MEASURING LOCAL SPACETIME CURVATURE WITH GPS

AND GRACE

A thesis submitted in partial fulfillment of the requirements
For the degree of Master of Science
in Physics

By

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ABSTRACT

MEASURING LOCAL SPACETIME CURVATURE WITH GPS AND GRACE

By

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Master of Science in Physics

In this thesis the sensitivity of GPS and GRACE satellites to the local spacetime curvature was examined. Annual oscillations in a perturbation variable, $\beta$, in the Schwarzschild metric, as measured by GPS, used to express local position invariance (LPI) of fundamental constants was studied [15]. In this thesis these annual variations are modeled by extraterrestrial potentials. It was found that the solar tidal potential was the dominant effect and the influence of other planets was negligible. Looking at the contributions in the Fourier domain, it was seen that there were similarities in the periodicities of peaks at 180 and 365 days. The yearly peaks were found to coincide within 61.1% in amplitude. This is suggestive that the yearly and 180 day periods in $\beta$ are attributed to the solar tidal potential. Plotting the predicted $\beta$ versus the measured $\beta$ shows there is a linear correlation between the two with $R^2 = 0.8980$ and a slope of 0.7978. This shows evidence that GPS is sensitive to the local spacetime curvature. Since the solar tidal potential was left unfiltered from $\beta$, this thesis must reach the preliminary conclusion that GPS currently does not have sufficient accuracy for geopotential mapping of the earth’s surface.

To support ongoing efforts for the use of GPS for geodesy, geopotential mappings of the earth’s surface were made using GRACE spherical harmonic coefficients. These were created using harmonics up to degree and order 50 and 100. The mappings represent a baseline for which to compare future GPS mappings against. Weekly GRACE data was averaged and analyzed in the time and Fourier domains, as well as a function of longitude and latitude, resulting in peaks that agreed favorably with the solar potential in periodicity, but not in amplitude. On average these peaks were found to be roughly one order of magnitude smaller than the solar tidal potential for GRACE. It is assumed this is due to the handling of the solar tidal potential in GRACE, but further study is needed.
Chapter 1

Introduction

Geodesy, the study of the earth’s features and geopotential, is an exciting field that has seen significant improvement in metrology over the past decade. There are currently two main methods of making geodetic measurements: local geodesy and satellite geodesy, each of which has its advantages and disadvantages. Local geodesy involves the use of global positioning system (GPS) receivers or gravimeters and, therefore, the user must be present in the location of the measurement and must possess the necessary hardware. Satellite geodesy is an expensive and massive undertaking including techniques and experiments such as satellite laser ranging (SLR), the Gravity Recovery and Climate Experiment (GRACE) experiment, and the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) experiment [5] [7] [18].

This thesis proposes an alternative to expensive single purpose satellite geodesy experiments and local ground-based geodetic measurement techniques using GPS. This thesis proposes that oversampling GPS data provides a level of sensitivity such that the detection of the local spacetime curvature, and therefore the earth’s geopotential, is now possible so that GPS can not only be leveraged for navigation but also for geodesy. Using archival GPS data, the Jet Propulsion Laboratory’s (JPL) HORIZONS planetary ephemerides, and GRACE data, it is shown that GPS is able to monitor long term spacetime curvature effects of the same periodicity as the sun. The JPL ephemeris data is then used to curve fit and filter the GPS data and, in principle, the local spacetime curvature can then be extrapolated to determine causal gravitational anomalies on earth.

While detection of the local spacetime curvature is indeed possible using GPS, refinements still must be made to achieve precise geodetic mapping of the earth comparable to GRACE results. In the analysis, perturbations to ideal GPS atomic clock rates, as observed by Kentosh and Mohageg, were attributed to different sources where the solar tidal potential term was the dominant effect and terrestrial gravitational undulation terms were assumed to be of higher order [15]. This can be of particular use in the accuracy of GPS. Because of the speed of signal propagation, GPS receiver positioning is very sensitive to inaccuracies in the onboard atomic clocks. This can be improved by subtracting out the leading terms so that not only will GPS navigation accuracy improve but also the ability to leverage GPS for geodesy. Because of the amount of GPS data available over a nearly twenty year period, it will provide geologists and geophysicists access to much more data than compared to other satellite geodesy experiments. New GPS satellites are being launched regularly into the constellation with improved atomic clocks adding to the overall coverage and availability of the system, making it a prime candidate for use in geodesy.

According to Ashby, GPS satellites are indeed sensitive to the spacetime curvatures of the sun, but only due to the solar tidal potential [1]. Therefore, the solar tidal potential should be the most observable effect. When the observed perturbations in ideal clock behavior were compared with the theoretical tidal potential as given by JPL HORIZONS,
periodic effects with the same periodicity of the sun were found. While these effects agreed in frequency, they did not agree in amplitude. This suggests that, while the oscillations in clock frequency may not be conclusively attributed to the solar potential, the GPS clocks are sensitive to oscillations in the local spacetime curvature.

This thesis begins with a brief introduction into physical geodesy as it pertains to GPS and GRACE including comments about conventions and reference systems used by geodesists and astronomers. The link between physical geodesy and the GPS data will be made in a discussion about general relativistic effects and how they apply to GPS atomic clocks in motion about the sun and earth. This will include the background for a perturbation term, $\beta$, that is added to the Schwarzschild metric, which was studied by Kentosh and Mohageg to express the local positional invariance (LPI) of fundamental constants [15]. Here $\beta$ is instead attributed to gravitational sources in an attempt to explain annual variations in GPS clock corrections. The perturbation term will be analyzed in the time and Fourier domains and compared to known results. The intent of this is to show that GPS is sensitive to the local spacetime curvature and, therefore, able to detect local gravitational anomalies from earth, which is a key area of geodesy.
Chapter 2
Theory of Physical Geodesy

Geodesy, or the study of the figure and measurements of the earth, is a long standing field that began with the notion that the earth was a flat object at the center of the universe. As techniques and technologies improved, it was learned that this was indeed not the case and that the earth had a distinct shape and nuanced features. Now the focus is on studying physical geodesy, or the physical properties of the geopotential, as it pertains to the figure and dynamics of the earth.

2.1 Gravitation and the Potential of Point Masses

As stated by Newton, the force of attraction between two point masses can be expressed simply as [9] [12],

\[ \vec{F} = -\frac{Gm_1 m_2}{r^2} \hat{r}, \] (2.1)

where \( m_1 \) and \( m_2 \) are the masses of each object, \( r \) is the distance between the two, and \( G \) is Newton’s gravitational constant defined as \( G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \). The force is directed along the line connecting the center of mass of the two objects, hence the negative sign. Dividing through by \( m_1 \) results in the acceleration of the particle due to \( m_2 \) and is known as gravitation expressed as \( \vec{g} \) [9] [12]. Because this field is conservative and has no curl, this equation can be rewritten as the gradient of a potential [25]. The potential is the gravitational potential for point masses and is given by [25],

\[ \vec{g} = -\nabla \Phi, \quad \Phi = \frac{GM}{r} \] (2.2)

Here \( \Phi \) represents the potential energy per unit mass of a point particle in the gravitational potential of another point mass \( M \) a distance \( r \) away. It is of fundamental importance to introduce the gravitational potential because it reduces the force equation from a vector equation with three components down to a scalar equation. This is useful for the systems of particles or continuous solid bodies that are encountered in geodesy [12]. For any number of point particles this equation can be extrapolated out to a summation of the form,

\[ \Phi = \sum_i \frac{Gm_i}{r_i}, \] (2.3)

which is known as the principle of superposition [9] [12].

2.2 Gravitational Expansion and the Tidal Potential

Often times the second equation in (2.2) is rewritten out of convenience as,
\[ \Phi = -\frac{GM}{|\vec{r} + \vec{R}|}, \]  
(2.4)

where \( r \) has been decomposed as the magnitude of two different vectors \( \vec{r} \) and \( \vec{R} \). This is commonly done in astronomy where the radial distance is needed but is unknown or difficult to directly determine. Therefore the distance is constructed via a vector sum of two vectors whose components are well known. These are often referenced to different celestial bodies. In this thesis the potential of the sun at the GPS satellites is examined. In this case the sun to GPS distance will be calculated via the vector sum \( \vec{r} = \vec{R}_0 + \vec{r}_{GPS} \) where \( \vec{R}_0 \) is the vector from the center of the sun to the center of the earth and \( \vec{r}_{GPS} \) is the vector from the center of the earth to the GPS satellite. This potential can then be expanded as follows \[1\],

\[ \Phi = -\frac{GM}{|\vec{R}_0 + \vec{r}_{GPS}|} = -\frac{GM}{R_0} + \frac{GM(\vec{R}_0 \cdot \vec{r}_{GPS})}{R_0^3} - \frac{GM}{2R_0^5} \left( \frac{3(\vec{R}_0 \cdot \vec{r}_{GPS})^2 - R_0^2r_{GPS}^2}{2} \right) + \ldots \]  
(2.5)

The first term in (2.5) is the mean potential at \( R_0 \). The second term in front of \( \vec{r}_{GPS} \) is just the gravitational field strength at \( R_0 \). Finally, the last term is the tidal potential between the two bodies. It will be seen later that only the tidal potential is visible in GPS data and, therefore, is the term examined.

