only needs to be controlled manually, but needs to be programmable to control the prototype platform to navigate from one waypoint to another autonomously.
IV.1- Prototype motion and controls:

The motion for each of the four wheels of the prototype is provided by DC gear motors. Since most small scale DC motors have a high maximum rotation speed (usually between 5,000-10,000 rpm) it’s important to use a gear motor with a high gear ratio to provide maximum torque, and controllable speeds to the wheels. It’s also a good practice to purchase the motors and the control system that controls the rotation speed of the motors from a single source, since these components need to be able to communicate and function with one another to provide seamless speed control to the wheels. The two important names that dominate the world of small scale robotics are Lego NXT and Vex robotics. The Vex drive train (motor and gear reducer) is more compact, and the mechanical components are more robust than those found in the Lego system. The gears and the drive shafts of the Vex system, for example, are made out of steel, in comparison to the NXT motion drive parts that are made from polymer materials.

To choose the better microcontroller, which provides the logic to the motion control system of the platform, a comparison is drawn in Table-2 between the two systems, the Lego NXT and the Vex microcontroller, to determine which would be more suitable for the proposed prototype platform.

<table>
<thead>
<tr>
<th>Table-2</th>
<th>Comparison of Logic system microcontrollers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NXT Intelligent Brick</td>
</tr>
<tr>
<td>Supported Software</td>
<td>LabVIEW robotics module</td>
</tr>
<tr>
<td>Motor output channels</td>
<td>3-ports servo motors</td>
</tr>
<tr>
<td>Wireless Capability</td>
<td>Bluetooth single port</td>
</tr>
<tr>
<td>Power source</td>
<td>6- AA batteries</td>
</tr>
<tr>
<td>Microcontroller speed</td>
<td>48 MHz</td>
</tr>
<tr>
<td>Memory</td>
<td>256KB flash</td>
</tr>
</tbody>
</table>
The first problem encountered with the NXT Brick is the fact that it only has 3 motor ports, when the four-wheel-steering prototype requires that the four motors that control the motion of the wheels be controlled simultaneously. Another problem with the NXT Brick is that it’s a system that can only be programmed to run autonomously, and it does not have any manual control modules through which a user may control the platform remotely. The Vex system, on the other hand, has sufficient motor ports (8 ports). There are two types of manual controls offered with the Vex systems, the Vex Cortex microcontroller can receive manual control commands through Wi-Fi communication with a control transmitter similar to that found on the X-box systems, and the Vex PIC microcontroller receives manual command signals through an FM radio transmitter and receiver. The Vex system is found to overall be a preferable control system for the prototype, over the NXT system since it also has a faster microcontroller speed and a larger internal memory space than the NXT system.

The control system chosen for this project is pictured in Figure-30 below. The Vex PIC microcontroller and the FM radio transmitter/receiver are preferred over the Wi-Fi wireless system since the FM transmitter has more accurate control of the speed of the motors, and because the wireless transmission of the FM system is more interference free than the Wi-Fi system. The FM radio system also has a longer distance range then the Wi-Fi system.

Figure-30: Vex control system
IV.2- Analysis of drive motor speeds torques:

The Vex 393 gear motor along with the Vex 29 controller, which are used to drive each pair of wheels on the prototype platform, are shown in Figure-31, and the motor specifications are listed at the top of the Figure. The speed of the motor is further reduced by a ratio of 2.7 : 1 by the ring and pinion gears in the differential gear box (discussed in the next section), bringing the speed to 37 rpm and the stall torque to 40 in-lbs at the wheel. Since a total of four (4) motors are used to drive the platform, the total stall torque of the platform including a gear efficiency of 75% is 120 in-lbs. The output drive shaft on the Vex 393 motor mates to a 0.125” square drive shaft, and the motor casing has two threaded mounting holes as shown in Figure-31. The drive wheels, also made by Vex robotics, have a 4” overall diameter, and slide over the same drive shaft as the motors.

In order to determine if the torque supplied by the motors is sufficient to provide motion to the platform, the worst mobility case scenario is considered where the platform is traveling up an incline as shown in Figure-32, and the inclined angle θ measures 30 degrees. It follows that the forces:

\[ F_x = W \sin \theta, \text{ and } N = W \cos \theta \]
It is assumed that the weight of the platform \( W \) will not exceed 20 lbs. and thereby \( F_x = 10 \text{ lbs.} \), and \( N = 17.32 \text{ lbs.} \). Also assuming that the platform will be traveling on relatively low friction surfaces, and the static friction coefficient is 0.25, it follows that the static friction force is:

\[
F_s = \mu N = 4.33 \text{ lbs.}
\]

The total force required to move the platform up the incline:

\[
P = F_x + F_s = 14.33 \text{ lbs.}
\]

The total required torque:

\[
T = P r, \text{ where } r = 2 \text{ inch}, \text{ and } T = 28.66 \text{ in-lbs.}
\]

Considering the fact that 120 in-lbs. of torque are provided at the output wheel drive shaft by the Vex motors and the differential gear boxes, the system has a torque safety factor of 4.2.

The maximum speed of the platform with wheel diameters \( D = 4 \text{ inches} \) and a wheel speed = 37 rpm is:

\[
\text{Platform speed} = \pi * D * \text{(wheel speed)} = 465 \text{ In./min.} = 7.75 \text{ In./sec.}, \text{ which is a reasonably controllable speed.}
\]
IV.3- Differential gear boxes:

The objective of using differential gear boxes is to allow a difference in travel between the two wheels at each corner of the platform, and thereby creating a torque force for steering. In order for the differential gear box to function smoothly with minimal friction, its scale has to be proportional to the prototype model. If the gear box is too large for example, it will add additional weight to the platform, and the amount of friction between its gears is too large for the application, and a difference in the travel between the two wheels will be limited. At the same time, a gear box that’s too small for the application may have polymer gears, which may not handle the torque that’s applied to them. For the application of the prototype model of this project, differential gear boxes from the 1/10 scale model Radio Controlled (RC) car Vortex SS, made by Red Cat is used, and is shown in Figure-33. The gear boxes have an injection molded polymer outer housing, and die casted metal gears.

To adapt the Red Cat differential gear boxes to the Vex motors and wheels, a number of modifications had to be made to the gear boxes and the drive and output shafts. For the application of the RC car, the wheel output shafts of the gear boxes are not designed to withstand the radial load of the platform, and therefore did not have ball bearings between the drive shafts and the housing of the gear box. The housing of the spider gear assembly is machined, and ball bearings are added to the drive shafts of each wheel, as shown in Figure-34. The original input drive shaft to the gearbox and each of the original wheel drive shafts of the Red Cat gear boxes
are cut, and 0.125” square drive shaft couplers from the Vex system are welded to the end of each of the three drive shafts of each gear box (also shown in Figure-33), to adapt the motors and the wheels from the Vex system to the Red Cat gear box.

The final differential gear box assembly, with square 0.125” input and output drive shafts, is shown in the top picture of Figure-35. Since the gears and the housing parts of the gear boxes are mold casted, there are many imperfections in the gears and the housing parts that introduce binding when the drive shaft is rotated. To reduce binding friction, each gear box is subjected to a break-in run time, where a cordless hand drill is connected to the input drive shaft of the gear box, and a wheel is fit over one of the output shafts to keep it static (as shown in the bottom picture in Figure-35), and the drill is rotated at a speed of 1,500 rpm for a period of one hour.
IV.4- Machining custom designed parts:

The pivot blocks, one of which is shown in the top right picture of Figure-36, are one of the few parts on the platform assembly that require custom machining. The function of each pivot block is to hold the differential gear box and the motor together, and to mount the wheel assembly to one of the corners of the frame via a pivot, as shown on the top left picture in Figure-36. There are four screw holes on the top of the pivot block that mount it to the differential gear box, and two holes on the side that mount it to the motor. The pivot blocks are made from polycarbonate for the material’s great mechanical properties, such as its light weight, its ease to machine, its impact resistance, and its metal-like stress-strain relationship. Another great property of polycarbonate, being an acrylic, is that it is easy to bond. When the actual pivot blocks are fabricated, the design of the part is altered to facilitate machining. Each pivot block is machined from two separate parts, as shown in the bottom pictures of Figure-36, which are bonded together using the solvent Chloroform. A manual, Bridgeport type, milling machine is used to machine the pivot blocks from polycarbonate stock material.

