COMPUTER-ASSISTED INSTRUCTION

EASY, FIRST DEGREE EQUATIONS IN ONE UNKNOWN

A graduate project submitted in partial satisfaction of the requirements for the degree of Master of Arts in Education, Secondary

by

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ABSTRACT

COMPUTER-ASSISTED INSTRUCTION

EASY FIRST DEGREE EQUATIONS IN ONE UNKNOWN

by

Joan Rita Mintzer

Master of Arts in Education

Twenty years ago the use of computers as instructional machines was only an idea that was being considered by a handful of scientists and educators. Today, the idea has become a reality. Computer-assisted instruction (CAI) has undergone an amazingly rapid development. The development can be attributed to the potential of CAI for answering today's need in education for individualized instruction. This graduate project exemplifies fulfillment of the need by presenting useful programs for pre-algebra.

These programs were written in the belief that CAI offers students faster learning through individual pacing, personalized tutoring, and automatic measurement of progress by recording of responses. CAI relieves the teacher of much record keeping and repetitious drill. As such the project contains teaching, learning, and review programs.
The first program presents an easy equation of the first degree, $x + 2 = 6$. Then follows an explanation of the skill involved in solving the equation. Two further examples and eight problems complete the learning unit. After each problem the student types an answer into the computer. If the answer is correct, another problem is presented to the student. If the answer is incorrect, the same problem appears a second time. Should a problem be answered incorrectly two more times, the correct answer is given the student. Then the next problem is presented. At the end of the unit the student is informed as to how many problems were answered correctly. Mastery is set at 70% correct. If the student achieves less than this, he is directed to a sub-routine for more instruction. This same format frames all the units.

An overview of CAI introduces the reader to the project.
Innovations in educational practices since the 1950's are said to have been occasioned by Sputnik. Whether this is true or not - and the thoughtful historian will suspect that is is not - the launching of the Russian satellite is clearly a benchmark in educational progress. Among the significant innovations since the mid-50's is computer-assisted instruction (CAI).

The conditions were right for significant innovations. Apart from the "fear" induced in the American public by the Russian satellite, reasoning professionals were reevaluating theory and practice in education. The National Association of Secondary School Principals had commissioned J. Lloyd Trump to gather an investigative/evaluative group to assess the field and propose new ideas. While the ideas brought forward could hardly be called new, the emphasis on them was.

In particular, and germane to CAI was the need for individualizing instruction. Trump's report states:

The proposals for the school of the future call for a reduction of time students spend in group instructions. Classes do not meet five days weekly. This flexibility would permit broader study in a variety of fields, or deeper in few. This would hold true not only for talent-ed youth, but also for slower-learning students. ... The increase in length of the school day and week - when appropriate for selected students - would be carried out in the area of individual study.

The immediate impact of this recommendation was the restructuring of the school day, known as flexible modular scheduling. It required time
and materials for individual study. Already at hand was one form of individual study, programmed learning.

Programmed learning can be traced at least to Frederick Burk of San Francisco State College. In 1913 he created a series of lessons for individual students that they might follow and finish at their own rate of speed. The great advocate of programmed learning, however, is B.F. Skinner. In the use of technology in education, no matter what the technology may be, he insisted upon the incorporation of three principles of learning: active participation, immediate confirmation, and individual progression adapted to the capabilities of individual learners. The foundation was laid in the mid-50's and still supports the educational technology that serves the individual. This is what CAI is all about.

CAI is autoinstructional. Its origins are more recent. It arose from the happy conjunction of two phenomena: computer technology and programmed instruction. What may be credited with bringing the two together was the federal government. The National Science Foundation and various funding agencies which were created by the Elementary and Secondary Education Act of 1965 provided the funds - millions of dollars annually - to foster the growth and development of CAI.

Stanford University and its distinguished professor, Patrick Suppes, should probably be credited with laying the foundations for CAI. While the story has been told at length elsewhere, its high points merit recognition here. Beginning in 1963 with a grant from Carnegie Foundation, the Institute for Mathematical Studies in the Social Sciences at Stanford University began to develop a small tutorial
system. Students were bused in from nearby elementary schools to learn at six student stations hooked to a computer. They received instruction on a more or less regular schedule, in mathematics and the language arts.

The following year, the U.S. Office of Education made a sizeable grant to the Institute for the development and implementation of a computer-assisted instructional program in initial reading and mathematics for culturally disadvantaged children. At the same time, a second CAI system for drill-and-practice was being developed by members of the group. This system reinforced the concepts provided by the teacher through intensified drill and practice. In the 1967-68 school year, approximately 3,000 students from California to Kentucky received daily lessons in reading, arithmetic, logic, and Russian, all from one central computer located at Stanford. CAI had become an integral partner of the teacher in educating American youth.

The picture today displays much progress. CAI refers to a wide range of educational techniques that rely on a computer to assist in the presentation of learning material. The system may be simple or complex, as educational demands and budgetary considerations allow. In general, a computer will present the material to be learned or the drill and practice problems on a TV type screen. The student responds to the presentation by typing a response or by "marking" on the TV screen with a light pen. The computer then reacts to the response or mark by signaling that the answer was correct or wrong. Depending upon the quality of the response the computer withdraws from its data bank whatever material is appropriate for the student's next step. In short,
the computer has become a most sophisticated assistant to the teacher.