2.3 Gravitation of Continuous Bodies and the Expansion of Spherical Harmonics

The idealization of point masses is an important starting point to the problems in geodesy. Since the bodies encountered in geodesy are generally large solid masses it is important to extrapolate the ideas of the discrete masses to the continuous. In this case it is useful to recast the idea of mass in terms of a continuous distribution over a given volume. The result is a density of the form \[12\] \[25\],

\[ \rho = \frac{dm}{dv}, \]  
(2.6)

where \( dm \) is the differential mass and \( dv \) is the differential volume. The sum in (2.3) then becomes the integral \[9\] \[12\] \[25\],

\[ \Phi = \iiint_V \frac{dm}{r} = \iiint_V \frac{\rho}{r} dv, \]  
(2.7)

where \( r \) is the distance between masses. In principle this is all that is necessary to solve for the potential in geodetic problems, but it is very difficult because it requires direct knowledge of the mass density at every point within the volume and is therefore impractical. One solution to this problem is to consider the second derivative of the potential in (2.1) and (2.2). Revisiting (2.1) in terms of \( \vec{g} \) and rewriting \( \hat{r} = \frac{\vec{r}}{|\vec{r}|} \) and combining it with (2.6)
results in the integral [9] [12] [25],

\[ \vec{g}(\vec{r}) = -G \int_V \rho(\vec{r}') \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} d^3r', \]  

(2.8)

where \( \vec{r} \) points to the test point and \( \vec{r}' \) integrates over the entire mass distribution \( \rho \). Taking the divergence of both sides and noting that \( \nabla \cdot \left( \frac{\vec{s}}{|\vec{s}|^3} \right) = 4\pi \delta(\vec{s}) \) results in [9] [12] [25],

\[ \nabla \cdot \vec{g}(\vec{r}) = 4\pi G \rho(\vec{r}), \]  

(2.9)

which is known as Gauss’ law of gravitation. The process is mathematically identical to the derivation of Gauss’ law for electrostatics and, in fact, much of geodesy can be viewed as mathematically analogous to electromagnetism with mass serving as a charge analog and like charges attracting each other. Using (2.2), rewriting \( \vec{g} \) as the gradient of a potential, yields Poisson’s equation for gravitation. This is [9] [12] [25],

\[ \nabla^2 \Phi = 4\pi G \rho(\vec{r}), \]  

(2.10)

which is still dependent on the mass density, but reduces to Laplace’s equation outside of the distribution. In this case, the potential can be expressed by an expansion of orthogonal harmonic functions. Geodesists generally use an expansion of spherical harmonic functions. This means that the geopotential takes the usual form [9] [12],

\[ \Phi = \sum_{l,m}^\infty A_{l,m} \frac{Y_{l,m}(\theta, \phi)}{r^{l+1}}, \]  

(2.11)

where, in geodesy, the spherical harmonics \( Y_n \) are expressed as a sum of the Legendre polynomials \( P_{lm} \) [14] [25],

\[ Y_{lm}(\theta, \phi) = \sqrt{\frac{(l-m)!(2l+1)(2-\delta_{0,m})}{(l+m)!}} P_{lm}(\cos \theta) e^{im\phi}. \]  

(2.12)

The normalization chosen is to normalize power to unity over the entire sphere and is known as the Kaula standard for geodesy [14]. This is consistent with the standard in signal analysis as well. The benefit to this convention is that a nominal value of the geopotential can be assigned and any perturbations to the geopotential can be described by a summation of these spherical harmonics. This, as will be discussed later, is the standard adopted by researchers operating the GRACE experiment for their potential mappings [2] [24]. Upon combining (2.11) and (2.12) the conventional spherical harmonic expansion equation is found [14] [25],

\[ \Phi = \frac{Gm}{r} \left[ 1 + \sum_{l,m}^\infty \left( \frac{q}{r} \right)^l \left( C_{lm} \cos m\phi + S_{lm} \sin m\phi \right) P_{lm}(\cos \theta) \right]. \]  

(2.13)
This is the fundamental equation used in the GRACE experiment and the subsequent analysis for this thesis. The normalization conditions described in (2.12) are implicitly defined to be contained in the Legendre polynomials. These Legendre polynomials are then said to be the fully normalized Legendre polynomials. It is important to remember this expansion is an approximation that is only valid when the integrating surface contains no mass distribution. This approximation inevitably fails to describe the geopotential inside the earth because the solution for the potential is then the solution to Poisson’s equation rather than the solution to Laplace’s equation and therefore cannot be expanded in terms of spherical harmonics. This equation can still be used, though, to accurately approximate the earth's geopotential at the surface by allowing \( \vec{r} \) in (2.10) to extend just beyond the radius of the earth [25]. This is considerably easier to solve than (2.3) because no mass or density distribution is present and therefore needs to be known. The solution to the geodesy problem then becomes determining the coefficients \( C_{lm} \) and \( S_{lm} \) experimentally by fitting spherical harmonics. As a final check of the perturbative expansion, it is seen that the geopotential due to the earth converges to zero at infinity as is expected.

2.4 Earth Gravity Model 1996 and World Geodetic System 1984 Conventions

2.4.1 World Geodetic System 1984 Coordinates

The previous discussion describes the general practice of solving the geodesy problem in order to determine the geopotential, but does little to help define how the coordinates theta and phi are constructed on an oblate spheroid such as the earth. For this reason and others to be discussed shortly, the World Geodetic System 1984 (WGS84) was created. The WGS84 was last revised in 2004 and provides a standard coordinate system, reference ellipsoid, and reference geoid for all geodetic measurements and models to be compared against [13] [19]. This coordinate system is geocentric and has its meridian of zero longitude located along the Greenwich meridian [13] [19]. This means that the \( x \)-axis is pointing along 0° longitude and the \( y \)-axis is aligned with 90°E longitude with the \( x-y \) plane coincident with the earth’s equatorial plane. The \( z \)-axis is then orthogonal to that plane and pointing towards the geographical North Pole, completing a right handed coordinate system, as seen in Figure 2.1 [13] [19]. Because the coordinate axes are fixed with respect to positions on the earth, the coordinate system is rotating in space and is considered to be earth-centered earth fixed (ECEF) and is therefore non-inertial. The advantage of ECEF coordinates over earth center inertial (ECI) coordinates for local measurements is the fact the earth's equatorial plane is in constant motion due to the sun and moon, and therefore the ECI coordinates must be continually adjusted to compensate for this, making them in theory slightly non-inertial themselves [13]. Therefore, the ECEF coordinate system is the most convenient for GPS and GRACE measurements as well as GPS ephemerides. It will be seen later that ECEF coordinates must be abandoned when in the context of general relativity [11]. This is because velocities in general relativity are referenced to inertial coordinates, most commonly this is to a stationary observer at infinity [11].
2.4.2 World Geodetic System 1984 Reference Ellipsoid

In addition to the coordinate system, it is important to define a reference system from which local coordinates such as longitude, latitude, and surface height can be defined. The earth itself is not a perfect sphere, but rather and ellipsoid, which is reflected in WGS84. The reference ellipsoid defined by WGS84, with a cross-section along the $z$-axis normal to the equatorial plane, has semimajor axis $a = 6378.12$ km and semiminor axis $b = 6356.75$ km [13] [19]. The eccentricity of this cross-section is then given by [13],

$$e^2 = 1 - \left(\frac{b}{a}\right)^2,$$

which results in $e^2 = 0.006694$ [13]. This suggests that while the earth is mostly spherical, there is some minor flattening that proves to be important when considering local coordinates and the geopotential. These features, including $a$ and $b$, can be seen in Figure 2.1.

2.4.3 Earth Gravity Model 1996 and the Geoid

In addition to a set of coordinates and reference ellipsoid, WGS84 also provides a reference geoid for the earth. This reference geoid is an equipotential surface that represents what the shape of the earth would look like if it was all fluid experiencing only the earth’s gravitation and rotation [13]. It represents the mean sea level over the surface of the earth. While the geoid is a smooth surface it is also highly irregular. This means that the changes in geoid heights represent gradual undulations of the earths gravitational field [13]. The difference between the geoid and the reference ellipsoid can be seen in Figure 2.2. This
geoid model was eventually replaced by Earth Gravity Model 1996 (EGM96), which was complete up to degree and order 360 with corresponding resolution of 15 arc-minutes [13]. While EGM96 is still the reference geoid for WGS84, there are currently better models becoming available such as the GRACE Gravity Model 02 (GGM02) [23]. This model combines GRACE experiment data along with EGM96 data to create another set of spherical harmonics of degree and order 360.

Figure 2.2: The undulations of the geoid along with the actual figure of the earth and the reference ellipsoid [13].

2.4.4 Gravity Anomalies

If (2.13) is revisited, it is evident it provides the full potential in a spherical coordinate system to best fit a spherical shape. But as was seen in Figure 2.1, the reference surface and geoid take the general form of an ellipsoid. This means that certain terms in the spherical harmonic expansion will have artifacts of that ellipsoid. The terms that contain the artifacts are known as the even zonal harmonics and can be seen in Figure 2.3. While those terms are completely valid and are useful for determining the macroscopic form of the earth, it is often desirable to look at the undulation of the geoid. This allows researchers to look at features such as the time dependent changes in density indicative of phenomena like fluid flows for geology and meteorology [21] [22].

In order to obtain the gravity anomalies, or perturbing potential, it is first necessary to subtract out the reference geoid. This is done using the reference ellipsoid starting with the angular velocity, semimajor axis $a$, the semiminor axis $b$, and the eccentricity $e^2$ mentioned previously. From these we can derive the first dynamic form factor of the earth, $J_2$ [9] [25]. Other dynamic form factors, $J_{2n}$, are calculated from satellite geodesy [25]. The dynamic form factors are functions of the principal moments of inertia and are related to
the flattening of the ellipsoid [12] [25]. These values are measured to high accuracy and can be found in tables in the literature. The perturbing potential is then found by subtracting out the form factors $J_{2n}$ from the even zonal harmonics (the even cosine terms). This is expressed as [9],

$$
\Phi = \frac{GM}{r} \left[ 1 - \sum_n J_{2n} \left( \frac{a}{r} \right)^{2n} P_{2n} \cos \theta \right],
$$

(2.15)

where $J_{2n}$ is the $n$th dynamic form factor and $P_{2n}$ are the even zonal harmonics. In principle the first term with $\frac{GM}{r}$ can be expressed as the total potential summation of all the available spherical harmonics including the reference ellipsoid. In practice only the first few even zonal terms need be subtracted until the ellipsoidal reference geoid is sufficiently removed [9].
To then examine the gravity anomalies in greater detail, the mean earth field is subtracted out. This is calculated as $\Phi_0 = \frac{GM}{a}$ where $a$ is the average earth radius as defined by WGS84. The result is,

$$\Delta \Phi = \Phi - \Phi_0,$$

which are strictly just the deviations from the mean static gravitational field. This is more insightful than the field itself absent the reference ellipsoid because it is easier to see the areas that drive fluctuations from a uniform potential.
In the previous section it was seen how the problem of geodesy was originally written in terms of density functions, but was eventually recast in terms of the geopotential. This was important because knowing the density of the earth at all points is an unnecessarily difficult task. Instead, using known solutions to Laplace’s equation reduced the difficulty of the problem and allowed the geopotential to be expanded as a linear combination of harmonic functions. In this section it will be seen how potentials can be linked to the atomic clock rates onboard the GPS satellites through the principles of general relativity. This is at the core of the idea that GPS can be leveraged for physical geodesy and is the foundation of this thesis.