Figure-36: SolidWorks model (top), and actual machined pivot blocks (bottom)
IV.5- Welded dynamic steel space-frame:

Since the control over both the linear travel and the rotation of the overall platform is performed through the travel of each of the four wheels, it’s important for all the wheels to maintain contact with the driving surface at all times. The purpose of a flexible or a dynamic frame, as opposed to a rigid frame, is to allow each wheel to move in the vertical direction independently of the remaining three wheels. The flexibility in the frame guarantees that all four wheels remain in contact with the driving surface by compensating for uneven surfaces. The pictures in Figure-37 show the SolidWorks model of the frame for the prototype of the four-wheel steering design, which consists of a six bar link design, and has one degree of freedom. The main components of the frame include the center section, and the two side sections which pivot on the center frame, and are linked through the center link and the ball joint links, as shown in the center picture. The frame is fabricated using 0.500” square 1020 cold rolled steel tubing with 0.063” wall thickness. The Mig-welding process is used to fuse the different members of the frame together, and silver color enamel is applied as a
top coat to prevent oxidations of the frame. The pivot points at each corner of the frame feature machined steel bushings that house two (2) 10 mm x 5 mm ball bearings separated by a metal bushing, as shown in Figure-38. The purpose of using ball bearings is to maintain smooth steering at each of the four wheels when the load of the platform is resting on the pivot joints.

Figure-38: Ball bearing wheel pivot joints
IV.6- Wheels and tires:

The original Vex robotics rims, shown in Figure-39, are made from a rigid polymer and the square center opening measures 0.125 inches to fit over the Vex drive shafts. The original Vex tires are made from a more flexible polymer, but are not soft enough to comply with the driving surface given the light weight of the platform. When the first iteration prototype platform of the four-wheel-steering design is first tested, the original tires made by Vex created a great deal of slip, especially on slippery surfaces like glass. The original tires are removed from the Vex rims, and the profiled outer surface of the rim is machined to a smooth flat surface. A soft synthetic foam rubber strip, originally intended for weatherproofing doors and windows, is found to have the ideal elasticity to conform to the driving surface based on the weight of the platform. The rubber strips have an adhesive backing that’s bonded to the outer surface of each rim. Two narrower strips of rubber are used to form the tire rather than one wider strip to allow more flexibility within the tire since, in addition to deformation, a narrow and tall beam is also vulnerable to buckling. A great improvement in traction is observed after replacing the original Vex tires with foam rubber tires.

Figure-39: Soft foam rubber tires vs. Vex tires
IV.7- Prototype component layout:

In the “machined custom parts” section of this report, it is highlighted how the wheel drive components such as the motor and differential gear box are assembled and attached to the chassis. A top view of the final prototype platform is shown in Figure-40, and each component on the platform is labeled. The batteries, being the heaviest component on the platform are placed in the middle of the center chassis to distribute an even load on each of the four corners of the platform. The microcontroller is mounted in the front of the chassis and the motor controllers closer to the middle on each side of the center chassis so that the wires reach all four motors. Finally, the two units that communicate signals to the microcontroller, the FM radio receiver and the programming hardware are mounted to the rear of the chassis since the antenna of the FM receiver gets a better signal at the rear of the platform, when most of the time the user is behind the platform. The USB port of the programming hardware faces the back of the platform, where it is easily accessible by the user.

Figure-40: Component layout
IV.8- Platform weight and battery run time analysis:

During the torque analysis, it is assumed that the overall weight of the platform would have an upper limit of 20 lbs. A calculation of a more realistic estimation of the platform weight is made by adding the weights of the individual components of the prototype platform, as shown in Table-3, and the total estimated weight is 12.1 lbs. When the torque analysis is repeated for the new realistic weight of the platform the new safety factor is found to be 6.9. Based on the torque safety factor, and the fact that the maximum stall current specification of each motor is 4.8A, we can divide the maximum stall current of each motor by the safety factor to obtain the actual current usage of each motor under the load of the platform, which is 0.7A. The 7.2V Nickel Metal Hydride battery has a 3000 mAh or 3 Amp hours of capacity, and only one battery powers the platform at any one time. Considering the fact that all four (4) motors will be active most of the time during operation of the platform, the overall battery run time is calculated by dividing the battery capacity by the total current usage by all four motors, which is 1.1 hours of run time. The second on-board battery is used as a back-up power source, in case the first battery is depleted during testing.

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight (lbs.)</th>
<th>Quantity</th>
<th>Sub-Total weight (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chassis, chassis links and ball joints</td>
<td>3.70</td>
<td>1</td>
<td>3.70</td>
</tr>
<tr>
<td>Differential Gear box</td>
<td>0.4</td>
<td>4</td>
<td>1.60</td>
</tr>
<tr>
<td>Motors and motor controllers</td>
<td>0.3</td>
<td>4</td>
<td>1.20</td>
</tr>
<tr>
<td>7.2V Ni-Metal Batteries</td>
<td>0.8</td>
<td>2</td>
<td>1.60</td>
</tr>
<tr>
<td>Vex PIC microcontroller, and FM receiver</td>
<td>0.5</td>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td>Wheels and tires</td>
<td>0.2</td>
<td>8</td>
<td>1.60</td>
</tr>
<tr>
<td>Pivot blocks, Vex gears, steering links</td>
<td>0.3</td>
<td>4</td>
<td>1.20</td>
</tr>
<tr>
<td>Ball bearings and other hardware</td>
<td>0.7</td>
<td>1</td>
<td>0.70</td>
</tr>
</tbody>
</table>

**Total Platform weight**: 12.10 lbs
IV.9- Cost analysis of the prototype:

An accurate bookkeeping on the financial and labor costs of the prototype is maintained throughout the fabrication. The main items that are purchased for the prototype are listed in Table-4, along with the source of where they were purchased from, the cost per piece, and the quantity of each item. Some items are purchased from local retail sources like Ehobbyhouse store, Fry’s electronics and the Home Depot, and others are purchased online, and the total financial cost of the prototype is $945.46 US. In addition to the financial cost, a log book is kept with separate log inputs for each session of time spent on the prototype fabrication. The total hours spent on the prototype are 97 hours, which includes time spent on machining and assembling the different components.

<table>
<thead>
<tr>
<th>Item</th>
<th>Source</th>
<th>Cost per piece ($US)</th>
<th>Quantity</th>
<th>Sub-Total Cost ($US)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vex Robotics Design Kit</td>
<td>Ebay.com</td>
<td>$342.54</td>
<td>1</td>
<td>$342.54</td>
</tr>
<tr>
<td>Wheels and tires</td>
<td>Vexrobotics.com</td>
<td>$7.25</td>
<td>8</td>
<td>$58.00</td>
</tr>
<tr>
<td>Motors and motor controllers</td>
<td>Vexrobotics.com</td>
<td>$34.48</td>
<td>4</td>
<td>$137.92</td>
</tr>
<tr>
<td>Differential gear box assemblies</td>
<td>Ehobbyhouse.com</td>
<td>$22.00</td>
<td>4</td>
<td>$88.00</td>
</tr>
<tr>
<td>10mm x 5mm Ball bearings</td>
<td>McMaster.com</td>
<td>$6.00</td>
<td>16</td>
<td>$96.00</td>
</tr>
<tr>
<td>Raw material (steel, Aluminum, polycarbonate)</td>
<td>McMaster.com</td>
<td>$86.00</td>
<td>1</td>
<td>$86.00</td>
</tr>
<tr>
<td>7.2V Ni-Metal Batteries</td>
<td>Fry’s Electronics</td>
<td>$22.00</td>
<td>2</td>
<td>$44.00</td>
</tr>
<tr>
<td>Hardware (screws, washers, springs)</td>
<td>Home Depot</td>
<td>$93.00</td>
<td>1</td>
<td>$93.00</td>
</tr>
<tr>
<td><strong>Total Prototype Cost:</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>$945.46</strong></td>
</tr>
</tbody>
</table>
IV.10- Manual testing and analysis:

The initial manually controlled test of the prototype failed to prove that the design of the mechanically automated four-wheel-steering can be successful. The steering of each wheel did not appear to be controlled in the desired direction, in order for the platform to travel in straight path or to turn. To conduct a deeper analysis of why each wheel is not automatically steered in the desired direction, the prototype is analyzed as described in the following paragraphs.

Before starting the first test, the links between the front and rear wheels on each side of the platform are temporarily removed, to allow the passive steering pivot at each corner of the platform to be independent of the rest of the pivots. The prototype is then placed on a smooth glass table top to allow some controlled slip in the wheels when necessary. Using the manual controls, a signal is sent to the motors on the right side of the platform while keeping the left motors static, to simulate a left turn. The steering motion and rotational direction of each of the front and rear wheels on the right side, which are being powered, are then analyzed and recorded. The test is repeated a dozen times to look for a pattern in the direction of steering rotation of the powered wheels, but no pattern is detected. It is also observed that during many trials of this test, one or both of the powered wheels did not rotate at all to steer the wheels. When the pair of wheels in the right front or right rear randomly steered either in the correct or incorrect direction, the differential gear box allowed one of the wheels to slip and rotate freely while the other gained traction, which resulted in the steering of the pair of wheels.
To be able to explain the results theoretically, the front right corner pair of wheels is analyzed in a free-body diagram, as shown in Figure-41, and the reason why the steering rotational direction is random is determined to be as follows. The restriction force by the frame at the pivot point on the driving axle, which is at 45 degrees during pure rotation, is shown with the light green color vector. The dark green color vector represents the component of the restriction force perpendicular to the driving axle. The large blue and red vectors represent the traction forces of each wheel, which are equidistant from the pivot point on the drive axle, shown by the red and blue crossed double arrows. From the free-body diagram, it’s easy to determine that so long that the there is a restriction force perpendicular to the driving axle at the pivot point, and the pivot point lies in the center of the two driving wheels, the traction forces of each wheel create equal amounts of steering torque at the pivot, but in opposite directions. The analysis from the free-body diagram confirms the observed results that the only way the wheel steering rotates is by one of the wheels slipping, and the traction force of the other wheel creating the steering rotational torque. Since either the right or left wheel may slip on slippery glass at any one time, the steering rotation direction is uncontrollable. Also, when there is no slip in either wheel, steering rotation does not take place.

