PROPERTIES OF VERNIER ANTENNA

A graduate project submitted in partial fulfilment of requirements
For the degree of Master of Science in
Electrical Engineering

By

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California State University, Northridge
I wish to thank project chair and mentor Dr. Sembiam Rengarajan for his immense support for this project. I am also grateful to Dr. Radmanesh and Prof. Mallard for their suggestion, insights and review on this project. I am also thankful to California State University, Northridge for providing all facilities needed as a part to complete this project. This accomplishment would not have been possible, had I not received help for different complexities involved in this project from all esteemed faculties mentioned above.
DEDICATION

This graduate project is dedicated to my parents who have always supported me on every step.
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ABSTRACT

Properties of Vernier antenna

By

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Master of Science in Electrical Engineering

Antennas are structures that can either transmit or receive electromagnetic radiation. Either a single antenna or multiple antenna elements (antenna array) are used to transmit and receive electromagnetic energy. There are various types of designs available for antennas array. This project aims to design and study properties of a unique array design called Vernier array. This array consists of transmitting and receiving antennas. Spacing between two elements in transmit array is not equal to the spacing between two elements in receive array. Due to this, the multiplication of their radiation patterns would result in narrow main lobe with low side lobes. To obtain an optimized Vernier array design, key parameters contributing to the radiation pattern are varied. The Vernier antenna design presented in this project is a sparse array design. Sparse array has an advantage over other array designs; the number of elements required for an application is far less. Vernier array design is useful for applications, which involve both, transmit and receive array functionalities.
1 INTRODUCTION

A Vernier scale is an instrument that allows a user to have a precise measurement compared to a linear, undivided scale. It contains two identical reading scales that are offset to each other by a small amount. Due to this small offset, a user can calibrate the scale such that a precise reading is marked on the Vernier scale. Figure 1.1 clearly shows how the overlapping of two scales marks a reading. An advantage of this scale is that we get higher resolution because of the two scales i.e. in effect changing spacing between the two scales. An antenna array comprises of two or more antenna elements. Factors that contribute to radiation pattern are -the spacing between two consecutive elements, antenna type and antenna size or aperture. Different array designs are existent; each with their individual advantages and disadvantages. It is desirable to develop an optimized structure for a given application.

1.1 Objective and application

This project aims to study the Vernier type of array structure and perform simulation on the same to verify its usability in real world applications. The distance between two adjacent elements of transmit array and that between two adjacent elements of receive array are varied to improve resolution of reading. These transmit and receive arrays have uniformly spaced elements. Non-uniformly spaced arrays are not explored in this project. In principle, \( \lambda/2 \) spacing between array elements is necessary for prevention of any unwanted grating lobes. In the Vernier array, one can design an array with not only lower grating lobes but also less number of elements with a spacing greater than \( \lambda/2 \). The overall array factor of transmit array and receive array is the product of the two. Such array designs can be linear, two or even three-dimensional. However, this
project addresses linear array only. In the next few chapters, fundamentals are
discussed and design examples are simulated on Matlab software. The aim of this
project is to observe if grating lobes are sufficiently suppressed when the transmit and
receive patterns are multiplied. Some of the parameters that affect pattern
characteristics are

number of elements in an
array,

spacing between two
elements,

current excitations,

scan angle,

inter-element phase
difference.

After performing these
simulations and analyzing data, possible applications of Vernier array will be explored.

1.2 Introduction to antenna array

Arrays have gained popularity because they can control the shape and size of
the main beam as well as the sidelobe levels. Figure 1.2 is a diagram of the spherical
coordinate system for an antenna. For a linear array, radiation pattern depends on the
θ angle only; there is no dependency on the φ angle if array is along z-axis.

Antenna arrays are widely used in applications such as cellular
communication, radio astronomy, ultrasound, radar, satellite and deep space
communication and many more. The type of antenna used, number of antenna
elements and arrangement of these elements are application-specific. Different array
arrangements produce different radiation pattern. A single antenna element has a fixed
aperture size and directivity. Producing larger aperture is often desirable but this is
difficult with a single antenna due to size constraints. Hence, to achieve large
are utilized. Overall pattern of an array is calculated with the spatial coordinate system. Chapter 2 explains the formulae used for pattern calculation. These antenna arrays are directive, provide high gain and have a narrow beam width. Arrays can scan digitally, by phase shifting and by time gating.

Arrays are classified into different types based of various parameters, as shown in Table 1.1 below

Table 1.1 Classification of antenna array

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacing</td>
<td>Dense array</td>
</tr>
<tr>
<td>Current excitation</td>
<td>Uniform excitation</td>
</tr>
<tr>
<td>Arrangement</td>
<td>Linear</td>
</tr>
<tr>
<td>Periodicity</td>
<td>Periodic</td>
</tr>
</tbody>
</table>

Table 1.1 lists the classification of array based on different factors. This Uniform and non-uniform current excitations in linear array are explored and results
are compared against one another. Some important terminologies that we come across in this project are discussed below

**a) Radiation pattern**

The radiation pattern of an antenna consist of a main lobe, side or secondary lobes and back lobe. It is a graphical representation of the directional characteristic of the radiation from an antenna. Main lobe appears in the direction of highest radiation energy. Radiation pattern depends on parameters such as – the number of elements, spacing, excitation, type of element and the medium surrounding the antenna. The radiation pattern represents the beam width and directivity of the antenna array.