### 3.1 Proper Time and the Schwarzschild Metric

It was Albert Einstein in his paper on the geometry of the universe published in 1916 that marked the beginning of general relativity as it stands today [10]. The main ideas driving general relativity are the invariance of the speed of light in inertial frames and the treatment of spatial and temporal coordinates as space-time. The result is a continuum of different effects such as time dilation and length contraction that can be neatly expressed in a construct known as the metric tensor. These tensors are the solutions to the Einstein field equations of general relativity [11].

The tensor most applicable to geodesy is the Schwarzschild metric [11]. This metric describes the warping of space-time around a spherical body with no charge, no angular momentum, and a cosmological constant of zero. Because most astronomical objects rotate slowly, including the earth, the Schwarzschild metric is a useful approximation compared to the complex Kerr metric for rotating bodies [11]. The Schwarzschild line element in full is defined to be [11],

\[
ds^2 = c^2 \left( 1 - \frac{2Gm}{c^2 r} \right) dt^2 - \left( 1 - \frac{2Gm}{c^2 r} \right)^{-1} dr^2 - r^2 d\Omega^2,
\]

(3.1)

where \(ds\) is the infinitesimal line element between two points, \(dt\) is the coordinate time defined to be the time of an at rest observer at infinity, and \(d\Omega\) is the unit of solid angle. Using the expression \(c^2 dr^2 = ds^2\) for proper time, (3.1) can be rewritten in terms of the elapsed proper time between two spacetime events [11]. Given a path \(\alpha(t, \vec{r}(t))\) between events as a function of coordinate time and spatial coordinates, it is then possible to integrate this equation to solve for the proper time experienced by the local observer along that path. Dividing through by the differentials and assuming the path is parameterized by time, \(\alpha = \alpha(t, \vec{r}(t))\), (3.1) can be rewritten as [11],

\[
c^2 d\tau^2 = \left[ c^2 \left( 1 - \frac{2Gm}{c^2 r} \right) - \left( 1 - \frac{2Gm}{c^2 r} \right)^{-1} \left( \frac{dr}{dt} \right)^2 - r^2 \left( \frac{d\Omega}{dt} \right)^2 \right] dt^2.
\]

(3.2)
Assuming that the path $\alpha$ is the path of a GPS satellite around earth allows (3.2) to be simplified. Because the earth has relatively little mass, the $\frac{Gm}{c^2r}$ term in front of $(\frac{dr}{dt})^2$ is much less than unity, for $r \approx r_{GPS}$. Therefore, the weak field approximation can be invoked. In this approximation, the coefficient in front of $(\frac{dr}{dt})^2$ will vanish and the resulting equation has the form [11],

$$d\tau = \sqrt{1 - \frac{2\Phi}{c^2} - \frac{v^2}{c^2}} \, dt,$$

(3.3)

where $\frac{Gm}{r}$ was expressed as the potential $\Phi$ from (2.2) and $(\frac{dr}{dt})^2 - r^2 (\frac{d\Omega}{dt})^2$ was rewritten as the velocity along the path as seen by an interital observer at infinity. Lastly, we can use the Taylor expansion on (3.3) to arrive at [11],

$$d\tau = \left(1 - \frac{\Phi(\vec{r}, t)}{c^2} - \frac{v(\vec{r}, t)^2}{2c^2}\right) \, dt,$$

(3.4)

which gives the difference in clock rates between two clocks. In (3.4) it is the assumption that $dt$ measures the coordinate time and the clock is therefore stationary located at infinity where there is zero potential [11]. Therefore, to calculate the difference in clock rates between two clocks in motion in different potentials, say atomic clocks on board GPS and clocks at the center of mass of the earth, (3.4) would be invoked twice. The first calculation would find GPS clocks referenced to a stationary observer at infinity and the second calculation would find earth clocks referenced to the same observer at infinity. Upon substitution, the relationship between GPS clocks and earth clocks was found to be [11],

$$dt_{GPS} = \left(1 - \frac{\Phi_{GPS} - \Phi_{earth}}{c^2} - \frac{(v_{GPS}^2 - v_{earth}^2)}{2c^2}\right) \, dt_{earth}.$$

(3.5)

Here the velocities are still with respect to a distant inertial observer and are therefore absolute velocities. If the point of observation is boosted to a frame that is comoving with the center of mass of the earth, then the earth velocity term in equation (3.5) vanishes. It is also assumed that the GPS velocity term vanishes as well. The remaining potentials can be rewritten as the difference $\Delta\Phi$. Therefore, equation (3.5) becomes,

$$dt_{GPS} = \left(1 + \frac{\Delta\Phi}{c^2}\right) \, dt_{earth}.$$

(3.6)

The justification for the GPS velocity to vanish is inherent in the GPS and GPS clock correction data. In the GPS clock correction data GPS motional affects are removed so that term is effectively zero. Also, in geocentric coordinates the earth is given no velocity, so both terms in (3.5) will vanish.

It is important to note that this is an idealized equation in the sense that the potential of the earth is modeled simply as a spherically symmetric mass. As was seen in Chapter 2 (2.13), this is not the case. In this analysis, perturbations to the idealized model will be quantified by a dimensionless factor $\beta$. This is expressed as a perturbation to the ideal
general relativity equations (3.6) as [15],

\[ dt_{GPS} = \left( 1 + (1 + \beta) \frac{\Delta \Phi}{c^2} \right) dt_{\text{earth}}. \]  

(3.7)

Here it is seen how the clock rates of GPS atomic clocks is then related to the potential at that particular point in the orbit. If the perturbations \( \beta \) tend to zero the equation reduces back to the original Schwarzschild solution seen in (3.6). The perturbations \( \beta \) can in practice originate from an infinite number of sources, but it is the assumption that these perturbations originate from the gravitational tidal potential of the sun and the potentials of other planets in the solar system. Since each potential acts independently, the \( \beta \) term can be written as a summation of each of the perturbations,

\[ \beta = \beta_{\text{Sun}} + \beta_{\text{Jupiter}} + \ldots \] 

(3.8)

To get a better understanding of how the magnitudes of the perturbations compare with each other, simple order of magnitude estimates for a few celestial bodies are calculated. The unitless perturbations are calculated using \( \frac{\Delta \Phi}{c^2} \) where \( \Delta \Phi \) is the same potential as defined in (2.2). On average, the estimated perturbation of the sun is on the order \( 10^{-8} \), while for Jupiter it is on the order \( 10^{-12} \), and the estimated perturbation of the earth’s irregularities are \( 10^{-13} \). This confirms when averaging over many days of data, the sun and Jupiter are the strongest perturbing potentials acting on the GPS atomic clocks with the features of the earth one of the higher order terms. Leveraging GPS atomic clock correction data for geodesy then becomes an exercise in filtering lower order perturbation terms and uncovering the factor responsible for earth’s geopotential anomalies. This equation is then the central expression for the GPS analysis in this thesis.
In the preceding sections the relevant theoretical foundations were established for geodesy and general relativity. It will then be seen how these principles govern the GRACE and GPS data and how they are leveraged in geodesy.

In this section the Jet Propulsion Laboratory (JPL) HORIZONS database will be leveraged to build time dependent potentials for each planet in the solar system. It is believed that the potentials, mainly the solar tidal, have an effect on the GRACE and GPS satellites. Therefore, these potentials will be expressed in the Fourier domain and will be used as the model to compare and identify different contributions in the data.

4.1 Planetary Ephemerides

The JPL HORIZONS project seeks to build ephemerides for every sort of body within the solar system including planets, asteroids, comets, and moons [8]. This provides position vectors, velocity vectors, and various Keplerian elements such as right ascension and declination as functions of time for all the catalogued bodies. The HORIZONS database uses laser ranging data along with numerical solutions to the orbital equations of motion to construct position and velocity vectors that reflect recent states as well as give the system the ability to project orbits forward or backward in time up to several thousands of years [8].

The HORIZONS database gives several different options for building the ephemerides. The dataset that is most pertinent for this analysis are the geocentric position vectors in cartesian coordinates. For ease of calculation, these are chosen in an inertial reference frame that corresponds to the International Celestial Reference Frame (ICRF). This reference frame is usually centered at the barycenter of the solar system, but in this case is translated to the center of the earth so distances to the different planets can be readily calculated. The ICRF is as close to an inertial reference frame as possible in general relativity and has its axes referenced to a number of different radio sources such as quasars [8]. The two main orientations of the $x$ and $y$ axes in this reference frame have the axes aligned with either the ecliptic or equatorial plane [8]. For the equatorial orientation the $z$ axis is aligned with the mean north pole at the reference epoch J2000, which corresponds to the 12th hour on January 1, 2000. The $x$ axis then points along the ascending node of the ecliptic and equatorial plane, also known as the vernal equinox, at the reference epoch. The $y$ axis then completes the right handed coordinate system. In the ecliptic orientation the $z$ axis is tilted so that the $x$-$y$ plane aligns with the ecliptic plane. For the purposes of calculating the potentials of each solar system element at the earth, only the radial distances are needed as seen in (2.2). The choice of coordinates is therefore moot. For the orbitat epicycles in Figure 4.1 the ecliptic plane was chosen for aesthetic purposes. As will be seen later, GPS ephemerides, once converted to an inertial reference frame, use equatorial coordinates. At that point a change in ephemeris coordinates will be made.
Figure 4.1: Figure of the geocentric planetary ephemerides from JPL HORIZONS in astronomical units (AU). These show the planetary epicycles as described by Ptolemy. From these ephemerides the radial distance and subsequent potential can be found.