Based on the free-body diagram analysis, the only way the steering rotation direction of each wheel can be controlled is if the pivot point is offset along the driving axle, to be closer to
one wheel versus the other. In this case, the traction force from the wheel that’s further from the pivot will create a larger torque at the pivot point than the wheel that’s closer to the pivot point, thereby rotating the steering of the pair of wheels in the same direction that the pivot point is offset. In order to confirm this new concept, the following test is performed on the prototype.

Both the right front and rear pair of wheels are modified by replacing the drive axles of the outer wheels with longer drive axles, which offsets the pivot points of the right corners of the platform to be closer to the inner wheels as shown in the top picture of Figure-42. The platform is retested by only powering the right two wheels, similar to the first test, and the test is repeated a dozen times. During every trial of the test, and as expected, both the front and rear powered wheels rotated counter clockwise as shown in the bottom picture of Figure-42. The second test confirmed the free-body diagram analysis, and the fact that when the pivot point is placed offset along the driving axle to be closer to one wheel versus the other, the steering rotation direction is always consistent with the offset direction.
V- Second-iteration of the prototype:

From the analysis of the first iteration of the prototype, it was concluded that in order for the steering rotation direction to be controllable, the pivot point of the steering needs to shift relative to the two wheels. At the same time, the pivot cannot be fixed at a point closer to one wheel versus the other otherwise steering will only be controllable in one direction of rotation. It follows that the only way steering rotation of each pair of wheels can be controllable in both directions is if the location of the pivot point relative to the two wheels is dynamic.

During the design of the second iteration of the mechanically automated four-wheel-steering prototype, an additional degree of freedom is added to each pair of wheels to allow the steering pivot point to shift in both directions along the drive axle. The mechanism, consisting of a spring loaded four bar link, is illustrated in the pictures of Figure-43 and provides a 0.4 inch travel of the pivot point in each direction from the center of the two wheels. Though easier to fabricate than a ball bearing linear slide, the disadvantage of using a four bar link to achieve the shift in the steering pivot is that as the vertical links rotate, they create a difference in gravitational potential energy between the top and bottom horizontal links. To compensate for the difference in gravitational potential energy, a torsion spring is added to each vertical link to keep the pivot point at the center of the two wheels when the wheel is not being steered.
On a platform level, and when the vehicle is making a left turn as shown in Figure-44A, the chassis experiences a counter-clockwise rotational motion due to the difference in rotational speed or direction between the right and left wheels. The rotational motion of the chassis causes the front two wheel pivots to shift towards the left and the rear two wheel pivots to shift to the right, through the newly added lateral degree of freedom in each wheel. Observing the reaction of the front right set of wheels after the shift in the steering pivot to the left in Figure-44A, the steering rotational torque due to the forward traction force of the left wheel is much more significant than that of the right wheel since the left wheel is further away from the pivot. The front right set of wheels are expected to steer counter-clockwise as shown in Figures-44B and C. Between the rear right set of wheels, the steering rotational torque from the forward traction force of the left wheel is more significant, and therefore this set of wheels will steer clockwise. The steering rotational direction of the left front and rear set of wheels depends on the rotational direction of the wheels. If the traction force is forward but slower than the right front and rear wheels, the
front left set of wheels are expected to steer counter-clockwise, and the rear left set of wheels clockwise, as shown in Figure-44B. On the other hand, if the left front and rear set of wheels are rotating in the reverse direction, the front left set of wheels are expected to steer clockwise and the left rear set of wheels counter-clockwise, as shown in Figure-44C. As a conclusion, whether the vehicle is moving forward while making a right turn or while going through a pure counter-clockwise rotation, all four sets of wheels steer in the desired direction. The reader can verify that the wheels also steer in their desired directions when making right turns or going through pure clockwise rotational motion.

It has been shown that the new degree of freedom that is added to each set of wheels helps the automated four-wheel-steering platform achieve steering while the vehicle is turning, but it’s just as important to analyze how each wheel steering will rotate from its steered position back to its neutral or straight position when it is desired that the vehicle moves in a straight path. In the example shown in Figure-45A, the vehicle has completed going through a pure rotational motion, and all four wheels begin to rotate at the same speed to move the vehicle forward. The shift of the front and rear left steering pivots are offset to positions so that the significant wheel traction forces steer them back to their desired positions. The right front and rear wheel pivots, however, are not offset so that the wheels steer
in the desired direction. But, due to the steered angle of the right wheels, as the platform travels forward, the right front steering pivot shifts to the right while the rear right steering pivot shifts to the left until the traction force of the inner right front wheel and the outer rear right wheel become significant enough, and the right wheels steer back to their neutral positions as shown in Figure-45B. In addition, if any over-steer takes place during the steering straightening process, the shifts in the pivots of each wheel allow the steering of each wheel to correct itself.
V.1- Manually testing of the second-iteration prototype:

The prototype platform is controlled manually through the FM transmitter pictured in Figure-46. The differential drive control is established by the right joystick’s x and y axis coordinates, while the left joystick is inactive. The Y+ coordinate moves the platform forward, while the Y- coordinate moves it in reverse. The combination of Y+ and X+ moves the platform forward while making a right turn, while Y+ and X- moves the platform forward while making a left turn. Using the X+ coordinate exclusively rotates the platform clockwise without any travel, and using the X- coordinate alone rotates the platform counter-clockwise. Each potentiometer’s progressive control translates to the platform’s speed control, meaning that the further the joystick is moved from its neutral position, the faster the platform moves in the desired direction.

The objective of the manual control test is to test the vehicle on straight paths, on turns in a switchback, and while going through pure rotation. The results from the test must show that any user can control the prototype platform to travel through the course similar to the one highlighted in Figure-47. The platform must travel from the starting position to the end of
the course, go through a 180 degree pure rotation, head back to its starting position and rotate for a second time to face its starting orientation.

The manual control test of the second design iteration of the prototype is repeated at least for ten runs on the course, and eight out of the ten runs are completed successfully to prove that the concept of mechanically automating the steering of each wheel does not only work in theory, but also functions in the real world. A 45 second video footage demonstrates that the platform can be manually controlled to travel through the course. The video shows the platform going through the switchback, pure rotation at a dead-end, and repeating the course while travelling in the opposite direction to return to its starting position and orientation. For the duration of the video all four wheels automatically steer in their desired directions throughout the course.

A flaw or a disadvantage of the mechanical design of the platform, which is observed in two out of the ten manual control test runs, is the fact that having two closely placed wheels driven by an open differential gear box causes undesired slip in one of the wheels due to uneven travel surfaces, or imperfections in the fabrication of the links in each wheel. For example, as the four bar link mechanism allows the steering pivot to shift along the drive axle, it also causes a slight rotation in the drive axle, which in turn causes a lift and therefore slip in one of the wheels. As a result of this design flaw, the vehicle loses power in one or numerous wheels due to wheel spin, and the platform becomes immobile.
VI- Third iteration design of the prototype:

The purpose of the last iteration of the prototype is to optimize the mechanical design of the automated four-wheel-steering mobile platform and to minimize wheel slip. The main objective is to eliminate the differential gear boxes and the additional wheels, and replace the two wheel design by a single wheel. By eliminating the open differential gear boxes, we also hope to minimize wheel slip. The mechanical design of the third iteration wheel consists of a base, to which the motor is mounted in-line with the wheel drive shaft, a pivot block that mounts to the chassis, and two vertical links that connect the pivot block to the base, as shown in Figure-48. The assembly of the four components results in a four bar link mechanism which allows the wheel to be offset laterally relative to the steering pivot. When the wheel is offset from the steering pivot, in addition to creating a forward travel motion, the wheel travel force also generates a torque at the steering pivot, and thereby steering the wheel. On a platform level, the concept of the steering pivot offset functions in a very similar fashion to the second iteration design, where the difference in wheel travel between the two sides of the platform causes the steering pivot offset in each wheel.

As a result of eliminating the differential gear boxes, the gear ratio of the motor decreased and therefore the speed of the platform increased from 7.75 to 21 inch/second, the torque FOS
decreased from 6.9 to 2.6. Finally the weight of the platform decreased from 12.1 to 10.5 lbs due to the elimination of the differential gear boxes and the additional wheels.