As the science has developed, several distinctions have arisen in the theory. These differentiate between CAI, computer-assisted learning (CAL) and computer managed instruction (CMI). Their common denominator is the computer. The differences between them, however, are several. CAL is used to solve problems. The student feeds data into the computer to obtain results. What the slide rule and calculator used to do for the student, the computer now does. This is CAL. CMI is a computer-based management information system. It stores data relevant to resource management and prepares useful reports for decision making by the community, faculty, and students. The community representatives are concerned with budget, the faculty with the curriculum, and the students with learning. The interaction of these three groups is assisted by CMI. CAI, as described in this overview, is a learning technology. The computer is programmed to present certain material to the student who reacts to it. There are both tutorial levels on which a student learns and drill-and-practice levels on which the student finds reinforcement for what has been learned. This graduate project is concerned with CAI on the tutorial level.

The amount of information available on CAI is extensive. Any of the references used in the preparation of this overview supply more sources than the user of CAI may need. One source, however, deserves particular attention because it is readily and regularly available - the journal *Creative Computing*. This is a bi-monthly magazine of
recreational and educational computing. The reader is kept current with the state of the science.

For instance the November-December 1977 issue offers an article, "Final Exams - Let the Computer Write Them", that is within the scope of CMI. The same issue continues with part three of a five part series on CAI that looks at incorporating a sliding grade level into programs that generate several types of problems. The March-April 1978 issue offers "CAI: Interaction Between Student and Computer", a closer look at the more subtle areas. The September-October 1978 issue discusses "Computers and Early Learning". This article speaks to such topics as drill and practice systems, tutorial systems, creative activity and student assessment. In short, the teacher who uses computers can keep current with computer science.

Within the perspective offered by this overview, this graduate project was prepared.
THE GRADUATE PROJECT

An Overview

This CAI project offers four learning programs and one review program in pre-algebra for junior high school students. The programs constitute the principal portion of the project and will be found after this overview. Here, three items will be discussed: the target population, the use of the computer, and the content of the programs.

The target population is found in junior high schools. These students are those the author has been teaching and whom she knows best. Hence, the choice of population was practical. This is not to say that other students, high school or higher, may not profit from the programs. Undoubtedly there are others who prefer the easy approach to solving algebraic equations offered in these programs.

It is well known that the computer - rather, the use of the computer by students - strongly motivates students to learn. Hatfield and Kieren (1972) report on their work with seventh-grade students who learned to program a computer and used this skill in studying mathematics. The seventh-graders studied numeration systems, the properties of whole numbers, elementary number theory, and finite mathematical systems. During approximately 100 instructional days in each of two years, these students wrote and processed computer programs. The results of the experiment suggest that not only do low achievers develop better attitudes toward the learning of mathematics when they work with computers, but that there is evidence that they learn more than their peers who do not work with computers. The reason for this is offered by Zinn:
The student should benefit from a prompt evaluation or self-check of his response. Automated feedback is important wherever it is difficult for him to judge whether his response is correct . . . . Another factor which can promote student learning is the availability of records and a summary of his performance for his own use in planning further study.

Both of these factors - automated feedback and summary of performance - have been built into this graduate project.

Pre-algebra is really an introduction to algebra. The improvement of the teaching of mathematics, which was occasioned by the "New Math", has lowered the readiness-for-algebra age of students. Whether the relationship between the "New Math" and readiness is real or not, does not alter the fact of what is being taught in junior high schools today. This graduate project offers five programs that assist in the learning of pre-algebra.

The first four programs teach the students to solve easy, first degree equations. In order to solve these equations, they must learn and apply the four algorithms which here are called properties: addition, subtraction, multiplication, and division properties. In short, the same number can be added to or subtracted from both sides of an equation; both sides can be multiplied or divided by the same number; regardless of which operation is used, the result is an equivalent equation. The appropriate operation, therefore, produces the solution to the equation; that is, the correct operation uncovers the value of the unknown.

After an introduction to each property or operation, the student is offered eight drill-and-practice exercises. If the answer to each is correct, the next problem is offered. If incorrect, the computer
writes something like, "Careful! You better do the same problem over."
If the student cannot do it correctly after two more tries, the computer writes, "Sorry! \( x = 9 \)" (or whatever); then, the next problem is produced. At the end of the exercises, the computer tells the student how well he fared; e.g., "You scored 6 out of 8 right on your first try. Nice job, Jack."

The fifth and final program reviews with the student what has been learned. There are 16 exercises in the review program, four for each property. After the student has solved a problem, the computer admonishes the student to check the answer. This may be done mentally or with paper and pencil. After a pause, the computer flashes the correct answer on the screen. A simple "Goodbye, Jack" completes the run.