4.1.1 Deriving the Extraterrestrial Potentials

The data used for this analysis came from a 20 year period starting from November 14, 1993 and ending December 10, 2011. This, as will be seen later, coincides with the GPS data to allow for seemless comparison between the two. Using the standard gravitational parameters $GM$ from the planetary ephemeris tables, the $(x, y, z)$ position vectors were scanned into MATLAB and the potentials were calculated using (2.2). Each data point was taken a day apart from each other. This again was to preserve apples-to-apples comparisons with the GPS data and the calculated $\beta$'s that will be examined in the next section. The calculated potentials are seen in Figure 4.2.

Here it is seen that the sun is the dominant potential in the solar system and should therefore be the largest observed effect in the GRACE or GPS data. The average potential
is roughly four orders of magnitude larger than the next potential, which is due to Jupiter. Comparing these to the potential of the earth at earth’s surface from Figure 5.6, it is seen that the potential of the sun at the earth is larger than the earth’s potential itself. The potential due to Jupiter at the earth’s surface is then roughly two orders of magnitude smaller than the earth’s geopotential. While these potentials have independent sources, it is the superposition of all these potentials that acts on the earth. Therefore, the effects of each individual source cannot be discerned directly by GRACE or GPS data in the time domain.

![Potentials of the Planets at the Earth](image)

Figure 4.2: The potential of the different planets in the solar system at the earth using the HORIZONS ephemerides. It is important to note that the solar tidal potential here is at the GPS satellite.

To identify the contributions to GRACE and GPS, the HORIZONS potentials are transformed into the Fourier domain. The result of this analysis can be seen in Figure 4.3. Here it is seen that the sun is responsible for peaks at 366.7, 183.4, and a small contribution at 120 days. It is the amplitudes of these peaks that will be compared directly to the peaks in the GPS and GRACE data.

If the expansion of the potential is reexamined from (2.5),

$$\Phi = \frac{-GM}{|\vec{R}_0 + \vec{r}|} = \frac{-GM}{R_0} + \frac{GM(\vec{R}_0 \cdot \vec{r})}{R_0^3} - \frac{GM}{2R_0^5} + \frac{3(\vec{R}_0 \cdot \vec{r})^2 - R_0^2\vec{r}^2}{2R_0^5} + \ldots$$

(4.1)

where $\vec{R}_0$ is the earth-sun vector provided by HORIZONS and $\vec{r}$ is the vector from the center of the earth to the GRACE or GPS satellite, it is seen which contributions cancel.
The first two terms of the expansion are the mean field and the gravitational field strength. The final term in the expansion is the tidal potential. The first term affects all objects in the vicinity of earth the same and is therefore unobservable. The second term is equal and opposite to a Doppler shift with respect to the sun and is therefore also unobservable [1]. The remaining potentials are then the solar tidal potential and the second order terms for the rest of the solar system bodies. Therefore, all the curves in Figure 4.2 will be shifted down considerably since they scale with $R_0^2$. This means that the solar potential is the dominant effect in the solar system and its tidal potential will be the most observable on the GPS and GRACE satellites.

Figure 4.3: The potential of the different planets in the solar system at the earth in the Fourier domain. Here it is seen that the solar tidal potential is responsible for yearly oscillations as well as 180 day oscillations.
Chapter 5
Gravity Recovery and Climate Experiment

In the previous section JPL HORIZONS planetary ephemeris data was examined to construct a model to the variations that will be seen in the GRACE and GPS data. It was seen which terms in the gravitational expansion cancel and which potentials were most likely to be observed. It was found that the solar tidal potential is the most observable potential and will be the one compared to GPS and GRACE data.

In this section the Gravity Recovery and Climate Experiment (GRACE) is examined to build a geopotential model that will be used as a baseline to compare to future work on GPS mappings. Time dependent geopotential solutions are also examined and modeled with the solar tidal potential.

5.1 How Does GRACE Work

The GRACE experiment, launched March 17, 2002, is a low earth orbit satellite geodesy mission currently being operated jointly by the National Aeronautics and Space Administration (NASA) and the German Aerospace Center (DLR) to map the time-dependent and mean gravity field of the earth [23]. The two GRACE satellites are in near polar orbit at 89° from the equator, following each other at an average altitude of 500 km with 220 km of separation between them [18]. They are linked by a K-band microwave ranging signal that monitors the distance between them sensitive to displacements up to 10 µm [4] [18]. The ranging horn onboard the satellites transmit and receive K-band carrier signals of 24 GHz and 32 GHz. The satellite ranges are then calculated based on relative phase information between the two signals [18]. The change in distance between the two satellites then reflects the changes in geopotential of the earth due to changes in centripetal acceleration.

The GRACE satellites are also equipped with onboard accelerometers to monitor non-gravitational accelerations, GPS receivers for accurate positioning and timing, as well as star trackers to monitor the angle and orientation of the satellites [4] [26]. Laser reflectors outfitted on the bottom of the satellites also allow for laser range finding, enhancing the positional tracking capabilities of the system. These all work in conjunction to isolate the gravitational anomalies from other effects on phase.

The resulting gravity field solutions, or geopotential models, are then created using each phase measurement. Changes in the centripetal acceleration are mapped to regions of earth below the affected satellite. Regions with larger centripetal accelerations correspond to a larger gravitational attraction on the earth below. Then using (2.2), the changes in acceleration can be attributed to changes in mass on the earth. The gravity field data are then fit to spherical harmonics in a least-squares sense to produce each gravity model [20] [23].
5.2 GRACE Gravity Model 02

The first GRACE gravity model (GGM01) was based on approximately 100 days of early GRACE data [23]. This model proved to be a substantial improvement over pre-GRACE gravity models. Following the creation of GGM01, fourteen monthly earth gravity models were released using GGM01 as the base gravity model spanning April 2002 to December 2003. These gravity models were combined to create GGM02 which could show gravity field variability to resolutions as small as 600 km [23]. This gravity model has since been expanded to include additional GRACE data and combined with surface information from EGM96 to create GRACE products that extend to spherical harmonic coefficients of degree and order 360. This represents one of the best geopotential models to date [20].

5.3 GRACE Data Products

The GRACE data used in this thesis was made available by the Helmholtz Centre Potsdam German Research Centre for Geosciences (GFZ). They have made available a vast number of different products ranging from gravity field solutions to individual data sets from each onboard satellite transducer. These fall into distinct categories. Level 0 data is the raw machine data that comes directly from the different instruments onboard the GRACE satellites. This data is not available for public access. Level 1A data is also unavailable to the public and includes the raw data along with some processing. Level 1B data is the first set of data available for public use. This data consists of raw machine data that has been converted into a usable format for researchers with some post processing. The purpose of this data set is to provide users the necessary tools to extract their own time dependent gravity field solutions. It also contains data for GRACE orbit determination and mean gravity field mapping [3]. The concept is to provide advanced users the ability to derive their own results from raw data using their own standards and conventions.

The data used in this thesis falls under the category of level 2. These are derived solutions to the geopotential expressed as best fit spherical harmonic coefficients. These come in different flavors including terms from tidal potentials, other background sources, solutions augmented with terrestrial measurements, or simply solutions from only the GRACE satellites [2]. For this thesis three different products were analyzed. The products analyzed include weekly and monthly static field geopotential spherical harmonic coefficient products estimated from satellite data only, known as GX-OG-2-GSM, as well as the GRACE Gravity Model 02 (GGM02C and GGM02S) spherical harmonic coefficient product. The GGM02C model leverages terrestrial data from EGM96 as well as months of satellite data to achieve solutions of degree and order 360. GGM02S is the same gravity model using months of GRACE data sans the terrestrial data and therefore is a GRACE only model. The weekly geopotential products are available up to degree and order 30, while the monthly geopotential products are available up to degree and order 120 [2]. The GGM02C and GGM02S models were used as the baseline gravitational field mapping, while the weekly products were used to look at the average overall gravitational field magnitude as a function of time and to see if there were any artifacts of other worlds in the Fourier domain that would affect GPS clocks. The monthly products were not extensively used because there
was less data available and there were many missing months in between each datum.

5.4 Error Estimates in GRACE GGM02 Harmonics

While both GGM02C and GGM02S are available up to harmonic order and degree 360, it is not advised to use the complete set to model geopotential. This is because there is considerable uncertainty in the higher harmonics [23]. With each additional harmonic the model of the geopotential becomes more defined, but at the higher harmonics there is a tradeoff in perceived resolution and added uncertainty. Currently, GRACE GGM02C data comprises the majority of the harmonics up to degree 200 while higher harmonics are augmented considerably with EGM96 data [23]. The relative errors between GGM02, GGM01, and EGM96 are seen in Figure 5.1.

The errors for GGM02S, GGM01S and EGM96 are calculated in terms of the root-sum-square variances [17] [23]. It is seen that there is substantial improvement in GGM02 over GGM01 and EGM96. From the run off in GGM01 and GGM02 in Figure 5.1 it is seen that at higher spherical harmonic orders the majority of errors is strictly from GRACE [23]. This is why it is necessary to constrain the GRACE data with the terrestrial EGM96 data. From this figure, it is seen that using degrees above 100 for GGM02S is inadvisable. Likewise, using degrees 80 for GGM01S is not recommended. For these reasons the spherical harmonic data for the GGM02 models will be truncated at degree 100 in this thesis.

![Figure 5.1: The relative errors referenced to the geoid for different gravity models as a function of spherical harmonic degree [23]](image-url)
The difference between GGM02S and GGM02C errors are found at higher frequency [23]. While the GGM02S uncertainties are unbounded, GGM02C converges to EGM96 because it is augmented with terrestrial data. At the higher frequencies there is no discernible difference between GGM02C and EGM96 [23].

It is also interesting to associate the spherical harmonic degrees with wavelength of geopotential undulations. In this case, lower indices represent longer wavelengths and higher indices represent shorter wavelengths. Keeping this in mind, Figure 5.1 then becomes a plot of relative error versus undulation wavelength. It is seen that GRACE does a better job at approximating the low frequency effects on the geopotential, while local terrestrial measurements from EGM96 fair much better approximating the high frequency effects. This will be an important fact to keep in mind in future endeavors where GPS data is used for geopotential mapping as it is unreasonable to expect detection of the higher frequency harmonics.