**b) Array factor**

This is a factor used to calculate radiation pattern of isotropic elements. By multiplying the array factor of an array with the element pattern of a single element one can obtain the entire radiation pattern plot of an array antenna. Array factor is independent of the type of antenna used as it is calculated by replacing each element by an isotropic element. Chapter 2 lists formulas for array factor.

**c) Grating lobes**

Grating lobes are undesired lobes that appear in direction other than that of the main lobe. Figure 1.3 illustrates the lobes in a radiation pattern. The issue with grating lobe is that they can have amplitude almost equal to that of the main lobe. In an application such as the radar, the chances of false detection are high due to false signal that the grating lobe might be detecting. Grating lobes occur if the spacing between two adjacent elements is more than $\lambda/2$. In the Vernier antenna design, the spacing between two elements in transmit and receive arrays is greater than $\lambda/2$ as it is a sparse array design. Grating lobes are expected to be reduced in the multiplication of the transmit and receive array patterns.
1.3 Concept of Vernier array

In many applications such as cell phones or deep space communication, signals are sent and/or received over one or many antenna elements. If ‘N’ is the total number of transmit and receive elements, then a method needs to be developed so that the array performs one function at a given time. Either time gating or phase shifting the array excitations achieves transmit-receive functions. The spacing between adjacent elements in such arrays ≤ λ/2. These arrays suffer from disadvantage of ‘mutual coupling’, which is caused by electromagnetic interaction between elements. Mutual coupling affects the pattern of array and causes mismatch at the load.

Some research has been conducted on the concept of Vernier array [3]. This project aims to study the characteristics of Vernier arrays in which the spacing between transmit elements is p*d and the spacing between receive array is (p-1) * d.

The value of p is an integer greater than 2 and d is the Vernier resolution.

As discussed previously, to avoid grating lobe conditions, d= λ/2 will be assumed for all future calculations and simulations.
Figure 1.4 illustrates the general design of a Vernier array. The pattern of the transmit array of spacing p*d is multiplied by the pattern of the receive array of spacing (p-1) *d to obtain an effective array of spacing d. A well-designed Vernier array should have radiation pattern similar to that of a dense array of λ/2 spacing.

If these conditions are satisfied, then it can be proved that sparse array designs can replace applications using dense arrays. A designer can utilize advantages of Vernier array such as less number of elements compared to dense array design, less mutual coupling and no need for time gating. These advantages and disadvantages are weighed in detail, along with design simulation to support the results in following chapters.

Aperture distributions such as cosine distribution, cosine squared distribution, uniform distribution and cosine over pedestal distribution are studied. Combinations of these distributions are attempted to design an optimized array.
OVERVIEW OF ANTENNA ARRAYS

Some general discussion of antenna arrays, including Vernier array, was presented in Chapter 1. The formulas needed for simulation of array factor are explained in this chapter.

2.1 Design of arrays

Figure 2.1 shows the geometry of elements in an array based on which pattern calculation for each element is derived. From Figure 2.1, the general expression for array factor for N elements is constructed. Expression for array factor is -

\[ \text{AF} = 1 + I_1 e^{i(kd \cos \theta + \beta)} + I_2 e^{i2(kd \cos \theta + \beta)} + ... + I_n e^{i(N-1)(kd \cos \theta + \beta)} \]  \[ [2.1] \]

where,
AF = array factor
k = wave number = \((2\pi)/\lambda\)
$I_n =$ currents as per distribution type ($n=1, 2, 3...$)

d= spacing between two consecutive elements

$\theta =$ range of angle (0 to 180$^\circ$)

$\beta =$ phase difference between two consecutive elements

A simplified expression for uniform distribution is

$$AF = \sum_{n=1}^{N} e^{i(n-1)\Psi}$$  \hspace{1cm} [2.2]

where,

$\Psi = kd \cos \theta + \beta$

Since these array elements lie on z-axis, the array factor will depend on $\theta$ and not on $\phi$. For a Vernier array, the array factor equation for transmit and receive arrays is different because of different values of spacing and excitations.

calculation:

$N_t = 7$ (Transmit array),

$N_r = 3$ (Receive array),

$p=2$ and

d=$\lambda/2$

Transmit array spacing= $p*d = \lambda$ and

receive array spacing= $(p-1)*d= \lambda/2$.

The array factor is normalized to a peak value of one and then a polar plot is obtained using Matlab.
The main lobe is in the broadside direction if θ=90º whereas it will be in the end fire direction if θ = 0 or 180º. We can clearly observe grating lobes and secondary lobes in both the transmit and receive array patterns in Figure 2.2 (three elements) and Figure 2.3 (seven elements).

An important observation here is grating lobes appear at different directions in each plot. Therefore, the multiplication of these two plots would result it suppressed grating lobes. This is a desired result for a Vernier array.

Figure 2.4 show the two plots in dB scale where the grating lobes are seen.

These array plots are representative plots to explain how grating lobes occur in transmit and receive arrays. Aim of this project is to eliminate these grating lobes seen in Figure 2.2 and Figure 2.3.
2.2 Weighting method

Tapered currents distributions are applied to elements in an array to reduce side lobes. This is referred as ‘apodizing’ the array and the method is called ‘weighting method’. By weighting the elements, one can have control on beam width and sidelobe level of an array.