The completed SolidWorks assembly and a rendering of the assembly are shown in Figure-49. A modification to the center section of the frame had to be made by raising it up by approximately one inch, to accommodate enough clearance space for the horizontally mounted motors. Purchasing additional material and hardware in the amount of $120.00 brings the total cost of the three iteration prototypes to $1,065.46. Also, an additional 45 hours are spent between machining and assembling parts for the last two design iterations bringing the total hours spent on the fabrication of the prototype to 142 hours.

During the manual test, the third iteration prototype performed better than the second iteration. The platform experiences less wheel slip, thanks to the direct drive motor mounting configuration, and the elimination the differential gear boxes. The steering response is also quicker due to the single wheel design. Overall, the prototype platform is lighter, mechanically simpler and costs less, since it consists of fewer components than the previous design. It is probably also more efficient since it has less wheel friction and weighs less than previously.
V.1- Steering pivot offset torsion spring force analysis:

It is discussed in the second iteration design that the four-bar-link mechanism used to offset the steering pivot requires torsion springs to counter the gravity force when the vertical links rotate. To calculate the torque required by the torsion spring, the component of the gravity force $F_g$ in a direction normal to the vertical link, and the calculated torque $T$ generated by $F_g$ according to the length of the vertical link $R$, versus the angle of rotation of the vertical link $\theta$ as shown in Figure-50, are listed in Table-5 and plotted in Plot-3. According to the maximum tilting angle of the vertical link of 10-degrees, a torsion spring with an approximate torque of 1.0 lbs-inch is required to counter the gravity force.

<table>
<thead>
<tr>
<th>$\theta$ (Degrees)</th>
<th>$F_g$ (lbs.)</th>
<th>$T$ (lbs-inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.04581257</td>
<td>0.091625134</td>
</tr>
<tr>
<td>2</td>
<td>0.09161118</td>
<td>0.18322358</td>
</tr>
<tr>
<td>3</td>
<td>0.13738189</td>
<td>0.27476377</td>
</tr>
<tr>
<td>4</td>
<td>0.18311074</td>
<td>0.366221487</td>
</tr>
<tr>
<td>5</td>
<td>0.22878382</td>
<td>0.457567649</td>
</tr>
<tr>
<td>6</td>
<td>0.27438722</td>
<td>0.548774432</td>
</tr>
<tr>
<td>7</td>
<td>0.31990703</td>
<td>0.639814053</td>
</tr>
<tr>
<td>8</td>
<td>0.36532939</td>
<td>0.73065878</td>
</tr>
<tr>
<td>9</td>
<td>0.41064047</td>
<td>0.821280941</td>
</tr>
<tr>
<td>10</td>
<td>0.45582647</td>
<td>0.911629333</td>
</tr>
<tr>
<td>11</td>
<td>0.50087361</td>
<td>1.001747226</td>
</tr>
<tr>
<td>12</td>
<td>0.54576819</td>
<td>1.091536377</td>
</tr>
<tr>
<td>13</td>
<td>0.59049652</td>
<td>1.180993035</td>
</tr>
<tr>
<td>14</td>
<td>0.63504046</td>
<td>1.270089952</td>
</tr>
<tr>
<td>15</td>
<td>0.67939999</td>
<td>1.358799987</td>
</tr>
<tr>
<td>16</td>
<td>0.72354806</td>
<td>1.447096118</td>
</tr>
<tr>
<td>17</td>
<td>0.76747572</td>
<td>1.534951454</td>
</tr>
<tr>
<td>18</td>
<td>0.81116961</td>
<td>1.62233922</td>
</tr>
<tr>
<td>19</td>
<td>0.85461641</td>
<td>1.709232811</td>
</tr>
<tr>
<td>20</td>
<td>0.89780288</td>
<td>1.795605752</td>
</tr>
<tr>
<td>21</td>
<td>0.94071587</td>
<td>1.881431735</td>
</tr>
<tr>
<td>22</td>
<td>0.98334231</td>
<td>1.966684615</td>
</tr>
<tr>
<td>23</td>
<td>1.02566921</td>
<td>2.051338425</td>
</tr>
<tr>
<td>24</td>
<td>1.06768369</td>
<td>2.135367376</td>
</tr>
<tr>
<td>25</td>
<td>1.10937294</td>
<td>2.218745874</td>
</tr>
<tr>
<td>26</td>
<td>1.15072426</td>
<td>2.301448521</td>
</tr>
<tr>
<td>27</td>
<td>1.19172506</td>
<td>2.383450124</td>
</tr>
<tr>
<td>28</td>
<td>1.23236285</td>
<td>2.464725705</td>
</tr>
<tr>
<td>29</td>
<td>1.27262525</td>
<td>2.545250506</td>
</tr>
<tr>
<td>30</td>
<td>1.3125</td>
<td>2.625</td>
</tr>
</tbody>
</table>

Figure-50: Tilting angle of vertical link

Plot-3: Tilting angle of vertical link vs. gravity force, torque
VII- The autonomous test plan

The purpose of the autonomous test is to obtain scientific measurements of the controllability of the mechanically automated four-wheel-steering platform. Two specific parameters can be calculated from an autonomous test, which are the accuracy and the precision of how controllable the platform design is. From a mobility standpoint, the accuracy of the platform is defined as the degree to which a platform is able to minimize error in distance when navigating from one point of reference to another. The precision, however, is a measure of how consistently the platform can repeat an autonomous run. Each term is mathematically defined in the results section of the report.

A university laboratory spanning approximately 25 feet squared is chosen as the setting of the autonomous test, since the 12 inch x 12 inch tiles covering the floor make it easy to take distance measurements between two waypoints or between the platform’s final position and the actual waypoint. The test consists of a goal waypoint, and the starting position and the end position coordinates in a two dimensional workspace are given. The test is repeated for four different starting orientations of the platform. During the first starting orientation, the platform faces the waypoint, and for every new starting orientation, the platform is rotated 90 degrees to face away from the endpoint. Once the vehicle completes each run, measurements of the x and y coordinate parameters of the ending position of the platform are measured. The data from all the runs is shown in Table-6 of the results section of the report, and is used to calculate the accuracy and the precision of the mobile platform design. The final orientation of the platform once it reaches the endpoint is not measured.
VII.1- Kinematics of the automated four-wheel-steering design:

One of the main reasons why the differential drive motion control is chosen for the proposed mobile platform is because it is the simplest drive mechanism, where the travel and rotation motion of the mobile platform can be defined in terms of wheel speed. The similarity, in terms of motion control, between the differential drive platform and the mechanically automated four-wheel-steering design is the fact that the motion of both platforms is dependent on the rotation speeds and directions of the wheels on each side of the platform. Where each drive wheel on the differential drive platform only has one controlled degree of freedom, each wheel of the third iteration of the proposed design has a total of three degrees of freedom, a controlled degree of freedom in the travel direction of the wheel, and two passive degrees of freedom, one for steering and one in the lateral offset of the steering pivot. Since all four wheels are controlled, the motion of each wheel has a certain contribution to the overall motion of the platform, but also imposes certain restrictions to the motion of the platform depending on the travel direction of the wheel. However, to establish a solid reference for the motion contribution of each wheel, it’s important to start by defining a local reference frame for the rigid body of the mobile platform, and a global or inertial reference frame for the workspace of the robot. The rigid body of a mobile platform can have up to three degrees of freedom in the inertial reference frame, in the $X_R$, $Y_R$, and the $\theta$ directions, as shown in Figure-48, but the proposed platform only travels in the $X_R$ direction and rotates about $\theta$. The pose of the mobile platform
(\xi) and the travel and rotation speeds \((\xi'_{1})\) in the inertial reference frame are defined by the following vectors, with the reference coordinates and speeds as the elements of the vectors:

\[
\xi_{I} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \quad \xi'_{I} = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}
\]

In order to map the motion of the mobile platform from its local reference frame to the inertial reference frame, the matrix that describes the pose or speeds of the mobile platform is transformed using the orthogonal rotation matrix as follows:

\[
\xi'_{I} = [R(\theta)]^{-1} \xi'_{R}, \quad \text{where } R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

The theoretical kinematic relationships between each wheel and the motion of the mobile platform are based on the following assumptions. The plane of the wheel always remains perpendicular to the travel surface, and there is only one point of contact between the wheel and the travel surface. There is no slip between the wheel and the travel surface in the direction of travel, and the wheel only undergoes a rolling motion without any sliding. Slip or sliding also does not occur in a direction normal to the travel direction between the wheel and the travel surface.

