Control of Fully-Wetted and Artificially Supercavitating
Underwater Vehicles using Transverse-Mounted Synthetic Jets

A thesis submitted in partial fulfillment of the requirements
For the degree of Master of Science in Mechanical Engineering

By
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Dedication

The support of my friends and family were pivotal to my efforts with this project, but more than anyone I would like to recognize my mother. In conducting this research, I have grown to deeply appreciate just how strongly her efforts as a parent spurred my thirst for learning from an early age. My memories of her were a constant inspiration in the course of this work, as they will be for years to come.
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### Nomenclature

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<tr>
<td>$A$</td>
<td>area of jet port</td>
</tr>
<tr>
<td>$A_p$</td>
<td>area of piston</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>cavitation number</td>
</tr>
<tr>
<td>$V_0$</td>
<td>characteristic velocity of synthetic jet</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of water</td>
</tr>
<tr>
<td>$D_p$</td>
<td>diameter of synthetic jet piston</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of synthetic jet port</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency of synthetic jet actuation</td>
</tr>
<tr>
<td>$Fr$</td>
<td>Froude number</td>
</tr>
<tr>
<td>$L_x$</td>
<td>length of $M_x$ moment arm</td>
</tr>
<tr>
<td>$L_z$</td>
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</tr>
<tr>
<td>$L_c$</td>
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</tr>
<tr>
<td>$L_s$</td>
<td>length of synthetic jet slug</td>
</tr>
<tr>
<td>$m_p$</td>
<td>mass of synthetic jet piston and solenoid plunger</td>
</tr>
<tr>
<td>$\delta_s$</td>
<td>precompression of return spring</td>
</tr>
<tr>
<td>$M_x$</td>
<td>roll moment about load cell center</td>
</tr>
<tr>
<td>$T^*$</td>
<td>scaled thrust</td>
</tr>
<tr>
<td>$k_s$</td>
<td>spring constant of return spring</td>
</tr>
<tr>
<td>SDOM</td>
<td>standard deviation of mean</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number</td>
</tr>
<tr>
<td>$t^*$</td>
<td>time of synthetic jet outstroke</td>
</tr>
<tr>
<td>$\bar{T}$</td>
<td>thrust averaged over time</td>
</tr>
<tr>
<td>$F_y$</td>
<td>transverse force</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>tunnel speed</td>
</tr>
<tr>
<td>$R$</td>
<td>velocity ratio</td>
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<tr>
<td>$\bar{Q}$</td>
<td>ventilation number</td>
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<tr>
<td>$M_z$</td>
<td>yaw moment about load cell center</td>
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Abstract

By
Bradley Ayers

Master of Science in Mechanical Engineering

A set of water tunnel experiments were conducted to investigate the effects of crossflow and supercavitation on the thrust of a synthetic jet. This research was motivated by the desire to generate significant turning moments on a fully-submerged, supercavitating vehicle without the use of control fins or canards. The water tunnel model was a sting-mounted, 3-inch diameter cylindrical body interfaced to a 6-axis waterproof load cell. The synthetic jet actuator was contained within the model, with the jet orifice located near the aft end and oriented perpendicular to the mean flow. The actuator consisted of an externally controlled solenoid driving a piston within a cylindrical cavity. The jet thrust was measured over a range of synthetic jet operating parameters including the actuation frequency and the jet-to-crossflow velocity ratio. For ventilated cavitation experiments, ventilation rates were also varied.

When experimental measurements of time-averaged transverse forces and moments were compared with theoretical predictions, it was found that jet force decreased in a near linear manner with increasing crossflow. A flow behavior was hypothesized which credits the decrease in thrust with increasing tunnel speeds to secondary vortex structures which created low pressure fields across the hull surface. Differences in jet thrust and behavior between fully-wetted cases and ventilated partial supercavitation cases were also analyzed discovering decreased thrust under particular ventilation cases.
Section 1: Introduction

Maximum speed is an important consideration in the design of underwater vehicles. Due to the third-order relationship between velocity and power, the development of underwater vehicles which can exceed moderate speeds (~40 m/s) had been a long-standing challenge within naval engineering [1]. In recent decades, it had been proposed and demonstrated that employing the use of supercavitation is a viable means of drastically reducing total drag on underwater vehicles, thus allowing for considerably higher top speeds [2]. Significant research had been carried out relating to supercavitation, and there are presently numerous real world cases of supercavitating vehicles which have established the practicality of the concept. The Russian VA-111 Shkval, for example, reaches speeds over 100 m/s using natural supercavitation. However, there are still challenges associated with effectively and reliably controlling these vehicles. The main goal of this project is to explore the viability of transverse jets as a maneuvering actuator in place of conventional fins and canards.

Background

According to Bernoulli’s principle, a fluid undergoing acceleration experiences a drop in pressure. When the geometry of a body in a liquid causes the passing fluid to increase its velocity sufficiently, this pressure drop can bring the fluid below its vapor pressure and cause a localized phase change. This phenomenon is called cavitation. While cavitation had been extensively studied, it had been largely considered detrimental; most research had been related to the negative consequences of cavitation and dedicated to its prevention in machinery. Supercavitation is a concept which had been studied extensively only for the last few decades. This term refers to the use of cavitation to virtually eliminate skin friction and greatly reduce total drag of high speed underwater vehicles. In supercavitation, a sharp-edged bluff body called a cavitator, or cavitation plate, is placed at the front of the vehicle. The acceleration of water around the edges of the cavitation plate is sufficient to produce a trailing vapor cavity which encompasses the remainder
of the vehicle, resulting in a minimal wetted area. Below is an example of supercavitation in a laboratory experiment.

![Supercavitating Vehicle in Water Tunnel](image)

**Figure 1:** An example of a supercavitating vehicle in water tunnel. [3]

Early works by Logvinovich, Epshtein, and others dealt with theory relating to the shape and behavior of these cavities as a function of the cavitation number and Froude number [4]. More recently, considerable research had been done to expand upon the understanding of supercavitation and compile empirical data relating to forces and moments associated with cavitation plates at a non-zero angle of attack. Today there is a sizable body of information relating to cavity shape, distortion, trailing end behavior, and the planing forces associated with slender bodies in contact with the cavity wall. In addition, work has been done using ventilated gas to make supercavitation practical at lower speeds; this is called artificial or ventilated supercavitation. Artificial cavitation deals with the additional variable of gas feed rate, as well as the leakage of gas at the trailing end of the cavity. It also is significantly more concerned with the effects of buoyancy, as it is employed at lower speeds typical of laboratory water tunnels. The supercavitation discussed in this work is artificial/ventilated in order to be practical in the laboratory where the experiments took place.

Most research relating to the implementation of controls in supercavitating underwater vehicles had relied on conventional fins at the aft section of the vehicle for maneuvering. Work done with this configuration suggests that fins are an acceptable means of control, capable of achieving steady travel in the face of perturbations; however, there are a number of drawbacks associated with their implementation which make the development of control algorithms difficult.
The resultant force on the fins is a complicated matter. At shallow pitch, this relationship is linear, but the fin interacts with the boundary of the cavity, collapsing the cavity on the high pressure surface and often propagating cavitation along the low-pressure surface, especially near the leading edge and behind the base of the fin. At greater angle of attack, the entire fin may be enveloped by cavitation, thus rendering it less effective and its resultant forces less predictable. The result is a non-linear relationship between force and pitch under more extreme conditions shown in Figure 2 [1,2]. These problems can be compounded in real-world applications, where ocean conditions are not reliably quiescent. Due to these complications a search for a more elegant means of maneuvering is warranted.

![Figure 2: Resultant force from fins as a function of angle of attack. [2]](image)

It was supposed that if the forces required to generate a turning moment for a cavitating underwater vehicle could be achieved using transverse continuous jets, there would likely be several benefits. Unlike fins, conventional continuous jets would not rely on interaction with the freestream, so there would be no relationship between the vehicle’s velocity and the generated reaction force. Instead, the force would be a function of mass flow rate and velocity of the jet. Additionally, while fins extend through the cavity and pierce the cavity boundary in order to be effective, transverse jets do not. This means that the effectiveness of jets would not depend on
knowledge of cavity thickness, nor would fluctuations of cavity shape prove detrimental to the accuracy of the control system. This is an especially valuable assurance; while lift force at the front of a real world supercavitating underwater vehicle may be supplied by a pitched cavitation plate, the lift force on the aft section (required for stable, level flight) is provided principally by planing forces [2, 5, 6]. This means that the jets, especially those responsible for diving, would need to be effective when the surface of the vehicle is locally wetted. The effectiveness of jets should not be significantly affected by this on the assumption that their force is principally a function of momentum flux.

It was suggested by the Office of Naval Research that the use of synthetic jets (as opposed to continuous jets) be investigated as actuators in laboratory-scale experiments using lower-speed ventilated supercavitation. A synthetic jet is a zero-net-mass-flux actuator that periodically draws fluid into a chamber and then quickly expels it. The ingestion phase draws fluid from all directions through the jet orifice while expulsion generates a discrete vortex ring which is convected normal to the jet port. Repeating this cycle thus uses zero-net mass flux to generate non-zero time-averaged momentum [7, 8]. This process is illustrated below in Figure 3, where image A shows initial ingestion, B shows expulsion of the fluid, and C and D show subsequent ingestion and expulsion respectively [7]. These devices have been successfully used to control small test vehicles in laboratory research [7, 9, 10].
Figure 3: Conceptual depiction of synthetic jet operation [7]

Synthetic jets in research have been driven in a number of different ways. One method uses piezoelectric elements. These offer simple, compact designs which can operate at high frequency; however, displacement is minimal which limits thrust. Piezoelectric-driven synthetic jets are primarily used to impact flow separation in problems involving air, and are thus not suitable for this application [10]. Another method of actuating synthetic jets involves electromechanically driving pistons or bellows. This had been done in previous works by motors and cams as well as electromagnetic hardware to produce tangible thrust where the fluid medium is water [9, 12]. It was decided that a synthetic jet driven by a high-impulse solenoid would be most suitable for this application. Its implementation into design is discussed in further detail in later sections.

Objective

The purpose of this project was to investigate the viability of transverse-mounted synthetic jets as an alternative means of controlling both fully-wetted and artificially, partially supercavitating underwater vehicles. The primary benefit of the use of transverse-mounted synthetic jets was that it potentially permitted significant reduction of wetted surface area by eliminating fins, thus minimizing drag. While it was hypothesized that this could allow simplification of flight controls, it was understood that the effects of crossflow had the potential to
negatively impact the synthetic jets performance. With this in mind, understanding the impact of crossflow is important, as it may compromise the independence of thrust as a function of vehicle velocity (one of the primary benefits of transverse jets). Data was collected relating reaction force of the jets with a variety of synthetic jet parameters, ventilation conditions, and tunnel speeds. Discrepancies between load cell data and theoretical resultant forces warranted investigation into the development of the flow structure, specifically vortex rings at the synthetic jet orifice throughout the test matrix. High-speed video and Particle Imaging Velocimetry (PIV) techniques were used to investigate synthetic jet flow fields in depth for the fully-wetted (no artificial supercavitation present) cases.

This research also aims to show what impact, if any, the proximity of partial supercavitation may have on the effectiveness of a synthetic jet in crossflow. Conclusions drawn from this research may have real world applications for the use of synthetic jets in crossflows, the use of synthetic jets in proximity to free-boundaries, and the impact of synthetic jets on nearby supercavity characteristics. Data was collected to accomplish the following specific objectives:

- Determine the dependency of transverse thrust on jet frequency using load cell measurements of forces on the vehicle under quiescent conditions
- Determine the dependency of transverse thrust on crossflow by load cell measurements of forces on the model under fully-wetted (no cavitation) conditions, where jet frequency and freestream velocity were varied
- Examine and determine the effects of artificial supercavitation on the thrust of the synthetic jet using load cell measurements of force for partial supercavitation where the ventilation rate, jet frequency, and freestream velocity were varied.
- Analyze the behavior of the cavitating core of ring structures using high-speed video with varied jet frequency and freestream velocity for both the fully-wetted and partially supercavitation cases.
- Analyze flow fields using PIV velocity and vorticity fields for all fully-wetted cases with the synthetic jet operating at $St=0.051$, in an effort to understand dependency of thrust on freestream velocity.
Section 2: Experimental Setup

NUWC High-Speed Water Tunnel Facility

All experiments presented here were conducted at the Naval Undersea Warfare Center’s high speed water tunnel in Newport, Rhode Island. Tests were conducted at freestream velocities up to 7.5 m/s. A correlation between the tunnel pump RPM and freestream velocity was established by preliminary tests to approximate the necessary pump RPM to achieve the desired freestream velocity. Actual freestream velocity during the experiments was recorded using a pitot tube located in the freestream below the model at the same cross-sectional plane as the jet orifice. In this manner, the increase in localized freestream velocity caused by partial tunnel blockage was accounted for, and it was thus ensured that recorded crossflow velocities would accurately reflect the conditions at the synthetic jet port (the primary area of interest). All velocity-based calculations within this work are based on the velocity as calculated by the pitot tube-reported differential pressure.

Figure 4: Schematic of NUWC high-speed water tunnel
Model Design and Construction

The aim of the model design and construction was to produce an analog of a high speed underwater vehicle. In doing this, the model was built similarly to the model shown in Figure 5; a conceptual CAD rendering of a model used in previous research at NUWC. The real world application called for the model to be a streamlined, axisymmetric body of length roughly ten times its diameter. The model required ventilated supercavitation hardware, including a cavitator plate at the bow and a means of delivering gas to a reservoir within the nosepiece. The experimental actuator was positioned near the stern of the model and was only required to operate about one axis for the scope of this project.