After accounting for the weights in an array, the expression for array factor is

\[
AF = w^T v(k) = \sum_{n=0}^{N-1} e^{i(kd \cos \theta - \cos \theta_0)}
\]  \[2.3\]

where,

\( w^T = \) weight of an element at position \( n = e^{i \pi \cos \theta_0} \)

\( v(k) = \) Steering vector

If an array were steered by 45° then the matrix of weights would be

\[
w = \begin{bmatrix}
1 & e^{j \pi \sqrt{2}/2} & e^{j \pi \sqrt{2}} & e^{j3 \pi \sqrt{2}/2} & e^{j4 \pi \sqrt{2}/2}
\end{bmatrix}
\]
In the Vernier array, different side lobe levels are obtained for different tapered distributions.

2.3 Types of aperture distributions

Performance of Vernier design is studied by using four different types of current distribution. These distributions are – uniform, cosine, cosine squared and cosine over a pedestal distribution. The choice of number of transmit and number of receive elements, for all distribution types, is based on ‘sparse array design’ US patent paper by G. Lockwood [3]. Appendix C details the Matlab code for the same.

a) Uniform distribution

Simulation results for uniform distribution are presented in this section. Expected side lobe level for uniform distribution is -13.3 dB below the main lobe. For uniform distribution, current \( I(z) \) in each element is

\[
I(z) = \begin{cases} 
1 & \text{for } -\frac{L}{2} \leq z \leq \frac{L}{2} \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} [2.4]

\textit{calculation:}

For \( p=3 \)

\( N_t = 3 \)

\( N_r = 8 \)

Stem plot for uniform distribution Figure 2.5. The plot shows current value of each element as a function of position.

\textit{calculation:}

For \( p=3 \)

\( N_t = 13 \)

\( N_r = 9 \)

Simulation result for \( N_t = 13 \) and \( N_r = 9 \) is shown in Figure 2.6.
Figure 2.5 Convolution plot for array A = 3 elements, array B = 8 elements

Figure 2.6 Convolution plot for array A = 13 elements, array B = 9 elements
Interpretations of Vernier array uniform distribution plots:

i. In Figure 2.5, one element is missing at both ends of the convolution plot. This missing element causes increase in secondary lobes in radiation pattern.

ii. In Figure 2.6, one element is missing at both ends of the convolution plot. In addition to that, the overall convolution plot is irregular in shape. This further contributes to formation of large secondary lobes and grating lobes.

iii. The spacing between two adjacent transmit elements is \( p*d \) and the spacing between two adjacent receive elements is \( = (p-1)*d \).

iv. The difference between Figure 2.5 and Figure 2.6 is number of elements. In the first case number of transmit elements \( \leq p \); in second case transmit elements \( \geq p \).

v. The two cases illustrated in Figure 2.5 and Figure 2.6 show how selection of number of transmit-receive elements and spacing between two adjacent elements affects the convolution pattern.

b) Cosine distribution

Current distributions such as cosine, cosine squared and cosine over a pedestal are called ‘tapered distributions’. Figure 2.7 shows the stem plot for cosine distribution. In tapered current distribution, the side lobes and the grating lobes are lowered because amplitude of current tapers down at the ends of the array.
For cosine current distribution

\[ I(z) = \begin{cases} 
\cos \left( \frac{\pi z}{L} \right) & \text{for } -\frac{L}{2} \leq z \leq \frac{L}{2} \\
0 & \text{otherwise} 
\end{cases} \]  \hspace{1cm} [2.5]

calculations:

For \( p=3 \)

\( N_t = 13 \)

\( N_r = 9 \)

**Interpretations of Vernier array cosine distribution plots:**

i. Compared to uniform distribution, the cosine pattern has a smooth aperture and it does not have any missing elements at the end of the array. Hence, occurrence of side lobes and grating lobes is reduced.

ii. For a single array, the side lobe level desired for this type of current distribution is at = -23 dB level below the main lobe.
c) **Cosine squared distribution**

Cosine squared distribution has severe tapering at the ends of array as seen in Figure 2.8, thereby reducing the side lobe levels further.

For cosine squared type distribution

\[
I(z) = \begin{cases} 
\cos^2 \left( \frac{\pi z}{L} \right) & \text{for } -\frac{L}{2} \leq z \leq \frac{L}{2} \\
0 & \text{otherwise}
\end{cases}
\]  

[2.6]

Example of Vernier array:

Calculations:

Number of transmit and receive elements are the same as the ones used to calculate the results of cosine distribution (where \( N_t = 13 \) and \( N_r = 9 \)). The results for cosine squared distributions are shown in Figure 2.8

![Figure 2.8 Convolution of cosine squared pattern](image)
Interpretations from Vernier array cosine squared distribution plots:

i. The side lobe level expected, for a single array, for this distribution is even lower than cosine distribution at = -31.7 dB.

ii. Side lobe level depends on the type of current distribution and not on the number of elements present in an array.

iii. A trade-off of severely tapered distribution is that even though the side lobe levels decrease, the beam width increases. For applications such as radar, if a narrow directional beam were required, then this type of distribution would prove to be ineffective.

d) Cosine over a pedestal distribution

This type of distribution is a cosine distribution added on a uniform distribution. Slight elevation of uniform distribution is seen in Figure 2.9.