An outcome of the project that remains to be explored is this: Will the experience with these programs inspire students to write their own programs?
"ADDITION PROPERTY FOR EQUATIONS"
9& "THE ADDITION PROPERTY STATES THAT ADDING THE SAME NUMBER TO BOTH"
10& "SIDES OF AN EQUATION GIVES AN EQUIVALENT EQUATION. IN ORDER TO"
11& "OBTAIN X" WE MUST ELIMINATE THE INTEGER WITH X. THIS NUMBER IS"
12& "ELIMINATED BY SUBTRACTING IT."
13&
14& "HERE ARE TWO EXAMPLES."
20& "X-7=9"
30& "X-7+7=9+7 (ADDITION PROPERTY)"
40& "X=16"
50& "X-3=8"
60& "X-3+3=8+3 (ADDITION PROPERTY)"
70& 
80& "NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. USE THE ADDITION"
81& "PROPERTY TO SOLVE. IF YOU ANSWER A PROBLEM CORRECTLY YOU WILL"
82& "PROCEED"
83& "TO THE NEXT ONE. IF YOU ANSWER INCORRECTLY YOU WILL GET ANOTHER"
84& "CHANCE."
85& "TYPE IN YOUR RESPONSE WHEN THE COMPUTER STOPS."
120& FOR J=1 TO 8
121 LET Q=0: LET X=B+A
140&
150& "X="; F
160 INPUT "X=": F
170&
180 IF X=F GOTO 220
190 GOSUB 300
200 & "SORRY: WRONG ANSWER. PLEASE TRY AGAIN."
210 GOTO 150
220 & "RIGHT! TRY ANOTHER."
230 NEXT J
240 & "YOU SCORED "W" OUT OF 8 RIGHT ON YOUR FIRST TRY."
250 IF W>2 THEN 500
260 & "GOOD WORK! "N" IS A MATH HERO."
270 GOTO 9999
300 LET Q=Q+1
310 IF Q>1 THEN 400
320 LET W=W+1
330 RETURN
400 & "SORRY: X="; F
410 GOTO 230
500 & "ADDITIONAL PRACTICE."
505 REM THIS IS A SUBROUTINE USING THE ADDITION PROPERTY.
510 FOR K=1 TO 10
513 RANDOMIZE
514 N = INT(10*RND(X)+11)
515 M = INT(10*RND(X)+11)
530 &
532 LET X = M + N
533 & X = "M" + "N"
550 INPUT "X" = "F"
560 & X = "X" &
570 NEXT K
580 & "GOOD BYE."
590 DATA 7,3,4,-3,9,18,3,5,2,-2,10,25,50,6,-18
9999 END

READY
RUN
EQADD
THIS IS HELPFUL HARRY SIGNING IN. PLEASE IDENTIFY YOURSELF.
? JOAN

ADDITION PROPERTY FOR EQUATIONS
THE ADDITION PROPERTY STATES THAT ADDING THE SAME NUMBER TO BOTH SIDES OF AN EQUATION GIVES AN EQUIVALENT EQUATION. IN ORDER TO OBTAIN X, WE MUST ELIMINATE THE INTEGER WITH X. THIS NUMBER IS ELIMINATED BY SUBTRACTING IT.

HERE ARE TWO EXAMPLES.

\[ x - 7 = 9 \]
\[ x - 7 + 7 = 9 + 7 \] (ADDITION PROPERTY)
\[ x = 16 \]

\[ x - 3 = 8 \]
\[ x - 3 + 3 = 8 + 3 \] (ADDITION PROPERTY)
\[ x = 11 \]

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. USE THE ADDITION PROPERTY TO SOLVE. IF YOU ANSWER A PROBLEM CORRECTLY YOU WILL PROCEED TO THE NEXT ONE. IF YOU ANSWER INCORRECTLY YOU WILL GET ANOTHER CHANCE. TYPE IN YOUR RESPONSE WHEN THE COMPUTER STOPS.

\[ x - 7 = 3 \]
\[ x = ? \ 10 \]
RIGHT! TRY ANOTHER.

\[ x - 4 = -3 \]
\[ x = ? \ 1 \]
RIGHT! TRY ANOTHER.

\[ x = 9 = 18 \]
\[ x = ? \ 27 \]
RIGHT! TRY ANOTHER.

\[ x - 3 = 5 \]
\[ x = ? \ 8 \]
RIGHT! TRY ANOTHER.

\[ x - 2 = -2 \]
\[ x = ? \ 0 \]
RIGHT! TRY ANOTHER.
\[ x - 10 = 25 \]
\[ x = ? \quad 35 \]

RIGHT! TRY ANOTHER.

\[ x - 25 = 50 \]
\[ x = ? \quad 75 \]

RIGHT! TRY ANOTHER.

\[ x - 6 = -18 \]
\[ x = ? \quad 9 \]

SORRY! WRONG ANSWER. PLEASE TRY AGAIN.

\[ x - 6 = -18 \]
\[ x = ? \quad 3 \]

SORRY! \( x = -12 \)

YOU SCORED 7 OUT OF 8 RIGHT ON YOUR FIRST TRY.
GOOD WORK. JOAN IS A MATH HERO.
READY
HELLO, THIS IS HELPFUL HARRY AGAIN. WHAT IS YOUR NAME?

THE SUBTRACTION PROPERTY STATES THAT SUBTRACTING THE SAME NUMBER FROM BOTH SIDES OF AN EQUATION GIVES AN EQUIVALENT EQUATION.

IN ORDER TO OBTAIN X, WE MUST ELIMINATE THE INTEGER WITH X. THIS NUMBER IS ELIMINATED BY ADDING IT.

HERE ARE TWO EXAMPLES.

X+70=94:
X+70-70=94-70 (SUBTRACTION PROPERTY):
X=24:
X+3-3=7-3:
X=4:

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. USE THE SUBTRACTION PROPERY TO SOLVE. IF YOU ANSWER A PROBLEM CORRECTLY YOU WILL PROCEED TO THE NEXT ONE. IF YOU ANSWER INCORRECTLY YOU WILL HAVE ANOTHER CHANCE. TYPE IN YOUR RESPONSE WHEN THE COMPUTER STOPS.

FOR J=1 TO 8
READ A,B
Q=0:X=B-A
PRINT"X+"A="B
INPUT"X~";F
IF X=F GOTO 230
GOSUB 300
ELSE"CAREFUL: YOU BETTER DO THE SAME PROBLEM OVER."
GOTO 160

IF W>2 THEN 500
NICE JOB. "NICE".
GOTO 9999
Q=Q+1
RETURN

YOU SCORED "8-W" OUT OF 8 RIGHT ON YOUR FIRST TRY.

