CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

Phase-Locked Loop

Receiver Characteristics

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by

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LIST OF SYMBOLS

A	rms signal amplitude into loop, volts.
A*	receiver attenuation factor.
B_L	one sided noise bandwidth.
C, C(t)	AGC control signal, volts.
C	capacitance, farads.
C(s)	AGC closed-loop transfer function.
d(t)	doppler phase function on input signal, rad.
D(s)	doppler phase function in s-domain.
e	VCO tuning bias, volts.
E[ ]	statistical mean value operation.
f	frequency variable, cps.
f_1	first IF frequency, cps.
f_2	second IF frequency, cps.
f_{h_1}	first-mixer hetrodyne frequency, cps.
f_{h_2}	second-mixer hetrodyne frequency, cps.
F	d-c gain of loop filter.
F(s)	linear loop-filter transfer function.
H(s)	feedback transfer function of servo control.
j	$\sqrt{-1}$, the imaginary unit.
K	overall loop gain, volts^{-1}sec^{-1}.
K_1	rms VCO signal output, volts.
K_A	receiver AGC attenuation, dB/volt.

K_{AGC}	AGC-loop equivalent loop gain.
K_c	AGC amplifier gain.
K_d	phase detector gain, volts/rad.
K_D	AGC detector gain, volts peak out/volts rms in.
K_f	F(s).
K_{I_1}	first IF gain.
K_{I_2}	second IF gain.
K_m	mixer gain, volts^{-1}.
$K_n(s)$	programmable counter transfer function.
$K_o(s)$	VCM transfer function.
K_R	adjusted receiver attenuation with no AGC, dB.
K_v	VCO gain constant, rad/sec-volt.
K_{rec}	actual receiver attenuation with no AGC, dB.
ℓ	limiter rms output level, volts.
L	loop order.
m(t)	modulation phase function, rad.
M	VCO output frequency multiplication factor.
M_1	internal oscillator frequency multiplication.factor.
$n_o(t)$	receiver input noise waveform, volts.
$n_i(t)$	loop input noise waveform, volts.
n(t)	signal loop baseband noise waveform, volts.
$\bar{n}'(t)$	narrowband noise waveform, volts.

N	noise power out of linear filter, volts^2.
N_o	noise (two sided) spectral density, volts^2/cps
p	Heaviside operator d/dt.
$p(\,)$	probability density of ( ).
r	loop damping parameter.
R_1, R_2	resistance, ohms.
$R_{n_1}(\tau)$	time autocorrelation of the function $n_1(t)$, volts.
$R_{n_2}(\tau)$	time autocorrelation of the function $n_2(t)$, volts.
$S_{n_1}(f)$	two sided spectral density of stationary process $n_1(t)$.
t	time, sec.
t_1, t_2	specific instants of time, sec.
t_{acq}	first-order loop phase-acquisition time, sec.
$t_{freq\ acq}$	second-order loop frequency-acquisition time, sec.
$v(t)$	VCO output, volts.
W_L	two-sided noise bandwidth of a linear loop whose phase transfer function is $H(s)$, cps.
$Y(s)$	AGC loop-filter response.
α	signal voltage suppression factor.
w_n	second-order loop natural frequency, rad/sec.
δ_{lock}	first-order loop "in lock" constant, rad.
$\theta(t)$	θ, input signal phase function, rad.
$\hat{\theta}(t)$	$\hat{\theta}$, loop estimate of $\theta(t)$, rad.

ρ_y	signal-to-noise ratio of y(t).
$\sigma_x^2(t)$	variance of non-stationary random process x(t) at time t.
$\sigma_\Phi(t)$	variance of loop phase error.
τ	$t_1 - t_2$, time difference, variable of $R_n(\ )$, sec.
$\phi(t)$	ϕ, two heterodyne loop detector error, rad.
ϕ_{ss}	steady-state loop detector phase error, rad.
Φ	stationary equivalent phase error process, rad.
w	angular frequency variable, rad/sec.
w_o	VCO short-circuit output frequency, rad/sec.
w_{h_1}	first mixer heterodyne frequency, M times VCO output frequency, rad/sec.
w_{h_2}	second mixer heterodyne frequency, from internal oscillator, rad/sec.
Λ_o	initial value of doppler phase-rate, rad/sec^2.
$\Omega(t)$	Ω, $\phi(t)$, loop frequency error, rad/sec.
Ω_o	initial value of loop frequency offset, rad/sec.
λ	lagrange operator
$I_o(\ )$	zeroth order modified Bessel function.

ABSTRACT

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In this paper an attempt is made to obtain practical design techniques for carrier-tracking loops using the linear assumption. The bulk of the work is based on a statistical analysis of a linear model, but a non-linear model is also provided for the first order loop. Suppressed carrier-tracking loop analyses are discussed in detail.

1. INTRODUCTION

The basic concept of a phase-locked loop is not new. The origins of phase-locked control date back to the 1920's and 30's. The first widespread use of phase lock, however, was in TV receivers to synchronize the horizontal and vertical sweep oscillators to the transmitted sync pulses. Shortly thereafter, Jaffee and Rechtin [1] showed how a phase-locked loop could be used as a tracking filter for a missile beacon, and how the loop parameters could best be specified.

Lately, narrow-band phase-locked receivers have proved to be of considerable benefit in tracking weak satellite signals because of their superior noise immunity [5, 6].

Before the advent of monolithic integration, cost and complexity considerations limited its use to precision measurements requiring very narrow bandwidths. In the past few years, the advantages of monolithic integration have changed the phase-locked loop from a specialized design technique to a general-purpose building block [7, 21].

Today, over a dozen different integrated phase-locked loop products are available from a number of IC manufacturers. Some of these are designed as "general purpose" circuits suitable for a multitude of uses.

1-A. Basic Phase-locked Loop Principle. - A phase-locked loop

(PLL) contains three basic components (see Fig. 1-1): a phase comparator, a low pass filter, and a voltage-controlled oscillator (VCO). Detailed mathematical analysis of the phase-locked loop is discussed in the next chapter.

The basic principle of loop operation can be explained as follows: with no signal input, the VCO operates at a free-running frequency (VCO frequency), and the error voltage e(t) is zero. If an input signal s(t) is applied at the loop, the phase comparator compares the phase, and generates an error voltage that is the difference of phase and frequency between two signals. This voltage is then filtered and applied to the VCO. The VCO converts voltage to frequency in a linear fashion. This voltage applied to VCO forces the VCO frequency to vary in a direction that reduces the frequency difference between VCO and the input signal. If the input frequency is sufficiently close to the VCO frequency, the servo nature of the PLL constrains the VCO to lock with the incoming signal, and track it over small frequency variations.

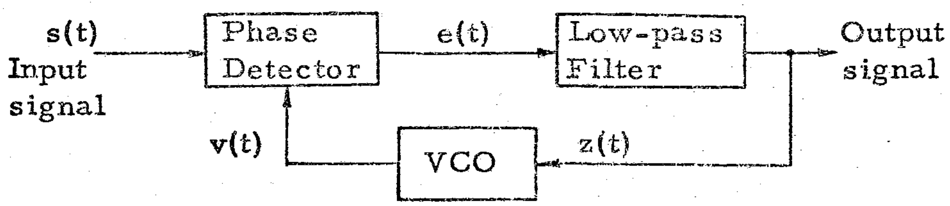


Fig. 1-1. Block Diagram of Phase-locked Loop.

The band of frequencies over which the PLL can acquire lock with an incoming signal is the capture range (see Fig. 1-2).

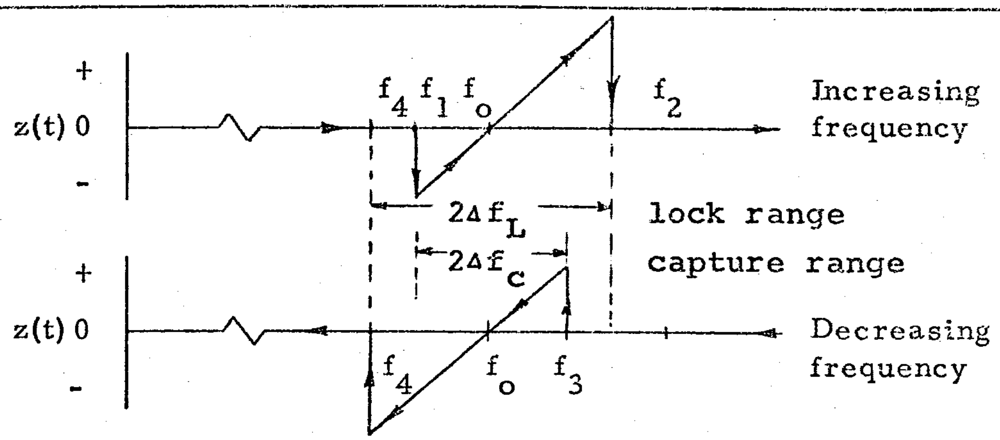


Fig. 1-2. The typical phase-locked loop frequency-to-voltage transfer characteristics for increasing and decreasing input frequency.

1-B. Application. - As a versatile building block, the PLL can be used for a wide range of applications. Some of the more important are the following:

a. The PLL is utilized in the space transponder, which coherently transmits to Earth an RF carrier whose frequency is at a fixed ratio relative to that received from Earth. For example, in the S-band system, the frequency transmitted from Earth to the spacecraft (in the 2110 to 2120 MHz band) is tracked by means of a phase-locked loop, multiplied by the ratio 240/221 and transmitted to Earth [5, 6].

b. The PLL can be used as a frequency demodulator, to obtain quality demodulated out superior to that of a conventional discriminator [2, 6].

c. Frequency-shift keyed (FSK) signals are used to transmit digital information over telephone lines. When the PLL is locked on the input signal, tracking the shifts in the input frequency, converts the frequency shifts back to binary logic pulses [5, 6, 7, 10, 13].

d. Bit synchronization of PCM telemetry transmission is typically obtained by phase lock methods [5, 6].

e. Signal conditioning can be obtained by the PLL, since VCO output duplicates the frequency of the desired input by greatly attenuating the noise, undesired sidebands and interference present at the input [6, 7, 20, 21].

f. The PLL can be used to generate new frequencies from a stable reference source by either frequency multiplication and division, or by frequency translation [4].

g. The PLL can be converted to a synchronous AM detector with the addition of a non-critical phase-shift network, an analog multiplier and a low pass filter [7, 8].

h. The PLL can also be used to achieve very precise speed control required in many electromechanical systems, such as magnetic tape drives and disk or drum head drivers [7].

Presently, PLL circuits are used in the relatively wide applications.

1-C. Terminology of Phase-locked Loop. - The following are the basic terms encountered in PLL literature.

Capture Range - The band of frequencies in the vicinity of center frequency where the PLL can acquire lock with an input signal. The capture

	range is always smaller than the lock range, and is related to the low-pass filter bandwidth (see Fig. 1-2). It is also known as the lock-in range or acquisition range.
Damping Factor -	The ability of the PLL to respond quickly to an input frequency set without excessive overshoot.
Free-running Frequen-cy (center frequency) - cy	This is the frequency at which the loop VCO operates when not locked to an input signal.
Lock Range -	The range of frequencies in the vicinity of center frequency, over which the loop can remain locked with an input signal (see Fig. 1-2). It is also known as the tracking range, or hold-in range.
Loop Gain -	The product of the d-c gains of all the PLL.
Loop Noise Bandwidth -	A loop property related to damping and natural frequency which describes the effective bandwidth of the received signals.
Low-Pass Filter (LPF) -	A LPF in the loop which allows only d-c and low frequency voltages to travel around the loop. It controls the capture range and the noise and out-band signal rejection characteristics.