For cosine over pedestal distribution

\[
I(z) = \begin{cases} 
C + (1 - C) \frac{\cos \pi z}{L} & \text{for } -\frac{L}{2} \leq z \leq \frac{L}{2} \\
0 & \text{otherwise}
\end{cases}
\]  \quad \text{[2.7]}

where

\( C = 0.31, 0.1778, 0 \)

calculations:

For \( C = 0.3162 \) and \( N_t = 13, N_r = 9 \)
Interpretations from Vernier array cosine over a pedestal distribution plots:

i. Cosine over pedestal is as an intermediate case between cosine and cosine squared distribution. A side lobe level of -20 dB (for $C = 0.3162$) is expected for a single array.

ii. While calculating the array factor, the element pattern contributes to total calculation, but it is not accounted for here as it becomes insignificant for broadside case.

In this section, different current distribution patterns and convolution of different patterns were considered. In the next section of this chapter, array factor of line source is simulated on Matlab. In addition to that, next section verifies the expected side lobe value for uniform, cosine, cosine squared and cosine over a pedestal type distribution.
2.4 Array factor and side lobe levels for current distributions

Table 2.1 lists the formulas for array factor, side lobe and power beam-width. Table 2-1 is cited from ‘Antenna Theory and Design’ – Stutzman [1; H.6: Hansen, Vol. I, Chap. 1; H.3: Silver, p. 187].

Table 2.1 Uniform and tapered distributions

<table>
<thead>
<tr>
<th>n</th>
<th>HP (rad)</th>
<th>(dB)</th>
<th>D/D_o</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.886 L</td>
<td>-13.3</td>
<td>1.00</td>
<td>Uniform line source</td>
</tr>
<tr>
<td>1</td>
<td>1.19 L</td>
<td>-23.0</td>
<td>0.810</td>
<td>Cosine taper</td>
</tr>
<tr>
<td>2</td>
<td>1.44 L</td>
<td>-31.7</td>
<td>0.667</td>
<td>Cosine-squared taper</td>
</tr>
</tbody>
</table>

\[ I(z) = C + (1-C) \cos \frac{\pi z}{L} \]

\[ f(u) = \frac{C \sin \frac{u}{2} + (1-C) \frac{2}{\pi} \frac{\cos u}{1-(2u/\pi)^2}}{C + (1-C) \frac{2}{\pi}} \]
Table 2-1 Uniform and tapered distributions (Continued)

<table>
<thead>
<tr>
<th>Edge Illumination</th>
<th>$-20 \log C$ (dB)</th>
<th>HP (rad)</th>
<th>Side Lobe Level (dB)</th>
<th>$D/D_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3162</td>
<td>-10</td>
<td>$1.03 \frac{\lambda}{L}$</td>
<td>-20</td>
<td>0.92</td>
</tr>
<tr>
<td>0.1778</td>
<td>-15</td>
<td>$1.08 \frac{\lambda}{L}$</td>
<td>-22</td>
<td>0.88</td>
</tr>
<tr>
<td>0</td>
<td>$-\infty$</td>
<td>$1.19 \frac{\lambda}{L}$</td>
<td>-23</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Matlab simulation results:

**a) For uniform distribution**

Plotting the radiation pattern expression $f(u)$ from the Table 2-1, where

$$u = \frac{\beta L}{2} (\cos \theta - \cos \theta_0)$$

Figure 2.10 Array factor for uniform distribution

From Figure 2.10, the side lobe level of -13.3 dB approximately matches the expected value as mentioned in Table 2.1.
b) For cosine distributions

This section plots radiation pattern expression $f(u)$ for cosine distribution from Table 2-1.

![Array factor calculation for Cosine Distribution](image)

**Figure 2.11 Array Factor for cosine distribution**

From Figure 2.11, the side lobe value of -23.01 dB is approximately equal to the expected value given in Table 2.1.

c) **Cosine squared distribution**

Similar to the previous results, cosine squared distribution plot is verified for side lobe levels.
Simulation result is shown in Figure 2.12. Severe tapering observed in the distribution plot for cosine squared distribution.

**d) Cosine over a pedestal distribution**

Plotting the pattern $f(u)$ from Table 2-1. Figure 2.13 displays results as below.

Figure 2.12 Array factor for cosine squared distribution

Figure 2.13 Array factor of cosine over a pedestal distribution
3 SIMULATING VERNIER ARRAY

To develop a robust design, this project considers combinations of current distributions. Testing for different scan angles further examines the usefulness of the array.

![Vernier array design](image)

**Fixed Aperture Design**
- i. Uniform distribution
- ii. Cosine distribution (n=1)
- iii. Cosine squared distribution (n=2)
- iv. Cosine over a pedestal distribution

Number of elements calculated as per aperture length and value of spacing (dependent on ‘p’)

**Variable Aperture Design**
- i. Uniform distribution
- ii. Cosine distribution (n=1)
- iii. Cosine squared distribution (n=2)
- iv. Cosine over a pedestal distribution

Case 1: 3 Transmit and 8 receive element
Case 2: 13 Transmit and 9 receive element

Figure 3.1 Vernier arrays simulated

Figure 3.1 describes different simulations studied in this work. Fixed aperture and variable aperture design facilitates the change of the aperture length and/or number of elements. Many combinations of inter-element spacing and current distribution are studied. Matlab codes for variable and fixed aperture design are given in Appendix A and Appendix B respectively. More features and details of the design plan are discussed.
3.1 Array factor of fixed aperture length

In fixed aperture design, the length of transmit array and receive array aperture are equal but the number of elements in the array vary as a function of spacing between two adjacent transmit or receive elements.