YOU MUST ELIMINE THE INTEGER WITH X.

NICE JOB. "NICE".
GOTO 9999
Q=Q+1
RETURN

SORRY! X="X:
GOTO 230

ADDITIONAL PRACTICE.

REM THIS IS A SUBROUTINE USING THE SUBTRACTION PROPERTY.
FOR K=1 TO 10
RANDOMIZE
N=INT(10*RND(X)+11)
M=INT(10*RND(X)+11)
X=N-M
540"X+"M"="N
550INPUT"X=";F
560:&"X=";F
580NEXT K
590"GOOD BYE FOR NOW."
990 DATA3,12,2,-4,5,-9,1,1,4,-4,12,3,10,-4,12,10,-4,13,-9
9999END

READY
Hello, this is Helpful Harry again. What is your name?

John

Subtraction Property for Equations

The subtraction property states that subtracting the same number from both sides of an equation gives an equivalent equation. In order to obtain x, we must eliminate the integer with x. This number is eliminated by adding it.

Here are two examples.

\[ x + 70 = 94 \]
\[ x + 70 - 70 = 94 - 70 \] (Subtraction Property)
\[ x = 24 \]

\[ x + 3 = 7 \]
\[ x + 3 - 3 = 7 - 3 \]
\[ x = 4 \]

Now you will be presented with 8 problems. Use the subtraction property to solve. If you answer a problem correctly you will proceed to the next one. If you answer incorrectly you will have another chance. Type in your response when the computer stops.

\[ x + 3 = 12 \]
\[ x = ? 9 \]

\[ x + 2 = -4 \]
\[ x = ? -6 \]

\[ x + 5 = -9 \]
\[ x = ? -14 \]

\[ x + 1 = 1 \]
\[ x = ? 0 \]

\[ x + 4 = -4 \]
\[ x = ? -8 \]

Careful! You better do the same problem over.

\[ x + 12 = 3 \]
\[ x = ? 15 \]

Sorry! \[ x = -9 \]

\[ x + 10 = -4 \]
\[ x = ? -9 \]

Careful! You better do the same problem over.
x + 10 = -4
x = -14

SORRY! x = -14

x + 13 = -9
x = -22

YOU SCORED 6 OUT OF 8 RIGHT ON YOUR FIRST TRY.
NICE JOB. JOAN.

READY
HI THERE! THIS IS HELPFUL HARRY AT YOUR SERVICE. TO WHOM AM I SPEAKING?

MULTIPLICATION PROPERTY FOR EQUATIONS

THE MULTIPLICATION PROPERTY STATES THAT MULTIPLYING EACH SIDE OF THE EQUATION BY THE SAME INTEGER GIVES AN EQUIVALENT EQUATION.

TO OBTAIN X, WE MUST ELIMINATE THE INTEGER WITH X. THIS NUMBER IS ELIMINATED BY DIVIDING BOTH SIDES OF THE EQUATION BY IT.

HERE ARE TWO EXAMPLES:

X/3 = 4
3*(X/3)*4 = 12 (MULTIPLICATION PROPERTY)

X = 12

X/6 = 42
6*(X/6) = 6*42 = 256 (MULTIPLICATION PROPERTY)

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. IF YOU ANSWER THEM CORRECTLY YOU WILL BE PRESENTED WITH A NEW PROBLEM.

IF YOU ANSWER INCORRECTLY YOU WILL GET ANOTHER CHANCE TO DO THE SAME PROBLEM. TYPE IN YOUR RESPONSE WHEN THE COMPUTER STOPS.

LET W = 0
FOR J = 1 TO 8
READ A, B
LET Q = 0: LET X = B*A
DATA 8, 48, 3, 90, 5, 105, 4, 96, 7, 56, 2, -12, -3, 98, -4, -10
LET "A" = B
INPUT "X"; F
IF F = X GOTO 220
GOSUB 300
GOTO 150
NEXT J
YOU SCORED "8-W" OUT OF 8 RIGHT ON YOUR FIRST TRY.
IF W > 2 THEN 500
NICE JOB. "NICE" .
GOTO 9999
LET Q = Q + 1
IF Q > 1 THEN 400
LET W = W + 1
RETURN
SORRY: X = "X: &
GOTO 230
ADDITIONAL PRACTICE.
505 REM THIS IS A SUBROUTINE USING THE MULTIPLICATION PROPERTY.
510 FOR K=1 TO 10
511 READ M,N
512 LET X=M*N
520 DATA 1,5,4,7,9,4,-3,10,4,-2,7,-4,-2,-2,7,0,9,-9,-3,-2
530 "$X/"M"="N
540 INPUT "$X=";F
550 $X="X";F
560 NEXT K
570 "$SO LONG."  
9999END

READY
RUN
EQMULT
HI THERE! THIS IS HELPFUL HARRY AT YOUR SERVICE. TO WHOM AM I SPEAKING?
? JOAN

MULTIPLICATION PROPERTY FOR EQUATIONS
THE MULTIPLICATION PROPERTY STATES THAT MULTIPLYING EACH SIDE OF THE EQUATION BY THE SAME INTEGER GIVES AN EQUIVALENT EQUATION. IN ORDER TO OBTAIN X, WE MUST ELIMINATE THE INTEGER WITH X. THIS NUMBER IS ELIMINATED BY DIVIDING BOTH SIDES OF THE EQUATION BY IT.