Natural Frequency -	The characteristic frequency of the PLL.
Phase Detector -	A device which compares the input and output of PLL signals, and produces an error voltage which depends upon their relative phase differences. It is also called phase comparator. A multiplier, or mixer, is often used as a phase detector.
Quadrature Phase Detector (QPD) -	A phase detector operated in quadrature (90 degrees out of phase) with the loop phase detector. It is used primarily for AM demodulation and lock detection.
VCO Conversion Gain -	The conversion factor between VCO frequency and control voltage in radians/second/volt.
Voltage Controlled Oscillator (VCO) -	An oscillator whose frequency is determined by an applied control voltage.

2. FUNDAMENTAL CONCEPT

In this chapter, the basic equation governing the PLL is developed. The first analysis is based on the noise present in a sinusoidal input signal. This essentially follows the work of Jaffee and Rechtin and also Viterbi [3, 10].

2-A. Basic System. - A basic block diagram of a typical PLL is shown below. The input is assumed to be a sinusoidal of the form [3, 4, 6]

$$s(t) = A\sqrt{2}\sin[w_o t + \theta(t)] \qquad (2-1)$$

where A = rms voltage amplitude of x(t).

w_o = frequency of the VCO when its input is shorted.

$\theta(t)$ = the input signal phase process.

If the input signal is contaminated with white Gaussian noise (WGN) with zero mean and statistically independent to each other, the input signal and the noise become additive.

$$x(t) = s(t) + n(t) = A\sqrt{2}\sin[w_o t + \theta(t)] + n(t) \qquad (2-2)$$

where WGN has the following statistical properties:

$$n(t) = \sqrt{2}n_1(t)\cos w_o t + \sqrt{2}n_2(t)\sin w_o(t) \qquad (2-3)$$

and the spectral density is given by [9]

$$s_{n_1}(f) = s_{n_2}(f) = N_o/2 \qquad (2-4)$$

and the autocorrelation is given by (see Fig. 2-2)

$$R_{n_1}(\tau) = R_{n_2}(\tau) = N_o\delta(\tau)/2 \qquad (2-5)$$

The signal input is multiplied by the VCO waveform*

$$v(t) = K_1 \sqrt{2}\cos[w_o t + \hat{\theta}(t)] \qquad (2\text{-}6)$$

where $\hat{\theta}(t)$ is the loop estimate of $\theta(t)$ and K_1 is the rms output of the VCO.

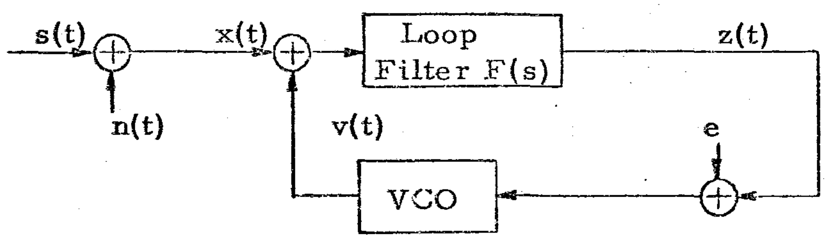


Fig. 2-1. The Basic PLL.

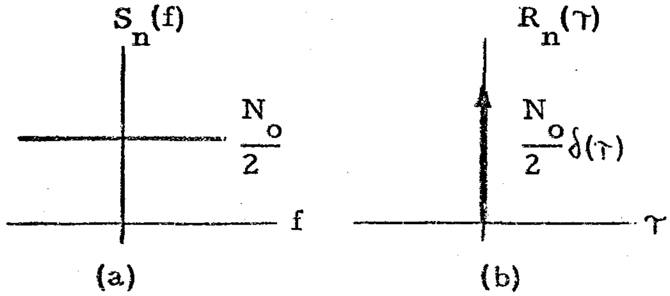


Fig. 2-2. White Noise. (a) Power spectrum; (b) Correlation function.

The results of this multiplication, the error voltage is

$$\begin{aligned} e(t) = s(t)v(t) &= 2AK_1 \sin(w_o t+\theta(t))\cos(w_o t+\hat{\theta}(t)) \\ &\quad + 2K_1(n_1(t)\cos w_o t+n_2(t)\sin w_o t)\cos(w_o t+\hat{\theta}(t)) \\ &= AK_1 \quad (\sin(\theta(t)-\hat{\theta}(t))+\sin(2w_o t+\theta(t)+\hat{\theta}(t)) \\ &\quad + K_1 n_2(t)\cos\hat{\theta}(t)-K_1 n_1(t)\sin\hat{\theta}(t) \\ &\quad + K_1 n_2(t)\sin(2w_o t+\hat{\theta}(t)) + K_1 n_2(t)\cos(2w_o t+\hat{\theta}(t)) \qquad (2\text{-}7) \end{aligned}$$

The multiplier and the loop filter remove the double frequency terms. For clarity, the filter and multiplier are shown separately. The multiplier has some gain K_m, and hence, the actual output of the phase detector is

$$e(t) = AK_1K_m \sin \phi(t) + K_1K_m n'(t) \quad (2\text{-}8)$$

where $n'(t)$ is also WGN and based on

$$n'(t) = n_1(t)\cos \hat{\theta}(t) + n_2(t)\sin \hat{\theta}(t). \quad (2\text{-}9)$$

The quantity $\phi(t) = \theta(t) - \hat{\theta}(t)$ is called the true phase error.

The VCO output frequency is a linear function of its input.

$$w_v(t) = w_o + K_v z(t) + K_v e \quad (2\text{-}10)$$

where K_v is the VCO gain in radians/volt-sec.

Consequently, VCO output can be written, omitting e for the present,

$$\hat{\theta}(t) = \int_o^t K_v z(u)\, du \quad (2\text{-}11)$$

One can write differential equations (DE) in a compact form by introducing the Heaviside operator $p = d/dt$**,

$$\hat{\theta}(t) = \frac{K_v}{p} z(t). \quad (2\text{-}12)$$

Substitution for $z(t)$ yields

$$\hat{\theta}(t) = K_1K_mK_v\frac{F(p)}{p}\left\{ A \sin \phi(t) + n'(t) \right\}. \quad (2\text{-}13)$$

It is convenient to define the open-loop gain of the loop,

$$K = K_1K_mK_v \quad (2\text{-}14)$$

and to substitute $\hat{\theta} = \theta - \phi$. This produces the characteristic equation which describes the dynamic behavior of the loop in the absence of

---

* Note that $x(t)$ and $v(t)$ are really 90 degrees out of phase with one another. The input has been written as a sine, and the VCO output as a cosine. The two phase $\theta(t)$ and $\hat{\theta}(t)$ are referred to the quadrature references.

** Example: $\frac{1}{p}x(t) = \int_o^t x(t)du$ then $px(t) = \dot{x}(t)$.

noise;

$$\theta(t) = \phi(t) + \frac{AKF(p)}{p}\left\{\sin\phi(t) + \frac{n'(t)}{A}\right\} \qquad (2-15)$$

or $$\hat{\theta}(t) = \frac{AKF(p)}{p}\left\{\sin\phi(t) + \frac{n'(t)}{A}\right\}$$

where once again

$\theta(t)$ = Input signal phase.

$\phi(t) = \theta(t) - \hat{\theta}(t)$, phase error signal.

$\hat{\theta}(t)$ = Phase estimate generated by the VCO.

A = Input signal amplitude.

K = Loop gain.

F(p) = Transfer function of the loop filter.

p = Heaviside operator, p = d/dt.

The loop order, L, is:

L = m + 1

where m = number of poles in the loop filter transfer function.

Under the assumption that the phase error, $\theta_1 - \theta_2$, is very small n(t) may be set equal to zero*, and then

$\sin\theta \approx \theta$.

The linearized version of (2-15) becomes

$$\theta(t) = \frac{p + AKF(p)}{p}\,\phi(t). \qquad (2-16)$$

* Whenever the phase error goes through 2π radians, it is said that the loop has skipped a cycle, and whenever the phase error never varies outside an internal of size 2π radians, it can be said that the loop has locked. If the loop is capable of reducing the phase error to a small enough value, say $|\phi| < \pi/6$, then $\sin\phi \approx \phi$.

Typically, the input process consists of modulation, and of Doppler ** due to the radial motion of the spacecraft relative to the ground tracking station [8,15].

$$\theta(t) = m(t) + d(t) \tag{2-17}$$

where

$m(t)$ = modulation, stationary with zero mean.

$d(t)$ = Doppler shift, non-stationary.

Rearrangement of (2-16) results

$$\phi(t) = \frac{p}{p + AKF(p)}\,\theta(t)$$

and substitution for $\theta(t)$ yields

$$\phi(t) = \frac{p}{p + AKF(p)}\,m(t) + \frac{p}{p + AKF(p)}\,d(t). \tag{2-18}$$

The above equation states that there are two kinds of errors present in the loop: the error due to the modulation is called phase distortion, and the remaining error, due to a doppler shift on the incoming carrier, is called tracking error.

The steady state tracking error can be determined by the final value theorem of Laplace transform theory.

$$D(p) = \mathcal{L}[d(t)].$$

$$\phi_{ss} = \lim_{p \to 0} \left[p \left\{ \frac{pD(p)}{p + AKF(p)} \right\} \right]. \tag{2-19}$$

To be effective, the loop must be designed to track whatever

** Doppler is the change in the observed frequency of a radio wave caused by a time rate of change in the effective length of the path of travel between the source and the point of observation.

variations d(t) may have, so the filter F(p) must be properly chosen.

The doppler shift can be expanded in a Taylor to obtain [8,15].

$$d(t) = \phi_o + \Omega_o t + \frac{\Lambda_o t^2}{2} + \text{------} + \frac{X_n t^n}{n!} \qquad (2\text{-}20)$$

where ϕ_o = Initial phase offset of the incoming signal from the free-running VCO phase.

Ω_o = Frequency offset of the incoming signal from the free-running VCO frequency.

Λ_o = Doppler rate of the incoming signal frequency.

X_o = n the time derivative of the incoming carrier phase.

$$D(p) = \phi_o/p + \Omega_o/p^2 + \Lambda_o/p^3$$

This produces a steady-state tracking error given by

$$\phi_{ss} = \lim_{p \to o} \left\{ \frac{p^2}{p+AKF(p)} \left[\frac{\phi_o}{p} + \frac{\Omega_o}{p^2} + \frac{\Lambda_o}{p^3} \right] \right\}.$$

For a first-order loop F(p) = 1.

$$\phi_{ss} = \lim_{p \to o} \left\{ \frac{1}{p + AK} \left[\phi_o p + \Omega_o + \Lambda_o \right] \right\}. \qquad (2\text{-}21)$$

It is interesting to note that (2-21) becomes

$$\phi_{ss} \begin{cases} = \infty, \ \Lambda_o \neq 0 \\ = \Omega_o/AK, \ \Lambda_o = 0 \\ = 0, \ \Lambda_o = \Omega_o = 0 \end{cases}$$

The error is not bounded if $\Lambda_o \neq 0$.