Features of fixed aperture array

a) The length of fixed aperture selected for simulation is $= 20 \lambda$.
b) As the value of ‘p’ changes, the distance between two adjacent elements changes. Due to this change in spacing, the number of elements in transmit and in receive array vary over aperture length.
c) As an effect of change in spacing, the radiation pattern of the array is affected.
d) Ideally, the total array factor should have low side lobe values for varying scan angles.
e) Transmit and receive pattern were generated from the current distribution results which were discussed in previous chapters.

Simulation:

Some representative simulations are discussed in this section. These results are significant due to their current distribution properties and radiation pattern. Polar plots for both fixed and variable aperture show plot of array factor as a function of theta.

Value of ‘p’ and scan angle is the input in Matlab code.

a) For p=2, Cosine over pedestal current distribution, scan angles from 0 to 90°, length of aperture = 20 $\lambda$. 
Figure 3.2 Cosine over pedestal distribution, multiplied array factor, p=2, fixed aperture length = 20*lambda, scan angle = 0°

Figure 3.3 Cosine over pedestal distribution, polar plot p=2, fixed aperture length = 20*lambda, transmit and receive array, scan angle = 0°
Figure 3.4 Cosine over pedestal distribution, side lobe levels, p=2.5, fixed aperture length = 20*lambda, transmit and receive array, scan angle = 0°

Results for cosine over pedestal are from Figure 3.2 through 3.4.
b) For $p=3$, cosine current distribution, scan angles from 0 to 90°. Length of aperture = 20 $\lambda$.

Figure 3.5 Cosine current distribution, polar plot, $p=3$, fixed aperture length = 20$\lambda$, transmit and receive array, scan angle = 60°

Figure 3.6 Multiplied array factor, cosine current distribution, polar plot, $p=3$, fixed aperture length = 20$\lambda$, scan angle = 60°

Figure 3.5 and Figure 3.6 show scanning condition for cosine distribution.
c) For $p=4$, uniform current distribution, scan angles from $0$ to $90\,^\circ$. Length of aperture $= 20\lambda$.

Figure 3.7 Uniform current distribution, polar plot $p=4$, fixed aperture length $= 20\lambda$, transmit and receive array, scan angle $= 60^\circ$.

Figure 3.8 Uniform current distribution, multiplied array factor $p=4$, fixed aperture length $= 20\lambda$, transmit and receive array, scan angle $= 60^\circ$. 
Figure 3.7 and Figure 3.8 plots for uniform current distribution. Large secondary lobes are seen in Figure 3.8.

d) For p=4, cosine squared distribution, scan angles from 0 to 90°. Length of aperture = 20 \lambda

![Figure 3.9 Cosine squared distribution, polar plots, p=4, fixed aperture length = 20\lambda, transmit and receive array, scan angle = 30°](image)

Figure 3.9 Cosine squared distribution, polar plots, p=4, fixed aperture length = 20\lambda, transmit and receive array, scan angle = 30°

![Figure 3.10 Cosine squared distribution, multiplied array factor, grating lobe, p=4, fixed aperture length = 20\lambda, transmit and receive array, scan angle = 30°](image)

Figure 3.10 Cosine squared distribution, multiplied array factor, grating lobe, p=4, fixed aperture length = 20\lambda, transmit and receive array, scan angle = 30°
3.2 Inference from fixed aperture simulation

a) Given a type of current distribution, as the value of ‘p’ increases, the level at which secondary lobe occurs also increases. This means that the results worsen as the value of ‘p’ increase.

b) Cosine squared distribution show lowest side lobe level in the multiplication pattern. Uniform distribution shows highest side lobe level. Cosine over a pedestal falls as an intermediate current distribution type between cosine and uniform distribution.

c) Individual array patterns i.e. transmit array and receive array, are compliant with the expected side lobe level as calculated in Chapter 2.

d) A threshold of -26 dB is set determine if a distribution type is suitable for practical applications.

3.3 Array factor of variable aperture length

For fixed aperture, the length of the aperture of transmit-receive array is ‘L’. This value ensures that the aperture length is same. However, in this section, the length of the aperture varies. This is because number of elements in transmit-receive array are now given as input in Matlab code, unlike aperture length from previous case. This section verifies performance of array over wide range of aperture lengths. These side lobe levels crosscheck with the threshold values calculated from chapter 2. Scan conditions calculated from angles between 0 and 90°. Comparison between variable and fixed aperture reveals which method provides lower side lobes and grating lobes.
a) For $p=2$, cosine over pedestal current distribution, scan angles from 0 to $90^\circ$, number of transmit elements = 9, number of receive elements = 13

Figure 3.11 Cosine over a pedestal distribution, polar plot, $p=2$, transmit = 9 elements, receive= 13 elements, scan angle =$45^\circ$