HERE ARE TWO EXAMPLES.

\[
\frac{x}{3} = 4 \quad \Rightarrow \quad 3 \times \frac{x}{3} = 3 \times 4 \quad \text{(MULTIPLICATION PROPERTY)} \]
\[x = 12\]

\[
\frac{x}{6} = 42 \quad \Rightarrow \quad 6 \times \frac{x}{6} = 6 \times 42 \quad \text{(MULTIPLICATION PROPERTY)} \]
\[x = 256\]

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. IF YOU ANSWER THEM CORRECTLY YOU WILL BE PRESENTED WITH A NEW PROBLEM. IF YOU ANSWER INCORRECTLY YOU WILL GET ANOTHER CHANCE TO DO THE SAME PROBLEM. TYPE IN YOUR RESPONSE WHEN THE COMPUTER STOPS.

\[
\frac{x}{8} = 48 \quad \Rightarrow \quad x = 384\]

TRY AGAIN.

\[
\frac{x}{8} = 48 \quad \Rightarrow \quad x = 384\]

GOOD WORK! HERE IS ANOTHER PROBLEM.

\[
\frac{x}{3} = 90 \quad \Rightarrow \quad x = 270\]

GOOD WORK! HERE IS ANOTHER PROBLEM.

\[
\frac{x}{5} = 105 \quad \Rightarrow \quad x = 525\]

GOOD WORK! HERE IS ANOTHER PROBLEM.

\[
\frac{x}{4} = 96 \quad \Rightarrow \quad x = 384\]

GOOD WORK! HERE IS ANOTHER PROBLEM.
\[
x / 7 = 56
\]
\[
x = ? 392
\]

GOOD WORK! HERE IS ANOTHER PROBLEM.

\[
x / 2 = -112
\]
\[
x = ? 1
\]

TRY AGAIN.

\[
x / 2 = -112
\]
\[
x = ? 1
\]

SORRY! \( x = -224 \)

\[
x /= 3 = 98
\]
\[
x = ? -294
\]

GOOD WORK! HERE IS ANOTHER PROBLEM.

\[
x / -4 = -10
\]
\[
x = ? 40
\]

GOOD WORK! HERE IS ANOTHER PROBLEM.

YOU SCORED 6 OUT OF 8 RIGHT ON YOUR FIRST TRY.

NICE JOB. JOAN.

READY
HELLO. HELPFUL HARRY IS READY AND WAITING. TYPE IN YOUR NAME.

INPUT Name:

DIVISION PROPERTY FOR EQUATIONS

THE DIVISION PROPERTY STATES THAT DIVIDING BOTH SIDES OF THE EQUATION

BY THE SAME INTEGER GIVES AN EQUIVALENT EQUATION IN ORDER TO

OBTAIN

BY X, WE MUST ELIMINATE THE INTEGER WITH X. THIS NUMBER IS ELIMINATED

BY MULTIPLYING BOTH SIDES OF THE EQUATION BY IT.

HERE ARE TWO EXAMPLES.

4X=24: 4X/4=24/4 (DIVISION PROPERTY)

X=6: 2X-12 : 2X/2=12/2 (DIVISION PROPERTY)

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. IF YOU ANSWER THEM

CORRECTLY YOU WILL BE PRESENTED WITH A NEW PROBLEM. IF YOU

ANSWER INCORRECTLY YOU WILL GET ANOTHER CHANCE TO DO THE SAME

PROBLEM.

TYPE IN YOUR RESPONSE WHEN THE COMPUTER STOPS.

LET W=0

FOR J=1 TO 8

READ A,B

LET Q=0: LET X=B/A

IF F=X GOTO 220

GO SUB 3

IF W>=2 THEN 500

NICE JOB "N"

GOTO 9999

LET Q=Q+1

IF Q>=1 THEN 400

LET W=W+1

RETURN

SORRY! X="X:

ADDITIONAL PRACTICE.

REM THIS IS A SUBROUTINE USING THE DIVISION PROPERTY.

FOR K=1 TO 10

READ M,N

LET X=N/M

DATA 2,12,3,9,-2,6,2,-6,7,14,-7,49,-6,36,-2,100,50,100,9,-81

LET "X="N
540 INPUT "X=":F
550 &"X=":X:
560 NEXT K
570 &"GOOD Bye"
990 DATA 3, 63, 5, 25, 10, -100, -7, 56, 9, -63, 4, 36, 2, -62, -3, -63
999 END

READY
DIVISION PROPERTY FOR EQUATIONS

The division property states that dividing both sides of the equation by the same integer gives an equivalent equation. In order to obtain \( x \), we must eliminate the integer with \( x \). This number is eliminated by multiplying both sides of the equation by it.

Here are two examples.

\[
4x = 24
\]
\[
\frac{4x}{4} = \frac{24}{4} \quad \text{(DIVISION PROPERTY)}
\]
\[
x = 6
\]

\[
2x = 12
\]
\[
\frac{2x}{2} = \frac{12}{2} \quad \text{(DIVISION PROPERTY)}
\]
\[
x = 6
\]

Now you will be presented with 8 problems. If you answer them correctly you will be presented with a new problem. If you answer incorrectly you will get another chance to do the same problem.

Type in your response when the computer stops.

1. \( 2x = 12 \)
   \( x = ? \) 6
   Yes. Try another.

2. \( 3x = 9 \)
   \( x = ? \) 3
   Yes. Try another.

3. \( -2x = 6 \)
   \( x = ? \) -3
   Yes. Try another.

4. \( 2x = -6 \)
   \( x = ? \) 3
   Yes. Try another.