![Conceptual CAD model of complete water tunnel fixture](image)

Figure 5: Conceptual CAD model of complete water tunnel fixture

The model was designed to occupy no more than approximately five percent of the cross sectional area of the test section of the water tunnel to keep tunnel blockage issues at bay. With the nominal test section dimensions of 12 in x 12 in, this limited the model diameter to 3 in. It was considered optimal from the design standpoint to make the vehicle this maximum diameter to offer as much space as possible for the internal synthetic jet components. The model length was scaled appropriately to have the supercavity close upstream of the synthetic jet port for most of the cases to be tested. This is an important consideration, as the synthetic jet must be wetted in order to produce useful thrust. The model was sting mounted to keep the strut from influencing supercavity behavior.
Figure 6 illustrates the basic construction of the model. The foremost section consists of the 1.75 in diameter cavitation plate bolted to the front surface of the nosepiece at a height 0.75 in offset from it. This surface featured a radial pattern of 48 ~1mm diameter holes to allow ventilation to the gap space behind the cavitator. During ventilation experiments, a reservoir within the nosepiece was pressurized by a gas line that ran the length of the model, through the sting mount and strut, and through a regulator attached to an external high pressure nitrogen tank. The bulk of the vehicle length was a simple hollow aluminum tube. Aft of the tube section was the synthetic jet subassembly, a watertight and self-contained module featuring a bypass channel for the delivery of ventilation gas. The aft-most portion of the model was the tailpiece, a ~3.5 in long cone shape intended to minimize the wake. An example of the model in operation with moderate ventilated cavitation is presented in Figure 7.
Synthetic Jet Design and Construction

The synthetic jet was designed to produce the greatest force possible, given tight size constraints due to a limited hull diameter (3 in) and the need for wiring and ventilation gas passages. With the axial length of the synthetic jet subassembly being non-critical, it was decided that the driving mechanism should be oriented axially within the model. This was a departure from most synthetic jet setups discussed in literature wherein jets are typically driven by bellows or pistons actuated in the same direction as the jet port. It was assumed that if the ratio of piston to jet port area was sufficiently large, flow within the chamber across the jet port will be sufficiently small that it will not significantly influence the velocity profile of exiting fluid.

It was decided that the synthetic jet should be actuated by a solenoid rather than by a motor and cam as done in previous research [13, 14]. Perhaps the most significant advantage to a solenoid is its compactness, which allows it to fit into the hull with relative ease. It is also simple to operate insofar as it requires minimal signal processing to control its motion; all that is needed is a square wave of the desired voltage, frequency, and duty cycle.

The solenoid selected for the synthetic jet was a Ledex Low Profile 6EC. This was the largest of a class of high-force solenoids by this manufacturer which would fit in the available space in the model. Because the associated data sheet showed a sharp decrease in force as the plunger is pulled farther from the de-energized position, there was added incentive to keep the stroke length of the plunger as short as possible. It was presumed by consulting the technical specifications associated with the solenoid shown in Figure 8 that a stroke length less than 5.0 mm would allow for an outstroke time on the order of 10 ms.
Figure 8: Force and responsiveness of Ledex 6EC solenoid.

With the solenoid selected and thus a rough estimate of outstroke force known, an ordinary differential equation was developed to assist in determining optimal design parameters, including stroke length, piston diameter, jet port diameter, and qualities of the spring used to return the piston and plunger to the de-energized position.

In development of this ODE model, three separate forces were analyzed. The first of these was the inertial force of fluid within the chamber being driven by the piston, as well as the mass of moving parts of the assembly. The second force considered was the loss term associated with flow restriction of the fluid exiting the jet orifice. This was modeled approximately as resistance to flow through an orifice plate. Finally considered is the force of the return spring in opposition to the pushing action of the solenoid. For all cases, an attempt was be made to find the relationship between behavior of the piston in terms of position, velocity, or acceleration, and force. Once these relationships were known, they were combined to produce the ordinary differential equation used to approximate the piston motion through one cycle of the shortest possible waveform. Each term within the ordinary differential equation is briefly discussed below.

1) Inertia within the jet chamber.
Inertia of the fluid within the jet and moving solenoid parts was modeled by Newton’s second law, \( F = m \ddot{x} \), where mass was determined by the density and volume of fluid in the jet at the instant
being analyzed. The mass of the solenoid plunger and piston was also estimated and included, thus expanding to the form in equation 1 where \( x_p = 0 \) describes the resting (most retracted) position of the piston.

\[
F = \left( \frac{\rho n (L_c - x_p) D_p^2}{4} + m_s \right) \ddot{x}_p
\]  \hspace{1cm} (1)

2) Resistance through the orifice.

For a hydraulic system, the relationship between flow rate and pressure drop is as follows in equation 2.

\[
Q = \alpha_d A \sqrt{\frac{2}{\rho}} (P_c - P_\infty)
\]  \hspace{1cm} (2)

Here \( \alpha_d \) refers to the discharge coefficient, a term accounting for energy lost in transit through the orifice. Given the highly dynamic and chaotic nature of the problem considered here, it was assumed that flow through the orifice is always turbulent. In conjunction with the assumption that the orifice plate is very thin, this will allow use of the theoretical value \( \alpha_d = 0.611 \). Assuming gauge pressure and an ambient pressure of 1 atmosphere, we can rearrange and rewrite the expression solely in terms of upstream pressure, defined by the solenoid force over area of the piston. This is shown in equation 3 below and simplified in equation 4. It is then solved for \( F \) and simplified further in equations 5 and 6.

\[
\dot{x}_p = \frac{Q}{A_p} = (0.611) \frac{A}{A_p} \sqrt{\frac{2}{\rho}} \left( \frac{4F}{n D_p^2} \right)
\]  \hspace{1cm} (3)

\[
\dot{x}_p = (0.975) \frac{D^2}{D_p} \sqrt{\frac{F}{\rho}}
\]  \hspace{1cm} (4)

\[
F(x) = (1.052) \rho \left( \frac{D_p^3}{D^2} \right)^2
\]  \hspace{1cm} (5)

\[
F(x) = (1.052) \rho \frac{D_p^6}{D^4} \dot{x}_p^2
\]  \hspace{1cm} (6)

3) Return spring forces

The return spring of the solenoid contributes a force based on its compression. This force is governed by Hooke’s Law, \( F = k \Delta x \), where \( \Delta x \) is distance of compression. Since the jet may rest
at its home position with some compression in the spring, a variable for this is necessary (δ_s), and since x was earlier designated as position relative to the piston’s fully-retracted position, the equation can be rewritten as equation 7.

\[ F(x) = k_s(δ_s + x_p) \]  \hspace{1cm} (7)

The total force/velocity/acceleration relationship will be the sum of the previous functions (shown below in equation 8). To facilitate finding a solution of the differential equation, x_p can be dropped from the first term (Eq. 1) with negligible impact on the accuracy of the ODE as piston position plays very little role in the mass of fluid constituting moving parts in the system. This will reduce the accuracy of the calculations, but only slightly as the displacement of the piston should be small with respect to the total volume of the chamber. In addition, the error produced will tend to oversize the solenoid so it is not critically important. The ODE can thus be rewritten as equation 9.

\[ F(x) = \left( \frac{\rho \pi (L_c - x_p) D_p^4}{4} + m_p \right) x_p + \left( 1.052 \right) \rho \frac{D_p^6}{D^2} x_p^2 + k_s (δ_s + x_p) \]  \hspace{1cm} (8)

\[ F(x) = \left( \frac{\rho \pi L_c D_p^4}{4} + m_p \right) x_p + \rho \frac{D_p^6}{D^2} \left( \frac{x_p}{0.975} \right)^2 + k_s x_p + k_s δ_s \]  \hspace{1cm} (9)

With the initial conditions:

\[ x = 0 \text{ when } t = 0 \]
\[ \dot{x} = 0 \text{ when } t = 0 \]

Unfortunately, even in its simplified form, none of the solution tools employed were able to produce an analytical solution to the differential equation, so an approximate method was used to assess jet performance for different jet parameters. Using an Excel spreadsheet with an input field for the basic jet dimensions, the coefficients of the ODE were solved directly as follows:
A function was then constructed in VBA that directly pulled the coefficient values from the spreadsheet and solved the following:

$$F = C_1 \ddot{x} + C_2 \dot{x}^2 + C_3 x + C_4$$

With the initial conditions:

$$x(t = 0) = 0$$
$$\dot{x}(t = 0) = 0$$

Using Euler’s method with a $1.0 \times 10^{-6}$ s time step, the discretized ODE was advanced in time until $x$ met or exceeded the designated stroke length. The calculated time to reach this position was defined as the *outTime*, $F$ was redefined to equal zero to reflect the solenoid shutoff, the sign of the second coefficient was made negative (necessary as $\dot{x}_p^2$ fails to reverse the sign for the return), and the position and velocity were redefined as follows:

$$x(t = outTime) = 0$$
$$\dot{x}(t = outTime) = 0$$

The simulation then continued for the return of the piston to its home position with the second leg of the simulation calculating the *inTime*. The sum of the out time and return time serve as an approximation of the shortest possible period for the synthetic jet under the designated parameters.

The percentage of the period dedicated to the outward stroke was representative of the ideal duty cycle for which the solenoid should be powered. Numerical integration of the outstroke velocity through the simulation aided in approximation of the synthetic jet thrust. Under the parameters selected for further design, the predicted motion appeared as shown in Figure 9 below.

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$(0.785)\rho L_c D_p^2 + m_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_2$</td>
<td>$(1.052)\rho \frac{D_p^6}{D^4}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$k_z$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$k_z \delta_s$</td>
</tr>
</tbody>
</table>
The design parameters defined by our theoretical analysis yielded a short stroke length of only 4mm and a cylinder bore of 45mm. This produces a slug with a length to diameter ratio (formation number) of 3.92, nearly the optimal number reported by [8] in the interest of keeping the stroke length short for best solenoid functionality. The cylinder bore was set to exactly 45mm to accommodate a precision machined piston ring. The dimensions of the synthetic jet chamber and slug are as shown in Figure 10 C. Also noteworthy is that the synthetic jet port had a 30 degree internal bevel to help reduce the possibility of flow reversal on the outstroke.

Figure 10 below shows a CAD cutaway of the synthetic jet subassembly. The collinear piston, plunger, and return spring bolt near the centerline of the assembly constitute the moving parts. In this image, the solenoid (blue) would be holding the piston in the fully extended position by keeping the plunger pulled tightly in the energized position. When the solenoid circuit is opened, the return spring housed between the backstop plate and return spring bolt would pull the moving parts downward until the plunger contacts the backstop plate and the fully de-energized position is reached. Also visible in Figure 10 A is the passage for ventilation gas along the left side. Figure 10 B shows the pre-anodized construction of the synthetic jet subassembly, with a threaded flange.
piece for the sting mount on one end and threading to attach the remainder of the hull on the other. Figure 10 C offers a visual of relative critical dimensions of the synthetic jet.

Figure 10: A, B & C: CAD cutaway of synthetic jet assembly, completed synthetic jet assembly, and conceptual cutaway with dimensions.

**Load Cell Experiments**

Two sets of experiments were conducted using a 6-axis load cell to record and analyze forces and moments for various conditions. The load cell was an ATI Delta IP68 waterproof transducer located within a flooded box above the water tunnel test section. The model was mounted such that drag forces were aligned in the positive X-direction, transverse jet forces in the negative Y-direction, and buoyancy in the negative Z-direction. Since the model was sting mounted below and forward of the load cell, there were also moments produced by operation of the synthetic jet; one about the Z-axis with a 10.53 inch lever arm; and one about the X-axis with a 7.00 inch lever arm. All data relating to the transverse thrust of the jet has had the sign reversed for convenience.

The first series of tests using the load cell were conducted without the use of ventilated supercavitation, thus making the hull fully-wetted. Average forces and moments over four second long samplings were recorded for synthetic jet frequencies of 0, 4, 8, 12, 16, 20, and 24 Hz, with
nominal tunnel freestream speeds of 0, 1, 2, 3, 4, 5, 6, 7, and 8 m/s. Four sets of data were collected to ensure consistency of readings.

The second series of tests included the use of ventilated supercavitation. For this series the synthetic jet frequency was limited to 0, 16, 20, and 24 Hz due to constraints on operating time and ventilation gas supplies. Ventilation rates were 50, 100, and 200 slpm and tunnel freestream speeds ranged from 2.0 to 8.0 m/s by 0.5 m/s increments. Tunnel speeds for particular ventilation rates were limited by excessive tunnel blockage at impractically low speeds and cavity instability at the higher speeds. Thus, data over the 2.0 to 8.0 m/s velocity range were broken into three overlapping tests, where the 50 SLPM dataset covered the low speeds, 100 SLPM covered the mid range speeds, and 200 SLPM covered the high speeds. As with the fully-wetted data, all samples were time-averaged over four-second intervals. In the ventilated case, however, the complete test matrix was repeated six times to account for increased noise in the data.

**Particle Imaging Velocimetry** Two-dimensional Particle Imaging Velocimetry (PIV) was used to examine the velocity fields of the synthetic jet in the XY-plane. A laser sheet was projected into the tunnel from the positive Y-direction at a height where it bisected the synthetic jet port. The sheet was created such that it covered roughly 50 mm upstream and downstream of the jet. The PIV camera was placed below the tunnel looking upward. With this configuration, PIV data captured the behavior of the leading-most and trailing-most extremes of the toroidal vortices produced by the synthetic jet. The image covers the area of interest as it captures the leading edge of the vortex ring and the trailing wake. A schematic showing the PIV setup is displayed in Figure 11.
The PIV data were collected with the synthetic jet operating at 20 Hz, with tunnel speeds varying from 0 to 5 m/s in 1 m/s increments. Data sets were collected over 16 phases of the synthetic jet cycle from $t = 0$ ms to $t = 46.88$ ms by 3.13 ms increments, with $t = 0$ corresponding to power being applied to the solenoid. 50 instantaneous velocity fields were taken for each set of conditions and ensemble-averaged. An attempt was made to collect PIV data for the ventilated supercavitation cases; however, bubbles from ventilation were entrained in the bulk flow of the tunnel to such a degree that the PIV software could not distinguish them from the seed particles, thus producing inaccurate data. For this reason, the PIV data were limited to the fully wetted cases.