5.5 Mean Static Field GRACE Analysis

5.5.1 Deriving the Accepted Geopotential

The GGM02C and GGM02S models were the first models to be analyzed. The data file included sine and cosine spherical harmonic coefficients up to degree and order 200 as defined in (2.13) as well as time derivative data. The time derivative data was not necessary in the calculations because this analysis was only concerned with the mean static field. This calculation gives the best estimate of the local geopotential anomalies on earth and is the baseline to reference GPS results against.

These spherical harmonic coefficients were read from file using a MATLAB script and then the WGS84 reference ellipsoid was removed in order to calculate the undulations in the geopotential. This was done by subtracting out the $J_{2n}$ factors from the $C_{2n,m}$ even zonal spherical harmonic coefficients seen in Figure 2.3. The reason only even terms are subtracted is because the form of the reference ellipsoid is that of an even function, which is symmetric about the equator. The even terms are then much larger in magnitude than the odd terms. These values are found in Table 5.1 and are available in the literature.

<table>
<thead>
<tr>
<th>Dynamic Form Factors of Earth</th>
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<tbody>
<tr>
<td>$J_2$</td>
</tr>
<tr>
<td>$J_4$</td>
</tr>
<tr>
<td>$J_6$</td>
</tr>
<tr>
<td>$J_8$</td>
</tr>
<tr>
<td>$J_{10}$</td>
</tr>
</tbody>
</table>

Table 5.1: Table of dynamic form factors describing the earth’s oblateness. [12] [25]
Modifications also had to be made to the MATLAB function that supplied the Legendre polynomials. According to the MATLAB literature, the Schmidt normalization routine needed to be modified in order to fit the Kaula standard defined in equation (2.12) for geodesy. This meant multiplying the Legendre polynomials by a factor of $\sqrt{2l + 1}$ to complete the normalization. The mean earth radius of $a = 6378.13$ km and the earth's standard gravitational parameter, $GM = 3.986 \times 10^{14}$ m$^3$ s$^{-1}$, were used in equation (2.13). Figure 5.2 then shows the resulting geopotential at the earth's surface up to degree 50 and 100 respectively.

From the second plot in Figures 5.2 and 5.3 it is apparent that when more terms from (2.13) are present, there is more definition in the geopotential due to the addition of high frequency terms. Because of this, features such as the Andes mountains in Chile and the Himalaya mountains in Nepal are more defined in the higher index mapping. The 100 degree harmonic expansion then represents the best trade off between added resolution and uncertainty in the higher harmonics.

According to (2.2) and (2.13) the gravitational potential is related to mass. This means that the plots in Figures 5.2 and 5.3 are not only geopotential mappings of the earth, but they are also directly related to the mass or density of the earth. Regarding the figures then as mass or density mappings of the earth provides additional information that is useful for geologists and meteorologists. In this way the GRACE data can be exploited by researchers to study fluid flows over various regions of the earth that are not otherwise visible normally.
Figure 5.2: Figure of the geopotential for GGM02C with index up to 50 and 100. This is used as the baseline for GPS analysis.
Figure 5.3: Figure of the geopotential for GGM02S with index up to 50 and 100. From a qualitative standpoint GGM02S and GGM02C are indistinguishable.
5.5.2 Reducing the Resolution to Compare with GPS

In Figures 5.2 and 5.3 both GGM02C and GGM02S are qualitatively indistinguishable. Strictly speaking, GGM02S would be the most appropriate geopotential model to compare to a GPS model because GPS data does not include any supplemented terrestrial data. To compare the differences between two such models and evaluate a residual, GGM02S should be used instead of GGM02C.

Because both the GGM02C and GGM02S models were developed from several years worth of GRACE data, the resolution of the models are substantial. In order to make comparisons between the GGM02S geopotential model and potential GPS models, the GRACE data must be degraded. As will be discussed in further detail later, the GPS satellites make two revolutions around earth each day broadcasting clock correction data once every 15 minutes. The result of this is that the earth will be divided up into a 24 x 24 grid where each GPS signal will be assigned a bin depending on where the satellite was at that particular moment when data was broadcast. In order to reduce the GGM02S model down to 24 x 24 resolution, each bin is averaged to produce a mean geopotential for that particular region of the earth. This is seen in Figures 5.4 and 5.5. The original GGM02C and GGM02S models were derived in a standard 720 x 360 pixel mapping. These pixels were then averaged in such a way as to produce the necessary 24 x 24 resolution.

Figure 5.5 represents the absolute best mapping the GPS constellation could produce. As mentioned in Section 5.4, GRACE is most sensitive to longer wavelength undulations in the geopotential. This suggests that GPS atomic clocks would be most sensitive to the longer wavelength changes in the potential as well. In addition, since the GPS satellites are higher in altitude it can be expected that the variations in geopotential will not be as pronounced as the GRACE data. In which case, it is more likely that the GPS mapping will take a muted form of Figure 5.4
Figure 5.4: Figure of GGM02S up to degree 50 with the 24 x 24 degraded model to correspond to GPS mapping.
Figure 5.5: Figure of GGM02S up to degree 100 with the 24 x 24 degraded model to correspond to GPS mapping.
5.6 Weekly GRACE Solutions

5.6.1 Fluctuations of the Mean Geopotential

Weekly GRACE gravity solutions are only provided by GFZ in the GSM format. This means that the gravity solutions are derived only using GRACE data. They are not augmented with terrestrial measurements like some of the mean field solutions. This is unimportant in this analysis though, because the interest is in the overall change in gravitational field strength as a function of time. The high frequency information provided by the terrestrial measurements are therefore negligible. This is to measure the effect and sensitivities of different celestial bodies on the GRACE and GPS results.

This data set is provided up to degree and order 30. The data used was from August 14, 2002 to December 22, 2010. While there is not much detail in the actual geopotential mapping itself, as is seen in Figure 5.6, it does not matter because the average value of the geopotential is mostly determined by the low frequency terms.

![GSM Weekly Data up to index 30 in MJ/kg](image.png)

Figure 5.6: One example of the weekly GRACE data provided by GFZ. The weekly geopotential mappings are qualitatively indistinguishable.

Taking the average of each of the 430 geopotential mappings and plotting it versus its Modified Julian Date (MJD) gives the overall time evolution of the geopotential over the entire earth. Doing such removes any fluctuations that would originate from earth dynamics like weather or oceanic current effects. Furthermore, the seven day data collection period also removes any extraneous high frequency noise. The highest frequency sources the data is sensitive to are those equal to the Nyquist frequency, in this case it corresponds to a 14 day period. There is also a limit on the longest wavelength this analysis can consider,
namely 3010 days. This leaves fluctuations in geopotential due to primarily extraterrestrial sources. These variations, as seen in Figure 5.7, are on the order $10^{-8}$. This is found by subtracting the nominal value of the geopotential via $\Phi - \frac{GM}{r_{\text{earth}}}$. This effectively removes the DC portion of the geopotential and makes the weekly fluctuations more visible in the plots.

While the changes in geopotential as a function of date are interesting to look at in the time domain, the curve itself does not reveal much about the factors causing the fluctuations. Much of the interesting physics can only be seen in the Fourier domain. Instead of plotting amplitude versus frequency, as is usually seen, the transform is plotted versus the period of oscillation. This is done because the driving forces, assumed to be other worlds in the solar system, have very small orbital frequencies and are usually expressed in term of orbital periods.

![Fluctuations in Potential versus Date](image)

**Figure 5.7:** Fluctuations of the geopotential with the nominal value of the potential removed.

Looking at the geopotential in the Fourier domain, seen in Figure 5.8, several prominent peaks are present. These are summarized in Table 5.2. Here the 334.4 day peak along with peaks at 188.1, 158.4, and 130.9 days are seen. Because this is a discrete Fourier transform, there is a finite amount of data points and therefore resolution in the frequency domain. This affects the way the peaks are found and displayed. Uncertainties in each peak were calculated by considering the spacing between adjacent data points and then averaging. Because there is a peak at 334.4 days does not necessarily mean there is a strong periodic effect at that particular frequency, it just means that was the nearest frequency data point because of the discreteness of the Fourier transform. The plot in the Fourier domain
complete with error bars is seen in Figure 5.9 and the uncertainties are also summarized in Table 5.2.

Figure 5.8: The geopotential plotted in the Fourier domain on a logarithmic $y$-axis and a linear $y$-axis. Here several peaks are seen, mostly at longer wavelengths. There appears to be a band of noise with a floor of $3 \times 10^{-10}$ up to a period of around 100 days. Periodic orbits greater than 100 days are more pronounced.
Figure 5.9: Here the plot of the variations in geopotential are displayed in the Fourier domain with the uncertainties in each peak. Due to the discreteness of the sampling, each peak has an associated error bar where the expected periodic effect could reside.

<table>
<thead>
<tr>
<th>Prominent Weekly Data Peaks and Uncertainties</th>
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<tbody>
<tr>
<td>Peak 1</td>
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<td>Peak 2</td>
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<tr>
<td>Peak 3</td>
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<td>Peak 4</td>
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<tr>
<td>Peak 5</td>
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<tr>
<td>Peak 6</td>
</tr>
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</table>

Table 5.2: Table of peaks found in the weekly GRACE data.

Because of the discreteness of the data points, it can be assumed that peaks 3, 4, and 5 constitute one broader peak with a periodicity of anywhere between 140 to 180 days. The same could be said about peak 6, which would encompass some of the surrounding points in Figure 5.8. This means there is some uncertainty in the periodicity of the peaks which must be kept in mind when comparing these peaks to the Fourier transforms of the solar
tidal potential. It is seen that there is good correspondance between the periodicities found in the solar tidal potential and the variations in the geopotential, within the errorbars.

5.6.2 Fluctuations as a Function of Latitude and Longitude

Before the connection between the peaks in the weekly spectral profile and extraterrestrial potentials is made, it is advantageous to look at the fluctuations in geopotential in another way. Instead of averaging the geopotential over the entire earth, the fluctuations in the potential were examined at specific latitudes and longitudes. This method has its advantages and drawbacks. Because no averaging is performed, there is no removing fluctuations due to oceanic currents and other atmospheric effects. While the geopotential as a function of time is more sensitive to terrestrial noise sources it is also more sensitive to extraterrestrial potentials as well.