5. \( 7x = 14 \)
   \( x = ? \) 2
   Yes. Try another.
-7 \(x = 49\)
\(x = ? \ -7\)

YES. TRY ANOTHER.

-6 \(x = 36\)
\(x = ? \ 9\)

CAREFUL! YOU BETTER DO THE PROBLEM AGAIN.

-6 \(x = 36\)
\(x = ? \ 9\)

SORRY! \(x = 6\)

-2 \(x = 100\)
\(x = ? \ 9\)

CAREFUL! YOU BETTER DO THE PROBLEM AGAIN.

-2 \(x = 100\)
\(x = ? \ -50\)

YES. TRY ANOTHER.

YOU SCORED 6 OUT OF 8 RIGHT ON YOUR FIRST TRY.

NICE JOB, JOAN

READY
EQONE

READY

LIST

EQONE

5&"HI, THIS IS HARRY. WHAT IS YOUR NAME?"
6INPUT N$

7&
10&"EQUIVALENT EQUATION: ONE":&
11&"LOOK AT THIS EQUATION:"&
12&"2X+3X+5X=20":&
13&"WE ALREADY KNOW THAT 2X+3X+5X CAN BE SIMPLIFIED TO 10X."
14&"THEREFORE, THE ABOVE EQUATION CAN BE WRITTEN AS":&
15&"10X=20"
16&"AND X=2"
17&
18&"HERE ARE TWO MORE EXAMPLES:"&
20&"3X+X=64":&"8X=64 (SIMPLIFIED)"&"X=8":&
21&"5X+2X+2X=36":&"9X=36 (SIMPLIFIED)"&"X=4":&
26&"NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. SIMPLIFY EACH"
91&"EQUATION AND SOLVE. IF YOU ANSWER A PROBLEM CORRECTLY YOU WILL"
92&"PROCEED TO THE NEXT ONE. IF YOU ANSWER INCORRECTLY YOU WILL HAVE"
93&"ANOTHER CHANCE. TYPE IN YOUR ANSWER WHEN THE COMPUTER STOPS:"&
133=0
134FOR J=1TO8
135READD,B,C,A
136DATA#0:LET X=D/(A+B+C)
137Q=O:LET X=D/(A+B+C)
138DATA28,3,4,7,81,4,3,2,48,2,4,110,72,2,5,2,24,5,5,2,36,2,2,5,6
139DATA1,1
160&"X=":B+"X=":C+"X="D
170INPUT "X=":F
175&
180IF X=F GOTO 230
190GOSUB 300
200&"CAREFUL! YOU BETTER DO THE PROBLEM OVER."
210GOTO 160
220&"YES. TRY ANOTHER."&
230NEXT J
240&"YOU SCORED "&4-M" OUT OF 8 RIGHT ON YOUR FIRST TRY."
250IF W>2 THEN 500
260&"NICE JOB. "&N$ ."
270GOTO 9999
280LET Q=Q+1
310IF Q>1 THEN 400
320LET W=W+1
330RETURN
400&"SORRY: X="&X
410GOTO 230
500&"ADDITIONAL PRACTICE."&
505REM THIS IS A SUBROUTINE USING EQUIVALENT EQUATIONS.
510 FOR K=1TO 10
511 READ P,N,0,M
512 LET X=P/(M+N+O)
520 DATA 14,2,5,7,10,2,2,6,12,3,3,6,18,4,6,8,20,3,6,10,22,2,7,2,30
521 DATA 6,5,19,60,9,1,5
530 &M"X="P
540 INPUT "X=";F
550 &:"X=";&
560 NEXT K
570 &"GOOD BYE FOR NOW."
9999 END

READY
RUN
EQTONE
HI, THIS IS HARRY. WHAT IS YOUR NAME?
? JOAN

EQUIVALENT EQUATION: ONE

LOOK AT THIS EQUATION:

2X + 3X + 5X = 20

WE ALREADY KNOW THAT 2X + 3X + 5X CAN BE SIMPLIFIED TO 10X.

THEREFORE, THE ABOVE EQUATION CAN BE WRITTEN AS

10X = 20
AND X = 2

HERE ARE TWO MORE EXAMPLES:

3X + 4X + X = 64
8X = 64 (SIMPLIFIED)
X = 8

5X + 2X + 2X = 36
9X = 36 (SIMPLIFIED)
X = 4

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. SIMPLIFY EACH EQUATION
AND SOLVE. IF YOU ANSWER A PROBLEM CORRECTLY YOU WILL PROCEED TO THE
NEXT ONE. IF YOU ANSWER INCORRECTLY YOU WILL HAVE ANOTHER CHANCE.
TYPE IN YOUR ANSWER WHEN THE COMPUTER STOPS.

7 X + 3 X + 4 X = 28
X = ? 2

2X + 4 X + 3 X = 81
X = ? 9

4 X + 6 X + 5 X = 45
X = ? 3

10 X + 2 X + 4 X = 48
X = ? 3

2 X + 2 X + 5 X = 72
X = ? 8

2X + 5 X + 5 X = 24
X = ? 2

5 X + 2 X + 2 X = 36
X = ? 5
CAREFUL! YOU BETTER DO THE PROBLEM OVER.

\[ 5x^2 + 2x + 2x = 36 \]
\[ x = 4 \]

\[ x + x + 1x = 6 \]
\[ x = 2 \]

YOU SCORED 7 OUT OF 8 RIGHT ON YOUR FIRST TRY.
NICE JOB. JOAN.

READY
HELLO. THIS IS HARRY. PLEASE IDENTIFY YOURSELF.