There is at most one steering pivot per wheel, and the steering axis is perpendicular to the travel surface. There are two equations that describe the forward kinematics of a differential drive mobile platform in terms of wheel radius \(r\), and with the distance of each wheel from the center of the platform measuring \(L\), and wheel speeds \(\varphi'_{1}\) and \(\varphi'_{2}\), one equation is for the translational speed of the mobile platform \(X'_{R}\), and the second for the rotational speed \(\theta'_{R}\), and they are defined as follows:

\[
X'_{R} = \frac{1}{2} r \varphi'_{1} + \frac{1}{2} r \varphi'_{2}, \quad \text{and} \quad \theta'_{R} = \frac{r \varphi'_{1}}{2L} - \frac{r \varphi'_{2}}{2L}
\]  \(1\)
Given the wheel speeds, the travel and rotation speeds of the mobile platform can be calculated using equations (1). However, to be able to program the mobile platform to navigate from one reference point in the workspace to another autonomously, we must calculate the wheel speeds based on the motion speeds and the path that the platform will take. This task requires the inverse kinematics. For example, when it is desired that the mobile platform go through a turn with a known radius $R$ as shown in Figure-49, the relationship between the wheel speeds, the geometry of the platform and the radius of the path is as follows:

$$R = L \frac{\phi_2 + \phi_1}{\phi_2 - \phi_1} \quad (2)$$

Assuming a certain wheel speed $\phi' \pm$ based on the desired speed of the platform, the speed of the second wheel $\phi' \pm 1$ can be calculated using equation (2) so that the mobile platform follows the path with radius $R$. When the wheel rotation speeds of the two sides are close, it is obvious from equation (2) that the radius of the turn will be infinitely large, or the path is close to a straight line.

There are numerous methods to program a mobile platform to navigate autonomously. An easy and quick method is to pre-plan the desired motion path by straight path and arc segments, then the wheel speeds during each segment of the path can be calculated and timed according to the length of each segment path. This type of algorithm will require the user to develop a program specific to the path that the mobile platform will take. A second method to program the mobile platform to navigate from a known reference point to another is through an iterative process, where at each step of the iteration, the speed of each wheel is calculated based...
on the vehicles travel and rotation speeds, which in turn are calculated according to the vehicle’s instantaneous position and the angle of the vector joining the vehicles position to the position of the waypoint. The algorithm for the iterative method only requires a user to input the location parameters of the starting position and the waypoint position that the platform will reach at the end of the program, in addition to the starting orientation of the platform. The second programming method is shown and explained in 6 steps in Appendix-A. The iterative programming method is not used for this project due to the memory limitation of the PIC microcontroller, and its lack of support for floating point variables. The segment method is used both for calibration and autonomous testing of the proposed mobile platform.

If the steering of the wheels of the proposed mobile platform design were not mechanically automated, the design would require additional controls to steer the wheels in a position complying with the travel path of the platform. The equation describing the kinematics of the Ackerman steering, also known as the Ackerman condition is as follows:

$$\cot \delta_o - \cot \delta_i = \frac{w}{l} \quad (3)$$

Where $\delta_o, \delta_i$ are the angles of the inner and outer wheels, and $w$ and $l$ are the track and wheelbase of the vehicle respectively as shown in Figure-50. The radius of rotation of the center C of the platform is related to the geometry of the platform and the angles each of the front wheels by the following equations:
A typical four-wheel-steering platform would require that the front and rear wheels on each side of the platform be steered at the same angle but in opposite directions. In addition to calculating the angle of steering for each side of the platform, the steering system motion control would require at least one position control motor to steer each side of the platform independently of the other. In general, achieving controllability over speed is much easier than achieving position control. Automating the steering process mechanically not only simplifies the kinematics of the four-wheel-steering mobile platform, but also eliminates the need for position control over the steering of each wheel.

\[
R = \sqrt{a^2 + l^2 \cot^2 \delta}, \quad \text{where} \quad \cot \delta = \frac{\cot \delta_0 + \cot \delta_i}{2}
\]
VII.2- Optimizing the offset range for the steering pivot:

It is mentioned throughout this paper that the motion control of the mechanically automated four-wheel-steering mobile platform can be very closely represented by the differential drive motion kinematics described in the earlier section. It must be clearly stated, however, that there are minor differences between the motion control kinematics of the two systems. The main difference is due to the fact that the proposed design requires additional wheel travel, which is not accounted for in kinematic equations (1), to steer the wheel in the desired travel direction. When testing the third iteration design, and by adjusting the range or the distance of the offset of the steering pivots, it’s discovered that the shorter the distance of the offset is made, the shorter the steering response time and the travel distance for steering to take place. This is mainly due to the fact that when the radius R2 is shorter than R1 as shown in Figure-51, the resulting blue arc created from the 45 degree rotation of R1 is much shorter in circumference than that created from R2. Since the arcs represent the travel of each wheel dedicated to steering, therefore a shorter steering pivot offset results in less travel dedicated to steering, and a quicker steering response time. Since it shortens the extra travel dedicated to steering, a shorter range of the steering pivot offset also brings the motion control kinematics of the proposed design closer to that of the differential drive. Based on the 4 inch wheel diameter and the 0.25” optimized offset, it has been calculated that the latest proposed design requires only 0.2 inch of travel distance to achieve 45 degrees of steering angle, and requires less than 6 degrees of rotation of the wheel. Such minor differences in the motion control kinematics of the proposed
design from that of the differential drive can be accounted for by calibration while programming the microcontroller for the autonomous test as discussed in the next two sections.
VII.3- Programming the Vex microcontroller:

One of the basic programming languages that interface with the Vex PIC microcontroller is the C language, and the Robot C software is used in this project as the compiler that translates the basic C programming language to a machine language that the microcontroller can process.

A simple example of a Robot C algorithm is shown in Figure-52, where the mobile platform is programmed to travel in a straight path for 2 seconds, and then go through pure rotation for 400 milliseconds. The motor speeds are defined by integer parameters spanning from 127 to -127 and the duration of each segment of a pre-defined path is set by timers.

Once the program is debugged and compiled, it is uploaded into the microcontroller using the USB hardware pictured in Figure-53. One of the cables of the programming hardware plugs into the PIC microcontroller, while the USB end plugs to the PC computer running the Robot C software. Only one program can be stored and executed by the microcontroller at any one time, so the mobile platform can either be programmed to run in autonomous control mode, or in manual control mode. The manual control program and the four pre-planned

```c
task main()
{
    wait1Msec(2000);
    bMotorReflected[port2] = 1;
    motor[port3] = 63;
    motor[port2] = 63;
    wait1Msec(2000);
    motor[port3] = -63;
    motor[port2] = 63;
    wait1Msec(400);
}
```

Figure-52: Example of a RobotC Algorithm

Figure-53: Vex PIC programming hardware
autonomous control program algorithms that are used for testing the proposed mobile platform are each displayed in Appendix-B. The pre-planned paths of each of the autonomous tests are displayed in Appendix-D.
VII.4- Calibration:

When programming the microcontroller for an autonomous test, the motor speeds need to be converted from radians per second to units that the Vex system can recognize. In order to find a conversion factor between the two parameters, a number of simple tests are executed, the data from which is used to find a calibration factor as follows.

For each test, the mobile platform is programmed to run for a set time $\Delta t$ and a known Vex unit for the motor speed. The distance traveled by each run is then measured. Based on the 2.75 inch diameter of the wheel, the distance traveled, and the time $\Delta t$, the speed of the platform in inches per second, and the speed of the wheel in radians per second are calculated. The test is repeated for different Vex speed units, and the calculated motor speeds are plotted vs. the Vex speed units, as shown in the example in Plot-4.

Since the relationship revealed by the plot curve between the Vex speed units and the calculated wheel speeds is non-linear, the linear fit or the line with the equation $y = 0.0207x$ does not fully represent the curve as can be seen in the plot. An interesting observation that can be made from the curve in Plot-4 is the fact that the motor does not physically rotate at speeds less than 20 Vex units, and the physical wheel speed does not increase for values larger than 90 Vex units. A table listing all the data from the straight path calibration tests is shown in Appendix-C.
According to a pre-planned autonomous test path, the platform speed in inches per second and the wheel speed in radians per second can be calculated. Unlike the curve in Plot-4, a conversion equation is required for the Vex speed units, given the wheel speed in radians per second. Only the physically usable wheel variable speeds in radians per second from Plot-4 are plotted versus the Vex speed units from 20 to 90, and shown in Plot-5. Since an exponential curve is most representative of the relationship between the two variables, the conversion factor from radians per second to the Vex speed units has the equation: \( y = 10.442e^{1.009x} \).

A similar calibration test is performed to calibrate the turning radius of the proposed mobile platform, based on the turning radius calculated by equation (2) for the differential drive platform kinematics. Approximate measurements of the actual radius
from each run are taken and plotted vs. the radius defined by equation (2), as shown in Plot-6. The equation from the linear fit of the curve: \( y = 0.67x + 4.1833 \) is used as a radius conversion factor to calculate the actual turning radius of the proposed mobile platform. It’s interesting to note from Plot-6 that the turning radius of the proposed platform is always smaller than that of the differential drive platform, given the same wheel speeds on each side of both platforms. The smaller turning radius may be a result of over steering by the mechanically automated steering design.
VII.5- Autonomous test results:

The x and y coordinates of the final position of the mobile platform are measured after each autonomous run, and are listed in the third and fourth columns of Table-6 of the next page. The results are also displayed as a scatter of points in Plot-7, where the (x, y) parameters of the goal waypoint are (-50, 108). The actual location of the final position of the mobile platform from each run is marked in Plot-7 with a blue diamond, the goal waypoint with a red square, and the starting position with a purple triangle. In order to calculate the accuracy of the system, the accuracy of the coordinate values from each run is calculated using the following equation:

\[
\text{Accuracy} = 1 - \frac{|\text{error}|}{\nu}
\]

where \( \nu = \) true or reference value.