To perform this analysis 100 different points were taken equally spaced from each other forming a 10 x 10 mask over the weekly data. The geopotential at each point was examined as a function of time and the six points with the largest fluctuations were recorded. Again the mean earth potential was removed. These are the data points found in Figures 5.10 and 5.11. Looking at the collection of test points in Figure 5.10 shows no obvious correlation. The points are not grouped together nor do they lie along some common line or boundary. Analyzing Figure 5.11 shows a slightly different story. The location of the points in this plot appear to lie along a boundary between areas of higher potential and areas of lower potential. This is most likely the cause of the larger fluctuations in geopotential and hence these are the areas with the highest sensitivity to the physical factors that drive the changes.

Figure 5.10: These are the locations of the test points on the GRACE gravity model where the weekly fluctuations in geopotential were evaluated.
The evolution of these geopotentials is found in Figures 5.12. Looking at these in a time series, it can be seen that the magnitude of the fluctuations is of the order $10^{-7}$. In most of these plots there is also an obvious sinusoidal pattern that manifests itself in the Fourier domain. In the spectral plots there is a prevalent peak at 376.3 days and vague traces of another possible peak at around 180 days. This is the same periodicity that was found when evaluating the average potential of the weekly models and corresponds to the solar tidal potential. The fluctuations in the geopotential appear to be stronger in the six test points given that the 376.3 day peak is an order of magnitude larger than the averaged data. This suggests that these test points are much more sensitive to external perturbative potentials. Because the periodicity of the peaks have good agreement and the test points are situated all across the earth with no definitive pattern, it suggests that the variations of the geopotential are caused by the solar tidal potential.

While these variations are stronger than the averaged weekly data, they are still much smaller than the solar tidal potential described in Figure 4.3. The yearly peak of the solar tidal potential is of the order $10^{-7}$ while the geopotential is of the order $10^{-6}$. This is because it is the GPS solar tidal potential that is being calculated, which is much larger because it has a larger orbital radius. In order to reconcile these peaks, the GRACE solar tidal potential is being considered. The GRACE data is in terms of the local geopotential as a function of latitude and longitude. To then investigate the solar tidal potential effects on GRACE we consider the solar tidal potential at those points. Over a week of measurements the GRACE satellites sample a specific latitude and longitude many times. The position of those measurements maps out a ring at the specific latitude as the earth rotates. The
average vector in that time span then lies directly on the \( z \)-axis pointing towards either the north or south pole. It is that vector that will be used to calculate the average solar tidal potential over each data point. This is seen in Figure 5.13.
Figure 5.12: The six locations with the highest variations in the geopotential along with their spectral components.
Figure 5.13: The GRACE geopotential variations at longitude and latitude (320,-10) and (80,30) respectively compared to the solar tidal potential. Here it is seen that the solar tidal potential is slightly larger in amplitude, but there is agreement in the periodicity of the peaks.
<table>
<thead>
<tr>
<th>Prominent Weekly Data Peaks</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak 1</td>
<td>46.31 Days</td>
</tr>
<tr>
<td>Peak 2</td>
<td>71.67 Days</td>
</tr>
<tr>
<td>Peak 3</td>
<td>130.9 Days</td>
</tr>
<tr>
<td>Peak 4</td>
<td>158.4 Days</td>
</tr>
<tr>
<td>Peak 5</td>
<td>188.1 Days</td>
</tr>
<tr>
<td>Peak 6</td>
<td>334.4 Days</td>
</tr>
</tbody>
</table>

Table 5.3: Table of peaks found in the weekly GRACE data.
Chapter 6

The Global Positioning System

In the previous section GRACE data was analyzed to determine the mean static geopotential of the earth as well as the spectral components of the time dependent geopotential over roughly eight years. The static geopotential provided by GGM02 is the best known model of the earth’s potential and will be used as a baseline for future GPS comparisons. The average time dependent geopotential as well as the position dependent geopotential in the Fourier domain was compared to the solar tidal potential and it was seen there was good agreement in the periodicity of the peaks.

In this section the HORIZONS results will be applied in much the same way to Global Positioning System (GPS) clock correction data to analyze the dimensionless parameter $\beta$ that was examined by Kentosh and Mohageg in (6.6). Using this information the origins of $\beta$ will be explained by curve fitting the solar potential and examining the yearly peak.

6.1 How Does GPS Work

The GPS is a world-wide satellite navigation system operated by the United States government. The GPS was originally created for military use but is now widely used in consumer electronics such as cellular phones and car navigation systems.

Currently the GPS is operated from 12 control stations that monitor the 32 satellites as they orbit around the earth. Each satellite is in a near circular orbit in one of six orbital planes that are inclined 55° with the respect to the equator. These control stations manage the satellites maintaining a constant electronic link to send and receive relevant signals. They are able to adjust the satellite orbits as well as make changes to the onboard time and therefore keep highly accurate ephemerides of each satellite. Each satellite sends down a navigation signal that includes information regarding its position and time of its clock. In principle, the distance from the satellite to the receiver is given by the expression $d = c/\Delta t$, where $\Delta t$ is time it takes the signal to travel. This results in the navigation equation for $x$, $y$, and $z$. Four signals are then used to account for the three position variables and time. Therefore, GPS receivers need to have at least four satellites within their line of sight to calculate their position.

6.2 GPS Time and the Onboard Atomic Clocks

Because the signals travel at the speed of light, errors in $\Delta t$ result in very large errors in the calculated position. As a result time keeping is very important to the operation of accurate GPS. The GPS operates on what is known as GPS time. This is an average of many different extremely precise clocks located all over earth. The different clocks all have slightly different rates due to longitude, height above sea level, and the oblateness of the earth. All these effects are taken into account when the averaging is performed and the result is considered the official GPS time used for navigation.
There are also highly accurate clocks onboard the GPS satellites. As seen in (3.5) these clocks will experience differing degrees of time dilation when compared to GPS time. In principle, as the satellite moves throughout its slightly elliptical orbit its clock rate will change continuously according to (3.5). This means that the clock rate is a function of position and time. Because of these effects corrections to GPS time must be applied to each satellite in order to make comparisons between GPS time on earth and GPS time at the satellite. These corrections are uploaded to each satellite as part of the navigation signal and the correction is subtracted from the satellite’s time to relate to GPS time.

6.3 GPS Data

The GPS data analyzed in this thesis comes from the aforementioned clock corrections. The International GNSS Service (IGS) collects and analyzes data from all of its control stations then publishes refined daily data sets for all 32 satellites. These are called the IGS “final” product and are made available in SP3-c format. The files contain precise satellite ephemerides for each satellite as well as clock corrections published every 15 minutes of GPS time. For this thesis data starting from November 14, 1993 and ending December 10, 2011 was examined.

6.4 GPS Analysis

6.4.1 The Perturbative Term $\beta$

The variable $\beta$, as discussed in (6.6), manifests as a perturbation in the ideal Schwarzschild metric. As discussed by Kentosh and Mohageg, this term is introduced to express violations of general relativity routinely related to local positional invariance (LPI) [15]. In their work possible LPI violations were explored in GPS clocks. Variations in the fundamental physical constants, namely Planck’s constant $h$, were examined. These variations in $h$ would result in changes to the proper energy of atomic transitions and, therefore, would amount to changes in oscillator frequency in the GPS atomic clocks [15]. General relativity says nothing about the variations of fundamental constants, therefore, these clock changes do not appear in the original Schwarzschild metric and are solely attributed to $\beta$.

In the analysis of Kentosh and Mohageg, the IGS “final” product in SP3-c format was used. Seven of the 32 satellites were selected due to the stability of their clocks and the eccentricities of their orbits [15]. To find $\beta$, changes in the clock corrections over each 15 minute interval were examined. This was done over a one year period from April 2010 through May 2011. Changes in clock corrections as a function of radial distance for each day were examined and fit to a linear trendline. These slopes were then compared to the theoretical slopes due to ideal GPS orbits with 2% eccentricity. The difference between the two curves at either aphelion or perihelion was considered $\Delta B$, which could be rewritten as,

$$\Delta B = m(r_a - r_{\text{GPS}})$$

(6.1)

where $m$ was the slope of the trendline, $r_a$ was the aphelion radius, and $r_{\text{GPS}}$ was the...
idealized GPS radius. This was related to the maximum difference between clock rates, $\Delta T_{\text{max}}$, which is found at apogee, and $\beta$. As a result, each daily datum was collapsed into one $\beta$ and then examined over an entire year. Some of these $\beta$’s had an unexpected annual oscillation, which is far too high of a frequency to attribute to LPI violations.

In the work of Kentosh and Mohageg, these annual oscillations were tentatively attributed to the orientation of continental land masses. In this analysis external potentials such as the solar tidal potential not initially considered in the calculations of Kentosh and Mohageg will be used to try to explain and remove these annual oscillations.

Expanding on their work, data from all 32 satellites over the period November 14, 1993 to December 10, 2011 was examined. A MATLAB script was written by Kentosh and Mohageg to extract and calculate all the raw $\beta$’s. A sample of some of the raw $\beta$’s are seen in Figure 6.1. This raw data contains a number of issues that must be addressed. The first of which are the large spikes due to outlying data points. It is assumed that these data points were not due to any relevent physical factors under consideration. To handle the outliers, a filter was applied over the data that removed any points that were larger than three standard deviations away from the mean value. This can be seen in Figure 6.2. With the outliers removed the annual fluctuations can now be seen in a majority of satellites. A common feature in most of the $\beta$’s is a block of noisy data that exists for the first few years of operation.