Look at this equation:

5x + 3x = 19 + 5

We know that 5x + 3x = 8x. We know that 19 + 5 = 24.

Therefore

8x = 24

x = 3

Here is another example.

6x + 4x = 15 + 5 (simplified)

x = 2

Now you will be presented with 8 problems. Simplify each equation and solve. If you answer a problem correctly you will proceed to the next one.

Type in your answer when the computer stops.

FOR J = 1 TO 8
READ A, B, C, D
LET X = (C + D) / (A + B)
DATA 2, 1, 3, 2, 7, 2, 1, 3, 4, 7, 8, 1, 8, 1, 2, 5, 30, 5, 4, 7, 31, 2, 5, 5, 4, 26
DATA 6, 1, 22, 6
LET X = (C + D) / (A + B)
DATA 2, 3, 20, 3, 7, 2, 2, 2, 42, 4, 3, 3, 36, 6, 2, 3, 14, 6, 4, 2, 30, 6, 3, 7, 25, 5, 5, 5, 31

DO
  IF X = F GOTO 230
  GOSUB 300
  BE CAREFUL. DO THE PROBLEM AGAIN.
  YES. TRY ANOTHER.
  YOU SCORED 8-W OUT OF 8 RIGHT ON YOUR FIRST TRY.
  NICE JOB. "N".
  YOU SCORED 8-W OUT OF 8 RIGHT ON YOUR FIRST TRY.
  NICE JOB. "N".
  ADDITIONAL PRACTICE.

FOR K = 1 TO 10
  READ M, N, O, P
  LET X = (O + P) / (M + N)
  DATA 2, 1, 3, 7, 2, 2, 2, 42, 4, 3, 3, 36, 6, 2, 3, 14, 6, 4, 2, 30, 6, 3, 7, 25, 5, 5, 5, 31

REM THIS IS A SUBROUTINE USING EQUIVALENT EQUATIONS, NUMBER TWO.
FOR K = 1 TO 10
  READ M, N, O, P
  LET X = (O + P) / (M + N)
  DATA 2, 1, 3, 7, 2, 2, 2, 42, 4, 3, 3, 36, 6, 2, 3, 14, 6, 4, 2, 30, 6, 3, 7, 25, 5, 5, 5, 31

RETURN
400 "SORRY! X = " X &
410 GOTO 230
500 "ADDITIONAL PRACTICE.";
505 REM THIS IS A SUBROUTINE USING EQUIVALENT EQUATIONS, NUMBER TWO.
521DATA1, 7, 10, 6, 4, 3, 7, 7
530&"X=", 'N'X=", '0'P
540INPUT "X="P
550&"X="X:
560 NEXT K
570&"GOOD BYE."
9999END

READY
RUN
EQ'IWO
HELLO, THIS IS HARRY. PLEASE IDENTIFY YOURSELF.
? JOAN

EQUIVALENT EQUATIONS: TWO

LOOK AT THIS EQUATION:

5X+3X=19+5

WE KNOW THAT 5X+3X=8X. WE KNOW THAT 19+5=24.

SIMPLIFYING 8X=24
THEREFORE X=3

HERE IS ANOTHER EXAMPLE.

6X+4X=15+5
10X=20 (SIMPLIFIED)
X=2

NOW YOU WILL BE PRESENTED WITH 8 PROBLEMS. SIMPLIFY EACH EQUATION AND
SOLVE. IF YOU ANSWER A PROBLEM CORRECTLY YOU WILL PROCEED TO THE NEXT
ONE. IF YOU ANSWER INCORRECTLY YOU WILL HAVE ANOTHER CHANCE. TYPE IN
YOUR ANSWER WHEN THE COMPUTER STOPS

2 X+ 3 X= 20 + 5
X=? 5

7 X+ 2 X= 70 + 2
X=? 8

3 X+ 4 X= 7 + 7
X=? 2

8X+ 1 X= 80 + 1
X=? 7

BE CAREFUL! DO THE PROBLEM AGAIN.

8 X+ 1 X= + 1
X=? 9

2 X+ 5 X= 30 + 5
X=? 5

4 X+ 7 X= 31 + 2
X=? 3

5 X+ 5 X= 4 + 26
X=? 3
\[6 \times x + 1 \times x = 22 + 6\]
\[x=? 4\]

You scored 7 out of 8 right on your first try.
Nice job. Joan.
Ready
LIST
REVIEW
5&"NAME PLEASE."
6INPUT N$
7&
16&"LETS REVIEW. ":&"SOLVE FOR X. ":&
50 FOR J=1TO4
51READ A,B
52 LET X=B-A
70 DATA 7,10,2,-4,2,0,7,-4
80&"X+"A=="B
90INPUT "X=";F
100&
110&"CHECK YOUR ANSWER. X="X:&
120 NEXT J
130 FOR K=1TO4
131 READ C,E
132 LET X=E/C
140 DATA 8,56,5,35,10,50,-7,49
160&"X=":E
170 INPUT "X=";F
180&
190&"CHECK YOUR ANSWER. X="X:&
200 NEXT K
210 FOR L=1TO4
220 READ M,N,O,P
230 LET X=P/(M+N+O)
240 DATA 2,3,4,18,3,5,2,50,6,2,3,44,10,4,1,60
250&"X=":O"X="P
260 INPUT "X=";F
280&
290&"CHECK YOUR ANSWER. X="X:&
300 NEXT L
310 FOR R=1TO4
320 READ Q,S,T,V
330 LET X=(T+V)/Q+S
340 DATA 1,2,3,30,2,2,40,4,4,3,1,6,1,7,3,5,25
350&"X="S"X="V+.T
360 INPUT "X=";F
380&
390&"CHECK YOUR ANSWER. X="X:&
400 NEXT R
410&"GOOD BYE ","N$ "."