In addition to the accuracy of the system, the precision of the system is calculated using the following equation:

\[
\text{Precision} = \frac{\text{range}}{\sigma}
\]

Where the range is the distance from start to end positions, and \( \sigma = \) deviation.

The fifth and sixth columns of Table-6 are the calculated errors for each coordinate from each run, from which the accuracy of each coordinate is calculated and listed in the following
The percent accuracy values of each coordinate and from each run are then averaged, and the ultimate percent accuracy of the proposed mobile platform of 96% is calculated by averaging the average accuracy of the x and y coordinates.

The precision, on the other hand, requires the knowledge of the standard deviation of the results. The square mean value of each coordinate is calculated by squaring the difference between the average error of the coordinate and the error of the coordinate of each run. The standard deviation of each coordinate is calculated by taking the square root of the sums of the square mean values of that coordinate. The ultimate percent precision of the proposed mobile platform of 94.7% is the average of the precision values of the x and y coordinates.

<table>
<thead>
<tr>
<th>Orientation of starting position</th>
<th>Run X-axis</th>
<th>Y-axis</th>
<th>Error X-axis</th>
<th>Error Y-axis</th>
<th>% Accuracy X-axis</th>
<th>% Accuracy Y-axis</th>
<th>Square mean X-axis</th>
<th>Square mean Y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 radians</td>
<td>1 -50</td>
<td>113</td>
<td>0</td>
<td>5</td>
<td>95.37</td>
<td>6.76</td>
<td>3.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 -49</td>
<td>108.5</td>
<td>1</td>
<td>0.5</td>
<td>99.54</td>
<td>1</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 -50</td>
<td>110.5</td>
<td>0</td>
<td>2.5</td>
<td>97.69</td>
<td>0</td>
<td>6.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 -46.5</td>
<td>113.5</td>
<td>3.5</td>
<td>5.5</td>
<td>94.91</td>
<td>12.25</td>
<td>30.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 -51.5</td>
<td>109.5</td>
<td>1.5</td>
<td>1.5</td>
<td>98.61</td>
<td>2.25</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>π/2 radians</td>
<td>1 -46.5</td>
<td>102</td>
<td>3.5</td>
<td>6</td>
<td>94.44</td>
<td>12.25</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 -48</td>
<td>102</td>
<td>2</td>
<td>6</td>
<td>94.44</td>
<td>4</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 -49.5</td>
<td>103</td>
<td>0.5</td>
<td>5</td>
<td>95.37</td>
<td>0.25</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 -58</td>
<td>101</td>
<td>8</td>
<td>7</td>
<td>93.52</td>
<td>64</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 -50</td>
<td>106</td>
<td>0</td>
<td>2</td>
<td>98.15</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>π radians</td>
<td>1 -47</td>
<td>109</td>
<td>3</td>
<td>1</td>
<td>94</td>
<td>99.07</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 -42</td>
<td>108</td>
<td>8</td>
<td>0</td>
<td>84</td>
<td>100.00</td>
<td>64</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3 -47.5</td>
<td>107.5</td>
<td>2.5</td>
<td>0.5</td>
<td>95</td>
<td>99.54</td>
<td>6.25</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>4 -49</td>
<td>105</td>
<td>1</td>
<td>3</td>
<td>98</td>
<td>97.22</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>5 -51</td>
<td>103</td>
<td>1</td>
<td>5</td>
<td>98</td>
<td>95.37</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>3π/2 radians</td>
<td>1 -52.5</td>
<td>109</td>
<td>2.5</td>
<td>1</td>
<td>95</td>
<td>99.07</td>
<td>6.25</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2 -57</td>
<td>106.5</td>
<td>7</td>
<td>1.5</td>
<td>86</td>
<td>98.61</td>
<td>49</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>3 -48</td>
<td>106</td>
<td>2</td>
<td>2</td>
<td>96</td>
<td>98.15</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4 -48</td>
<td>106.5</td>
<td>2</td>
<td>1.5</td>
<td>96</td>
<td>98.61</td>
<td>4</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>5 -53</td>
<td>102.5</td>
<td>3</td>
<td>5.5</td>
<td>94</td>
<td>94.91</td>
<td>9</td>
<td>30.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orientation of starting position</th>
<th>Run X-axis</th>
<th>Y-axis</th>
<th>Error X-axis</th>
<th>Error Y-axis</th>
<th>% Accuracy X-axis</th>
<th>% Accuracy Y-axis</th>
<th>Square mean X-axis</th>
<th>Square mean Y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 radians</td>
<td>1 -50</td>
<td>113</td>
<td>0</td>
<td>5</td>
<td>95.37</td>
<td>6.76</td>
<td>3.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 -49</td>
<td>108.5</td>
<td>1</td>
<td>0.5</td>
<td>99.54</td>
<td>1</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 -50</td>
<td>110.5</td>
<td>0</td>
<td>2.5</td>
<td>97.69</td>
<td>0</td>
<td>6.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 -46.5</td>
<td>113.5</td>
<td>3.5</td>
<td>5.5</td>
<td>94.91</td>
<td>12.25</td>
<td>30.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 -51.5</td>
<td>109.5</td>
<td>1.5</td>
<td>1.5</td>
<td>98.61</td>
<td>2.25</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>π/2 radians</td>
<td>1 -46.5</td>
<td>102</td>
<td>3.5</td>
<td>6</td>
<td>94.44</td>
<td>12.25</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 -48</td>
<td>102</td>
<td>2</td>
<td>6</td>
<td>94.44</td>
<td>4</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 -49.5</td>
<td>103</td>
<td>0.5</td>
<td>5</td>
<td>95.37</td>
<td>0.25</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 -58</td>
<td>101</td>
<td>8</td>
<td>7</td>
<td>93.52</td>
<td>64</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 -50</td>
<td>106</td>
<td>0</td>
<td>2</td>
<td>98.15</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>π radians</td>
<td>1 -47</td>
<td>109</td>
<td>3</td>
<td>1</td>
<td>94</td>
<td>99.07</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 -42</td>
<td>108</td>
<td>8</td>
<td>0</td>
<td>84</td>
<td>100.00</td>
<td>64</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3 -47.5</td>
<td>107.5</td>
<td>2.5</td>
<td>0.5</td>
<td>95</td>
<td>99.54</td>
<td>6.25</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>4 -49</td>
<td>105</td>
<td>1</td>
<td>3</td>
<td>98</td>
<td>97.22</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>5 -51</td>
<td>103</td>
<td>1</td>
<td>5</td>
<td>98</td>
<td>95.37</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>3π/2 radians</td>
<td>1 -52.5</td>
<td>109</td>
<td>2.5</td>
<td>1</td>
<td>95</td>
<td>99.07</td>
<td>6.25</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2 -57</td>
<td>106.5</td>
<td>7</td>
<td>1.5</td>
<td>86</td>
<td>98.61</td>
<td>49</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>3 -48</td>
<td>106</td>
<td>2</td>
<td>2</td>
<td>96</td>
<td>98.15</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4 -48</td>
<td>106.5</td>
<td>2</td>
<td>1.5</td>
<td>96</td>
<td>98.61</td>
<td>4</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>5 -53</td>
<td>102.5</td>
<td>3</td>
<td>5.5</td>
<td>94</td>
<td>94.91</td>
<td>9</td>
<td>30.25</td>
</tr>
</tbody>
</table>

The ultimate accuracy of the proposed mobile platform is 95.96%, and the standard deviation is 3.58%. The ultimate precision is 94.73%.
VIII- Conclusion

The three iterations of mechanical design and prototype fabrication of the mechanically automated four-wheel-steering mobile platform proved to be necessary to achieve the ultimate performance in terms of functionality, quick steering response, minimal wheel slip, and energy efficiency due to minimal wheel friction. The second iteration of the prototype solved most of the functionality problems, as far as steering all four wheels in the desired directions is concerned, but suffered from wheel slip due to the differential gear box design and unnecessary wheel friction due to the additional four wheels. The last iteration of the design is the simplest in terms of the number of mechanical moving parts are concerned, lighter and more cost efficient than the first two design iteration, and delivers excellent steering response. Most importantly, however, is the fact that the third iteration design has minimal wheel slip and wheel friction since it does not use differential gear boxes and it consists of a four wheel design. Optimizing the steering pivot offset also gives the third iteration design quicker steering response.

The kinematic calculations for each starting orientation of the autonomous test, shown in Appendix-D, are a good starting point for the autonomous programming algorithm, but many adjustments to the algorithm had to be made during testing. Minor mechanical and control malfunctions are mainly the reasons behind the differences between the pre-planned path algorithm, and the final algorithm which was developed to navigate the mobile platform from its starting position to the final position accurately.