ϕ_{ss} can be made small by adjustment of the loop gain if $\Lambda_o = 0$. The loop will track any initial phase offset θ_o with no steady-state error if $\Omega_o = 0$.

2-B. Lock Acquisition [2, 3, 12, 14]. - Assume only the d(t) term is present. For the first-order loop, the loop can track a function of the type

$$\theta(t) = \phi_o + \Omega_o t. \tag{2-22}$$

Whenever r_o is small enough that $\phi(t) \approx 0$;

$$\left| \frac{\Omega_o}{AK} \right| < \frac{\pi}{6} \quad . \tag{2-23}$$

For Ω_o is large, one can go back to the loop equation (2-15) with F(p) = 1.

$$\theta(t) = \phi(t) + \frac{AK}{p} \sin \phi(t)$$

Differentiation yields the following:

$$\dot{\theta}(t) = \dot{\phi}(t) + AK \sin \phi(t)$$

$$\theta(t) = \Omega_o \tag{2-24}$$

$$\Omega_o = \phi(t) + AK \sin \phi(t) \tag{2-25}$$

The frequency error is given by $\dot{\phi}(t) = \Omega(t)$. A phase-plane diagram of first-order lock-in can be constructed from (2-25) (see Fig. 2-3). ϕ will have reached its steady-state value when $\Omega = 0$. This point is stable, since after small perturbation of ϕ in either direction the system will tend to return to the stable point.

It is clear from Figure 2-3 and (2-25) that $\Omega(t) = 0$ at any of the following values of ϕ_{ss}:

$$\phi_{ss} = n\pi = \sin^{-1} (\Omega_o / AK) \qquad n = \pm 0, \pm 2, ---$$

$$\phi_{ss} = (n+1)\pi + \sin^{-1}(\Omega_o/AK) \qquad n = \pm 1, \pm 3, ----$$

The steady-state phase error is written merely [15,16]

$$\phi_{ss} = \sin^{-1}(\Omega_o/AK) \tag{2-26}$$

$$\Omega_{max}\ (\text{rad/sec}) = AK = 2\,W_L\ (\text{cps})$$

where W_L is the loop noise bandwidth.

The pull-in time may also be determined from (2-25) as follows: one can write

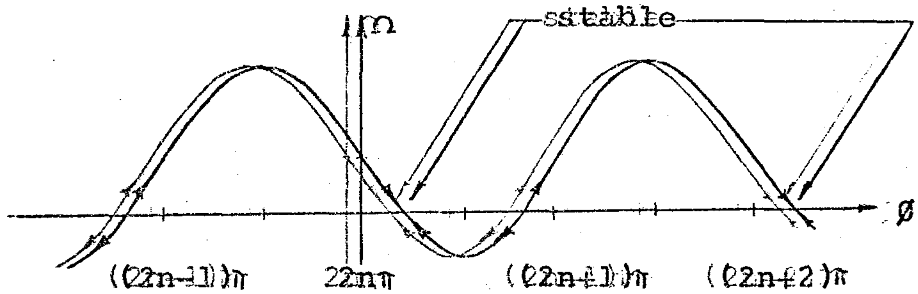


Fig. 2-3. First-order Loop Pull-in Behavior.

$$dt/d\phi = 1/(\Omega_o - AK\sin\phi) \tag{2-27}$$

This can be integrated to give the pull-in time to any particular value of ϕ, but, since the denominator above vanishes at the lock-in point, an infinite time is required before $\Omega_o = 0$. However, if it is agreed that the loop is in lock whenever $|\phi - \phi_{lock}| < \delta_{lock}$, the corresponding time is finite. The maximum time required to achieve lock, designated the acquisition time, will be that time required for the system to pass from $-\pi + \delta_{lock} - \sin^{-1}(\Omega_o/AK)$ to $\sin^{-1}(\Omega_o/AK) - \delta_{lock}$. When the value of δ_{lock} is small, the answer reduces to approximately

$$t_{acq.} = \frac{2}{AK\cos\phi_{ss}}\ln\frac{2}{\delta_{lock}} \quad (\text{sec}) \tag{2-28}$$

For example, if a loop is designed so that $\phi_{ss} = 5$ deg. when $_o$ is 100 Hz, and lock is taken to be 5 deg., then the required pull-in time is approximately 1 msec.

2-C. Acquiring Lock in The Second-order Loop With Passive Loop Filter.

The second-order loop in most widespread use is one in which the loop filter takes the form (see Fig. 2-4, below).

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s} \quad . \tag{2-29}$$

The equation (2-15) becomes

$$\theta(t) = \phi(t) + \frac{AK(1 + \tau_2 s)}{s(1 + \tau_1 s)} \sin \phi(t) \tag{2-30}$$

$$(\tau_1 s^2 + s)\theta(t) = (\tau_1 s^2 + s)\ \phi(t) + AK(1 + \tau_2 s) \sin \phi(t).$$

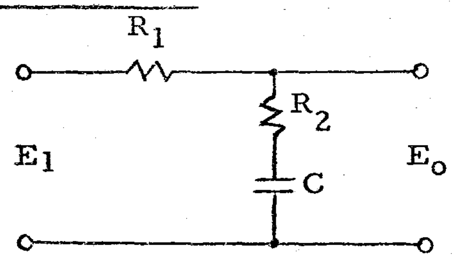
A second-order filter can follow a phase function of the type

$$\theta(t) = \theta_o + \Omega_o(t)$$

$$\Omega_o(t) = \dot{\phi} + \tau_1 \ddot{\phi} + AK \sin \phi + AK\,\tau_2 \cos \phi \cdot \dot{\phi}$$

$$= \tau_1 \ddot{\phi} + (1 + AK\tau_2 \cos \phi)\ \dot{\phi} + AK \sin \phi \ . \tag{2-31}$$

The steady state phase error ϕ_{ss} is the same as for the first loop, since $\dot{\phi} = \ddot{\phi} = 0$.



$\tau_2 = R_2 C$
$\tau_1 = (R_1 + R_2)C$

Fig. 2-4. Passive Integration Loop Filter.

$$\Omega_o = AK \sin \phi_{ss}$$

$$\phi_{ss} = \sin^{-1})\Omega_o /AK) \qquad (2\text{-}32)$$

The loop can lock if $AK > \Omega_o$, but the phase plane trajectory that takes ϕ to ϕ_{ss} is not the simple sinusoid, and it will take the solution to (2-31).

To simplify, the substitution can be made:

$$\Omega = d\phi/dt$$

$$\ddot{\phi} = \frac{d\Omega}{dt} = \frac{d\Omega}{d\phi} \cdot \frac{d\phi}{dt} = \Omega \frac{d\Omega}{dt}. \qquad (2\text{-}33)$$

This produces the phase plane trajectory equation

$$\Omega_o = (\tau_1 \frac{d\Omega}{d\phi} + 1 + AK\tau_2 \cos \phi)\Omega + AK \sin \phi. \qquad (2\text{-}34)$$

By integrating (2-34), Viterbi has derived a necessary condition on Ω_o: If lock-in occurs for all initial conditions of the VCO, then Ω_o is bounded by

$$|\Omega_o| < 2 [(\frac{AK}{\tau_1}) (1 + 1/2\ AK\tau_2)]^{1/2} \qquad (2\text{-}35)$$

and the time required to achieve lock is given by [8, 14]

$$t\ freq.\ acq. = (2\pi^2 \frac{\tau_2}{\tau_1})^{1/2} (\frac{r+1}{r}) \frac{\Omega_o^2}{WL} \quad (sec) \qquad (2\text{-}36)$$

where $r = AK\, \tau_2^2/\tau_1$, and WL is the loop bandwidth.

2-D. Tuning The VCO. - One can write the effect of the tuning voltage from (2-10),

$$W_v(t) = W_o + K_v e + K_v Z(t). \qquad (2\text{-}37)$$

The equation, above, can be treated as (2-15), except for a term $K_v e\, t$ subtracted from $\theta(t)$.

$$\dot{\theta}(t) - K_v e = \Omega(t) + AKF(p) \sin \phi(t) \tag{2-38}$$

The lock-in points now occur at

$$\phi_{ss} = \sin^{-1} \frac{(\Omega_o - K_v e)}{AK} \tag{2-39}$$

By the proper choice of e , the steady-state error can be made zero. This value of e is clearly

$$e = \Omega_o / K_v \tag{2-40}$$

In the first-order loop, theacquisition time with the foregoing value of e becomes

$$t\ acq. = \left(\frac{2}{AK}\right) \quad \ln \left(\frac{2}{\delta Lock}\right) . \tag{2-41}$$

3. THE LINEARIZED ANALYSIS OF PHASE-LOCKED SYSTEMS WITH STOCHASTIC INPUTS

If the level of $n'(t)$ is sufficiently low, and if the loop is designed properly, the phase error $\phi(t)$ should be very small ($\leq 30^\circ$). Then one can justify that the loop can be analyzed linearly.

3-A. Linear Loop Analysis. - In such a case, the approximation

$$\sin \phi = \phi, \tag{3-1}$$

when inserted into (2-15), yields a linear equation relating the loop input and output

$$\hat{\theta}(t) = \frac{AKF(p)}{p}\left[\phi + \frac{n'(t)}{A}\right] \tag{3-2}$$

since $\phi = \theta - \hat{\theta}$

$$\hat{\theta}(t) = \frac{AKF(p)}{p}\left\{\theta - \hat{\theta} + \frac{n'(t)}{A}\right\}$$

$$\hat{\theta}(t)\left\{1 + \frac{AKF(p)}{p}\right\} = \frac{F(p)}{p}\left[AK\theta + Kn'(t)\right].$$

A rearrangement of the above yields

$$\hat{\theta}(t) = \frac{AKF(p)}{p+AKF(p)}\left[\theta(t) + \frac{n'(t)}{A}\right]. \tag{3-3}$$

The output phase is the result of a linear filter acting upon the input phase process $\Theta(t)$ immersed in a normalized version of the loop noise, with the normalized factor in this case being equal to the rms signal amplitude A. The linearized model of PLL may be represented by the block diagram of Fig. 3-1, on the following page.

The closed-loop transfer function H(p) of the PLL relative to the input signal $\theta(t)$ may be written by inspection from Figure 3-1 as

$$H(p) = \frac{\hat{\theta}(p)}{\theta(p)} = \frac{AKF(p)}{p+AKF(p)} \tag{3-4}$$

The overall loop transfer-function is related to the loop-filter F(p) by the relations

$$F(p) = \frac{p\,H(p)}{AK\,[\,1 - H(p)]} \,. \tag{3-5}$$

As with any linear filter, $H(p) = H(jw)$ has some effective noise bandwidth WL, which can be written as [9]

$$WL = 2BL \triangleq \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(w)|^2 dw \tag{3-6}$$

where WL is the two-sided loop bandwidth (in Hz) and BL is the single-sided loop bandwidth. Note that the carrier passband has width 2WL.

A rearrangement of (3-2) relates the phase error to the input processes:

$$\phi(t) = [\,1 - H(p)]\,\theta(t) - H(p)\,\frac{n(t)}{A} \,. \tag{3-7}$$

The first term of (3-7) represents distortion due to filtering. $\theta(t)$ is affected by the doppler phase shift d(t) and the modulation m(t) as shown in the (2-17). The last term represents the phase error due to the presence of noise at the input of the loop.