READY
RUN
REVIEW
NAME PLEASE.
> JOAN

LETS REVIEW.

SOLVE FOR X.
X + 7 = 10
X = ? -3

CHECK YOUR ANSWER. X = 3
X + 3 = -4
X = ? -7

CHECK YOUR ANSWER. X = -7
X + 2 = 0
X = ? -2

CHECK YOUR ANSWER. X = -2
X + 7 = -4
X = ? -11

CHECK YOUR ANSWER. X = -11
8 X = 56
X = ? 7

CHECK YOUR ANSWER. X = 7
5 X = 35
X = ? 7

CHECK YOUR ANSWER. X = 7
10 X = 50
X = ? 5

CHECK YOUR ANSWER. X = 5
-7 X = 49
X = ? -7

CHECK YOUR ANSWER. X = -7
2 X + 3 X + 4 X = 18
X = ? 2

CHECK YOUR ANSWER. X = 2
3 X + 5 X + 2 X = 50
X = ? 5
CHECK YOUR ANSWER. X = 5

6 X + 2 X + 3 X = 44
X = ? 4
CHECK YOUR ANSWER. X = 4

10 X + 4 X + 1 X = 60
X = ? 4
CHECK YOUR ANSWER. X = 4

1 X + 2 X = 30 + 3
X = ? 4
CHECK YOUR ANSWER. X = 11

2 X + 2 X = 4 + 40
X = ? 11
CHECK YOUR ANSWER. X = 11

4 X + X = 6 + 1
X = ? 1
CHECK YOUR ANSWER. X = 1

1 X + 7 X = 5 + 3
X = ? 1
CHECK YOUR ANSWER. X = 1

GOOD BYE JOAN.

READY
THIS IS A TEST ON YOUR FIRST SIX PROGRAMS. SOLVE EACH PROBLEM FOR X.

40 LET W = 0
50 FOR J = 1 TO 3
51 READ A, B
52 LET X = B - A
70 DATA 7, 12, 3, -5, 2, 8
80 & "X + " & A & = " & B
90 INPUT "X" ; F
95 IF X = F THEN 100
96 LET W = W + 1
100 &
110 & "THE CORRECT ANSWER IS THAT X = " & X & : &
120 NEXT J
130 FOR K = 1 TO 3
131 READ D, E
132 LET X = E / D
140 DATA 8, 64, 5, 25, 10, 60
160 &D & "X = " & E
170 INPUT "X" ; F
175 IF X = F THEN 130
176 LET W = W + 1
180 &
190 & "THE CORRECT ANSWER IS THAT X = " & X & : &
200 NEXT K
210 FOR L = 1 TO 3
220 READ M, O, P
230 LET X = P / (M + N + O)
240 DATA 2, 4, 3, 27, 3, 2, 1, 36, 5, 1, 2, 48
250 &M & "X + " & N & "X + " & O & "X = " & P
260 INPUT "X" ; F
265 IF X = F THEN 210
270 LET W = W + 1
270 &
290 & "THE CORRECT ANSWER IS THAT X = " & X & : &
300 NEXT L
310 FOR R = 1 TO 3
320 READ Z, S, T, V
330 LET X = (T + V) / (Q + S)
340 DATA 1, 4, 20, 5, 2, 6, 2, 7, 3, 6, 14
350 &Q & "X + " & S & "X + " & V & "X = " & T
360 INPUT "X" ; F
365 IF X = F THEN 310
370 LET W = W + 1
380 &
THE CORRECT ANSWER IS THAT X = X:

YOU SCORED "12-W" OUT OF 12 RIGHT.

GOOD BYE NEW.

END

READY
RUN
TEST
NAME PLEASE.
  ? JOAN

THIS IS A TEST ON YOUR FIRST SIX PROGRAMS. SOLVE EACH PROBLEM FOR X.

X+ 7 = 12
X = 5

THE CORRECT ANSWER IS THAT X = 5

X+ 3 =-5
X = -8

THE CORRECT ANSWER IS THAT X = -8

X+ 2 = 8
X = 6

THE CORRECT ANSWER IS THAT X = 6

8 X=64
X = 6

THE CORRECT ANSWER IS THAT X = 8

5 X= 25
X = 5

THE CORRECT ANSWER IS THAT X = 5

10 X= 60
X = 6

THE CORRECT ANSWER IS THAT X = 6

2 X+ 4 X+ 3 X= 27
X = 3

THE CORRECT ANSWER IS THAT X = 3

3 X+ 2 X+ 1 X= 36
X = 6

THE CORRECT ANSWER IS THAT X = 6

5 X+ 1 X+ 2 X= 48
X = 6

THE CORRECT ANSWER IS THAT X = 6
1 \( x + 4 = 5 + 20 \)
\( x = ? \) 

THE CORRECT ANSWER IS THAT \( x = 5 \)

2 \( x + 2 = 2 + 6 \)
\( x = ? \) 

THE CORRECT ANSWER IS THAT \( x = 2 \)

7 \( x + 3 = 14 + 6 \)
\( x = ? \) 

THE CORRECT ANSWER IS THAT \( x = 2 \)

YOU SCORED 10 OUT OF 12 RIGHT.
GOOD BYE JOAN.
READY
Footnotes


2 Hahn & Bidna, op. cit., p. 251.


8 Creative Computing, P.O. Box 789-M, Morristown, N.J. 07960.