**High Speed Video**

While conducting load cell experiments it was observed that cavitation was occurring in the vicinity of the synthetic jet port when the jet was operating. This was most readily observable in the core of the ejected vortex rings, but also existed as a then unexplained secondary structure at higher crossflows. To analyze the behavior of these cavitation bubbles, high speed video of the synthetic jet operation was carried out over a variety of conditions. All high-speed video was recorded at 3,000 frames per second and captured several cycles of the synthetic jet. The camera was positioned looking upward (negative Z-direction) similar to the position of the PIV camera in figure 11. For operation in the absence of ventilated supercavitation, video was produced for 4, 8, 12, 16, 20 and 24 Hz from 0 to 6 m/s by 1 m/s increments. Video was also produced for a limited
number of supercavitation cases. For all of these latter cases, synthetic jet frequency was limited to 20 Hz. The tunnel speeds were 3, 4, and 5 m/s with 50 slpm of ventilation; 4, 5, and 6 m/s for 100 slpm, and 5, 6, and 7 m/s for 200 slpm ventilation rates.

**Uncertainties:**

The data collected in these experiments includes systematic uncertainty based on the accuracy and resolution of the diagnostic equipment. For example, calculable uncertainty exists in the PIV, force, and moment measurements. Errors associated with the PIV velocity field data, as well as force and moment data, are discussed in the following sections.

**Load cell instrument bias error**

The load cell used for load experiments registered data with some intrinsic error as a result of limited resolution. As shown in Table 2 below, several load cell calibrations exist with different resolutions. US-50-150 was used for these experiments, as the forces and moments involved did not exceed maximum loads for this calibration. The loads of interest were force in the Y direction and moments about the X and Z axes. For these, the respective manufacturer-listed accuracy of measurements were ±1/128 lbf, ±3/128 in-lbf, and ±1/64 in-lbf. To determine the force and moment uncertainty, time-averaged values for a 20 Hz, fully-wetted case were used for a 0 m/s and 4 m/s crossflow case displayed below. Uncertainty is expressed as a percentage by use of equation 11.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>$F_x, F_y$</th>
<th>$F_z$</th>
<th>$M_x, M_y$</th>
<th>$M_z$</th>
<th>$F_x, F_y$</th>
<th>$F_z$</th>
<th>$M_x, M_y$</th>
<th>$M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US-50-150</td>
<td>50 lbf</td>
<td>250 lbf</td>
<td>150 lbf-in</td>
<td>150 lbf-in</td>
<td>1/128 lbf</td>
<td>1/64 lbf</td>
<td>1/128 lbf-in</td>
<td>1/64 lbf-in</td>
</tr>
<tr>
<td>US-75-300</td>
<td>75 lbf</td>
<td>225 lbf</td>
<td>300 lbf-in</td>
<td>300 lbf-in</td>
<td>1/64 lbf</td>
<td>1/32 lbf</td>
<td>3/64 lbf-in</td>
<td>1/32 lbf-in</td>
</tr>
<tr>
<td>US-150-600</td>
<td>150 lbf</td>
<td>450 lbf</td>
<td>600 lbf-in</td>
<td>600 lbf-in</td>
<td>1/32 lbf</td>
<td>1/16 lbf</td>
<td>3/32 lbf-in</td>
<td>1/16 lbf-in</td>
</tr>
</tbody>
</table>

Table 2: Manufacturer sensing ranges and resolution of ATI Delta IP 68 load cell.
uncertainty = \left( \frac{\text{resolution}}{2 \times \text{load}} \right) \times 100\% \quad (11)

Table 3: Load cell uncertainty error percentages.

<table>
<thead>
<tr>
<th>Load Cell Accuracy</th>
<th>Fy</th>
<th>Mx</th>
<th>Mz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_r = 0 )</td>
<td>±2.41%</td>
<td>±0.32%</td>
<td>±0.20%</td>
</tr>
<tr>
<td>0 SLPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( U_r = 4 )</td>
<td>±8.23%</td>
<td>±1.88%</td>
<td>±16.47%</td>
</tr>
<tr>
<td>0 SLPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( U_r = 5 )</td>
<td>±16.28%</td>
<td>±0.75%</td>
<td>±0.22%</td>
</tr>
<tr>
<td>50 SLPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( U_r = 5 )</td>
<td>±0.82%</td>
<td>±0.17%</td>
<td>±0.14%</td>
</tr>
<tr>
<td>100 SLPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( U_r = 5 )</td>
<td>±0.73%</td>
<td>±0.19%</td>
<td>±0.35%</td>
</tr>
<tr>
<td>200 SLPM</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Standard Load**

<table>
<thead>
<tr>
<th>T* (Fy)</th>
<th>T* (Mx)</th>
<th>T* (Mz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>average</td>
<td>min.</td>
<td>max.</td>
</tr>
<tr>
<td>0 m/s 0 SLPM</td>
<td>0.234</td>
<td>0.207</td>
</tr>
<tr>
<td>2 m/s 0 SLPM</td>
<td>0.144</td>
<td>0.084</td>
</tr>
<tr>
<td>4 m/s 0 SLPM</td>
<td>0.068</td>
<td>0.016</td>
</tr>
<tr>
<td>5 m/s 0 SLPM</td>
<td>0.043</td>
<td>-0.229</td>
</tr>
</tbody>
</table>
Additionally, standard deviations were calculated for load cell data within individual sampling sets. A script was created that analyzed transverse thrust measurements (Appendix C)

\[
\sigma_{\text{mean}} = \sqrt{\frac{1}{N} \sum_{j=1}^{N} (x_j - \bar{x})^2}
\] (12) below. The impulse of individual synthetic jet cycles was calculated by integration of the data over the respective period of actuation for the 4 second record. In this manner, the data for frequencies 4, 8, 12, 16, 20 and 24 Hz, are based on 16, 32, 48, 64, 80, and 96 impulses respectively. Table 5 below shows the standard deviation of trial 4 of the fully-wetted case for the tested actuated frequencies and tunnel speeds. It was revealed that for low tunnel speeds, even in the case of 4 Hz operation where there were only 16 samples, the agreement of samples was satisfactory. At higher crossflows, large variation were observed. This is because of the synthetic jet’s inability to reliably create the same flow field with each actuation at low R-values. This is discussed in greater detail in later sections.

![Image](image-url)

Table 5: Percent standard deviation at various actuation frequencies and tunnel speeds

<table>
<thead>
<tr>
<th>f (Hz)</th>
<th>(U_\infty) (m/s)</th>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>2.16%</td>
<td>6.40%</td>
<td>19.48%</td>
<td>74.62%</td>
<td>268.16%</td>
<td>158.73%</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>5.62%</td>
<td>5.60%</td>
<td>24.45%</td>
<td>35.99%</td>
<td>113.64%</td>
<td>100.29%</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>3.41%</td>
<td>5.44%</td>
<td>22.50%</td>
<td>34.45%</td>
<td>81.09%</td>
<td>90.34%</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>13.49%</td>
<td>17.01%</td>
<td>28.25%</td>
<td>31.01%</td>
<td>83.50%</td>
<td>94.15%</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>1.62%</td>
<td>4.88%</td>
<td>25.04%</td>
<td>34.86%</td>
<td>134.34%</td>
<td>225.49%</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>12.71%</td>
<td>4.99%</td>
<td>33.27%</td>
<td>58.09%</td>
<td>397.59%</td>
<td>214.54%</td>
</tr>
</tbody>
</table>

**PIV Uncertainty**

As with the load cell measurements, PIV uncertainty exists in two types. The first is based on the resolution of pixel displacement; the discretization of particle positions creates an
uncertainty in distances traveled between exposures. The second uncertainty is due to variance in highly turbulent fluid behavior from sample to sample.

The pixel displacement uncertainty of the PIV system was 0.1 pixels, according to the manufacturer. The Δt for PIV acquisition system was adjusted such that pixel displacement was within the suggested range of 2 and 7 pixels. The associated error ranges from 1.4% to 5% for the displacement range.

Each case recorded by PIV was sampled 50 times. Data generated contains both average and RMS values. With this information, uncertainty was calculated using equation

\[
SDOM = \frac{\text{RMS}}{\text{Mean}} \quad (13)
\]

The pixel displacement uncertainty was calculated for the 6th phase of the 16 phases recorded for each case. The 6th phase was chosen because it captures the most dramatic velocity profile for the fluid being expelled from the synthetic jet. Data from the second row above the mask (user-defined blackout of data in the space occupied by the model) at the streamwise position that corresponds to the highest RMS value was used to arrive at the SDOM for the freestream velocities of 1 to 5 m/s. The SDOM ranged from 1.0% to 2.5%.
Section 3: Synthetic Jet Parameters

Dimensionless Criteria

When discussing the behavior of the synthetic jet within this work, there are numerous references to measurements that warrant explanation. The dimensionless parameters include the Reynolds number of the jet, the Strouhal number of jet operation, the velocity ratio, and the formation number. The Reynolds number is displayed in equation (13 below. The Strouhal number is shown in equation (14) and served as a dimensionless measurement of frequency. The velocity ratio is the ratio of characteristic velocity to freestream and is shown in equation (16). Lastly, the formation number is simply the ratio of slug length to slug diameter. Two of these dimensionless measures require a definition for the term $V_0$, which is discussed in later sections.

\[
Re_j = \frac{V_0 D}{\nu} \quad (14)
\]

\[
St = \frac{fD}{U_\infty} \quad (15)
\]

\[
R = \frac{V_0}{U_\infty} \quad (16)
\]

Slug Model

The slug model is a simple means of estimating the thrust of a synthetic jet design which has been employed in previous research [8]. In this model, the fluid ingested and ejected during a single synthetic jet cycle is treated as a slug which can be cleanly extruded through the jet port (see Figure 10 C). Thrust is defined as the time-averaged outward momentum flux of the fluid, which depends on the velocity and dimensions of the slugs. The thrust calculation can be performed with exact integration of the waveform as shown in equation (17) for a sine wave [8], or estimated using a characteristic velocity and the slug dimensions as shown in
equation \( (18) \) The characteristic velocity of a synthetic jet is generally defined by the slug length and time devoted to the outstroke, as shown in equation \( (19) \).

\[
\bar{T} = \frac{1}{16} \rho \pi^3 D^2 L_s^2 f^2
\]

\[
T = \frac{1}{4} \rho \pi D^2 L_s V_0 f
\]

\[
V_0 = \frac{L_s}{t^*}
\]

A sizeable body of prior research has compared experimental measurements of synthetic jet thrust to the theoretical values predicted by the slug model. \( L_s:D \) ratios between 4 and 4.5 have been shown to produce thrust values closest to the theoretical model. A formation number of 4 was used in this research.

**Derivation of \( V_0 \)**

Many of the results presented in the following subsections refer to a characteristic velocity, denoted as \( V_0 \). In related research where synthetic jets follow simple sinusoidal motion, the characteristic velocity is defined as the average jet velocity over the expelling phase of the synthetic jet \([7, 9, 12]\). This definition allows for simple calculation of \( V_0 \) as a function of operating frequency, slug size, and jet port diameter.

In the case of the synthetic jet created for this research, the expulsion phase was driven by sudden, high impulse of a solenoid operating on a high-power square wave. With this type of operation, one can assume a constant \( V_0 \) value for all operating frequencies and tunnel speeds, as other forces on the piston (such as ambient pressure and local flow behavior) would be negligible compared to the substantial driving force. To validate this assumption, PIV data with 50 phases per synthetic jet cycle were used in the quiescent and 2 m/s tunnel speed case with the goal of
showing consistency of outstroke times across different tunnel speeds. Using this data, \( V_0 \) can be calculated as the ratio of the slug length over the expulsion time.

\[
V_0 = \frac{L_s}{t_{\text{out}}}
\]  

(20)

Figure 12 shows two curves generated using the phase-locked PIV data at 0 m/s and 2 m/s tunnel speeds. It can be seen that the initiation and termination of the expulsion phase are virtually identical in the two cases. From these curves, an expulsion time of 10 ms was measured. With the slug length as a known jet parameter (49.8 mm, from Figure 10 C), \( V_0 \) is calculated to be 4.98 m/s. This value is used in all subsequent derivations of dimensionless numbers such as jet ratios, Strouhal numbers, and scaled thrust. The Reynolds number was also calculated using this data; its value for all experiments presented here is \( \text{Re}_j = 63,000 \).

**Momentum Analysis**

An algorithm was developed to analyze PIV measurements and estimate the average jet momentum flux over a single period. This algorithm employs the theoretical model of synthetic
jet momentum shown in polar form in equation

modified to estimate momentum using v-values from the ascii files associated with various crossflow cases. The algorithm examined a user defined row of cells, located the maximum v-velocity within the most intense phase of the PIV ensemble-averaged sets ($\varphi = 6$), and treated this x-position as the centerline of the jet. Numerical integration of the velocity over finite areas upstream and downstream of the centerline were carried out up to a user defined radius from the centerpoint. This process was repeated for all phases of the PIV data to integrate with respect to time. The discrete model is expressed in equation

\[
\pi \rho \int_0^{1/f} \int_0^R v(x)^2 r dr
\]

\[
\frac{\pi}{f^2} \rho \sum_{\varphi=0}^{15} \sum_{x_{min}}^{x_{max}} v(x)^2 (x - x_0)^2
\]
Section 4: Results

Quiescent Case

Initial experiments involved the operation of the synthetic jet with zero crossflow at the jet port. This allowed for observation of behavioral changes in thrust and flow characteristics, with frequency as the only variable.

Figure 13: Comparison of direct and derived synthetic jet thrust.