The inconsistency in the speed of the wheels has the most significant effect on the difference between the theoretical and actual motion of the platform. As an example, when the same wheel speeds are assigned to all four wheels and the platform is expected to travel on a straight path, the platform never does so during testing. In fact, the third iteration of the
prototype platform tends to pull to the right as it travels forward, mainly due to a small difference between the physical wheel rotation speeds and the assigned speed values in the algorithm. During calibration, it has been observed that when small wheel speed values are assigned in the algorithm, the physical wheel speeds on all four wheels are not consistent. When the Vex wheel speed values are increased starting from 0, three out of the four wheels begin rotation when the value reaches 16 Vex units, but the fourth wheel will not rotate until 18 Vex units. When one wheel of the mobile platform rotates slower than the other three, the platform turns towards the side of the slower wheel as it travels forward.

Another factor that affects the straight path of the mobile platform is the performance of the mechanically automated steering. Even though each wheel appears to always be steering in the direction of the travel path throughout the test course, it may not be doing so very accurately. Even a small misalignment between the steering angle of each wheel and the direction of the travel path results in a deviation of the mobile platform from the desired path over a long distance.

In addition to compensating for the mobile platform’s deviation from a straight path, the final autonomous algorithms also need to account for acceleration and deceleration. At the start of each autonomous run, and when the algorithm begins to execute, the wheels go through a limited amount of slip before the platform is set to motion, mainly due to large acceleration that the wheels experience during the time before the platform reaches its steady-state speed. Also, when the algorithm stops executing, the mobile platform does not come to a sudden stop, but instead its momentum carries it for up to six inches of additional travel.

Finally, the battery charge level also weighs in to make a difference between the original pre-planned path algorithm and the final algorithm used for each autonomous test run. When the
same test is repeated over many trials, it has been observed that the stopping position of the mobile platform becomes shorter of reaching the goal position with every run, even though the algorithm is not mended from one run to the next. However, when the batteries are replaced with a set of fully charged batteries, the mobile platform will overshoot the goal position. As the batteries are depleted, the speed of the motors decrease even when the settings are unchanged, and thereby affecting the final stopping position of the platform.

A final observation that’s made during testing is undesired steering oscillation at high travel speeds. The problem is mostly apparent when all four wheels are assigned the same speed, and when the assigned speed is above 80 Vex units or close to the maximum value. The mechanically automated steering design delivers such quick response in both steering directions that the wheels tend to over-steer and overcorrect the over-steer repeatedly, causing a high frequency oscillation in steering. The oscillation is mainly due to the minimal friction between the wheels and the travel surface and the ball bearing steering pivot joints, and is easily reduced by increasing friction in the steering pivot joints which dampens and reduces any oscillation in steering.

The mobile platform’s lack of ability to travel along a perfectly straight path is accounted for in the algorithm by increasing or decreasing the duration of the circular path, and therefore correcting for the deviation from the straight path. Similarly, the duration of the straight path motion of the platform is amended to compensate for the acceleration and deceleration of the platform. Before running the autonomous tests, two new batteries are purchased and fully charged. One of the batteries is always charged while the second one is used during testing, and the batteries are alternated for every couple of runs to minimize errors in the results due to a low charge of the batteries.
The results from the final autonomous tests are relatively consistent after the aforementioned compensation and correction, and the proposed mechanically automated four-wheel-steering design performs impressively. Considering the length of the average path being approximately 15 feet, the stopping position of the platform is always within inches of the goal position resulting in an overall accuracy of 96 percent. The platform also delivers consistent performance with a standard deviation of less than 4 inches, and the overall precision of 95 percent.

On a full scale prototype model of the proposed mechanically automated four-wheel-steering, it is recommended that an automotive steering damper be added to the steering pivot joints to minimize any steering oscillation. Even though the design of the mechanically automated steering functions well and is responsive under most motion conditions, a disadvantage of using the design for a full scale autonomous mobile platform is in its inaccuracy in controlling the angle of steering, which may be a problem for long distance travel. However, most autonomous robots have real-time feedback about their environment through sensors which will correct for small inaccuracies in steering control.
References


2- Robot C programming software for Vex PIC 0.5 version 3.62, build date Sept. 5th 2013, Manufactured with intellectual properties from Carnegie Mellon University, copy right 2012 Robomatter LLC.

3- Reference websites for figures and pictures used in this report:
   a- E Hobby house.com
   b- Robot platform .com
   c- Institute for Dynamic Systems and Control.com (IDSC)
   d- Apex Dynamics USA.com
   e- Wikipedia.com
   f- Airtrax.com
   g- Kaneko Higashimori Laboratory, Osaka University, Japan.
   h- MIT.edu
   i- Spherical Drive Systems (SDS).com
   j- Vex robotics.com
   k- Logo NXT.com
   l- Red cat racing.com
Appendix-A

The following six steps calculate the wheel speeds of a differential drive mobile platform to navigate from a known reference point to a given waypoint. The steps need to be iterated at a rate of at least 200 milliseconds, depending on the speed of the platform and the processing power of the microcontroller.

The given parameters are the starting position and orientation noted by \((x_0, y_0, \theta_0)\), the final position \((x_f, y_f)\), the travel speed of the platform \(\rho'\), the time interval between iterations \(\Delta t\), and the radius \(r\) and distance \(L\) of each wheel from the center of each axle.

1- Calculate: 

\[
\rho = \sqrt{(x_f - x_0)^2 + (y_f - y_0)^2}, \quad R = \frac{(x_f - x_0)^2 + (y_f - y_0)^2}{2(x_f - x_0)},
\]

\[
k = \frac{(y_f - y_0)^2 - (x_f - x_0)^2 + 2R(x_f - x_0)}{2R(y_f - y_0)}, \quad \alpha = (\tan^{-1}\frac{y_f - y_0}{x_f - x_0}) - \theta_0.
\]

2- Calculate speed in inertial reference frame: 

\[
x' = \rho' \cos \alpha, \quad y' = \rho' \sin \alpha,
\]

\[
\theta' = \frac{1}{\sqrt{1 - k^2}} \left[ (2(x_f - x_0)y' - 2(x_f - x_0)x' + 2Rx')(2R(y_f - y_0)) - (2Rx')(y_f - y_0)^2 - (x_f - x_0)^2 + 2R(x_f - x_0) \right]
\]

3- Transform speed to local reference frame: 

\[
x'_R = (x' \cos \theta + y' \sin \theta), \quad y'_R = (-x' \sin \theta + y' \cos \theta), \quad \theta'_R = \theta'
\]

4- Calculate wheel speeds: 

\[
\phi'_1 = \frac{x'R - L\theta'R}{r}, \quad \phi'_2 = \frac{x'R + L\theta'R}{r}
\]

5 - Calculate next true position: 

\[
x'_T = \frac{1}{2} r \phi'_1 + \frac{1}{2} r \phi'_2, \quad y'_T = 0, \quad \theta'_T = \theta_0 + \theta'_R \Delta t
\]

6 - 

\[
x'_1 = x'_T \cos \theta_1, \quad y'_1 = x'_T \sin \theta_1, \quad x_1 = x_0 + x'_1 \Delta t, \quad y_1 = y_0 + y'_1 \Delta t
\]
Appendix-B

```c
// C language algorithm for Manual control testing

#include <std.h>

int main()
{
  while (1 == 1) // Infinite loop to keep the program executing while the controller is on.
  {
    hIsAutonomousMode = false; // Turning off a pre-programmed autonomous mode
    hMotorReflected[port2] = true; // Reversing the rotation direction of right motors
    hMotorReflected[port7] = true;

    motor[port2] = vexRT[ch2]; // Assigning the transmitter channels to the motors.
    motor[port8] = vexRT[ch3];
    motor[port2] = vexRT[ch2];
    motor[port7] = vexRT[ch2];
  }
}
```

```c
// C language algorithm for the first starting

#include <std.h>

int spd1 = 47; // Right side wheel speed
int spd2 = 20; // Left side wheel speed
int spd3 = 20; // Straight path speed

int main()
{
  wait3Sec(2000); // Pause for the user to move their hands away from the robot
  hMotorReflected[port2] = true; // Reversing the rotation direction for the right side
  hMotorReflected[port7] = true;

  motor[port1] = spd1; // Assigning wheel speeds and time for the circular path
  motor[port8] = spd2;
  motor[port2] = spd1;
  motor[port7] = spd1;
  wait3Sec(2000);

  motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
  motor[port6] = spd3;
  motor[port2] = spd3;
  motor[port7] = spd3;
  wait3Sec(7700);
}
```
```c
// C language algorithm for the second starting
// orientation of the autonomous test. theta = 90 degrees

int spd1 = 32; // Right side wheel speed
int spd2 = 30; // Left side wheel speed
int spd3 = 29; // Straight path speed

test main()
{
    waitMsec(2000); // Pause for the user to move their hands away from the robot
    bMotorReflected[port2] = true; // Reversing the rotation direction for the right side
    bMotorReflected[port7] = true;

    motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
    motor[port0] = spd2;
    motor[port2] = spd1;
    motor[port7] = spd1;
    waitMsec(4000);

    motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
    motor[port0] = spd3;
    motor[port2] = spd3;
    motor[port7] = spd3;
    waitMsec(14000);
}

// C language algorithm for the third starting
// orientation of the autonomous test. theta = 180 degrees

int spd1 = 32; // Right side wheel speed
int spd2 = 30; // Left side wheel speed
int spd3 = 29; // Straight path speed

test main()
{
    waitMsec(2000); // Pause for the user to move their hands away from the robot
    bMotorReflected[port2] = true; // Reversing the rotation direction for the right side
    bMotorReflected[port7] = true;

    motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
    motor[port0] = spd2;
    motor[port2] = spd1;
    motor[port7] = spd1;
    waitMsec(18000);

    motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
    motor[port0] = spd3;
    motor[port2] = spd3;
    motor[port7] = spd3;
    waitMsec(18000);
}

// C language algorithm for the fourth starting
// orientation of the autonomous test. theta = 270 degrees

int spd1 = 32; // Right side wheel speed
int spd2 = 30; // Left side wheel speed
int spd3 = 29; // Straight path speed

test main()
{
    waitMsec(2000); // Pause for the user to move their hands away from the robot
    bMotorReflected[port2] = true; // Reversing the rotation direction for the right side
    bMotorReflected[port7] = true;

    motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
    motor[port0] = spd2;
    motor[port2] = spd1;
    motor[port7] = spd1;
    waitMsec(3000);

    motor[port3] = spd3; // Assigning wheel speeds and time for the circular path
    motor[port0] = spd3;
    motor[port2] = spd3;
    motor[port7] = spd3;
    waitMsec(9000);
}
```
Appendix-C