Figure 3.12 Cosine over a pedestal, multiplied array factor, $p=2$, transmit = 9 elements, receive= 13 elements, scan angle =$45^\circ$
Wide difference in length of transmit-receive array results in lower secondary lobes with cosine over a pedestal distribution. This is seen in Figure 3.11 through 3.12.

b) For $p=3$, cosine squared current distribution, scan angles from 0 to 90°, number of transmit elements = 9, number of receive elements = 13

![Figure 3.13](image1.png)

**Figure 3.13** Cosine squared distribution, polar plot, $p=3$, transmit = 9 elements, receive= 13 elements, scan angle =45°

![Figure 3.14](image2.png)

**Figure 3.14** Cosine squared distribution, multiplied array factor, $p=3$, transmit = 8 elements, receive= 2 elements, scan angle =45°
Figure 3.13 and Figure 3.14 show that results for cosine squared distribution. Very low side lobe level is observed from the results.

### 3.4 Inference from variable aperture simulation

a) Similar to fixed aperture, as the value of ‘p’ increases the value of secondary lobes increases i.e. it shows poorer performance.

b) Larger difference between transmit and receive aperture length results in formation of major side lobes. This is a disadvantage of Vernier array design.

c) Simulations results indicate that, for wide difference between aperture lengths between transmit and receive array, cosine over a pedestal distribution gives the lowest side lobe value. This distribution is suitable for applications where number of elements can be less, without having to compensate on side lobe levels.

### 3.5 Summary

Simulation results from fixed aperture design indicate that, the number of elements in an array is inversely proportional to spacing between two adjacent elements in an array. Figure 3.15 shows a graph showing this inverse trend.

Figure 3.16 plots a graph of side lobe values for different types of distribution as a function of variable ‘p’. All values of side lobe below -26 dB threshold are desired results for any Vernier antenna design. These results are measured for constant scan angle = 60°. All other scan angles show a similar trend as a function of ‘p’. Depending on application requirements, a user can select value of ‘p’ and side lobe level from Figure 3.16.
Figure 3.15 Number of elements \((N_R+N_T)\) as a function of ‘p’ for fixed aperture case.

Figure 3.16 Plot summarizing result of side lobe levels as a function of value of ‘p’.
4 APPLICATION OF VERNIER ANTENNA

4.1 Exploring Vernier array applications

The simulations performed in Chapter 3 summarize different current distribution combination for an array. Based on these results, appropriate type of current distribution and number of transmit/receive elements in an array can be determined for an application. Every design method should be tested for application usability. Inspiration for application design is taken from IEEE paper on GLISTIN type radar. This paper proposes to have a wide beam transmit and a narrow beam receive array. The Vernier array design method to design a GLISTIN type radar is presented in this chapter.

4.2 Vernier array design of GLISTIN type radar

To design an application similar to the GLISTIN radar, wide beam transmit array and a narrow beam receive array needs to be designed. Wide beam is observed in tapered distribution, while on the other hand narrow beam is observed in uniform distribution. Two tests cases are simulated to determine the appropriate combination of current distribution such that desired result is obtained.

Simulation: Test 1

In this test, pattern of two elements in transmit array is multiplied with the pattern of 20 elements in receive array. Uniform distribution is applied to the receive array and cosine squared distribution is applied to the transmit array. After testing for different scan angles, major grating lobes occur for all scan angles. Hence, this design is not useful.

Simulation: Test 2

The reason for occurrence of grating lobes in the first test is due to the differences in the aperture length of transmit and receive array and possibly due to the choice of current distribution. In this test, 20 receive and five transmit elements are chosen. After performing simulation at p=3, highest side lobe appears at -33.59 dB. For this test case, cosine over a pedestal type of distribution is applied to both transmit and receive array. The result of multiplication of transmit and receive pattern is illustrated in Figure 3.17.
Figure 3.17 Cosine over a pedestal distribution, total array factor, \( p=3 \), transmit = 5 elements, receive = 20 elements, scan angle = 60°

Wide ranges of test conditions were studied and different results were obtained. Cosine over a pedestal is an appropriate distribution pattern for this application. Results indicate that occurrence of grating lobes is high if two different types of current distributions are multiplied.
5 CONCLUSION

Vernier array design is successful in eliminating grating lobes which are otherwise seen in a single array. In order to produce adequately low side lobe levels, various combinations of inter-element spacing, current distribution and ‘p’ values were studied. To verify if this design can replace existing array design, a radar application is tested with Vernier design. Simulation results indicate that, as the value of ‘p’ increases the performance of side lobe level degrades. Cosine squared distribution shows lowest side lobe value while on the other hand uniform distribution shows poor (higher) side lobe values. Cosine over a pedestal distribution is most suited for wide difference in transmit-receive aperture length.