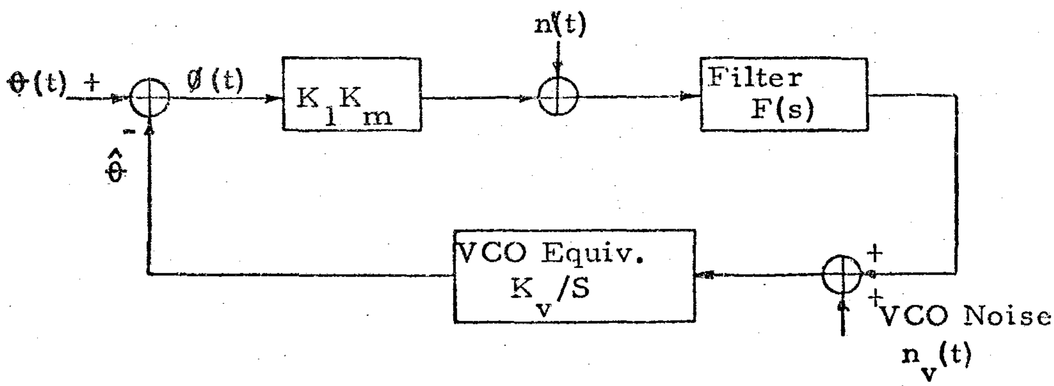


Fig. 3-1. The Linear Baseband Equivalent Model of PLL.

The total mean-square phase error $E[\phi^2]$ is given by

$$E[\phi^2] = \mu^2(t) + \delta\phi^2 + \delta\phi^2 \qquad (3\text{-}8)$$

The first term represents the transient distortion due to the Doppler shift, the second is modulation distortion, and the third is the mean-square phase noise. The last two, being stationary, can be computed by using inverse transformation of the power appearing at the filter output [9].

$$\delta_\phi^{\ 2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |1 - H(jw)|\ S\ x(jw)dw \qquad (3\text{-}9)$$

The mean-square phase error is defined as

$$\delta\phi^2 = \int_{-\infty}^{\infty} |H(w)| \quad S(\frac{n'}{A})(w)\frac{dw}{2\pi} \qquad (3\text{-}10)$$

where $S(\frac{n'}{A})(w) = N_o/2A^2$

$$\delta\phi^2 = \frac{N_o}{2A^2}\int_{-\infty}^{\infty} |H(w)|^2 \frac{dw}{2\pi}$$

The integration term is exactly the same form as (3-6).

$$\delta\phi^2 = \frac{N_o WL}{2A^2} = \frac{N_o BL}{A^2} \quad (\text{Radians})^2 \qquad (3\text{-}11)$$

Thus the mean square phase error due to thermal noise is equal to the noise power to signal power ratio in the loop bandwidth. Since it is known that A^2 is the power of the signal entering the loop, $N_o/2$ is the two-sided noise spectral density in the loop, and WL is the two-sided loop bandwidth.

3-B. Passive Integrator Loop Filter. - In practice, the carrier-tracking loop filter of most widespread use is one in which the filter takes the form (see Fig. 2-4).

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s}.$$

For this case, (3-4) becomes

$$H(s) = \frac{1 + \tau_2 s}{1 + (\tau_2 + 1/AK)s + (\tau_1/AK)s^2} \qquad (3\text{-}12)$$

If the loop damping parameter r is defined by

$$r = \frac{AK\tau_2^{\,2}}{\tau 1} = 4\zeta^2 \qquad (3\text{-}13)$$

where ζ is the loop damping, and it is assumed that $r\,\tau_1 \gg \tau_2$ (to make H(s) appear to have a pole at the origin), then the equivalent loop noise bandwidth can be computed from (3-6).

$$WL = \frac{r + 1}{2\tau_2(1 + \tau_2/r\tau_1)} \approx \frac{r + 1}{2\tau_2} \qquad (3\text{-}14)$$

The loop transfer function is then

$$H(s) = \frac{1 + ([r + 1]/2WL)s}{1 + ([r+1]/2WL)s + ([r+1]/2WL)^2 s^2} \qquad (3\text{-}15)$$

and the system damping can be computed from (3-12)

$$\zeta = 1/2\,(1 + \frac{\tau_2}{r\tau_1})\;\; r^{1/2} \approx \frac{r^{1/2}}{2}$$

$$Wn = (\frac{AK}{\tau_1})^{1/2} = \frac{2\sqrt{r}WL}{r+1} = \frac{r^{1/2}}{\tau 2} \qquad (3\text{-}16)$$

where Wn is the natural frequency of the loop.

For r = 4, critical damping (ζ = 1) occurs, and for $r > 4$, H(s) has two real negative poles. For $r < 4$, H(s) is underdamped. It is common practice to design the system with a damping factor

$\zeta = 0.707$, or $r = 2$, in which case the transient response phase error criteria would be optimum.

For example, consider a 1M-Hz clean-up loop for Mariner-Venus 67 ranging and occulation experiment design. It is assumed that the following parameters are set: $\tau_1 = 7600$ sec, $\tau_2 = 124$ sec, and $AK = 2\sec^{-1}$. The loop damping ratio, the natural frequency, and the loop bandwidth can be found by use of (3-13), (3-14) and (3-16).

$$\zeta = \frac{Wn}{2}\left(\tau_2 + \frac{1}{AK}\right) = \frac{1}{2}\sqrt{\frac{AK}{\tau_1}}\left(\tau_2 + \frac{1}{AK}\right) = 1.0 \text{ Critically damped}$$

$$Wn = \sqrt{\frac{AK}{\tau_1}} = 0.01622 \sec^{-1}$$

$$WL = \frac{r+1}{2\tau_2(1+\tau_2/r\tau_1)} = 0.02026 \sec^{-1}$$

where $r = AK\tau_2^{\,2}/\tau_1 = 4.0$.

3-C. Perfect Integrator. - Whenever an operational amplifier is used in the loop, the loop filter can be computed from (3-4) and the active filter transfer function (see Fig. 3-2).

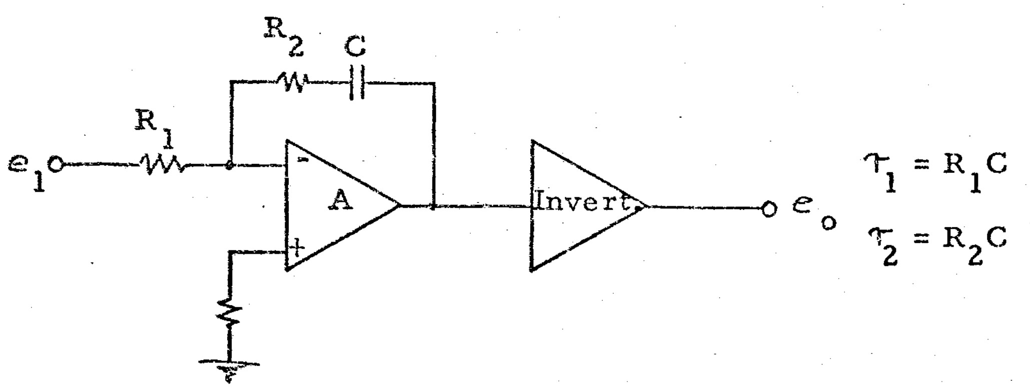


Fig. 3-2. Active Filter.

$$F(s) = \frac{A(s\tau_2 + 1)}{1 + \tau_1(1 + A)s + \tau_2 s^2} \qquad (3\text{-}17)$$

For large A

$$F(s) \approx \frac{1 + \tau_2 s}{\tau_1 s} \qquad (3\text{-}18)$$

This loop filter gives the same response as that given in (3-14), (3-15), and (3-16) when $r\tau_1 \gg \tau_2$, the loop with an imperfect integrating filter (3-12) performs very much the same as that with the perfect integrating filter (3-18).

4. LINEARIZED ANALYSIS OF TRACKING FILTER

The term "tracking filter" has come to describe a phase-locked receiver which is used either to track the carrier component of the incoming signal, or to demodulate its information, or both simultaneously.

4-A. Optimum Filter For Frequency and Random Phase Offset.

Jaffee and Rechtin defined the total phase error, denoted by $E[\phi^2_T]$, as

$$E[\phi_T^2] = \lambda^2 \epsilon_T^2 + \delta_\phi^2 + \delta_\phi^2 \qquad (4\text{-}1)$$

in which λ^2 is a Lagrange multiplier, a design parameter related to the bandwidth of the loop. The term ϵ_T^2 is the total transient distortion, which is given by the equation

$$\epsilon_T^2 = \frac{1}{2\pi} \int_{\infty}^{\infty} | 1 - H(jw) |^2 E[D(jw) D(-jw)] . \qquad (4\text{-}2)$$

Minimization of $E[\phi_T^2]$ by choice of the loop transfer function $H(s)$ specifies both AK and F(s). But the question now is, "How does one choose H(s) to minimize $E[\phi_T^2]$?" Jaffee and Rechtin recognized that $\lambda^2 \epsilon_T^2 + \delta^2$ could be computed by replacing Sx(s) in (3-9) by $\lambda^2 E[D(s)D(-s)] + Sx(s)$. This led them to the conclusion that $E[\phi_T^2]$ is minimized whenever L(s) is chosen in accordance with the Weiner optimization technique, which yields the Yavits-Jackson formula.

$$L_{opT}(S) = 1 - \frac{N_o^{1/2}/A}{[S(s)]^+} \qquad (4\text{-}3)$$

$$S(s) = \lambda^2 E[D(s)D(-s)] + Sx(s) + N_o/A^2 .$$

The bracket $[\ ]^+$ refers to a type of "square-root" factoring of the enclosed function, retaining in $[\ ]^+$ the left-hand-plane poles and zeros of the enclosed function only; singularities on the imaginary axis are equally divided between $[\ ]^+$ and its mirror image $[\ ]^-$.