Figure 13 compares direct thrust measurement to what has been dubbed “derived” thrust measurement. The latter is the force calculated based on the load cell measurement of moments divided by the appropriate component of distance between the origin and jet port. The direct measurement dataset is noticeably irregular, while the two derived forces (from Mx and Mz) agree closely with one another. It was determined that force derived from Mx measurements should be used in all subsequent results and discussion. Mx was selected over Mz because the z-axis was perpendicular to the direction of tunnel flow in non-zero crossflow experiments. This orientation could have produced inaccurate results if variations in upstream and downstream pressure fields were present in non-zero crossflow cases, as the effective jet port position would change slightly. This concern does
not exist with Mx because the x-axis is parallel to the flow direction.

The trend seen in the derived thrust \((M_\alpha/L_\alpha)\) curve is a linear regime followed by a decreasing relationship between frequency and thrust for Strouhal numbers above 0.041. By neglecting data points where the Strouhal number exceeds 0.041, a linear regression of the scaled thrust \(T^*\) with Strouhal number results in an \(R^2\) value of virtually 1 (as shown in Figure 14 below). The relationship between scaled thrust and Strouhal number is \(T^* = 5.51(St)\) throughout the linear regime for \(St \leq 0.041\).

![Figure 14: Linear regression of \(T^*\) vs. \(St\)](image-url)
Figure 15: Composite image of 6 instants of a vortex ring core at $U_\infty = 0$, $f = 4$ Hz.

Figure 15 shows a composite image of the cavitating core of a vortex ring produced by the synthetic jet with an operating Strouhal number of 0.010 ($f = 4$ Hz). The image shows the consistent celerity of the ring structure as it is convected from the synthetic jet at roughly 3.25 m/s. It also captures the transition from laminar (approximately one jet diameter) to turbulent (approximately 4 jet diameters) flow conditions for the ring structure. Turbulence is indicated by the lack of relative axisymmetry of the structure, with the emergence of irregularities in the third image superposition (15 ms).
Figure 16: Composite image of 6 instants of a vortex ring core at $U_\infty = 0$, $f = 20$ Hz.

Figure 16 shows a composite image of the same setup seen in Figure 15, but with an operating Strouhal number of 0.051 ($f = 20$ Hz). The general behavior of the vortex ring core appeared consistent between the two cases, supporting load cell measurements that suggest the behavior of individual jet cycles are independent of Strouhal number (within the linear regime).
Figure 17: Early formation of a vortex ring core for, \( U_\infty = 0 \), \( f = 20 \text{ Hz} \).

Figure 17 shows the early development of a vortex ring core, beginning with the first observable cavitation and ending with the ring roughly one jet port diameter from the hull. In the earliest stages of the ring’s development, it can be seen that cavitation occurs as a smooth, laminar ring. By the time the ring reaches just one jet port diameter, it is visibly bowing outward and showing slight asymmetry, which suggests it is in early transitional flow.

Review of all high-speed video of quiescent conditions shows obvious variations between individual vortex rings, but the general structure described here is consistent. Rings begin as a faint halo just above the hull, broaden as they approach one jet port diameter from the hull, and show slight outward bowing and asymmetry as the ring moves further from the jet port.
Figure 18: 16 phase PIV velocity field for full synthetic jet cycle at $U_\infty = 0$. 
Figure 18 shows the ensemble-averaged PIV velocity field for 16 phases. Despite the rising edge of the square wave beginning at $\phi=0$ and the falling edge occurring just before $\phi=4$, the outflow of fluid from the synthetic jet occurs mostly in phases $\phi=4, 5, & 6$. This is likely the result of inductance within the solenoid coil, causing it to operate out of phase; this phase delay is expected to be consistent with all phase-locked PIV data. The effective start of a cycle can be assumed to occur between $\phi=3$ and $\phi=4$.

In addition to the expulsion phase occurring in a different place than one might naturally expect, the ingestion of fluid back into the synthetic jet chamber begins immediately after the expulsion phase but continues through $\phi=15$ and wraps to the beginning, ending finally at $\phi=3$ (just before the cycle repeats). The unsteady nature of the ingestion phase should also be noted. Fluid appeared to be drawn back with the greatest intensity at $\phi=15$, but virtually ceases at $\phi=0$, only to draw in more fluid again at $\phi=1$.

The vorticity field from the ensemble-averaged velocity field data is presented in Figure 19. From $\phi=0$ the continued ingestion from the previous cycle is seen. The wake of the previous ring is visible above two small counter-rotating structures of fluid being drawn back into the jet port. These regions of vorticity associated with ingestion continue through $\phi=3$. At $\phi=4$, the expulsion of fluid begins with an intense reversal of vorticity in these regions. This vorticity increases drastically through $\phi=5$ and the generated ring is convected at $\phi=6$. At this point, peak vorticity of $\sim 2000 \text{ s}^{-1}$ is seen at the core of the rings. From $\phi=7$ to $\phi=11$, the ring structure moves out of the field of view with a relatively constant velocity, leaving behind two wakes of like vorticity. From $\phi=12$ to the conclusion of the cycle, the ingestion of fluid by vorticity counter-
rotational to the ring structure can be seen. The ingestion peaks at $\varphi=15$ but, again, continues and wraps to the start of the following cycle though $\varphi=3$. 
Figure 19: 16 phase PIV vorticity for full synthetic jet cycle at $U_\infty = 0$. 
Fully-Wetted Crossflow

Load Cell Measurements

Figure 20 A & B show the relationship between scaled thrust and Strouhal number for jet ratios ranging from 0.8 to infinity. The plots have been separated into low crossflow cases (A) and high crossflow cases (B). For all plotted cases, a linear correlation exists between scaled thrust and Strouhal number up to a critical point, where the relationship fails. The slope of this correlation generally becomes shallower as jet ratio decreases (crossflow increases). It is clear from Figure 20 B that the linear correlation becomes invalids between Strouhal numbers of 0.041 and 0.051, whereas in Figure 20 A the relationship is arguably maintained through the data points for St = 0.041 (and perhaps even St = 0.051).

Figure 20 A & B: Scaled thrust as a function of Strouhal number.

Figure 21 A & B present the same data, but with jet ratio on the x-axis and curves representing fixed values of the Strouhal number. From these plots it is seen that higher Strouhal
numbers typically correlate to higher scaled thrust. Also, it is observed that scaled thrust drops off sharply as jet ratios drop towards and below 1.

![Graph showing scaled thrust as a function of velocity ratios for various Strouhal numbers.](image)

**Figure 21 A & B**: Scaled thrust as a function of velocity ratios for various Strouhal numbers.

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**High-Speed Imagery**

Figure 22 presents a composite image similar to the quiescent case shown in Figure 15. Here, a cavitating ring structure was examined with 1 m/s crossflow (R=5.3). In this case, a vortex ring leaving the synthetic jet with a velocity comparable to the quiescent case can be seen, though it appeared to be in transitional flow by the time it reached 1 jet diameter from the hull. As the ring
core passed 2 jet diameters, its structure appeared mostly unchanged from ~5ms; however, it had washed slightly downstream due to the presence of crossflow. At the ~15 ms timestamp, the ring structure was bent and turned into the direction of crossflow. At this point, the ring appeared very turbulent and unstable. At ~20 ms, the ring is breaking up. In video format, the circulation of fluid is still evident, but there is no longer a continuous ring structure. By the final timestamp, only a few cavitation bubbles remained as the structure continued to be broken down and washed downstream.

![Composite image of vortex ring core at $U_\infty = 1$ m/s, $R = 5.30$.](image)

Figure 22: Composite image of vortex ring core at $U_\infty = 1$ m/s, $R = 5.30$.

Figure 23 shows a similar composite image for the 2 m/s crossflow case ($R=2.54$). As in Figure 22, early transitional flow of the vortex ring can be seen. By the time the ring passed 2 jet port diameters from the hull, it was already highly unstable. By the third timestamp, the vortex
ring core is pitching violently into the crossflow and being stretched dramatically in the streamwise direction. At this point, the ring structure quickly breaks up.

Figure 23: Composite image of vortex ring core at $U_\infty = 2$ m/s, $R = 2.54$.

Figure 24 shows a composite image for the 3 m/s crossflow case ($R=1.67$). Here, the leading and trailing ends of the ring structure are convected more quickly than the near and far edges. As this occurs, the ring structure quickly folds. By $\sim 5$ms the cavitation core has already bowed significantly inward. Additionally, a small amount of cavitation occurring in the transverse direction between the jetport and cavitation ring can be seen. By the following timestamp, this
transverse cavitation has collapsed. In most cases, this collapse disrupts the ring structure, causing it to break up before reaching 2 jet port diameters from the hull.

![Composite image of vortex ring core at $U_\infty = 3 \text{ m/s, } R = 1.67$.](image)

For lower jet ratios ($R<1.67$), the evolution and breakup of structures occurred very close to the jet port and over very short periods of time. For this reason, composite images were not feasible, as images from different timestamps would be overlapped and difficult to distinguish from one another.

In Figure 25 the development of a vortex ring core at $R=5.3$ is shown. Early cavitation is very subtle and thus difficult to see without tracing backwards though the images, but the ring structure appeared to grow from the rim of the jet port until it reached its sustained diameter at about 1 jet port diameter from the hull. The only observable difference in the early evolution of the ring structure between the quiescent $R=\infty$ and $R=5.3$ cases is that the upstream edge of the
crossflow case showed slightly more prominent cavitation. Additionally, for the crossflow case the entire structure is washed slightly downstream by the time it is shed.

Figure 25: Early vortex ring core generation at $U_\infty = 1 \text{ m/s}$, $R = 5.30$. 
The R=2.54 case shown in Figure 26 displays the same trends as Figure 25, but here they are more pronounced. Additionally, cavitation appeared slightly more intense, and there is a small pitch in the downstream direction while the ring structure is still within the boundary layer.

![Figure 26: Early vortex ring core generation at $U_\infty = 2$ m/s, $R = 2.54$.](image)

For the R=1.67 case shown in Figure 27, we begin to see the folding of the vortex in the composite image. Both the leading and trailing ends of the vortex rings appear to grow and convected quicker than the rest of the ring structure, beginning when the ring is about 0.5 jet port diameters from the hull. By the time the ring reaches a distance of 1 jet port diameter, the folding
is significant. For many rings, the transverse cavitation shown in the earlier composite occurs as the ring is shed.

Figure 27: Early vortex ring core generation at $U_\infty = 3$ m/s, $R = 1.67$.

In Figure 28, the synthetic jet is operating at $R=1.24$, and the transverse cavitation described for the $R=1.67$ case is becoming more prominent. Less cavitation within the vortex ring itself, and much earlier and more significant cavitation in transverse strands can be seen. This phenomenon is visible when the ring is as near as 0.5 jet port diameters from the hull. The folding
of the vortex ring and collapse of the transverse cavitation contribute to the breakup of the vortex ring within 1.5 jet port diameters of the hull.

Figure 28: Early vortex ring core generation at $U_\infty = 4$ m/s, $R = 1.24$.

Figure 29 A & B capture the early evolution of a vortex core at $R=0.96$. This image set is extended to show an emerging behavior in the ingestion phase of operation at this jet ratio. In keeping with the trend seen in the previous two crossflow cases, the transverse cavitation became more pronounced while the ring cavitation became fainter. A weak ring becomes visible only as the transverse cavitation begins to collapse, and this ring quickly breaks down. Once the transverse cavitation has completely dissipated, only a chaotic churning structure that is no longer a complete
ring can be seen. As this churning continued through Figure 29 B, a new bubble of cavitation occurs at the jet port.

Figure 29 A: Early vortex ring core generation at $U_\infty = 5$ m/s, $R = 0.96$ (image 1 of 2).
Figure 29 B: Early vortex ring core generation at \( U_\infty = 5 \text{ m/s}, R = 0.96 \) (image 2 of 2).

PIV Data

Figure 30 shows PIV velocity field images for the R=5.3 case. In these images, we see behavior highly consistent with the quiescent case presented in Figure 18. The vortex structure is convected at the same velocity and major events occur during the same phases as the quiescent case. The one exception to this is that peak ingestion appeared to occur at \( \varphi=0 \) rather than \( \varphi=15 \). Otherwise, the only observable change is the slight downstream washing of the vortex structure.
For the R=2.54 case shown in Figure 31, flow behavior begins to change. The vortex structure no longer vacates the field of view by φ=11. Instead, the structure breaks down and is washed away along the top of the field of view. The expulsion phase appeared to begin at φ=4 but ended by φ=6. Although it operated at the same frequency and duty cycle as other cases, the ingestion phase seems to be delayed slightly with respect to the quiescent case, with no fluid flowing in or out of the synthetic jet chamber until φ=12. The ingestion phase then appeared to
achieve peak intensity from $\phi=15$ to $\phi=1$. As in the quiescent case, no secondary ingestion is visible.

![Figure 31: Velocity field at $U_\infty = 2$ m/s, $R = 2.54$.](image)

The $R=1.67$ case shown in Figure 32 displays very similar behavior to the 2.54 case. The one noteworthy difference between the two cases is that the ring structure appeared to break up sooner and was washed out of the field of view downstream without having traveled as far from
the hull. The breakup is best captured in $\varphi=7$ and $\varphi=8$, where the downstream edge of the vortex ring is traveling significantly faster than the upstream edge.

Figure 32: Velocity field at $U_\infty = 3$ m/s, $R = 1.67$.

Figure 33 shows the $R=1.24$ case. Recalling that high-speed video revealed new and very pronounced cavitation at these crossflows, exact trends in the PIV are difficult to decipher, but we see that the expulsion and ingestion phases continue to be consistent with other $R$ values. The ring breakup, however, appeared to happen even earlier. In $\varphi=5$ and $\varphi=6$, a strong disagreement in the
upstream and downstream edges of the structure came seem. In $\varphi=7$ a region which appeared chaotic and amorphous emerges, which is washed out of the field of view by $\varphi=12$.