Table-A3-1, straight path calibration data:

<table>
<thead>
<tr>
<th>Run#</th>
<th>Vex Speed</th>
<th>Δt (sec)</th>
<th>Distance Travelled (inch)</th>
<th>Platform Speed (inch/sec)</th>
<th>Wheel Speed (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>30</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>30</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>30</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>30</td>
<td>209.8</td>
<td>6.99</td>
<td>0.81</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>30</td>
<td>274.6</td>
<td>9.15</td>
<td>1.06</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>25</td>
<td>268.2</td>
<td>10.73</td>
<td>1.24</td>
</tr>
<tr>
<td>8</td>
<td>35</td>
<td>20</td>
<td>241</td>
<td>12.05</td>
<td>1.39</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>20</td>
<td>259</td>
<td>12.95</td>
<td>1.50</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
<td>15</td>
<td>205</td>
<td>13.67</td>
<td>1.58</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>15</td>
<td>214</td>
<td>14.27</td>
<td>1.65</td>
</tr>
<tr>
<td>12</td>
<td>55</td>
<td>15</td>
<td>220</td>
<td>14.67</td>
<td>1.70</td>
</tr>
<tr>
<td>13</td>
<td>60</td>
<td>15</td>
<td>225</td>
<td>15.00</td>
<td>1.74</td>
</tr>
<tr>
<td>14</td>
<td>65</td>
<td>15</td>
<td>229</td>
<td>15.27</td>
<td>1.77</td>
</tr>
<tr>
<td>15</td>
<td>70</td>
<td>15</td>
<td>233</td>
<td>15.53</td>
<td>1.80</td>
</tr>
<tr>
<td>16</td>
<td>75</td>
<td>15</td>
<td>236</td>
<td>15.73</td>
<td>1.82</td>
</tr>
<tr>
<td>17</td>
<td>80</td>
<td>15</td>
<td>243.5</td>
<td>16.23</td>
<td>1.88</td>
</tr>
<tr>
<td>18</td>
<td>85</td>
<td>15</td>
<td>244</td>
<td>16.27</td>
<td>1.88</td>
</tr>
<tr>
<td>19</td>
<td>90</td>
<td>15</td>
<td>244.5</td>
<td>16.30</td>
<td>1.89</td>
</tr>
<tr>
<td>20</td>
<td>95</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
<tr>
<td>21</td>
<td>100</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
<tr>
<td>22</td>
<td>105</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
<tr>
<td>23</td>
<td>110</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
<tr>
<td>24</td>
<td>115</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
<tr>
<td>25</td>
<td>120</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
<tr>
<td>26</td>
<td>125</td>
<td>15</td>
<td>245</td>
<td>16.33</td>
<td>1.89</td>
</tr>
</tbody>
</table>

Table-A3-2, turning radius calibration data:

<table>
<thead>
<tr>
<th>Run #</th>
<th>Circle Radius</th>
<th>Δt (sec)</th>
<th>Distance Travelled (inch)</th>
<th>Platform Speed (inch/sec)</th>
<th>Wheel Speed (rad/sec)</th>
<th>Circle Radius</th>
<th>Δt (sec)</th>
<th>Distance Travelled (inch)</th>
<th>Platform Speed (inch/sec)</th>
<th>Wheel Speed (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>30</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>90</td>
<td>30</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>35</td>
<td>0.25</td>
<td>0.48</td>
<td>1.92</td>
<td>17</td>
<td>35</td>
<td>0.25</td>
<td>0.48</td>
<td>1.92</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>30</td>
<td>0.35</td>
<td>0.62</td>
<td>1.78</td>
<td>18</td>
<td>30</td>
<td>0.35</td>
<td>0.62</td>
<td>1.78</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>30</td>
<td>0.43</td>
<td>0.72</td>
<td>1.68</td>
<td>19</td>
<td>30</td>
<td>0.43</td>
<td>0.72</td>
<td>1.68</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>30</td>
<td>0.49</td>
<td>0.79</td>
<td>1.61</td>
<td>20</td>
<td>30</td>
<td>0.49</td>
<td>0.79</td>
<td>1.61</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>30</td>
<td>0.54</td>
<td>0.84</td>
<td>1.56</td>
<td>21</td>
<td>30</td>
<td>0.54</td>
<td>0.84</td>
<td>1.56</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td>30</td>
<td>0.58</td>
<td>0.88</td>
<td>1.52</td>
<td>22</td>
<td>30</td>
<td>0.58</td>
<td>0.88</td>
<td>1.52</td>
</tr>
</tbody>
</table>
Appendix-D

Path planning for 1\textsuperscript{st} starting orientation

[Note]: \(\alpha\) = the orientation angle of the platform with respect to the inertial reference frame.

- **Platform speed** = 10.3 inch/sec
- **Circular path**:
  - \(R = 40\) inch
  - \(\Phi'1 = 41\), \(\Phi'2 = 22\)
  - \(\Theta = 136.6\) deg
  - Circ. = 95.4 inch
  - \(\Delta t = \text{Circ.} / \text{speed} = 95.4 / 10.3 = 9.26\) sec.

- **Straight path**:
  - \(L = 110.6\) inch
  - \(\Phi'1 = \Phi'2 = 1.2\) rad/sec = 29 Vex.
  - \(\Delta t = L / \text{speed} = 110.6 / 10.3 = 10.74\) sec.

Path planning for 2\textsuperscript{nd} starting orientation

- **Platform speed** = 10.3 inch/sec
- **Circular path**:
  - \(R = 40\) inch
  - \(\Phi'1 = 22\), \(\Phi'2 = 41\)
  - \(\Theta = 195\) deg
  - Circ. = 136.1 inch
  - \(\Delta t = \text{Circ.} / \text{speed} = 136.1 / 10.3 = 13.21\) sec.

- **Straight path**:
  - \(L = 140.6\) inch
  - \(\Phi'1 = \Phi'2 = 1.2\) rad/sec = 29 Vex.
  - \(\Delta t = L / \text{speed} = 140.6 / 10.3 = 13.65\) sec.
Path planning for 3rd starting orientation

- Platform speed = 10.3 inch/sec
- **Circular path:**
  - R = 40 inch
  - Φ'1 = 22, Φ'2 = 41
  - Θ = 88.7 deg
  - Circ. = 61.9 inch
  - Δt = Circ. / speed = 61.9 / 10.3 = 6.01 sec.

- **Straight path:**
  - L = 109.4 inch
  - Φ'1 = Φ'2 = 1.2 rad/sec = 29
  - Δt = L / speed = 109.4 / 10.3 = 10.62 sec.

Path planning for 4th starting orientation

- Platform speed = 10.3 inch/sec
- **Circular path:**
  - R = 40 inch
  - Φ'1 = 41, Φ'2 = 22
  - Θ = 21.3 deg
  - Circ. = 14.9 inch
  - Δt = Circ. / speed = 14.9 / 10.3 = 1.39 sec.

- **Straight path:**
  - L = 141.6 inch
  - Φ'1 = Φ'2 = 1.2 rad/sec = 29
  - Δt = L / speed = 141.6 / 10.3 = 13.75 sec.