The optimum tracking loop design is to provide the best filter to track a given doppler-phase polynomial d(t) of degree N-1. It is assumed that modulation of the carrier is absent. The form of D(s) is then

$$D(s) = \frac{\theta_o}{s} + \frac{\Omega_o}{s^2} = --- = \frac{Q(s)}{s^N} \qquad (4-\ 4)$$

in which the degree of Q(s) is less than N. The filter specified by (4-3) is

$$H_{opT}(s) = 1 - \frac{}{\left[(-1)^N s^{2N} + \left(\frac{A^2\lambda^2}{N_o}\right) E[Q(s)Q(-s)]\right]^+} \qquad (4-\ 5)$$

The second-order example of loop optimization occurs for $d(t) = \theta_o + \Omega_o t$, where θ_o is a uniformly distributed random variable over $(-\pi, \pi)$ with variance $\pi^2/3$. To find $H_{opT}(s)$, one can insert N = 2 into (4-5): The denominator is (4-5) is

$$\left[S^4 - \frac{\lambda^2 A^2 \pi^2}{3\, N_o} s^2 + \frac{\lambda^2 A^2 \Omega_o^2}{N_o}\right]^+ = s^2 + \left(\frac{A^2\lambda^2\pi^2}{3\, N_o} + \frac{2\lambda A \Omega_o}{N_o^{1/2}}\right)^{1/2} s$$

$$+ \frac{\lambda A \Omega_o}{N_o^{1/2}} \qquad (4-\ 6)$$

The natural frequency is thus

$$Wn^2 = \frac{\lambda A \Omega_o}{(N_o/2)^{1/2}} \quad (4-7)$$

The optimum filtering function is given by

$$H_{opT}(s) = \frac{s\left(2Wn^2 + \frac{\pi^2 Wn^4}{3\Omega_o^2}\right)^{1/2} + Wn^2}{s^2 + \left[2Wn^2 + \frac{\pi^2 Wn^4}{3\Omega_o^2}\right]^{1/2} s + Wn^2} \quad (4-8)$$

Comparing (4-5) with (3-12) and (3-13), one can optimize the parameters:

$$\tau_2 = \frac{1}{Wn}\left[2 + \frac{\pi^2 Wn^2}{3\Omega_o^2}\right]^{1/2} = \frac{\pi r}{\Omega_o\left[3r(r-2)\right]^{1/2}}$$

$$Wn^2 = \frac{AK}{\tau_1} = \frac{3\Omega_o^2}{\pi^2}(r-2)$$

$$r = 2 + \frac{\pi^2 Wn^2}{3\Omega_o^2}$$

$$WL = \frac{(r+1)\Omega_o}{2\pi r}\left[3r(r-2)\right]^{1/2} \quad (4-9)$$

It is important here to note that r must exceed 2 in optimized loop design.

The Jaffee and Rechtin example using $\Theta_o = 0$ produces a result somewhat different from that above. This corresponds to the case in which the loop is initially tracking with no phase error.

$$\left.\begin{aligned} \tau_2 &= \sqrt{2}/Wn \\ Wn^2 &= AK/\tau_1 \\ r &= 2 \quad (\text{so } \zeta = 0.707) \\ WL &= \frac{3Wn}{2(2)^{1/2}} \end{aligned}\right\} \quad \text{when } \theta_o = 0 \quad (4-10)$$

If the loop is optimally designed to lock onto an Ω_o appearing at the bandwidth edge ($\Omega_o = 2\pi b_L$), the proper values of r can be computed numerically from (4-9)

$$r = 2.282$$

$$\zeta = 0.755$$

$$\tau_2 = 1.643/WL$$

$$\frac{\tau_1}{AK} = \frac{1.180}{WL^2} \quad . \tag{4-11}$$

The corresponding optimum loop transfer function is given by

$$H_{opT}(s) = \frac{1 + (1.643/WL)s}{1+(1.643/WL)s+(1.18/WL^2)s^2} \, . \tag{4-12}$$

If there is a small Doppler-rate term Λ_o (rad/sec^2) in d(t), the loop with $F(s) = (1+\tau_2 s)/\tau_1 s$ has a steady-state error

$$\phi_{ss}(\text{Doppler rate}) = \frac{\Lambda_o \tau_1}{AK} = \frac{\Lambda_o}{4r} \frac{(r+1)^2}{WL^2} (\text{rad}) \, . \tag{4-13}$$

When the imperfect-integrator loop filter $F(s) = (1+\tau_2 s)/(1+\tau_1 s)$ is used, there is a steady-state growth in phase error

$$\phi_{ss}(\text{Doppler rate}) \longrightarrow \frac{\Lambda o}{AK} t = \frac{\Lambda o(r+1)^2}{4rWL^2} \left(\frac{t}{\tau_1}\right) (\text{rad}) . \tag{4-14}$$

These values are minimized, for a fixed WL^2 and τ_1, by choosing r = 1. Parameters must be chosen to maintain less than about 30 degrees error in the loop. The r between 6 and 10 minimizes the effect S of VCO noise, where as a value of 2.282 minimizes the total phase error in tracking a frequency offset, and r = 1 provides the best Doppler-rate tracking capability. Representative types of transient behavior are illustrated in Fig. 4-1 [10,15].

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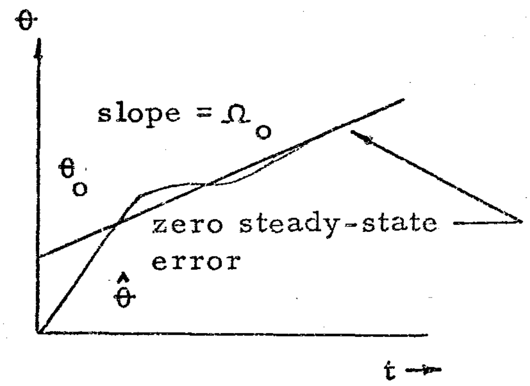
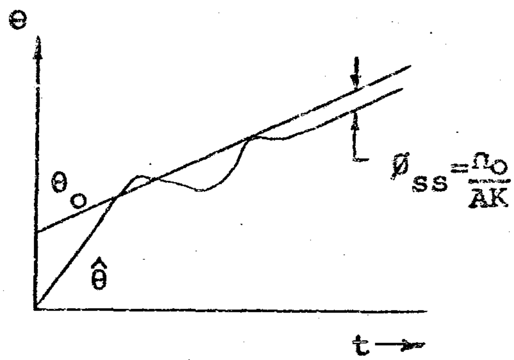
a). Ideal integrator, frequency step.



b). Passive integrator, frequency step.



c). Passive integrator, Doppler-rate input.

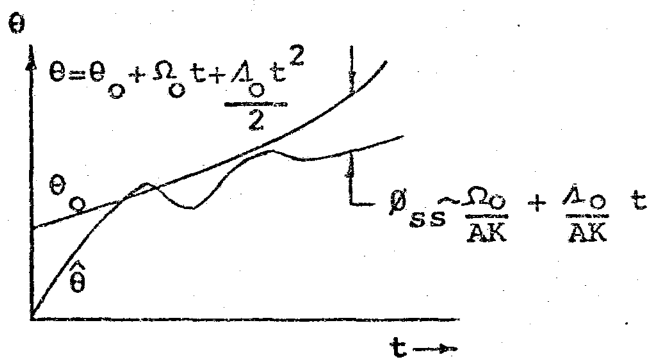


Fig. 4-1. Response of Second-Order Loop to Various Inputs.

5. NON-LINEAR ANALYSIS OF PHASE-LOCKED LOOP

If the loop gets sufficiently noisy, and if the linear representation of a loop is not predicated on high SNR's, the significant accuracy of linear analysis is lost. In such a case, the approximation sin $\emptyset$ = $\emptyset$ (3-1) is not made.

Tikhonov 11, 12 and Viterbi were able to solve for the exact steady state behavior of the First-order loop by Fokker-Plank techniques. It has been shown that for First-order loops, the steady-state phase error probability density function of the modulo 2 (see Fig. 5-1) is

$$p(\emptyset) = \frac{\exp(\rho \operatorname{los} \emptyset)}{2\pi \, Io(\rho)} \qquad /\emptyset/ \leq \pi \tag{5- 1}$$

where ρ is the SNR in the loop bandwidth and

$$\rho = \frac{1}{\delta \emptyset^2} = \frac{2A^2}{NoWL} \tag{5- 2}$$

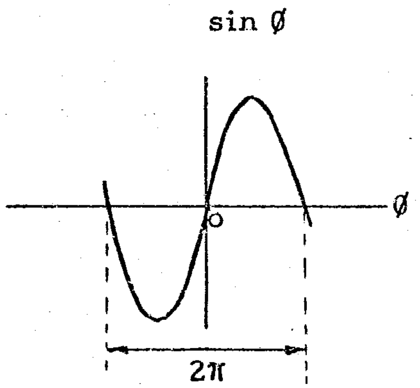


Fig. 5-1. The Modulo 2π.

where Io is the modified Bessel function of zeroth order. The equation (5-2) somewhat resembles a Gaussian distribution for large SNR, and becomes flat as ρ approaches zero (see Fig. 5-2).

The zeroth-order modified Bessel function is given by

$$Io(\rho) \sim \frac{\exp \rho}{(2\pi\rho)^{1/2}} \tag{5- 3}$$

and (5-1) becomes

$$p(\emptyset) \sim \frac{\exp[\rho(\cos \emptyset - 1)]}{(2\pi/\rho)^{1/2}}$$

Expanding cos Ø in a Taylor series, one obtains

$$p(\emptyset) \sim \frac{\exp[(-\rho\emptyset^2/2)(1-2\emptyset^2/4o^1 + 2\emptyset^2/6 \; (\ldots -1)]}{(2\pi/\rho)^{1/2}}$$

For large ρ, p(Ø) decreases rapidly with Ø

$$p(\emptyset) = \frac{e^{-\emptyset^2/2\,\delta\emptyset^2}}{(2\pi/\rho)^{1/2}} \tag{5- 4}$$

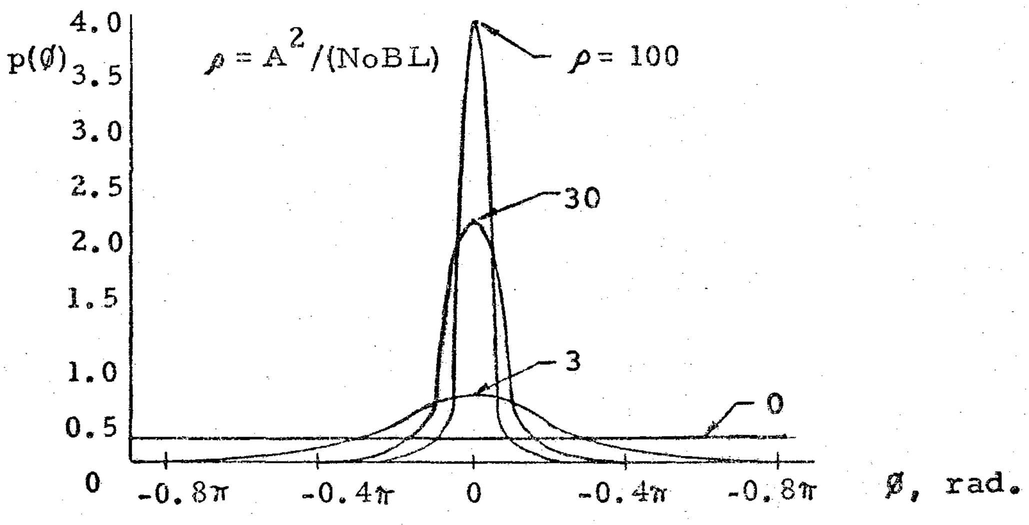


Fig. 5-2. First-order Loop Steady-state Probability Densities.

Viterbi has shown that the variance of $\emptyset$ can be obtained by

$$\sigma \emptyset^2 = \int_{-\pi}^{\pi} \emptyset^2 p(\emptyset)\, d\emptyset$$

$$= \frac{1}{2\pi I_o(\rho)} \int_{-\pi}^{\pi} \emptyset^2 \exp(\rho \cos \emptyset)\; d\emptyset$$

$$= \frac{1}{2\pi I_o(\rho)} \int_{-\pi}^{\pi} \emptyset^2 [\, I_o(\rho) + 2\sum_{n=1}^{\infty} I_n(\rho) \cos n\emptyset \,]\, d\emptyset$$

$$= \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \left[\frac{(-1)^n I_n(\rho)}{n^2 I_o(\rho)} \right] \qquad (5\text{-}6)$$

Note that as the SNR l approaches zero, the variance approaches $\pi^2/3$, which is the variance of a random variable that is uniformly distributed from $-\pi$ to $+\pi$.