Figure 33: Velocity field at $U_\infty = 4$ m/s, $R = 1.24$.

Figure 34 presents the highest crossflow PIV velocity field, which corresponds to $R=0.94$. In the $\varphi=1, 2$ & 3 images, where one would expect to see the end of the ingestion phase, we instead see that the downward component of velocity is dwarfed by the strong crossflow. In $\varphi=4$, where the first clear visual of the outstroke occurs, a velocity profile near the jet port that is similar to that of moderate crossflows is seen. However, by $\varphi=5$ a new behavior emerged; there appeared to be a
cessation of bulk outward flow. In fact, at the upstream end of the jet port, fluid is rushing inward. It is unclear from the PIV data whether this is an accurately reported field, or the confounded result of the software’s inability to analyze the image pairs due to increased cavitation.

![Velocity field at $U_\infty = 5$ m/s, $R = 0.96$.](image)

Figure 34: Velocity field at $U_\infty = 5$ m/s, $R = 0.96$.

Vorticity fields were also generated for crossflow cases up to $R=0.96$. In Figure 35, the outstroke phase appears to begin at $\varphi=3$, with negative vorticity on the upstream edge of the jet port and positive vorticity on the downstream edge. Vorticity increases in these regions through $\varphi=4$, and in $\varphi=5$ what is clearly an enlarging ring with its core approximately 0.5 jet diameters from the hull surface can be seen. In $\varphi=6$ this structure is approximately 1.0 jet diameters from the hull. At
this point, vorticity at the core of the ring is near its maximum of +/- 2000 s\(^{-1}\) and two trailing regions of moderate vorticity are observed. In \(\varphi=7\), the ring has been fully shed from the hull. The trailing vorticity had separated into a secondary structure which, by \(\varphi=8\), appeared to have pitched into the direction of the crossflow. At this point the vortex ring is still a stable structure, but the leading edge appeared slightly diffused. By \(\varphi=9\) the ring is slightly pitched in the direction of crossflow. The ring continues in this manner until it leaves the field of view in \(\varphi=11\).

The ingestion phase appeared to begin in \(\varphi=11\), where positive vorticity occurs near the leading edge of the jet port. This ingestion continues through \(\varphi=15\) and wraps to the beginning, finally ending in \(\varphi=2\). Ingestion steadily grows and then decreases in intensity, appearing to peak at \(\varphi=15\). During the ingestion phase, the secondary structure described earlier continues to linger 2 to 4 jet port diameters from the hull. It very slowly translates downstream and decreases in vorticity, but is ultimately disrupted by the subsequent vortex ring.
Figure 35: Vorticity at $U_\infty = 1 \text{ m/s, R} = 5.30$.

Figure 36 displays vorticity at $R=2.54$. As with the $R=5.30$ case, outflow appeared to begin in $\phi=3$ with negative vorticity on the upstream edge of the jet port. This vorticity increases through $\phi=5$. In $\phi=4$, it is worth noting that the negative vorticity region is surrounded by a region of opposing positive vorticity. In $\phi=5$, the core vorticity in the upstream edge of the vortex ring is in the neighborhood of 1800 s$^{-1}$, while downstream vorticity is approximately 2000 s$^{-1}$. By $\phi=8$ the
ring had pitched in the direction of crossflow. Beyond this point, the vortex structure quickly diminishes in vorticity and loses much of its velocity in the y-direction.

The phase in which ingestion begins is difficult to determine. Because the crossflow is sufficient to create a noticeable boundary layer, it is no longer possible to use the appearance of positive vorticity at the upstream edge to indicate an influx of fluid. Instead, negative vorticity at the trailing edge must be used. It is most likely that ingestion first occurs in φ=11; however, it is difficult to make this assertion confidently, as the region of negative vorticity could be a component of the wake produced from the previous ring. As with the R=5.30 case, the ingestion concludes at φ=2, with peak vorticity at φ=15.

One noteworthy behavior that was not observed in the quiescent and R=5.30 cases is a curious spike in positive vorticity at the leading edge of the jet port in φ=6. Also new in this case is a wake composed of alternating vorticities, which moves out of the field of view through almost purely translational motion.
Figure 36: Vorticity at $U_\infty = 2$ m/s, $R = 2.54$.

Vorticity fields for the $R=1.67$ case are shown in Figure 37. As with the previous case, the outstroke begins at $\varphi=3$, vorticity growing in $\varphi=4$, and the ring convecting in $\varphi=6$ can all be seen. In this case, the positive vorticity surrounding the leading edge in $\varphi=4$ is more pronounced. The vorticity of the leading edge from $\varphi=3$ through $\varphi=7$ also appeared considerably diminished in comparison to the higher R-value cases. The y-velocity of the ring structure is also considerably lower, appearing to be roughly 0.75 jet port diameters per phase until $\varphi=8$. Furthermore, the ring pitches in the upstream direction more dramatically than in the lower crossflow cases. At $\varphi=8$, the
ring appeared to be destabilized 3.0 jet port diameters from the hull surface. Following this, vorticity is amorphous and slow moving. By $\varphi=15$, it is nearly washed out of the field of view.

As previously observed in the $R=2.54$ case, the ingestion phase appeared to begin at $\varphi=11$ and concluded at $\varphi=2$, with maximum vorticity occurring at $\varphi=15$. During the early phases of ingestions, vorticity over the trailing edge of the jet port is less pronounced than in previous cases.

The wake for the $R=1.67$ case is more complicated than the higher $R$-value cases, showing stratification of vorticity in the flow direction as well as the $y$-direction. The trailing vorticity visible in $\varphi=5$ becomes separated from the ring structure as it pitches into the crossflow. As this happens in $\varphi=6$ and 7, there are two new regions of vorticity produced at the jet port, of the same sign as the ring itself. These two regions then also pitch towards the direction of the crossflow, and are washed away at a velocity comparable to the vortex ring.
In Figure 38, the vorticity field for the $R=1.24$ case is presented. As in previous cases, the beginning of the outstroke is indicated at $\phi=3$ by the appearance of negative vorticity at the upstream edge of the jet port. Vorticity strengthens through $\phi=5$, but the discrepancy between the two regions is greater still, with upstream vorticity approximately 60% of the downstream vorticity. By $\phi=5$ the vortex ring is pitching into the crossflow. By $\phi=6$ the ring had pitched approximately 20°. By $\phi=7$ the ring can be seen beginning to fail at roughly 2.0 jet port diameters from the hull. Beyond this, the ring had broken into two diffuse regions of opposing vorticity. The upstream
region assumed a crescent shape conforming to the circular downstream region. The two regions both wash out of the field of view in $\phi=13$, roughly 3 jet port diameters from the surface. The ring produced in this case is convected at approximately 0.5 jet port diameters/phase.

The start of the ingestion phase is unclear from vorticity data due to the presence of a boundary layer, but ingestion ceases at $\phi=2$ (just before the outstroke). Peak vorticity appeared to occur at $\phi=1$, departing from previous cases where it occurred at $\phi=15$.

The wake of the ring is similar to the $R=1.67$ case, where 2 pairs of counter-rotating regions can be seen. However, unlike the previous case, the final pair produced pitches downstream rather than upstream.
Figure 38: Vorticity at $U_\infty = 4 \text{ m/s}$, $R = 1.24$.

Figure 39 displays the vorticity field for $R=0.96$, which was the highest crossflow case examined with PIV. As with all other cases, the outstroke begins at $\phi=3$ with the first appearance of negative vorticity at the upstream edge. This vorticity increased through $\phi=5$, where the negative vorticity appeared to be approximately 50% of the positive vorticity. At this point, the ring is already pitched nearly 45°. By $\phi=6$, the ring is convected but already unstable with diminished
vorticity at the core. The ring is also moving a mere 0.5 jet port diameters per phase in the y-direction.

As with the R=1.24 case, it is difficult to determine when the ingestion phase begins, but it is likely in φ=11. This phase concludes in φ=2 as seen in previous cases, but here peak vorticity for the ingestion occurs in φ=1.

The wake for this case is fairly unremarkable. The reversal of secondary production of vorticity observed at the jet port in φ=6 for the R=1.67 and R=1.24 cases can also be seen here, but it is diffused and the trailing vorticity behind the vortex ring is never sheared away.
In figure 40, the locations of maximum vorticity for R-values as low as 1.24 are presented. Blue data point represent the upstream (left) portion of the ring structure \((\omega < 0)\) and red series represent the right portion of the ring structure \((\omega > 0)\). \(\varphi=6\) corresponds to the lowest \(y/D\) values with \(\varphi\) ascending with distance from the origin. These plots show that with increasing crossflow, there is a greater tendency for the ring structure to pitch into the crossflow. It is also observed, based on the spacing between phases, that there is considerable decrease in the convection of the structure with higher crossflow.
Figure 40: Locations of maximum vorticity relative to jet center for $\phi = 6$ to 9.
In Figure 41 below, transverse and downstream velocity were plotted as a function of vorticity. The velocity was calculated based on displacement between two phases of PIV data, vorticity for each point was calculated based on the average of the magnitude of the positive and negative vorticity, and averaged again between the two corresponding phases used in the velocity calculation. Transverse velocity shows a linear relationship with the average vorticity of the ring structure corresponding to approximately $0.0084\omega$. It appears that with lower vorticity, the lessened $u$-velocity was met with roughly inversely proportional downstream velocities.

![Figure 41: Plot of transverse and downstream velocity of the ring structure as a function of average vorticity.](image)

Ventilated Crossflow

**Load Cell measurements**

Load cell readings were taken for ventilated supercavitating cases at three different vent rates: 50, 100, and 200 slpm. Given the chaotic nature of the two-phase problem, the measurements were extremely noisy; in spite of this, they still offer insight into the lower crossflow cases.
Figure 42: Scaled thrust vs. jet ratio for low (50 slpm) vent rate.

In Figure 42, the derived transverse force is plotted for Strouhal numbers of 0.041, 0.051, and 0.061. Unfortunately, the data was subject to enough noise that these plots all have significantly overlapping error bars, but it is conceivable that the force measurements cumulatively support the efficacy of the transverse jet at low speeds with a minimal vent rate. At the lowest ventilation rate (50 slpm), scaled force in the neighborhood of 0.15 are seen; this is slightly less than the scaled force measured in the fully-wetted case. However, as the R-value decreases the scaled thrust climbs, plateauing around 0.3, until R drops below 1.4. At R=1.1, the $T^*$ drops to nearly zero. As R-values decrease further, $T^*$ begins to increase again, climbing to approximately 0.2 for R=0.63.

For the intermediate ventilation rate (100 slpm), Figure 43 below shows a noisy but consistently positive scaled thrust across all R-values. The lowest crossflow measured (corresponding to R=1.42) shows $T^*$ values that average to about 0.25 for the tested Strouhal numbers. This exceeds the closest tested R-values for the fully-wetted case. It is difficult to discern any subtle trends within the noisy data, but in general, the thrust appeared to increase for lower R-values. At the lowest R-value (R=0.80), $T^*$ is approximately 0.5.
Figure 43: Scaled thrust vs. jet ratio for intermediate (100 slpm) vent rate.

For the highest vent rate case (200 slpm) shown in Figure 44, largely flat trends in the scaled thrust are seen. In general, T* decreases as R approaches 0.

Figure 44: Scaled thrust vs. jet ratio for low (200 slpm) vent rate.

**High-Speed Imagery**

High speed video was also recorded for 20 Hz (St=0.051) for various R-values of the three ventilation rates. Some of these videos captured noteworthy events at cycle phases other than the outstroke, so side by side images were created in timesteps of 1 ms
(rather than the 0.33 ms used for the fully-wetted cases) to show greater spans of time in the same space.

The first case is shown in Figure 45 A & B, which represents R=2.34 for 50 slpm. From the image set it can be seen that the boundary layer is littered with bubbles shed from the ventilated cavity at the bow. Following the images closely, it is noted that the synthetic jet ingesting some of the ventilated gas (note the elongation of bubbles in the third image). When the vortex ring is produced, it is immediately turbulent with a prominent core. In Figure 45 B the gaseous ring pitches wildly into the crossflow. The ring is contorted and unstable as it leaves the field of view (in some cases, the ring breaks up completely within a few jet port diameters of the hull). After the ring had been shed, a churning wake reminiscent of the vorticity of the fully-wetted R= 2.54 case can be seen.
Figure 45: Early vortex ring formation with low ventilation at $U_\infty = 2 \text{ m/s}$ (1 of 2 images).
Figure 45 B: Early vortex ring formation with low ventilation at $U_{\infty} = 2$ m/s (2 of 2 images).

Figure 46 A & B shows the R=2.06 case with low ventilation. Similar to the previous case, air bubbles in the boundary layer are visible. These bubbles enter the jet port in the ingestion phase and contribute to a more prominent core in the produced vortex ring. The ring can first be seen cavitating on the downstream edge of the jet port. Milliseconds later it forms a complete ring, then quickly warps into a saddle shape as it crosses the boundary layer. Referring to Figure 46 B, it can be seen that as the ring reaches about 1.5 jet port diameters from the hull, it begins to pitch into the crossflow. At this same time, a haze of gas and/or cavitation appeared in the jet. Looking farther ahead it is seen that as this cloud collapses, the ring becomes unstable and is torn apart by the crossflow.
Figure 46: Early vortex ring formation with low ventilation at $U_\infty = 3$ m/s (1 of 2 images).
Figure 46 B: Early vortex ring formation with low ventilation at $U_\infty = 3$ m/s (2 of 2 images).