Further work on this design can be pursued. Cases such as – non-linear arrangement, two/three dimensional designs are suggested as a future scope to this project.
REFERENCES


APPENDIX A

Matlab code for variable aperture

%%ARRAY FACTOR FOR TRANSMIT AND RECEIVE ANTENNAS - VARIABLE APERTURE
clc;
clear vars;
clear all;
close all;
c0=3.*(10.^8); %SPEED OF LIGHT
f=1.*(10.^9); %FREQUENCY
lambda=c0./f; %WAVELENGTH
p=3.5;
d=lambda/2; %EFFECTIVE SPACING OF BOTH ARRAYS IN MULTIPLICATION
a=(p-1).*d; %SPACING FOR - Array A
b=p.*d; %SPACING FOR - Array B
round1=2;
round2=8;
%CALCULATION OF NUMBER OF ELEMENTS OVER 15*LAMBDA WAVELENGTH
x1=-((round1-1).*a)./2:a:((round1-1).*a)./2;
x2=-((round2-1).*b)./2:b:((round2-1).*b)./2;
%cOSINE PATTERN (n=1)
i1c=(cos((pi.*(x1))./((round1-1).*a)));
i2c=(cos((pi.*(x2)./((round2-1).*b))));
ic=conv(i1c,i2c,'full');
%cOSINE SQUARED CURRENT DISTRIBUTION(n=2)
i1cs=cos((pi.*(x1))./((round1-1).*a)).^2;
i2cs=cos((pi.*(x2))./((round2-1).*b)).^2;
ics=conv(i1cs,i2cs,'full');
%cOSINE OVER PEDESTAL CURRENT DISTRIBUTION
C=0.3162;
i1cp=C+(1-C).*(cos((pi.*x1)./((round1-1).*a)));
i2cp = C + (1 - C) .* (cos((pi.*x2)./(round2 - 1).*b));
icp = conv(i1cp, i2cp, 'full');

%% Plotting stem current patterns with convolution
figure(1)
subplot(3,1,1)
stem(x1,i1c)
title('ARRAY A COSINE CURRENT DISTRIBUTION')
subplot(3,1,2)
stem(x2,i2c)
title('ARRAY B COSINE CURRENT DISTRIBUTION')
subplot(3,1,3)
stem(1:length(ic), ic)
title('CONVOLUTION RESULT OF ARRAY A & B - COSINE CURRENT DISTRIBUTION')

figure(2)
subplot(3,1,1)
stem(x1,i1cs)
title('ARRAY A COSINE SQUARED CURRENT DISTRIBUTION')
subplot(3,1,2)
stem(x2,i2cs)
title('ARRAY A COSINE SQUARED CURRENT DISTRIBUTION')
subplot(3,1,3)
stem(1:length(ics), ics)
title('CONVOLUTION OF ARRAY A & B - COSINE SQUARED CURRENT DISTRIBUTION')

figure(3)
subplot(3,1,1)
stem(x1,i1cp)
title('ARRAY A COSINE OVER PEDESTAL CURRENT DISTRIBUTION')
subplot(3,1,2)
stem(x2,i2cp)
title('ARRAY A COSINE OVER PEDESTAL CURRENT DISTRIBUTION')
subplot(3,1,3)
stem(1:length(icp), icp)
title('CONVOLUTION OF ARRAY A & B - COSINE OVER PEDESTAL CURRENT DISTRIBUTION')

%% FOR RECEIVING ANTENNA - ARRAY FACTOR CALCULATION
beta=(2.*pi)./lambda;
theta0=45.*pi./180;    %SCAN ANGLE
AFr1=zeros(1,360);
AFr2=zeros(1,360);

%% CALCULATING ARRAY FACTOR FOR RECEIVING ARRAY
for theta=1:360
    for n=1:round1
        AFr1(theta)=AFr1(theta)+(i1cs(n).*exp(1j.*beta.*(n).*a.*(cos((theta.*pi)./180)-cos(theta0))));
    end
end

%% CALCULATING ARRAY FACTOR FOR TRANSMITTING ARRAY
for theta=1:360
    for n=1:round2
        AFr2(theta)=AFr2(theta)+(i2cs(n).*exp(1j.*beta.*(n).*b.*(cos((theta.*pi)./180)-cos(theta0))));
    end
end

AF=AFr1.*AFr2;
afr1_norm=AFr1./(max(abs(AFr1)));
afr2_norm=AFr2./(max(abs(AFr2)));
af_norm=AF./(max(abs(AF)));
test1=20+20.*log10(abs(afr1_norm));
test2=40+20.*log10(abs(afr2_norm));
test3=60+20.*log10((abs(af_norm)));
for theta=1:360
    if test1(theta)<=-1
        test1(theta)=0;
    end
    if test2(theta)<=-1
        test2(theta)=0;
    end
end
end
if test3(theta)<=-1
    test3(theta)=0;
end
AF_conv=conv(AF_{r1},AF_{r2},'full');
theta_plot=1:360;
figure
subplot(1,2,1)
polar(theta_plot.*pi./180,test1)
title('For array A - Receive Array')
subplot(1,2,2)
polar(theta_plot.*pi./180,test2)
title('For array B - Transmit Array')
figure
polar(theta_plot.*pi./180,test3)
title('total Array Factor for given current distribution')
figure
subplot(3,1,1)
plot(theta_plot,(20.*log10(abs(afr1_norm))))
ylim([-40 5])
title ('Array Factor Plot for Receive Distribution')
subplot(3,1,2)
plot(theta_plot,(20.*log10(abs(afr2_norm))))
ylim([-40 5])
title ('Array Factor Plot for Transmit Distribution')
subplot(3,1,3)
plot(theta_plot,(20.*log10(abs(af_norm))))
ylim([-70 5])
title ('Array Factor Plot for given Distribution')
Matlab code for fixed aperture