5-A. Cycle Slipping [16, 17]. - A PLL "slips a cycle" when the magnitude of its phase error process exceeds 2 radians (see Fig. 5-3). The occurrence of a cycle slip is a random event depending on the noise in the PLL and the deterministic phase error, and introduces errors in Doppler tracking. Actually, there are two different parameters describing this event; the mean time to first cycle slip, and the average number of cycle slips per second.

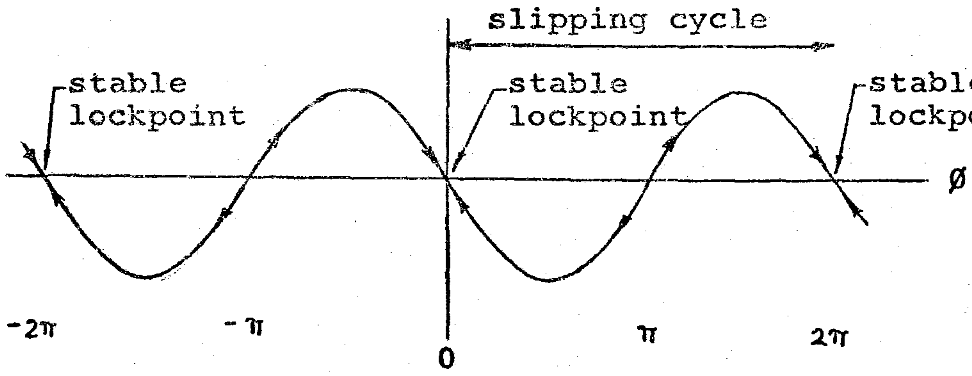


Fig. 5-3. Stable Lock Point and Slipping One Cycle.

The mean time to first cycle slip is defined as the average time the phase error takes to go from 0 to $\pm 2\pi$ radians. Lindsey[13] has shown that

$$\tau WL(2\pi/\phi_o) \approx \left(\frac{r+1}{r}\right)^2 \frac{\rho}{2} \int_{-2\pi}^{2\pi} \int_{-2\pi}^{\phi} [c-u(x)] \exp[u(x)-u(\phi)] dx d\phi \quad (5-7)$$

where

$$C = \frac{\int_{o}^{2\pi} \exp[u(x)] dx}{\int_{-2\pi}^{2\pi} \exp[u(x)] dx} \quad (5-8)$$

and

$$u(x) = -\left(\frac{r+1}{r}\right) \rho \operatorname{Cos} x \frac{-\rho}{2r} x^2 - \rho \frac{\Omega o}{AK} x \quad (5-9)$$

where τ is the mean time to first cycle slip, and u(x) is the unit step function.

The probability of losing lock in t seconds has been shown by Lindsey to be given by

$$P\{\text{slipping one or more cycles}\} = 1 - e^{-t/\tau} \quad (5-10)$$

$$P\{K \text{ slips in t sec}\} = \frac{(t/\tau)^k \exp(-t/\tau)}{K!} . \quad (5-11)$$

6. PHASE-LOCKED RECEIVERS

6-A. Double Heterodyne Phase-Locked Receiver. - Phase-lock loop receivers of the double-conversion heterodyne type are employed by the deep space instrumentation facility (DSIF) and compatible spacecraft. The DSIF receiver configuration is shown in Figure 6-1. These receivers operate at S- and X- band and utilize automatic gain control (AGC) and a PLL preceded by a bandpass limiter. The heterodyne design is employed to translate the RF signal down to a frequency for which stable phase detectrs can be built. AGC is required to provide a signal whose amplitude is within the dynamic range of the amplifier stages. The bandpass limiter minimizes the total mean square error of the loop over a wide range of input SNRs.

This receiver combines the advantages of two separate intermediate-frequency amplification(IF) to produce high gains with those of the PLL. Such a receiver can operate at very low signal levels with a great deal of stability, precision, and reliability.

The tracking portion of the receiver operates as follows: The input is assumed to be

$$x_o(t) = 2A(t)\cos(w_o t+\theta(t))+n_o(t) \qquad (6-1)$$

where A(t) is a slowly varying rms signal amplitude and $n_o(t)$ is a wide-band noise with density N_o. The input is mixed with a multiplied-up version of the VCO output, so the first IF output process is

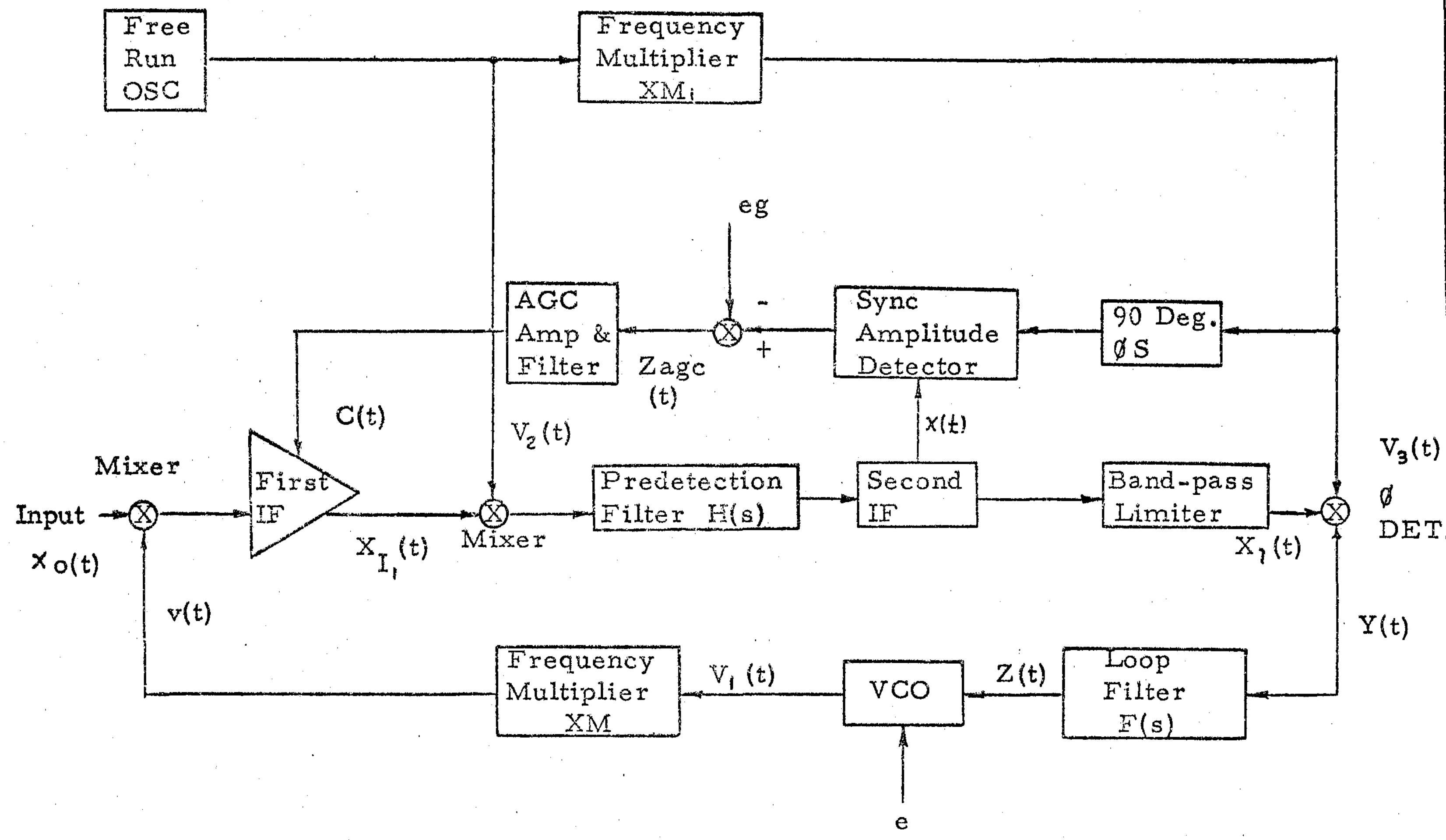


Fig. 6-1. DSIF Receiver Configuration.

$$x_{I_1}(t) = K_{I_1}[\sqrt{2}\cos(w_1 t+\theta-\theta)+n_1(t)] \quad (6-2)$$

where $w_1 = w_o - w_{h_1}$, K_{I_1} is the first IF gain and w_{h_1} is the first mixer heterodyne frequency, M times VCO output frequency. The first IF frequency is

$$f_1 = w_1/2\pi = (w_o - w_{h_1})/2\pi \quad (6-3)$$

The output of the second IF amplifier is

$$K_{I_1}K_{I_2}[\sqrt{2}A(t)\cos[(w_1-w_h)t+\theta-\hat{\theta}+\theta_1]+n_2(t)]$$

Again, the second IF frequency is

$$f_2 = w_2/2\pi = (w_1-w_{h_2})/2\pi \quad (6-4)$$

If each of the IF amplifier's bandwidth are known, the overall IF bandwidth can be computed by

$$W_N = \frac{\frac{1}{\pi}\int_0^\infty |H[j(w+w_h)]|^2 \cdot |F(jw)|^2 dw}{|H[j(w+w_h)]F(jw)|^2_{max}} \quad (6-5)$$

The first IF has much wider bandwidth than does the second, so W_H is essentially the W_H(second IF filter H(s) noise bandwidth).