Figure 47 presents the R=1.24 case at 50 slpm. At this speed and above, the shed ventilation gas began circulating with the flow of the tunnel which caused haziness in the water; however, under close inspection it is seen that the amount of air washing along the boundary layer from the cavity at the bow is minimal compared to lower crossflow cases at the low vent rate. In this case, the ring forms very similarly to the 4 m/s crossflow case (R=1.24) with fully-wetted flow. The ring is shed and quickly bent. At approximately 0.5 jet port diameters from the hull, two cavitating regions appear on the near and far edge of the jet port. These collapse as the ring begins to pitch into the crossflow. The ring appeared to completely break up before reaching 3.0 jet port diameters from the hull.
Figure 47: Early vortex ring formation with low ventilation at $U_{\infty} = 4$ m/s.

Figure 48 shows the R=1.24 case for the moderate (100 slpm) ventilation rate. The water is hazier than the low ventilation rate case, but the same behavior is witnessed. The only noteworthy differences are that the twin cavitation bubbles on the near and far edge of the jet port collapse slightly earlier, and a small and brief bubble of cavitation occurs on the close edge of the jet port as the ring breaks up. This bubble was not witnessed with every ring formed, but was not atypical.
Figure 48: Early vortex ring formation with moderate ventilation at $U_\infty = 4$ m/s.

Figure 49 shows the $R=0.97$ crossflow case with a moderate ventilation rate. In this image set, we begin to see the wake of cavitation rushing downstream above the model. The heavy light in the background makes early development of the ring impossible to observe, but once the ring becomes visible, it is pitched about 30° and moving very slowly in the y-direction. About 7 milliseconds after it is first visible, the ring appeared to have pitched further to about 45° and is quickly breaking up.
Figure 49: Early vortex ring formation with moderate ventilation at $U_{\infty} = 5$ m/s.

Figure 50 A & B show the $R=0.97$ case with a high ventilation rate. Very little information can be collected from the high speed video under these conditions, as the tunnel was dense with bubbles and the ventilation wake obscured early development of the vortex rings. The only visible phenomenon is the downstream side of a vortex ring, which is washed out of the field of view at approximately 1.2 jet port diameters from the hull.
Figure 50 A: Early vortex ring formation with high ventilation at $U_\infty = 5$ m/s (1 of 2 images).
Figure 50 B: Early vortex ring formation with high ventilation at \( U_\infty = 5 \) m/s (2 of 2 images).

Figure 51 A & B shows synthetic jet behavior for the R=0.80 case. This is the only case observed where the supercavity bisected the synthetic jet port. The supercavity seen in this image is in the process of washing upward along the hull. In doing so, it forms one of a pair of counter-rotating vortex tubes. Because of this, the large stream of air seen here appeared to be rolling towards the camera when viewed as video. Rather than seeing a deformation or localized growth of the ventilated cavity when the synthetic jet begins its outstroke, what appeared to be a secondary volume of air which quickly elongates downstream can be seen. The extruded slug appeared to be largely ejected air, which immediately began to rotate in the streamwise direction (counter to the ventilated cavity itself). The introduction of this second counter-rotating volume initially had little impact on the supercavity, but as the outstroke continues there is a sudden and radical disruption.
of the streamwise vorticity. As the two volumes wash over the tailpiece of the model, they chaotically mix and appear to have no bulk rotation about the streamwise direction.

Figure 51 A: Early vortex ring formation with high ventilation at $U_\infty = 6$ m/s (1 of 2 images).
Figure 51 B: Early vortex ring formation with high ventilation at $U_\infty = 6$ m/s (2 of 2 images).

Figure 52 shows the $R=0.67$ high ventilation rate case. This is the only case where complete supercavitation is observed. Because the entire vehicle is enclosed by the ventilated cavity, the synthetic jet is operating by ingesting and ejecting air. With such low density fluid, the effects of its operation are either non-existent or indiscernible on the highly featured free boundary.
Figure 52: Early vortex ring formation with high ventilation at $U_\infty = 7 \text{ m/s}$.

**Boundary Layer Measurements**

PIV ascii data was used to roughly calculate the boundary layer thickness near the jet port for fully-wetted cases. To perform this calculation, the $u$-velocity values were pulled from the column nearest to 2 jet port diameters upstream from the center of the jetport at $\phi=12$ (the quietest phase). The velocity at 3 jet port diameters from the hull was considered representative of the bulk velocity. Next, the two values nearest to 99% of the bulk velocity were found and linear interpolation was used to yield an approximate $\delta$-value. Table 4 on the following page displays approximate boundary layer thickness in millimeters, as well as jet port diameter as a function of crossflow and $R$. 
Table 6: Approximate boundary layer thicknesses.

<table>
<thead>
<tr>
<th>$U_\infty$ (m/s)</th>
<th>R</th>
<th>$\delta$ (mm)</th>
<th>$\delta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.3</td>
<td>15.3</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>2.54</td>
<td>17.0</td>
<td>1.3</td>
</tr>
<tr>
<td>3</td>
<td>1.67</td>
<td>17.4</td>
<td>1.4</td>
</tr>
<tr>
<td>4</td>
<td>1.24</td>
<td>16.7</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>21.9</td>
<td>1.7</td>
</tr>
</tbody>
</table>

**Momentum Analysis**

The calculated impulse of a single jet cycle for different R-values is displayed in Figure 53. The plot was created from the averages of the solutions for calculations on the second and third rows of PIV data (1.8 and 3.6 mm above the jet port) considering a radius equal to the jet port diameter (see Appendix B for script). $P$ represents the raw calculation, while $P_{adj}$ is an adjusted plot where the calculated momentum flux was scaled using the ratio of the apparent slug volume (as calculated from numerical integration of PIV data) to known slug volume (from design parameters). The plot for $P_{adj}$ describes a decreasing impulse with increasing crossflow. However, while load cell data reported transverse thrust at $R=0.96$ for $St=0.051$ to be an 85% reduction from the quiescent measurement, the impulse of a single jet cycle from momentum appeared to be only a 40% reduction from the quiescent case.

![Figure 53: Synthetic jet impulse at various crossflows.](image-url)
V. Discussion

The objective of this project was to establish the viability of transverse-mounted synthetic jets as actuators for fully-wetted and artificially supercavitating underwater vehicles. This involved experimental investigation of the flow field and thrust produced by operation of a synthetic jet in crossflow with an established boundary layer. While the synthetic jet was able to generate a measurable turning moment for velocity ratios up to R=1, the thrust produced was low enough that it would likely prove inadequate for real-world application in high-speed underwater vehicles without significant design improvement. Regardless, the project yielded relevant thrust data for a variety of operating frequencies. This data was used to explore correlations between frequency and thrust, as well as thrust and crossflow velocity. Additionally, high-speed video captured numerous cavitation phenomena occurring during operation of the synthetic jet. With the support of PIV measurements, we are able to examine and partially explain changes in thrust as a function of frequency and crossflow, based on observable changes in flow characteristics for the fully-wetted case. Load cell measurements and high-speed video for the cavitating cases are less conclusive, but suggest anomalous behavior which warrants future exploration.

Quiescent Case

Load measurements in the quiescent case (see Figure 13) show that synthetic jet thrust scales linearly with frequency for Strouhal numbers up to 0.04 (and perhaps 0.05). For higher Strouhal numbers, the drop-off in thrust is mainly attributed to failure of the synthetic jet machinery to complete the pumping action across the entire stroke length. The observed first-order scaling of thrust and frequency (rather than the second-order scaling mentioned in work by Glezer, Amitay, and Mohseni [7, 9, 12]) is a result of the driving mechanism used in this design. Previous models have been constructed using motors and cams, such that the pumping action follows sinusoidal (or near-sinusoidal) motion; in these designs, the velocity of fluid leaving the synthetic jet must increase to achieve higher frequencies as a result of compression of the waveform. The solenoid-
driven synthetic jet used here had an essentially fixed outstroke and instroke profile, with higher
frequency achieved by reducing the time spent at the fully energized and fully resting positions.
Figure 54 below illustrates this difference in operation.

![Comparison of motor driven and solenoid driven synthetic jet piston motions.](image)

Figure 54: Comparison of motor driven and solenoid driven synthetic jet piston motions.

This figure illustrates that in the case of sinusoidal motion, a doubling in frequency will
double the outward velocity at any point in the expulsion phase. From the slug model, the thrust of
a sinusoidal synthetic jet scales with $f^2$. This relationship exists because the momentum of the jet
is the product of ejection velocity, approximately $2Lf$; the number of ejections per time, $f$; and the
mass of a slug, $\rho LD^2/4$. This results in the scaling relationship $\bar{T} \sim D^2 L^2 f^2$. Also recall that the
characteristic velocity $V_0$ of a synthetic jet is $L/t^*$ which, as established, would be $2Lf$ for the case
of sinusoidal motion, but would be independent of frequency for a solenoid-driven synthetic jet.
For this reason, the thrust of a solenoid-driven jet is best expressed as $\bar{T} \sim D^2 L^2 f/t^*$, showing a
linear dependence on frequency. This relationship is intuitive when considering that in the case
of a solenoid driven jet, we see the repeated creation of discrete ring structures which will have the
same circulation and ejection velocity throughout the operational range of the mechanism (unless
influenced by the wake of previous rings). It is evident based on the data in Figure 19 that there is
virtually no influence from the preceding wakes for Strouhal numbers up to 0.041 or 0.051. For
these reasons, frequency appeared to be indicative only of the number of discrete, momentum-
producing events in a given timeframe.
PIV data collected for St = 0.051 supports the conclusion that vortex ring wakes are not influential in this flow regime. From the velocity field data it is seen that the ring structure is formed and exits the field of view between φ=5 and φ=11, which corresponds to 0.019 ms. Furthermore, we know that the velocity of the cavitating ring core is representative of the speed of the ring structure. By locating the cell on the velocity field nearest to this cavitating region, it is determined that the ring is being convected at approximately 3.5 m/s. Likewise, extrapolation from high speed composite images suggests a convecting velocity equal to roughly 264 jet port diameters per second (3.35 m/s). These velocities place the ring structures about 13.5 jet port diameters apart at St=0.051 operation. With this considerable spacing, it is intuitive that the ring structures would not influence one another.

The vorticity field data shown in Figure 19 also illustrates how comparably weak the wake seen from φ=7 is until its dispersion at approximately φ=3. Within the wake, a peak vorticity of roughly 200 s\(^{-1}\) is seen at φ=9; this is only 10% of the vorticity observed in the core of the ring structures themselves. The final example of frequency-independent ring behavior can be seen in the composite images in Figure 15 and Figure 16, which show nearly identical evolution of the cavitating core between low frequency (St=0.010) and higher frequency (St=0.051) cases. In both instances equal convecting velocities can be seen. We also note structures which appear laminar within one jet port diameter of the hull, transitional between two and four jet port diameters, and turbulent at distances greater than four jet port diameters.

The final component of data for the quiescent case is the image set showing the detailed evolution of the cavitating ring core within a single jet port diameter of the hull. This image set exists for all tested frequencies, but appeared virtually identical across cases; as a result, only the St=0.051 case was presented here (Figure 17). In this image set, a small cavitation ring forming very close to the jet port at the start of the expulsion phase can be observed. As the outstroke cycle continues, circulation increases and this cavitation becomes more prominent. During this time, the ring diameter grows until it is roughly twice that of the jet port at a distance of one jet port diameter.
from the hull. As discussed when the composite images were presented, no variation in behavior with frequency is observed; however discussion of this early development should be considered for comparison to the crossflow cases. The quiescent case may also be treated as a crossflow case with R=\infty.

To summarize our findings for the quiescent case, it was demonstrated by various means that vortex rings generated by our synthetic jet maintained their ring structure while being convected and did not influence subsequent rings in any significant manner. Thus, the time-averaged momentum of a solenoid-driven synthetic jet scales linearly with jet frequency until mechanical limitations cause a drop-off in jet performance. For the quiescent case, it was found that the relationship between scaled thrust and Strouhal number was $T^* = 5.51 \times St$ for the entirety of the linear regime. In engineering units, this relationship expands to equation (23 shown below.

\[
T = \frac{5.51}{4} \rho D^3 V_0 \pi \tag{23}
\]

**Fully-Wetted Crossflow**

With the introduction of crossflow, load cell data was collected in 1 m/s increments up to 5 m/s for the fully-wetted case. Using the recorded true stream velocity, this translated to velocity ratios from R=5.30 to R=0.96. The collected data displayed in Figure 20 shows a linear regime followed by a drop-off in thrust for all cases, which is consistent with the thrust curve for the quiescent case. For crossflow cases at high R-values, thrust was relatively unaffected. However, for low R-values there was a significant decrease in the slope of the linear regime, as well as an earlier drop-off. A summary of absolute maximum thrusts and maximum thrusts within the linear regime is provided below as Table 7.
The frequency limitations seen here are not necessarily indicative of a flow-based phenomenon inherent to synthetic jets of this type and used in this application. Rather, it is possible that the synthetic jet was influenced by the static pressure drop within the tunnel at higher speeds. Recall that the waveforms of this synthetic jet are rigid, and frequency merely determines the regularity of their occurrence. With adequate static pressure drop in the tunnel at higher crossflows, it is possible that a slight elongation of the return portion of the waveform may cause overlapping at higher frequencies. When this occurs, incomplete slugs are expelled from the synthetic jet and thrust is consequently reduced. For this reason, it is important to consider that maximum attainable thrusts achieved in the laboratory are not necessarily indicative of the conceptual limit for designs of this nature. Rather, emphasis should be placed on changes in $dT^*/dSt$ as a function of $R$ within the regions where correct operation is apparent.

Figure 55 below is a plot of scaled thrust as a function of Strouhal number for $R$-values as low as 0.96. The corresponding slopes of the linear regressions and their R-squared values can be found in Table 8. Strouhal numbers were limited to 0.041 to ensure that only the linear regime for all cases was considered.