One interesting feature with the block of high frequency noise is the fact that it ends in all satellites at the same date. The modified Julian Date corresponds to midnight of May 1, 2000. Prior to this date, the US military added random errors to the GPS signal to degrade the accuracy for civilians. The purpose of this was to deny enemies the use of these publically available signals for precision weapon guidance. Once the military developed another method for denying enemies access to GPS signals this Selective Availability (SA) was turned off, therefore, the period of data prior is useless for the purposes of this thesis.
Raw Beta Data for Satellite 8

Modified Julian Date

Beta (Unitless)

4.9 5 5.1 5.2 5.3 5.4 5.5 5.6

x 10

4

−15

−10

−5

0

5

Raw Beta Data for Satellite 10

Modified Julian Date

Beta (Unitless)

4.9 5 5.1 5.2 5.3 5.4 5.5 5.6

x 10

4

−20

−15

−10

−5

0

5

10

15

x 10^4
Raw Beta Data for Satellite 15

Modified Julian Date

Beta (Unitless)

4.9 5 5.1 5.2 5.3 5.4 5.5 5.6

Raw Beta Data for Satellite 18

Modified Julian Date

Beta (Unitless)

4.9 5 5.1 5.2 5.3 5.4 5.5 5.6

x 10^4
Figure 6.1: Some examples of the raw $\beta$ data from satellites 8, 10, 15, 18, 27, and 32. Part of the annual oscillations can be seen along with outliers and missing data points.
Beta Data for Satellite 16 Outliers Removed

Beta Data for Satellite 24 Outliers Removed
Figure 6.2: Here is some of the $\beta$ data with the outliers removed via a filter. Some of the fluctuations can be seen as well as a section of noisy data.

After the SA portion of data was removed from the $\beta$'s another outlier filter was applied. The resulting data contains only $\beta$ values as derived by Kentosh and Mohageg as well as days where no values were recorded. These are stored in MATLAB as not-a-number or NaN. To fill some of the voids in the data, interpolation via least-squares-curve fitting was
applied. This was done using the backslash operator available in MATLAB to solve a system of equations for the missing data points. The fitting curve had the functional form,

\[ y(t) = y_0 + y_1 \cos \left( \frac{2\pi t}{T} + \phi \right) e^{-\gamma t} \]  

(6.2)

where \( y_0 \) and \( y_1 \) are the fitted parameters by MATLAB. The variables \( \phi \) and \( \gamma \) are chosen on a case by case basis for each satellite with the period \( T \) given to be 1 year. The best fit curves are seen in Figure 6.3 while the \( \beta \)'s with interpolated data are seen in Figure 6.4. It is seen in some satellites there are two different regions. This is due to either IGS changing the settings on the onboard atomic clocks or adjusting the orbital plane of the satellite. To account for these discontinuities, multiple fits were made for each section of the data. This can be seen in a number of the satellites in Figure 6.4.

![Beta Data and Best Fit Curve for Satellite 6](image-url)
Figure 6.3: Select $\beta$ data along with the least-squares best fitting curve. In some satellites there are distinct regions separated by a discontinuity. A different fitting function was applied for each section of data.
Figure 6.4: Select interpolated \( \beta \) data compared with the \( \beta \) data with missing values. Here it is seen that the interpolations fit well with the existing data.

While there are some obvious periodic variations in \( \beta \) it is impossible to directly determine all of the spectral components from the time domain data. In order to determine what is driving the fluctuations in \( \beta \) the data must be examined in the Fourier domain. This way comparisons can be made between the different potentials and the changes in \( \beta \). These are
seen in Figure 6.5. In nearly all of the 32 satellites there was a strong peak at either 351.8 or 383.8 days. Due to the discrete nature of the Fourier transform and the uncertainties discussed earlier, it is concluded that both these peaks correspond to a strong yearly oscillation in the $\beta$'s. There were also strong peaks at or around 180, 120, and 70 days. The results are summarized in Table 6.1.
Figure 6.5: Some examples of the $\beta$ data from satellites 3, 9, 13, 20, 24, and 26 in the Fourier domain. Here the annual oscillations are seen as well as peaks with periods of 70, 120, and 180 days.
<table>
<thead>
<tr>
<th>Prominent $\beta$ Data Peaks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak 1</td>
<td>70 Days</td>
</tr>
<tr>
<td>Peak 2</td>
<td>120 Days</td>
</tr>
<tr>
<td>Peak 3</td>
<td>180 Days</td>
</tr>
<tr>
<td>Peak 4</td>
<td>365 Days</td>
</tr>
</tbody>
</table>

Table 6.1: Table of peaks found in $\beta$ from the GPS data.

### 6.4.2 Fitting the Solar Potential

In the calculation of $\beta$ from Kentosh and Mohageg, the clock corrections from actual GPS orbits were compared to what is expected from ideal GPS orbits [15]. In these calculations only the earth’s potential was considered in (3.6). Therefore, $\Delta \Phi$ only includes the earth rather than the full potential from every object in the universe as is expected in the Schwarzschild metric. $\Delta \Phi$ can be written as,

$$\Delta \Phi = \Delta \Phi_{\text{earth}} + \Delta \Phi_{\text{other}}$$

where $\Delta \Phi_{\text{other}}$ represents the potentials due to other worlds. Using this expression in (3.6) results in,

$$dt_{\text{GPS}} = \left(1 + \frac{\Delta \Phi_{\text{earth}} + \Delta \Phi_{\text{other}}}{c^2}\right) dt_{\text{earth}}$$

(6.4)

Rearranging terms and factoring out $\Delta \Phi_{\text{earth}}$, the above equation can be recast in a more helpful way. The above equation is equivalent to,

$$\frac{\Delta t}{t} = \frac{dt_{\text{GPS}} - dt_{\text{earth}}}{dt_{\text{earth}}} = 1 + \left(1 + \frac{\Delta \Phi_{\text{other}}}{\Delta \Phi_{\text{earth}}}\right) \frac{\Delta \Phi_{\text{earth}}}{c^2}$$

(6.5)

which shares similarities with (6.6). Matching terms and noting that in Kentosh and Mohageg’s calculations only the earth potential was considered ($\Delta \Phi = \Delta \Phi_{\text{earth}}$), then $\beta$ becomes,

$$\beta = \frac{\Delta \Phi_{\text{other}}}{\Delta \Phi_{\text{earth}}}$$

(6.6)

This in principle is the bridge that relates the “perturbing” potentials to the fluctuations in $\beta$. As was seen in Figure 4.2, the sun is the dominant potential in the solar system. To a first order approximation, $\Delta \Phi_{\text{other}}$ is then just the solar tidal potential. It is then left to determine exactly what $\Delta \Phi_{\text{other}}$ and $\Delta \Phi_{\text{earth}}$ are and how the quantity on the right hand side compares to each $\beta$. 

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Revisiting (4.1) sheds some light on how to handle $\Delta \Phi_{\text{solar}}$ for GPS satellites. Looking at the third term in the expansion, which is the term describing the solar tidal potential experienced by the GPS satellite, $\vec{r}$ is the distance from the center of the earth to the GPS satellite and $\vec{R}_0$ is the vector from the center of the sun to the center of the earth.

The vector $\vec{r}$ is provided in the IGS data files and is the GPS ephemeris. The vector $\vec{R}_0$ is provided by the JPL HORIZONS ephemerides. As mentioned previously it is most convenient for applications of geodesy to specify the GPS ephemerides in earth-centered earth fixed coordinates (ECEF). This means that the GPS ephemerides are specified in a non-inertial reference frame while the sun ephemeris is specified in an inertial reference frame. To reconcile the two different reference frames, the GPS ephemerides need to be converted from ECEF coordinates to earth-centered inertial coordinates (ECI). In order to do this the instantaneous velocities and accelerations of the GPS satellites need to be calculated. This was done by interpolating two consecutive position vectors as given by IGS and dividing by a 15 minute interval. This was done likewise with the new velocity vectors to find the accelerations. The transformation to ECI coordinates was then done following the procedures outlined in the references [6]. The transformation leaves the GPS ephemerides in ECI coordinates with the $z$ axis pointing toward the mean north pole, the $x$ axis pointing along the mean vernal equinox, and the $y$ axis completing the right handed coordinate system. This is the same equatorial reference frame that was described earlier for the JPL HORIZONS ephemerides. As a check to the proper conversion of GPS ephemerides from ECEF to ECI, all 32 satellite trajectories were plotted for one day around the earth. This is seen in Figure 6.6. If the conversion to ECI coordinates was not made properly and the ephemerides were still in an ECEF coordinate system the GPS orbits would be seen smeared around the earth as the coordinates would still be rotating even though the image has them assumed fixed.
Figure 6.6: Here are the GPS orbits for all 32 satellites in an ECI reference frame. The first image is in three dimensions while the second image is viewed from the north pole. In the second image the orbital planes can be seen more clearly.

Once both reference frames for the GPS satellites and the sun are aligned, $\Delta \Phi_{solar}$ can then be calculated. This was done using the third term in (4.1). For this calculation the
solar ephemeris was recalculated from JPL HORIZONS using 15 minute intervals in the equitorial coordinate system. Care must be taken when assessing the tidal potential because the ephemeris data were given in 15 minute intervals but only one $\beta$ was extracted per day. This means that the potential must be evaluated in such a way that only one data point is extracted per 24 hours.

Taking a daily average of the solar tidal potential gives a null result. The other options considered in this thesis are using the daily maximum value of the solar tidal potential and calculating the solar tidal potential using the daily apogee of satellite orbit. Using the daily maximum value of the solar tidal potential means maximizing the third term in (4.1). In order to do so requires maximizing the dot product between the earth-sun vector and the earth-GPS vector. This happens when both vectors are most closely aligned with each other. In principle, this occurs at different points in the GPS satellite’s orbit depending on the time of year and the orientation of the sun. Because of this, the potential will always be calculated with the GPS satellite on the far side of the earth furthest away from the sun. Therefore, this configuration is not the most appropriate to use in the calculation.

Calculating the solar tidal potential using the daily apogee of the GPS satellite ensures that each data point is taken at the same time in the same spot in the orbit. In this case the angle between the earth-sun and earth-GPS vectors will be constantly changing. In the calculation of $\beta$, the parameter $\Delta T_{\text{max}}$ was found by calculating the difference in clock rates when the GPS clock was at apogee [15]. Therefore, this configuration is the appropriate configuration to use in the calculation to compare to $\beta$. Sample calculations of the solar tidal potential for different satellites along with the angle between the two vectors is seen in Figure 6.7. Because $\beta$ was derived from the difference in clock rates between ideal orbits and actual GPS orbits, the mean solar tidal potential was taken as the ideal and was therefore removed from the actual solar tidal potential. This caused the solar tidal potential to oscillate about zero, which is consistent with the behavior of the $\beta$’s.
Figure 6.7: Here is the solar tidal potential for various satellites along with the angle between the earth-sun vector and the earth-satellite vector. It is seen that the magnitude of the solar tidal potential is $10^{-5}$, which is considerably smaller than $\beta$.