The first IF is controlled by an AGC voltage*, and the K_{I_1}, K_{I_2} can be set K_{I_1}, $K_{I_2} = 1/A^*$(adjusted receiver attenuation factor), so the second IF output is

$$x(t) = \sqrt{2}A\cos(w_2 t+\theta-\hat{\theta}+\theta_1)+n_i(t) \quad (6-6)$$

where $n_i(t)$ is the loop input noise, volts. This voltage will be applied to a band-pass limiter whose saturation limits are $\pm$ L volts. The limiter mean-square output is constant at L^2, of which $2(2L/\pi)^2$ lies in the frequency zone about f_2. A signal component is present whose mean-square value is $2(2L\alpha/\pi)^2$; the factor α^2 is then a signal power suppression factor, with $0 \leq \alpha \leq 1$. The remainder of the limiter output in the IF zone is noise; it has total power $2(2L/\pi)^2(1-\alpha^2)$. The limiter output SNR ρ_L is a function of α only [5,8,10,15,22]:

$$\rho_L = \alpha^2/(1-\alpha^2) \tag{6-7}$$

The limiter output feeds a phase detector in which reference input is the free-running oscillator used to produce the second IF frequency; i.e., for some fixed θ_2,

$$v_3(t) \sim \sqrt{2}\sin(M_1 w_{h_2}t + M_1\theta_1 - \theta_2). \tag{6-8}$$

The detector output is

$$y(t) = K_d(\alpha \sin\{[w_o - w_{h_1} - (1+M_1)w_{h_2}]t + \theta - \hat{\theta} + (1+M_1)\theta_1 + \theta_2\} + n(t)). \tag{6-9}$$

The constant K_d is the gain of the detector. When the loop is in lock, the multiplied-up VCO output must have as its frequency

$$w_{h_1} = w_o - (1 + M_1)w_{h_2}. \tag{6-10}$$

The value of the IF frequencies are thus directly related to f_{h_2}:

$$f_1 = (1 + M_1)f_{h_2} \qquad f_2 = M_1 f_{h_2}. \tag{6-11}$$

The heterodyne signal coming into the first mixer from the VCO through a multiplier (M) will be

$$v(t) = \cos\left\{ w_{h_1}t + MK_dK_v\frac{F(p)}{p}\sin(\theta - \hat{\theta} + (1+M_1)\theta_1 + \theta_2) + \frac{n(t)}{\alpha} + \frac{MK_v n_v(t)}{p} \right\} \tag{6-12}$$

where K_v and $n_v(t)$ are VCO gain and equivalent VCO input noise respectively. The receiver estimate of the incoming phase function $\theta(t)$ is given by

* See Appendix A for a synchronous detector AGC loop.

$$\hat{\theta} = \frac{\alpha K_d K_v MF(p)}{p}\left\{\sin[\theta-\hat{\theta}+(1+M_1)\theta_1(t)+\theta_2(t)]+\frac{n(t)}{\alpha}\right\}$$

$$+ \frac{MK_v}{p}n_v(t). \qquad (6\text{-}13)$$

If the detector output error $\emptyset=\theta-\hat{\theta}+(1+M_1)\theta_1(t)+\theta_2$ is small the loop estimate is

$$\hat{\theta}(t) = \frac{\alpha K_d K_v MF(p)}{p+\alpha K_d K_v MF(p)}\left\{\theta(t)+\frac{n(t)}{\alpha}+(1+M_1)\theta_1(t)+\theta_2\right.$$

$$\left.+\frac{n_v(t)}{\alpha K_d F(p)}\right\}. \qquad (6\text{-}14)$$

The term $(1+M_1)\theta_1+\theta_2$ represents a heterodyne noise present in the loop.

The AGC detector uses a 90 degree shifted version of $M_1 w_{h_2}$ to derive the stabilized amplitude of the signal; the input to the AGC filter and amplifier is

$$z_{AGC}(t) = -e_g + AK_D[\cos \emptyset + \frac{n_{AGC}(t)}{A}] \qquad (6\text{-}15)$$

where e_g is the AGC gain (volts peak out)/(volts rms in).

The loop transfer function H(p) is given by

$$H(p) = \frac{\alpha K_d K_v MFF(p)}{p+\alpha K_d K_v MFF(p)} \qquad (6\text{-}16)$$

where F is the loop filter d-c gain.

Note that the above equation is exactly the same form as (3-4) with the parameter AK replaced by

$$AK = \alpha K_d K_v MF \qquad (6\text{-}17)$$

So when the receiver is tracking, $\emptyset=\theta_1-\hat{\theta}+(1+M_1)\theta_1+\theta_2$ is small. There are, however, some other minor differences between the signal PLL and the double hetrodyne receiver, principally in the phase-detector outputs. It is necessary to distinguish between $e_{\phi} = \theta-\hat{\theta}$, the loop tracking error, and the $\phi=\theta-\hat{\theta}+(1+M_1)\theta_1+\theta_2$, the loop detector-error.

6-B. <u>The Costas or I-Q loop [5,18].</u> - In the Costas loop, the phase of the data carrier is extracted from the suppressed carrier signal plus noise by multiplying the input voltages of the two phase detectors with that produced from the output of the VCO and a 90 degrees phase shift of that voltage, filtering the results, and using this signal to control the phase and frequency of the loop's VCO output (see Fig. 6-2).

Let the receiver carrier be denoted by

$$y(t) = A\sqrt{2}\, m(t)\sin(w_o t + \theta) + n(t) \tag{6-18}$$

where $m(t) = \pm 1$ with symbol duration T seconds, but the wideband Gaussian noise process (two-sided spectral density of $N_o/2$) be represented by

$$n(t) = \sqrt{2}n_1'(t)\cos(w_o t + \theta) + \sqrt{2}\, n_2'(t)\sin(w_o t + \theta) \tag{6-19}$$

where the spectral density of $n_1'(t)$ and $n_2'(t)$ is equal to $N_o/2$. At the phase detector output one can write

$$\begin{aligned} z_c(t) &= \{n(t)+\sqrt{2}Am(t)\sin(w_o t+\theta_o)\}\{\sqrt{2}\cos(w_o t+\hat{\theta})\} \\ &= \{n(t)\sqrt{2}\cos(w_o t+\hat{\theta})+2Am(t)\sin(w_o t+\theta_o)\cos(w_o t+\hat{\theta})\} \\ &= 2n_1'(t)\cos(w_o t+\theta)\cos(w_o t+\hat{\theta})+ \\ &\quad 2n_2'(t)\sin(w_o t+\theta)\cos(w_o t+\hat{\theta})+ \\ &\quad 2Am(t)\sin(w_o t+\theta_o)\cos(w_o t+\hat{\theta}) \\ &= \frac{2n_1'(t)}{2}\{\cos(w_o t+\theta-w_o t-\hat{\theta}) + \cos(w_o t +\theta+w_o t+\hat{\theta})\} \end{aligned}$$

$$+ \frac{2n_2'(t)}{2}\{\sin(w_o t+\theta-w_o t-\hat{\theta}) + \sin(w_o t+\theta+w_o t+\hat{\theta})\}$$

$$= Am(t)[\sin(\theta_o-\hat{\theta}) + \sin(2w_o t+\theta_o+\hat{\theta})]+$$

$$n_1'(t)[\cos(\theta-\hat{\theta}) + \cos(2w_o t+\theta+\hat{\theta})]+$$

$$n_2'(t)\ \sin(\theta-\hat{\theta}) + \sin(2w_o t+\theta_o+\hat{\theta})]. \qquad (6\text{-}20)$$

The multiplier and the loop filter remove the double frequency terms. For this case (6-20) becomes

$$z_c(t) = [Am(t)+n_2(t)]\sin(\theta-\hat{\theta})+n_1(t)\cos(\theta-\hat{\theta}).$$

substitute the phase error $\emptyset = \theta - \hat{\theta}$ into above,

$$z_c(t) = [Am(t)+n_2(t)]\sin\emptyset + n_1(t)\cos\emptyset. \qquad (6\text{-}21)$$

Output of the LPF one can have, letting G(p) denote the Laplace transform of the impulse response of the LPF,

$$y_c(t) = [AG(p)m(t) + n_2(t)]\sin\emptyset + n_1(t)\cos\emptyset$$

where $n_1(t)$ and $n_2(t)$ are independent Gaussian processes with spectral density $N_o/2$ and noise bandwidth W (band limited by filter $n_1'(t)$ and $n_2'(t)$).

Now the output of the coherent amplitude detector (CAD) is given by

$$z_s(t) = [n(t) + \sqrt{2}Am(t)\sin(w_o t + \theta_o)]\cdot$$
$$[\sqrt{2}\sin(w_o t + \hat{\theta})]$$
$$= 2n_1'(t)\cos(w_o t + \theta)\sin(w_o t + \hat{\theta}) +$$
$$2n_2'(t)\sin(w_o t + \theta)\sin(w_o t + \hat{\theta}) +$$
$$2Am(t)\sin(w_o t + \theta_o)\sin(w_o t + \hat{\theta})$$
$$= n_1'(t)[\sin(\hat{\theta}-\theta) + \sin(2w_o t+\hat{\theta}+\theta)] +$$
$$n_2'(t)[\cos(\theta-\hat{\theta}) - \cos(2w_o t+\theta+\hat{\theta})] +$$
$$Am(t)[\cos(\theta_o-\hat{\theta}) - \cos(2w_o t+\theta_o+\hat{\theta})]. \qquad (6\text{-}22)$$

$$z_s(t) = Am(t)\cos\emptyset(t) - Am(t)\cos(2w_o t + \theta + \hat{\theta}(t))$$
$$- n_1'(t)\sin\emptyset(t) + n_1'(t)\sin(2w_o t + \theta + \hat{\theta}(t))$$
$$+ n_2'(t)\cos\emptyset(t) - n_2'(t)\cos(2w_o t + \theta + \hat{\theta}(t)).$$

After the data filter, one can write

$$y_s(t) = [AG(p)m(t) + n_2(t)]\cos\emptyset(t) - n_1(t)\sin\emptyset(t). \qquad (6\text{-}23)$$

The stability of the loop can be found from the Costas loop transfer function. Since stability is a function of the linear system, one can temporarily let all the noise terms equal to zero. In this case the control signal, $z(t)$, is given by

$$z(t) = A^2[G(p)m(t)\sin\emptyset(t)][G(p)m(t)\cos\emptyset(t)] \qquad (6\text{-}24)$$

The phase of the oscillator is given by

$$\hat{\theta} = \frac{K_m K_v F(p)}{p} z(t) \qquad (6\text{-}25)$$

let $K = K_m K_v$ and $\hat{\theta}(t) = \theta - \emptyset(t)$, one can write

$$\theta - \emptyset(t) = \frac{KF(p)}{p} z(t)$$

and $\dot{\theta} = \dot{\emptyset}(t) + KF(p)z(t)$ (6-26)

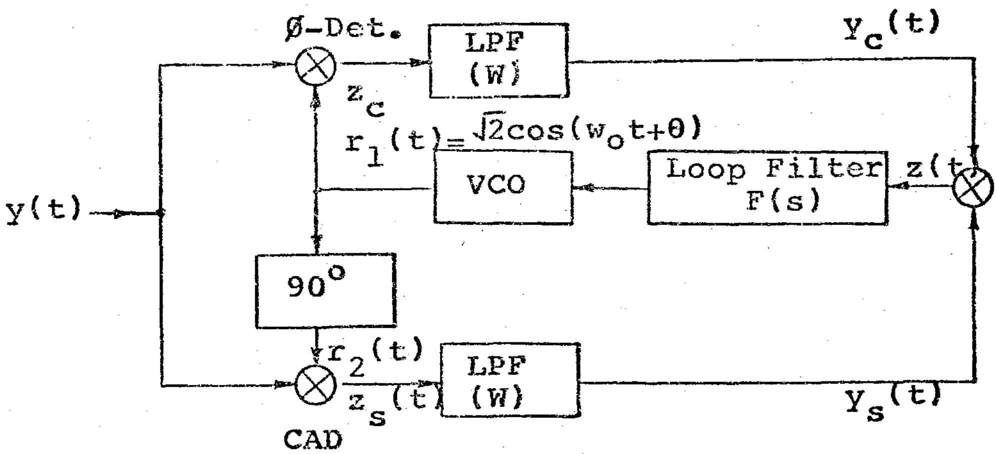


Fig. 6-2. The Costas loop.