### Table 7: Peak thrust values for various crossflows.

<table>
<thead>
<tr>
<th>$R$-value</th>
<th>Peak $T^*$</th>
<th>Corresponding St-value</th>
<th>Peak $T^*$ in linear regime</th>
<th>Corresponding St-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>0.28</td>
<td>0.061</td>
<td>0.25</td>
<td>0.051</td>
</tr>
<tr>
<td>5.30</td>
<td>0.28</td>
<td>0.061</td>
<td>0.26</td>
<td>0.051</td>
</tr>
<tr>
<td>2.54</td>
<td>0.22</td>
<td>0.061</td>
<td>0.19</td>
<td>0.041</td>
</tr>
<tr>
<td>1.67</td>
<td>0.20</td>
<td>0.051</td>
<td>0.20</td>
<td>0.051</td>
</tr>
<tr>
<td>1.24</td>
<td>0.14</td>
<td>0.041</td>
<td>0.14</td>
<td>0.041</td>
</tr>
<tr>
<td>0.96</td>
<td>0.12</td>
<td>0.041</td>
<td>0.12</td>
<td>0.041</td>
</tr>
<tr>
<td>0.80</td>
<td>0.12</td>
<td>0.041</td>
<td>0.12</td>
<td>0.041</td>
</tr>
</tbody>
</table>
Figure 55: Drop-off in thrust as a function of Strouhal number at various velocity ratios.

Table 8: Comparison of scaled thrust slopes with velocity ratios.

<table>
<thead>
<tr>
<th>V₀/U₀</th>
<th>dT*/dSt</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>inf</td>
<td>5.51</td>
<td>0.9996</td>
</tr>
<tr>
<td>5.30</td>
<td>5.68</td>
<td>0.9894</td>
</tr>
<tr>
<td>2.54</td>
<td>4.85</td>
<td>0.9933</td>
</tr>
<tr>
<td>1.67</td>
<td>4.22</td>
<td>0.9839</td>
</tr>
<tr>
<td>1.24</td>
<td>3.36</td>
<td>0.9773</td>
</tr>
<tr>
<td>0.96</td>
<td>2.70</td>
<td>0.8714</td>
</tr>
</tbody>
</table>
Using the linear regression slope values shown in Table 8, Figure 56 plots the slope of scaled thrust curves as a function of the inverse velocity ratio. A linear regression of this data shows a good fit for the relationship \( y \approx -3x + 5.9 \). This function (shown in equation 25) and suggests an empirical correlation between scaled thrust, Strouhal number, and velocity ratio for solenoid-driven synthetic jets. The expression can be written more explicitly by expanding quantities in terms of real engineering units. The final equation is shown in 25.

\[
T^* = \left(5.9 - \frac{3}{R}\right)St
\]

24

\[
T^* = \left(1.47 - \frac{3U_\infty}{4V_0}\right)\rho D^3 \pi f
\]

25

With an empirical correlation between variables established, we are left to explain what changes in behavior occur with crossflow to account for the observed decline in thrust. By inspecting the composite images in Figure 22, Figure 23, and Figure 24, the ring core evolution at R-values of 5.30, 2.54, and 1.67 respectively can be seen. In the 5.30 case, the ring is initially leaving the hull with velocity comparable to the previously discussed quiescent case. However, as
the ring moves beyond three jet port diameters the ring structure begins to pitch into the crossflow, and subsequently breaks up within the field of view. A more dramatic instance of the same behavior is seen in the R=2.54 case, where by 15 ms the ring had not only begun to pitch but had been stretched and deformed by three jet port diameters. After 15 ms the ring structure is consistently broken up. In the R=1.67 case, the ring core develops for only 10 ms before deteriorating. Subsequent times would show some residual cavitation, but outward velocity is so insignificant after 10 ms that images would be overlapped and cluttered. At this crossflow velocity a new behavior also began to occur; beginning at the near and far edges of the jet port, two long cavitation bubbles emerged, reaching towards and just inside the cavitating ring core. At higher crossflows, this phenomenon became so pronounced that further composites could not be created due to overlap. However, in the image sets showing early ring evolution (Figure 25, Figure 26, Figure 27, Figure 28, and Figure 29 A & B) this cavitation is more prominent at higher crossflows and is increasingly detrimental to the ring structure, resulting in lower convecting velocities and earlier deterioration of the structure.

It is hypothesized that the cavitation observed at the jet port near and far edges is a vortex structure created by a combination of local low pressure and vorticity. Based on the position of the ring core at the onset of this cavitation, it can be determined that for moderate crossflows (R=1.67 or R=1.24), this phenomenon is beginning between φ=5 and φ=6 of the PIV data. In this range, the velocity field data indicates that the time in question is in the middle of the outstroke of the synthetic jet. The beginning of the slug has already formed a vortex ring and the continued outflow of fluid is intensifying the ring as it accelerates away from the hull (see Figure 57). At this time, there was a brief window in which the cavitating ring was distant enough from the hull that freestream-influenced behavior may occur beneath it. It is supposed that the deformation of crossflow streamlines in the vicinity of the actively extruding slug created a localized drop in pressure at the interface of the crossflow and extruding slug.
Figure 57: Average of 50 PIV raw images at $\varphi = 5$, $R = 1.67$.

Image just before the emergence of described cavitation bubbles.

Because PIV raw images indicate that the latter portion of the extruded slug briefly shows a strong cylindrical structure before entrainment into the convecting vortex, it is asserted that streamlines between the vortex structure and hull must bow around the slug. Using the potential flow model shown in equations 26 and 27, the maximum velocity of the bypassing freestream will be close to $2U_\infty$ at $\theta = \pi/2$.

\[ V_r = U_\infty \left( r - \frac{R^2}{r^2} \right) \cos \theta \]

26

\[ V_r = -U_\infty \left( 1 + \frac{R^2}{r^2} \right) \sin \theta \]

27

By equation 28 below, the pressure at the points $\theta = \pi/2$ and $3\pi/2$ under the non-viscous flow assumption made in the potential flow model can be estimated. Due to the shallow submersion of the model, it is also assumed that the system was near atmospheric pressure under quiescent...
conditions. The pressures listed in Table 9 were calculated using the potential flow model with these assumptions. Given that the vapor pressure of water at 25°C is 3,170 Pa, the third column of the table displays ratios of pressure drop to critical pressure drop needed to see cavitation. While no case achieved the necessarily low pressure to explain the cavitation bubbles outright, the second-order scaling of pressure drop with respect to freestream velocity shows considerable decrease in pressure for all but the lowest speeds tested. This may help to explain the bowing of the ring structure, even at speeds low enough to preclude the formation of the secondary cavitating structure.

\[ p_{\theta=\pi/2} \approx p_{\text{atm}} - \frac{1}{2} \rho (2U_{\infty})^2 \]

Table 9: Estimated pressure at 0=\pi/2 and 3\pi/2

<table>
<thead>
<tr>
<th>R</th>
<th>p (Pa)</th>
<th>( \frac{dp}{dp_{\text{critical}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>25374</td>
<td>0.77</td>
</tr>
<tr>
<td>0.94</td>
<td>46313</td>
<td>0.56</td>
</tr>
<tr>
<td>1.24</td>
<td>69712</td>
<td>0.32</td>
</tr>
<tr>
<td>1.67</td>
<td>83896</td>
<td>0.18</td>
</tr>
<tr>
<td>2.54</td>
<td>93791</td>
<td>0.08</td>
</tr>
<tr>
<td>5.30</td>
<td>99595</td>
<td>0.02</td>
</tr>
</tbody>
</table>

In addition to the static pressure drop due to the freestream bypassing the slug, there are other events that potentially contributed to the formation of the observed cavitation bubbles. One factor was the sudden termination of the expulsion phase. Because the solenoid is an electromechanical device which pulled the moving parts into the energized position during the outstroke, the driving force increased with proximity of the plunger to the magnetic coil. This means that the true velocity of the slug increased near the end of the outstroke. At the conclusion of the outstroke, all momentum from the moving parts was taken up by the plunger striking an internal plate. This resulted in an immediate cessation of the piston stroke, causing the velocity profile across the jet port to quickly adjust to zero net flow. With no net flow across the jet port,
whatever remnant of the slug had not been entrained into the vortex ring must have experienced a dramatic drop in pressure due to the abrupt disappearance of an upstream source.

As a result of this sudden pressure drop, the remaining fragment of slug stagnated above the jet port. With the slug no longer traveling quickly in the transverse flow, there was a brief instant where the accelerated bypassing flow sheared against this stagnant region, creating vorticity along an axis normal to the jet port. It is apparent in the high speed video of the synthetic jet that, for \( R \leq 1.67 \), these secondary cavitation bubbles are consistently present to some degree and appear to be counter-rotating in a manner consistent with this proposed explanation. At the highest crossflows tested (\( R \leq 1.24 \)), the resulting cavitating vortex cores appear suddenly and prominently when the ring is less than one jet port diameter from the hull. In these cases, the bubbles collapse very quickly and often completely disrupt the vortex ring. For \( R \leq 1.67 \), the ring structure was generally maintained until the ring was 2.5 jet port diameters from the hull and appeared to be disrupted by this secondary structure as well. Even in the much lower crossflow case (\( R = 2.54 \)), these secondary structures were visible in about half of the experiments observed by high-speed video. They were, however, very filamentous in appearance and not visible until much later than in high crossflow cases. Extrapolating from the \( R \leq 2.54 \) cases, it is reasonable to assume that these structures exist even at \( R = 5.30 \).

This model serves as a partial explanation for the drop-off in synthetic jet thrust despite the unchanging synthetic jet parameter \( V_0 \). If the twin vortex structure was produced in all non-quiescent cases, there was a low pressure field that existed along the hull for some portion of each synthetic jet cycle which partially counteracted the produced momentum flux. The circulation of this vortex would have scaled with increasing freestream velocity, thus making this reduction in thrust more considerable at higher crossflows. Below in Figure 58 are examples of these transverse vortex structures captured by the raw PIV imaging. These structures appear considerably earlier at higher crossflows.
Figure 58: Raw PIV images of transverse cavitating structure at R=2.54, 1.67, 1.24 and 0.96 and ϕ=7, 6, 5, and 5 respectively.

Shedding trajectory of vortex rings may also have played a role in thrust drop-off with increasing tunnel speed. Trajectory was a function of the discrepancies in vorticity between the leading and trailing regions of the vortex ring, which become more pronounced at higher crossflows. This is illustrated in Figure 35, Figure 36, Figure 37, Figure 38 and Figure 39. As crossflow is increased, the vorticity of the upstream region (ω < 0) drastically decreased while vorticity on the trailing end (ω > 0) is increased. This occurs, in part, as a result of subtractive or additive vorticity from the boundary layer in the early formation of the vortex ring. If we assume
that the influence of the boundary layer is equal but opposite on the two ends of the ring structure, we could conclude that the average vorticity of the ring would be unchanged with increasing crossflow. However, even if this assumption is made, the convecting velocity of the ring is still heavily impacted. The circulation of the vortex ring is the underlying phenomenon that propels the ring structure through the surrounding medium. With greater circulation in the downstream half than the upstream half, the downstream half is driven away from the hull more quickly. This results in the ring structure pitching upstream. The ring structure is often maintained through this reorientation, but it consequently moves with a considerable upstream component of velocity, rather than being convected as intensely as possible in the transverse direction (which would produce the maximum jet thrust).

**Ventilated Cavitation**

Data collected for the ventilated cavitation case was primarily force data. For high ventilation rates and high crossflow speeds, systematic noise was too great to draw confident conclusions, but for the lowest ventilation rate the plots showed an interesting deviation from the fully-wetted case. In Figure 42, a plot of scaled thrust is presented as a function of velocity ratio for St=0.041, 0.051, and 0.061. In it, the highest thrust does not occur at the lowest crossflow; instead, thrust increases with increasing crossflow up to R=1.63, where the scaled thrust is 0.35 for St=0.061. It is supposed by observation of the high-speed video that the ingestion of air bubbles into the synthetic jet may cause lower thrust at low crossflows. This phenomenon is best seen in Figure 45. This two-phase ingestion not only lessens the mass of the slug to be extruded (thus decreasing momentum), but also introduces asymmetry to the ring structure, making its behavior more chaotic. Many of the vortex rings seen in low speeds ventilated cases break up significantly earlier than in the corresponding fully-wetted cases. At R≥1.63, this contamination of the boundary layer falls away.
It was considered that this behavior might be credited to differences in gas leakage types. Based on the Campbell-Hilborne criteria, the type of leakage is determined by whether the product of the Froude number and cavitation number is less than or greater than one \([\]\). In the case of \(\sigma_{Fr}>1\), buoyancy is considerable and drives the cavity leakage to shed as twin vortex tubes. If \(\sigma_{Fr}<1\), the cavity is nearly axisymmetric and gas is shed as toroidal vortices. However, calculating the cavitation numbers for all cases revealed that the entirety of the supercavitation experiment was conducted within the \(\sigma_{Fr}>1\) regime.

A simple, likely explanation for this behavior involves the closure location of the supercavity in very low speed cases. For both tests where \(R \geq 1.63\), the supercavity left a small portion of the nosepiece wetted along the underside of the model. This irregular geometry at the cavity closure likely caused churning which was absent in the other, more regular cases; this churning behavior entrained air bubbles into the flow just above the model surface. Even though it can be definitively stated that this was not the result of different cavity closure types in accordance with the Campbell-Hilborne criteria, the anomalous behavior is a strong indicator of the impact of air bubbles being ingested by the synthetic jet. From this finding, it is reasonable to assert that the synthetic jet would not operate effectively within the \(\sigma_{Fr}<1\) regime, as leaked ventilation gas would be periodically swept over the model hull rather than lifted over the tail end of the model in two counter-rotating vortex tubes.
VI. Conclusions

The work presented here represents an initial investigation into a solenoid-driven synthetic jet in crossflows up to velocity ratios of approximately 1:1. Jet parameters were chosen with the intention of optimizing time-averaged momentum flux, as this research was intended to investigate the potential use of the transverse jet as a maneuvering actuator. When applicable, supercavitation parameters were limited to appropriate cavitation, and ventilation numbers appropriate for the velocities investigated. Within the scope of this research, several conclusions can be drawn regarding generated loads, flow fields, and detrimental cavitation phenomena.