%ARRAY FACTOR FOR TRANSMIT AND RECEIVE ANTENNAS
clc;
clear all;
close all;
c0=3.*(10.^8);  %SPEED OF LIGHT
f=1.*(10.^9);   %FREQUENCY
lambda=c0./f;   %WAVELENGTH
L=20.*lambda;  %LARGEST ARRAY LENGTH
p=4;          %VARIABLE THAT DETERMINES SPACING
d=lambda./2;  %EFFECTIVE SPACING OF BOTH ARRAYS IN MULTIPLICATION
b=p.*d;       %SPACING FOR - Array A
a=(p-1).*d;   %SPACING FOR - Array B
no_elements1=round(L./a);
no_elements2=round(L./b);

%CALCULATION OF NUMBER OF ELEMENTS OVER 20*LAMBDA WAVELENGTH
x1=-L./2:(L./(no_elements1-1)):L./2;
x2=-L./2:(L./(no_elements2-1)):L./2;

%COSINE OVER PEDESTAL CURRENT DISTRIBUTION
C=0.3162;
i1cp=C +(1 – C) * (cos((pi.*x1)./L));
i2cp=C +(1 - C) * (cos((pi.*x2)./L));

%COSINE PATTERN (n=1)
i1c=(cos(pi.*x1./L));
i2c=(cos(pi.*x2./L));

%COSINE SQUARED CURRENT DISTRIBUTION(n=2)
i1cs=(cos((pi.*x1)./L)).^2;
i2cs=(cos((pi.*x2)./L)).^2;

%FOR RECEIVING ANTENNA - ARRAY FACTOR CALCULATION

%APPENDIX B
beta=(2.*pi)./lambda;
theta0=60.*pi./180;   %SCAN ANGLE
AFr1=zeros(1,360);
AFr2=zeros(1,360);
for theta=1:360;
    for n1=1:no_elements1
        AFr1(1,theta)=AFr1(1,theta)+(1.*exp(1j.*beta.*(n1).*a.*(cos((theta.*pi)./180)-cos(theta0))));
    end
end
for theta=1:360;
    for n2=1:no_elements2
        AFr2(1,theta)=AFr2(1,theta)+(1.*exp(1j.*beta.*(n2).*b.*(cos((theta.*pi)./180)-cos(theta0))));
    end
end
AF=AFr1.*AFr2;
theta_plot=1:360;
afr1_norm=AFr1./max(abs(AFr1));
afr2_norm=AFr2./max(abs(AFr2));
af_norm=AF./max(abs(AF));
test1=50+20.*log10(abs(afr1_norm));
test2=30+20.*log10(abs(afr2_norm));
test3=70+20.*log10((abs(af_norm)));
for theta=1:360
    if test1(theta)<=-1
        test1(theta)=0;
    end
    if test2(theta)<=-1
        test2(theta)=0;
    end
    if test3(theta)<=-1
        test3(theta)=0;
    end
end
figure
subplot(1,2,1)
polar(theta_plot.*pi./180,test1)
title('For Receive array')
subplot(1,2,2)
polar(theta_plot.*pi./180,test2)
title('For Transmit array B')
figure
polar(theta_plot.*pi./180,test3)
title('Total Array Factor for given current distribution')
figure
subplot(3,1,1)
plot(theta_plot,(20.*log10(abs(afr1_norm))))
ylim([-40 5])
title ('Array Factor for Transmit - Plot for given Distribution')
subplot(3,1,2)
plot(theta_plot,(20.*log10(abs(afr2_norm))))
ylim([-40 5])
title ('Array Factor for Receive - Plot for given Distribution')
subplot(3,1,3)
plot(theta_plot,(20.*log10(abs(af_norm))))
ylim([-70 5])
title ('Total Array Factor Plot for given Distribution')
Matlab code for stem plots for uniform distribution

```matlab
%% Clearing
clear vars;
close all;
clc;

%%% Array for A
nA1 = 5; A1 = zeros(nA1,1);
for i = 1:3
    A1(2*i-1,1) = 1;
end

nA2 = 25; A2 = zeros(nA2,1);
for i = 1:13
    A2(2*i-1,1) = 1;
end

%%% Array for B
nB1 = 22; B1 = zeros(nB1,1);
for i = 1:8
    B1(3*i-2,1) = 1;
end

nB2 = 26; B2 = zeros(nB2,1);
for i = 1:9
    B2(3*i-2,1) = 1;
end

%%% Convolution
C1 = conv(A1,B1);
C2 = conv(A2,B2);

%%% Plots
x1 = 0:1:size(C1,1);
figure(1);
subplot(3,1,1);
stem(x1(1:size(A1,1)),A1);
title('Stem plot for array A')
```

APPENDIX C
subplot(3,1,2);
stem(x1(1:size(B1,1)),B1);
title('Stem plot for Array B')
subplot(3,1,3);
stem(x1(1:size(C1,1)),C1);
title('Stem convolution of array A & B - elements missing at the end')
x2 = 0:1:size(C2,1);
figure(2);
subplot(3,1,1);
stem(x2(1:size(A2,1)),A2);
title('Stem plot for array A')
subplot(3,1,2);
stem(x2(1:size(B2,1)),B2);
title('Stem plot for Array B')
subplot(3,1,3);
stem(x2(1:size(C2,1)),C2);
title('Stem convolution of array A & B - step pattern observed')