Here it is seen that the magnitude of the solar tidal potential is very small when compared to the sample $\beta$'s seen previously. Looking at (6.6), it is seen that in order for the solar tidal potential to be of the same magnitude as $\beta$, $\Delta \Phi_{earth}$ must be of the order $10^{-5}$. 
Looking at the difference between the potential of the earth at the earth’s surface and the nominal value of the earth’s potential at an ideal GPS satellite as well as the difference in potential between an ideal satellite and a satellite at apogee reveals that these potentials are simply too large to be used to scale the solar tidal potentials to fit to $\beta$. The difference in potential between a satellite at the ideal GPS orbit of 26,600 km and one at apogee at 26,660 km is still of the order $10^{-2}$, which is three orders of magnitude too large to scale the solar tidal potential to the $\beta$’s. In order to make the quantity $\Delta \Phi_{solar}$ unitless and of the same magnitude as $\beta$, $\Delta \Phi_{solar}$ is divided by its average value to form the quantity $\frac{\Delta \Phi_{solar}}{\Phi_{solar}}$. These quantities are seen along with $\beta$ in Figure 6.8.
Figure 6.8: Dividing through by the average solar tidal potential of each satellite yields a unitless quantity that is the same order of magnitude as $\beta$.

It is seen that while the solar tidal potential has been converted into a unitless quantity that is the same order of magnitude as $\beta$, there is little agreement between the two curves. In order to reconcile these discrepancies the solar tidal potential term in (4.1) is revisited. Rewriting the dot product in terms of the angle between the two vectors yields,

$$\Phi = -GMr^2 \left(3\cos^2 \theta - 1 \right) \frac{1}{2R_0^3}$$

(6.7)

where $\theta$ is the angle between the earth-sun vector and the earth-GPS vector. As was seen in Figure 6.7 the angle between the two vectors is an oscillating function of time. Because $\cos^2 \theta$ is a non-linear function, adjusting the phase offset in $\theta$ will change the amplitude and phase of oscillation in the solar tidal potential. By adjusting this phase the solar tidal potential can be fit to the $\beta$’s. Again this is analyzed in the Fourier domain to see what periodic contributions the solar tidal potential has on $\beta$. In addition, the percent difference between the yearly peaks was calculated to see the correlations between $\beta$ and the solar tidal potential. The $\beta$’s and their solar tidal potential fits are seen in Figure 6.9.
Solar Tidal Potential and Beta for Satellite 1

Modified Julian Date

Potential (Unitless)

\(5.15\ \times 10^{-4}\)  \(5.2\)  \(5.25\)  \(5.3\)  \(5.35\)  \(5.4\)  \(5.45\)  \(5.5\)  \(5.55\)  \(5.6\)

\(\times 10^{-1}\)  \(\times 10^{-0.5}\)  \(\times 10^{0}\)  \(\times 10^{0.5}\)  \(\times 10^{1}\)

FFT Solar Tidal Potential and Beta for Satellite 1

Period in days

FFT (unitless)

5.15 5.2 5.25 5.3 5.35 5.4 5.45 5.5 5.55 5.6

\(\times 10^{-4}\)  \(\times 10^{-0.4}\)  \(\times 10^{-0.2}\)  \(\times 10^{0}\)  \(\times 10^{0.2}\)  \(\times 10^{0.4}\)

Solar Tidal Potential and Beta for Satellite 3

Modified Julian Date

Potential (Unitless)

\(5.15\ \times 10^{-4}\)  \(5.2\)  \(5.25\)  \(5.3\)  \(5.35\)  \(5.4\)  \(5.45\)  \(5.5\)  \(5.55\)  \(5.6\)

\(\times 10^{-10}\)  \(\times 10^{-5}\)  \(\times 10^0\)  \(\times 10^{5}\)  \(\times 10^{10}\)

FFT Solar Tidal Potential and Beta for Satellite 3

Period in days

FFT (unitless)

5.15 5.2 5.25 5.3 5.35 5.4 5.45 5.5 5.55 5.6

\(\times 10^{-10}\)  \(\times 10^{-5}\)  \(\times 10^0\)  \(\times 10^{5}\)  \(\times 10^{10}\)
Figure 6.9: Here is the fitted solar tidal potential for various satellites along with the associated $\beta$. It is seen that the magnitude of the solar tidal potential is similar to $\beta$ in the time domain, as well as the yearly peaks in the Fourier domain.

Here it is seen that there is some agreement in the time domain between the solar tidal potential and $\beta$. By adjusting the phase of the $\cos^2 \theta$ term in the solar tidal potential, a stronger 180 day periodicity was introduced as seen in the Fourier domain plots.
Fourier domain the yearly peak is seen in all plots and the 180 day peak is seen in most satellites in $\beta$ as well as in the solar tidal potential. While there is good agreement in amplitude of the yearly peak on most satellites, there is less agreement between the 180 day peaks. Because the GPS satellite orbits are so small compared to the earth-sun distance, the overwhelming effect on the solar tidal potential would come from the yearly variations in the earth-sun distance, with the changes in satellite position (and therefore changes in $\theta$) being of higher order. This is why there is better agreement in the yearly peak. Calculating the percentage differences in the amplitudes of the yearly peaks yields an average distance of 61.1%. While this does not completely confirm the variations in $\beta$ are driven by the solar tidal potential, it is strong evidence that there is a correlation between the two. The summary of the findings are found in Table 6.2.

<table>
<thead>
<tr>
<th>Prominent $\beta$ Data Peaks</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak 1</td>
<td>70 Days</td>
</tr>
<tr>
<td>Peak 2</td>
<td>120 Days</td>
</tr>
<tr>
<td>Peak 3</td>
<td>180 Days</td>
</tr>
<tr>
<td>Peak 4</td>
<td>365 Days</td>
</tr>
<tr>
<td></td>
<td>Solar Tidal</td>
</tr>
</tbody>
</table>

Table 6.2: Table of peaks found in the solar tidal potential data.

Another interesting way the data was examined is seen in Figure 6.10. In this graph the predicted $\beta$'s from the solar tidal potential are plotted against the measured $\beta$'s. According to (6.6) these two quantities should form a line with a slope of unity. As is seen in Figure 6.10, there is a linear relationship between the two different $\beta$'s, with a coefficient of determination of 0.8980, but the slope is 0.7978 rather than unity. This deviation from unity represents some level of noise that is present with a period of one year. It is unclear what exactly would cause the change in slope from what is expected, but this pink noise is another area of future research.
Figure 6.10: These are the locations of the test points on the GRACE gravity model where the weekly fluctuations in geopotential were evaluated.
Chapter 7

Future Work and Conclusions

7.1 GRACE Analysis

It was originally intended for this thesis to use the GRACE data to detect extraterrestrial potentials and explore the use of GRACE as a telescope for the solar system. This included examining the geopotential in the Fourier domain and comparing periodicities with known celestial bodies. While the initial sensitivity of GRACE exhibited evidence of the solar tidal potential, it is still possible in principle to filter the GRACE data to reveal other planets. More research is needed on the precise handling of the solar potential by GFZ to produce the GRACE gravity products. At this point there is only a preliminary understanding of how the different GRACE components come together to produce a gravity map. In order to fully understand how each potential acts on GRACE the higher levels of GRACE data need to be examined, such as the component data available in 1B.

There is also more research to be done on the sensitivity of various positions on the earth to external potentials. As was seen in the analysis, there is a definite dependency on the amplitude of time dependent geopotential variations with position on the earth’s surface. There is more interesting research to be had on the role these positions and fluctuations play on ground based clocks.

7.2 GPS and Solar Potential Analysis

Another initial goal of this thesis was to provide geopotential maps of the earth using only GPS clock correction data. This included accurately predicting $\beta$ using extraterrestrial potentials to remove lower order effects from the clock correction data. This mapping would be compared to the accepted GRACE plots. As was seen in the analysis of the solar tidal potential, work is still needed to conclusively identify the causes in the variations on $\beta$. One of the areas that needs more investigation is the equation relating $\beta$ to the external potentials. It was seen in Figure 6.10 that there was a relationship between the solar tidal potential and $\beta$, but it was unclear what was causing the slope to deviate from unity. More research needs to be done on the various sources of noise in GPS.

7.3 Conclusions

The sensitivity of GRACE and GPS satellites to local spacetime curvatures was examined. It was shown that GRACE satellites are sensitive to the local earth geopotential, and by evaluating the changes in the geopotential mappings in the Fourier domain, a correlation was found with the solar tidal potential as calculated from JPL HORIZONS ephemerides. Peaks were found in the Fourier expansion of the GRACE data that corresponded in periodicity and within two orders of magnitude with the HORIZONS results. It was seen that in the expansion of the potential that only the tidal potential would be observable on satellites that are in orbit around the earth. The GRACE data was found to be in agreement with this assertion. GRACE geopotential mappings were also degraded to support the ongoing
efforts of leveraging GPS clock correction data for geodesy.

The variations in GPS clock correction data, expressed through a parameter $\beta$ in the Schwarzschild equations, were also examined. Periodic yearly fluctuations in the LPI parameter $\beta$ were found by Kentosh and Mohageg [15]. These fluctuations were deemed to not be due to LPI violations and therefore were attributed to external potentials in the solar system. To first order these fluctuations were attributed to the solar tidal potential. Fitting of the solar tidal potential to $\beta$ yielded promising results in the yearly peaks in the Fourier domain, with a percentage difference of 61.1%. As a result of the fitting, the 180 day peak was overly pronounced in the Fourier domain, but nonetheless it existed in both the $\beta$ and solar tidal potential curves. The plot of the predicted and measured $\beta$'s showed there is a correlation between the two with a coefficient of determination of 0.8980. The deviation of the slope from unity is evidence that there exists observable pink noise with periods of one year.
Bibliography