Quiescent Case

1. In the case of quiescent flow, it was established that scaled thrust varied linearly with frequency within the mechanical limitations of the synthetic jet. This first-order relationship is a result of individual ring structures being discrete impulse-generating events, which are not influenced significantly by preceding cycles.

Fully-Wetted Crossflow

2. In the fully-wetted crossflow experiments, it was demonstrated that the impulse of each ring generation decreased with increasing crossflow. It was established that within the linear regime of thrust generated, scaled thrust and velocity ratio varied directly with one another for a particular Strouhal number.

3. For R-values as low 0.94, transverse thrust scaled inversely with R for the solenoid-driven synthetic jet considered in this work. Scaled thrust closely adhered to the linear relationship

\[ T^* = \left(1.47 - \frac{3U_\infty}{4V_0}\right) \rho D^3 \pi f \]

4. Bowing of vortex ring structures seen with increasing crossflow may be explained by low
pressure in the \( \theta=\pi/2 \) and \( 3\pi/2 \) regions, where secondary vortex structure spanned from the hull of the model to the inside edge of the vortex ring core.

5. Secondary vortex structures contributed to the early breakdown of the vortex ring structure in high-crossflow cases, where cavitation in the secondary structure was more intense.

**Ventilated Crossflow**

6. Synthetic jet performance was severely inhibited by the ingestion of air bubbles. For this reason, it is unlikely that the implementation of synthetic jets in the presence of supercavitation where \( \sigma Fr<1 \) would be effective.
VII. Future work

After analysis of the data collected for this research, several correlations on the behavior of pulsatile vortex rings in crossflow were made. A model which explains the dropoff of synthetic jet thrust with increasing crossflow was hypothesized. It also potentially explains the emergence of transverse secondary vortex structures. However, several questions have been raised by this project.

The most extensive and well-studied case within the scope of this research was in the absence of ventilated cavitation. This was in part because of the noise of data in the ventilated case, but also because of limitations of measurements in the bubble-filled water. The load cell data produced unintuitive trends which were inconclusive without any flow field data. For this reason, further work on pulsatile rings in crossflow and in proximity to an upstream free boundary may be warranted.

The hypothesized flow behavior which caused transverse cavitation in the fully-wetted cases was based in part on the drop in static pressure within the tunnel. This decreased pressure would not occur in real world cases where a vehicle is moving through quiescent fluid. For this reason, the functionality of the synthetic jet may not have been a fair assessment of its potential application. Further work could be undertaken to study the behavior of pulsatile vortex rings in crossflow under various static pressures rather than the \( p \leq 1 \) atm case studied here.
References


Appendix A

Ordinary Differential Equation VBA Model

Sub main
On Error Resume Next
ActiveSheet.ChartObjects.Delete

Force = ActiveSheet.Range("I14").Value 'Assignment of values for density = ActiveSheet.Range("D5").Value
Delta = ActiveSheet.Range("Q6").Value 'Important variables
Lstroke = ActiveSheet.Range("D13").Value 'pulled direction from Lstroke

C1_ = ActiveSheet.Range("Q5").Value 'Coefficients pulled
C2_ = ActiveSheet.Range("Q7").Value
C3_ = ActiveSheet.Range("Q8").Value 'Directly from the spreadsheet

C1 = C1_ * x ^ 2 + C2_ * x + C3_ / CL_n 'Updating

x = 0 Assigning initial conditions
v = 0 'to velocity, position, and
A = (Force + C2_ * v ^ 2 - C1_ * x - C3_ / CL_n) / acceleration

runtime = 0 Setting time to zero

While x < Lstroke 'Loop until piston reaches its stroke length
plots = runtime + 10
ReDim Preserve xArray(0 To plots)
ReDim Preserve timeArray(0 To plots)

A = (Force + C2_ * v ^ 2 - C1_ * x - C3_ / CL_n) / acceleration

s = 0 'Size of timestep (dt)
t = runtime Setting time to zero

totalTimeTimesteps = 0 Setting ThrustByTimeTimesteps = 0

Wend

End Sub

PISTON RETURNING TO HOME POSITION:

For x = Lstroke 'Setting initial conditions
v = 0 'the piston's return to its
A = (Force - C2_ * v ^ 2 - C1_ * x - C3_ / CL_n)

instantaneousThrust = 0 'Setting thrust=0 for piston's return

While x > 0
If countStep Mod plotSpacing <> 0 Then
    ReDim Preserve timeArray(LBound(timeArray) To UBound(timeArray) + 1)
    ReDim Preserve xArray(LBound(xArray) To UBound(xArray) + 1)
    timeArray(runtime) = runtime 'timeArray(runtime) / timeStep) = runtime
    xArray(runtime) / timeStep) = x
End If

runtime = runtime + timeStep 'Assignment of new time value
vNew = v + A * timestep 'Assignment of new values
sNew = s + s * timestep 'for position, velocity,
A = A / timeStep 'and acceleration to new values
v = vNew 'Setting current position, velocity,
x = xNew 'and acceleration to new values
A = A / timeStep 'for next iteration

If runtime > 2 Then 'Check performed to ensure time does
    MsgBox "Error. Solenoid failed to reach Lstroke" 'not exceed two seconds. If it
e does not, the scenario is assumed to fail.
End If

averageThrust = totalThrustByTimesteps / (runtime / timeStep) 'Calculating of ideal power required
to drive solenoid

totalThrustByTimesteps = totalThrustByTimesteps + instantaneousThrust 'Updating

End Sub

PISTON MOVING OUTWARD:

While x < Lstroke 'Loop until piston reaches its stroke length
If countStep Mod plotSpacing <> 0 And countStep <> 0 Then
    ReDim Preserve timeArray(LBound(timeArray) To UBound(timeArray) + 1)
    ReDim Preserve xArray(LBound(xArray) To UBound(xArray) + 1)
    timeArray(runtime) = runtime 'timeArray(runtime) / timeStep) = runtime
    xArray(runtime) / timeStep) = x
End If

runtime = runtime + timeStep 'Assignment of new time value
vNew = v + A * timestep 'Assignment of new values
sNew = s + s * timestep 'for position, velocity,
A = A / timeStep 'and acceleration to new values
v = vNew 'Setting current position, velocity,
x = xNew 'and acceleration to new values
A = A / timeStep 'for next iteration

If runtime > 2 Then 'Check performed to ensure time does
    MsgBox "Error. Solenoid failed to reach Lstroke" 'not exceed two seconds. If it
e does not, the scenario is assumed to fail.
End If

wavePeriod = runtime 'Setting initial conditions
returnTime = wavePeriod * outTime 'Sending data to output field

averageThrust = totalThrustByTimesteps / (runtime / timeStep) 'Calculating of ideal power required
to drive solenoid

solenoidPower = Lstroke * Force / outTime 'Calculating of ideal power required
to drive solenoid

End Sub

---

PLOT CREATION:

Dim MyNewSx As Series
MyChart.Select
With ActiveChart
    ChartTitle.Characters.Text = "expected motion"
    Axes(Category(1), Value).HasTitle = True
    Axes(Primary, Value).HasTitle = True
    Axes(Primary, Category).HasTitle = True
    Axes(Primary, Value).HasTitle = True
End With
End Sub

---
Appendix B

Momentum Analysis code (Python v2.7):

```python
import csv

radCutoff = float(input('nBoundary radius (mm): '))
rowInput = int(input('nRow to be analysed (1 = first row): '))

def readCSV(phaseNum):
    # load PIV data as array
    with open(pathA + str(phaseNum) + pathB, 'rb') as f:
        reader = csv.reader(f, delimiter = '	')
        a = list(reader)

    b = []
    for i in range(1,4097):
        if float(a[i][1]) < row + .01 and float(a[i][1]) > row - .01:
            b.append(float(a[i][3]))
    return b

def findAxis():
    global lowerBound
    global upperBound
    for i in range(16):
        pulledData.append(readCSV(i))
    cntLn = pulledData[5].index(max(pulledData[5]))
    lowerBound=cntLn-
    upperBound=cntLn+i

def createAveV():
    # executed after find Axis
    findAxis()
    radAndAreaValues()
    for j in range(16):
        v = []
        for i in range(lowerBound, upperBound + 1, 1):
            v.append(pulledData[j][i])
        v.insert(1, interpolate(rDim[1] - dx, rDim[1], v[0], v[1], rDim[0]))
        v.insert(len(v) - 1, interpolate(rDim[len(rDim) - 2] + dx, v[len(v) - 2], v[len(v) - 1], v[len(v) - 1] - 1, rDim[len(rDim) - 1]))
        v.pop(0)
        v.pop(len(v) - 1)
        for i in range(len(v) - 1):
            vAve[j].append((v[i]+v[i+1])/2)

def volumetricTotal():
    createAveV()
    global area
    global vAve
    Vdot = []
    Mdot = []
    totalVolumeB = 0
    totalMomentumB = 0
    for j in range(16):
        totalVolumeA = 0
        totalMomentumA = 0
        for i in range(len(vAve[j]) - 1):
            totalVolumeA = totalVolumeA + vAve[j][i]*area[i]/(16*20)
            totalMomentumA = totalMomentumA + (vAve[j][i]**2)*area[i]*980.0/(16*20)
        Vdot.append(totalVolumeA)
        Mdot.append(totalMomentumA)
        totalVolumeB = totalVolumeB + totalVolumeA
        totalMomentumB = totalMomentumB + totalMomentumA
    print totalVolumeB
    print totalMomentumB

def radAndAreaValues():
    for i in range(len(rDim)):
        if i == 0:
            rDim[i] = -radCutoff
        else:
            rDim[i] = (i - len(rDim))*dx
    rDim.append(0)
    L=len(rDim)
    for i in range(L - 1):
        rDim.insert(i, rDim[i])
    for i in range(lent(rDim)):
        area.append(abs(3.1416*(rDim[i]/1000)**2 - (rDim[i+1]/1000)**2))

def interpolate(rL, rR, vL, vR, r):
    v=vL+((r-
```

The Python code above is a part of the momentum analysis code. It includes functions for reading CSV files, analyzing PIV data, calculating radially averaged values, and computing volumetric and momentum quantities.
Appendix C

Standard Deviation of Individual Cycles within 4-Second Samples

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### Python 2.7.5

**Calculation of Standard Deviation for Discrete Cycles of the Synthetic Jet**

**Bradley Ayers**

5/11/2015

---

```python
import os, glob, csv

# Directory of loadcell data
path = "C:\Users\Brad\Desktop\sdev data"

os.chdir(path)

newTimes = []
newForces = []

u = str(input("nTest velocity (to 1 decimal): n"))
f = str(input("nFrequency to be examined: n"))
n = str(input("nTest number: n"))

properties = u + "-00-" + n + "-" + f

for file in glob.glob("*" + properties):
    filename = file
    print "nAnalyzing file named " + filename + "n"
    a = input('n')
    with open(path + filename) as files:
        reader = csv.reader(files, delimiter="t")
        d = list(reader)
        xSize = len(d[1])
        ySize = len(d)
        T = 1 / float(f)
        t0 = float(d[1][0])
        impls = [0 for x in range(int(4.0001 / T))]
        dataPts = [(round((float(d[y][0]) - t0) * 60, 6),
                    round(float(d[y][5]), 4),
                    int((float(d[y][0]) - t0) * 60 / T))
                    for y in range(0, ySize)]

        def interpolate(x1, x2, y1, y2, x):
            y = y1 + ((x - x1) / (x2 - x1)) * (y2 - y1)
            return y

        def addCycleDivisionPoints():
            n = 1
            for i in range(len(dataPts)):
                if dataPts[i][0] / T > n:
                    dataPts.append((T * n,
                                    round(interpolate(dataPts[i - 1][0],
                                                     dataPts[i][0],
                                                     dataPts[i - 1][1],
                                                     dataPts[i][1],
                                                     T * n), 4), n))
                n = n + 1

        def integrateStep(leftPt, rightPt):
            x1 = dataPts[leftPt][0]
            x2 = dataPts[rightPt][0]
            y1 = dataPts[leftPt][1]
            y2 = dataPts[rightPt][1]
            h = (y1 + y2) / 2
            dx = x2 - x1
            A = h * dx
            return A

        def integrateAll():
            x = 0
            for i in range(len(impls)):
                k = 0
                while k == 0:
                    impls[i] = impls[i] + integrateStep(x, x + 1)
                    x = x + 1
                    if dataPts[x][2] != i:
                        k = 1
                        print str(dataPts[x][0]) + 't' + str(impls[i]) + 't' + str(dataPts[x][1])
                        print ''

        def standardDev():
            a = 0.0
            b = 0.0
            for i in range(len(impls)):
                a = a + impls[i]
                mean = a / len(impls)
            for i in range(len(impls)):
                b = b + (impls[i] - mean) ** 2
            sigma = ((1.0 / float(len(impls) - 1)) * b) ** 0.5
            sdom = (sigma / float(len(impls))) ** 0.5
            print 'mean = ' + str(mean) + 'n' + 'b = ' + str(b) + 'n' + 'sigma = ' + str(sigma) + 'n' + 'percent dev. = ' + str(sigma / mean * 100) + '%' + 'n' + 'sdom = ' + str(sdom * 100.0) + 'n' + 'Impulse in N*s = ' + str(mean / 7 * 4.448) + 'n' + 'Sigma in N = ' + str(sigma / 7 * 4.448)

        addCycleDivisionPoints()
        dataPts.sort(key=lambda x: x[0])
        integrateAll()
        SN = standardDev()
        print impls

        with open("C:\Users\Brad\Desktop\testfiles\test.txt", "w") as f:
            f.writelines(str(SN) + 'n')
            f.writelines('t'.join(str(dataPts[j][i]) for i in range(3)) + 'n' for j in range(len(dataPts)))
```

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