For sufficiently small phase errors one can write

$$\dot{\theta} = \dot{\phi}(t)+KF(p)A^2(G(p)m(t)\phi(t))(G(p)m(t)). \qquad (6\text{-}27)$$

One can consider two cases

Case I: $B\ W_m \ll B_L$

With very low data rate compared to B_L, the loop noise bandwidth, one can have that

$$\dot{\theta} = \dot{\phi}(t) + A^2KF(p)G(p)\phi(t) \qquad (6\text{-}28)$$

and stability is determined by considering the closed loop transfer function

$$H(p) = \frac{AKF(p)G(p)}{p+A^2KF(p)G(p)} \qquad (6\text{-}29)$$

Case II: $B\ W_m \gg B_L$

When data is present and $\phi(t)$ is narrowband compared to the spectrum of m(t), one can have that (6-27) becomes

$$\dot{\theta} = \dot{\phi}(t) + A^2KF(p)[m'(t)]^2\phi(t) \qquad (6\text{-}30)$$

where

$$m'(t) = G(p)m(t) = \int_0^t G(t-\tau)m(\tau)d\tau . \qquad (6\text{-}31)$$

By approximation, replace $[m'(t)]^2$ with its mean

$$\dot{\theta} = \dot{\phi}(t) + A^2KF(p)\overline{[m(t)]^2}\phi(t) \qquad (6\text{-}32)$$

The Case I is less stable than Case II since G(p) adds at least one more pole to the open loop filter function. In the second case, the effect of the filter is to decrease the open loop gain by $m^2(t)$ which would be normally negligible. In addition, note that when data is present, if a sequence of all ones or zeros occurs, the Case I equation holds. The stability of the loop can be found by a root locus plot of (6-30).

One can now consider a phase error performance. Continuing with the noise analysis using (6-21) and (6-24), one can have that the control voltage is given by

$$z(t) = [(Am(t)+n_2)^2 \sin\phi + n_1\cos\phi][(Am(t)+n_2)\cos\phi - n_1\sin\phi]$$

$$= (Am(t)+n_2)^2 \sin\phi\cos\phi + n_1(Am(t)+n_2)\cos\phi\cos\phi$$

$$-n_1(Am(t)+n_2)\sin^2\phi - n_1^2\cos\phi\sin\phi$$

$$= \{(Am(t)+n_2)^2 \sin 2\phi - n_1^2 \sin 2\phi + n_1(Am(t)+n_2)(1+\cos 2\phi)$$

$$-n_1(Am(t)+n_2)(1-\cos 2\phi)\}/2$$

$$= A(t)^2 \sin 2\phi/2 + Am(t)n_2 \sin 2\phi + n_2^2 \sin 2\phi/2$$

$$- n_1^2 \sin 2\phi/2 + n_1(Am(t)+n_2)\cos 2\phi. \qquad (6\text{-}33)$$

From (6-25)

$$\hat{\theta} = \{(A^2/2 + Amn_2 + n_2^2/2 - n_1^2/2)\sin 2\phi + n_1(Am+n_2)\cos 2\phi\}KF(p)/p.$$

Since $\hat{\theta} = \theta - \phi$

$$\theta = \phi(t) + A^2KF(p)\sin 2\phi/2p + KF(p)[Amn_2 + n_2^2/2 - n_1^2/2]\sin 2\phi/p$$

$$+ KF(p)(Am+n_2)n_1\cos 2\phi/p.$$

From (2-33) $\Omega = \dot{\theta}$,

$$\dot{\theta} = \dot{\phi}(t) + A^2KF(p)\sin 2\phi/2 + KF(p)[Amn_2 + n_2^2/2 - n_1^2/2]\sin 2\phi$$

$$+ KF(p)(Am+n_2)n_1\cos 2\phi.$$

A rearrangement of the above yields,

$$\dot{\phi}(t) + A^2KF(p)\sin 2\phi/2 = KF(p)[(Amn_2 + n_2^2/2 - n_1^2/2)\sin 2\phi +$$

$$(Am+n_2)n_1\cos 2\phi] + \Omega. \qquad (6\text{-}34)$$

Now letting $2\phi = \Phi$, and $2\dot{\phi} = \Phi p$, one can write

$$\Phi p + A^2KF(p)\sin\Phi = KF(p)[(2Amn_2 + n_2^2 - n_1^2)]\sin\Phi +$$

$$2(Am + n_2)n_1\cos\Phi + 2\Omega. \qquad (6\text{-}35)$$

It is not difficult to show that all the noise terms to the right of the second F(p), call them N(t), have a correlation function given by

$$R_N(\tau) = 4[R_{n_1}^{\ 2}(\tau) + A^2R_{n_1}(\tau)] \qquad (6\text{-}36)$$

From (6-35) one can write

$$\Phi p + A^2KF(p)\sin\Phi = KF(p)N(t) \qquad (6\text{-}37)$$

Again for small Φ, say $\angle 1/6$, then (6-38) becomes

$$(p + A^2KF(p))\Phi = KF(p)N(t)$$

$$\Phi = \frac{KF(p)N(t)}{p+A^2KF(p)}$$

Multiply above A^2/A^2 and

$$\Phi = \frac{A^2KF(p)}{p+A^2KF(p)}\left(\frac{N(t)}{A^2}\right) \qquad (6\text{-}38)$$

(6-38) is exactly the same form as (3-4), so that the phase error can be computed from (3-11). Hence

$$\delta_\Phi^{\ 2} = \frac{N_o'B_L}{A^4} \quad \text{(rad.)} \qquad (6\text{-}39)$$

$$\text{where} \quad 2B_L = \int_{-j\infty}^{j\infty} \left| \frac{A^2KF(s)}{s+A^2KF(s)} \right|^2 \frac{ds}{2\pi} \qquad (6\text{-}40)$$

where $2B_L$ is the loop noise bandwidth of the equivalent loop defined by (6-35) under the assumption $W \gg B_L$. For narrow band loops, one need only consider the spectrum at zero, so

$$S_N(o) = \frac{N_o'}{2} = \int_{-\infty}^{\infty} R_N(\tau)d\tau \qquad (6\text{-}41)$$

For an ideal low pass data filter of W-Hz (one-sided), one can write

$$\frac{N_o'}{2} = 4\int_{-\infty}^{\infty} [R_{n_1}^{\ 2}(\tau) + A^2R_{n_1}(\tau)]d\tau$$

$$= 4\int_{-W}^{W}\left(\frac{N_o}{2}\right)^2 df + 2N_o A^2$$

and finally,

$$\frac{N_o''}{2} = 2N_o^2 W + 2N_o A^2 \qquad (6\text{-}42)$$

Hence $\quad \sigma_E^2 = B_L N_o''/A^4$

$$= \frac{4N_o B_L}{A^2}\left[1 + \frac{N_o W}{A^2}\right] \qquad (6\text{-}43)$$

Therefore, since $E = 2\phi$ one can write

$$\sigma_E^2 = E[E^2] \quad = E[4\phi^2] = 4E[\phi^2] = 4\sigma_\phi^2$$

where $E[\cdot]$ indicates mean square phase error. For this case (6-43) becomes

$$\sigma_\phi^2 = \frac{N_o B_L}{A^2}\left[1 + \frac{N_o W}{A^2}\right] \qquad (\text{ILPF}) \qquad (6\text{-}44)$$

For the case that the data filter is a low pass single pole RC filter with noise bandwidth W (W = 1/4RC). One can find that [2].

$$R_{n_1}(\tau) = N_o W e^{-4W(\tau)} \qquad (6\text{-}45)$$

so

$$\frac{N_o''}{2} = 4\int_{-\infty}^{\infty}[R_{N_1}^2(\tau) + A^2 R_{n_1}(\tau)]\, d\tau \qquad (6\text{-}46)$$

or $\quad \dfrac{N_o''}{2} = 2\, N_o A^2 + N_o^2 W \qquad (6\text{-}47)$

therefore,

$$\sigma_\phi^2 = \frac{N_o B_L}{A^2}\left[1 + \frac{N_o W}{2A^2}\right] \qquad (\text{RCLPF}) \qquad (6\text{-}48)$$

Comparison between (6-44) and (6-48) indicates that

RC low pass filter reduces the variance from the ideal low pass filter with the fixed noise bandwidth.

Note that the Costas loop can be implemented with integrate and dump circuits (matched filter with sample and hold) replacing the LPFs in the in-phase and quadrature-phase arms, thereby giving improved noise immunity, however, integrating time and sampling time become critical depending upon paticular data rate.

7. FUNDAMENTAL LOOP DESIGN

7-A. Application. - In this section, the design of phase-locked AM receiver using Costas loop is discussed. Consider the two-phase synchronous detection system (Costas loop) as shown in Figure 7-1 [19]. This is also called I-Q loop since this circuit uses two synchronous detection systems here for in-phase (I-channel) and (Q-channel) quadrature detection. The mathematical expression of this process is already shown in the previous chapter.

Basic operation of this system is as follows: with no phase error, the VCO is synchronized with the input signal and the in-phase channel output will be desired signal while the quadrature channel output will be zero, due to the quadrature relationship of the corresponding VCO and the incoming signal. If a phase error is present, the output signal will be reduced with some amplitude, but otherwise no change will take place. the output of the quadrature channel shows some voltage, and this voltage will be either in phase or exactly out phase

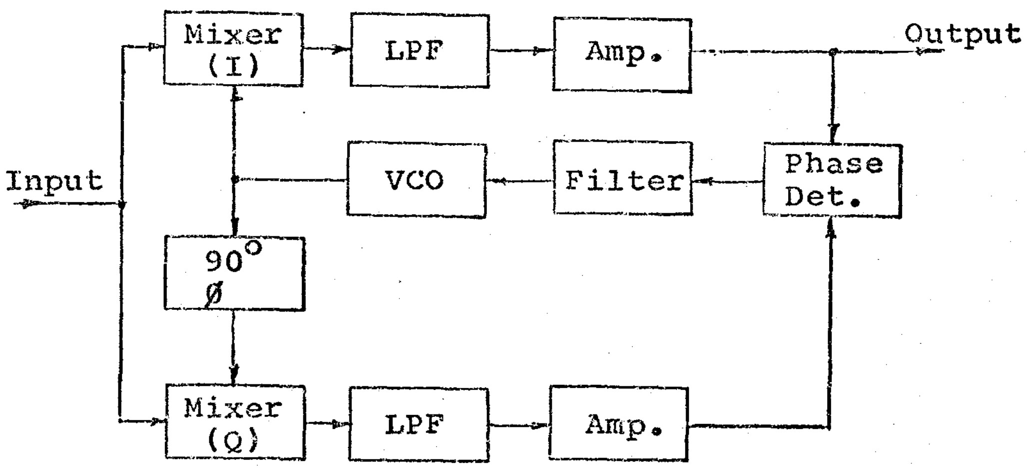


Fig. 7-1. AM receiver using Costas loop.

depending upon the sign of the phase error. The output of phase detector will produce d-c voltage. After passing through the low-pass automatic phase control (APC) filter, this d-c voltage is applied to the VCO and thereby effects the necessary correction of oscillator phase angle.

The experimental receiver obtained by Chu 11 is shown in Figure 7-2. This diagram is almost identical to the Figure 7-1 except for the addition of an RF amplifier to provide the RF sensitivity needed and a squelch cicuit to eliminate the heterodyne whistle. The problem with this type of system is obtaining a 90 degree constant phase shift. The technique he used here is to design two networks whose phase angles increase linearly with the logarithm of the frequency used. That is,

$$\emptyset_1 = C + \log f \qquad \emptyset_2 = C + \log Kf$$

$$\emptyset_1 . \emptyset_2 = C + \log K + \log f - C - \log f$$

$$= K$$

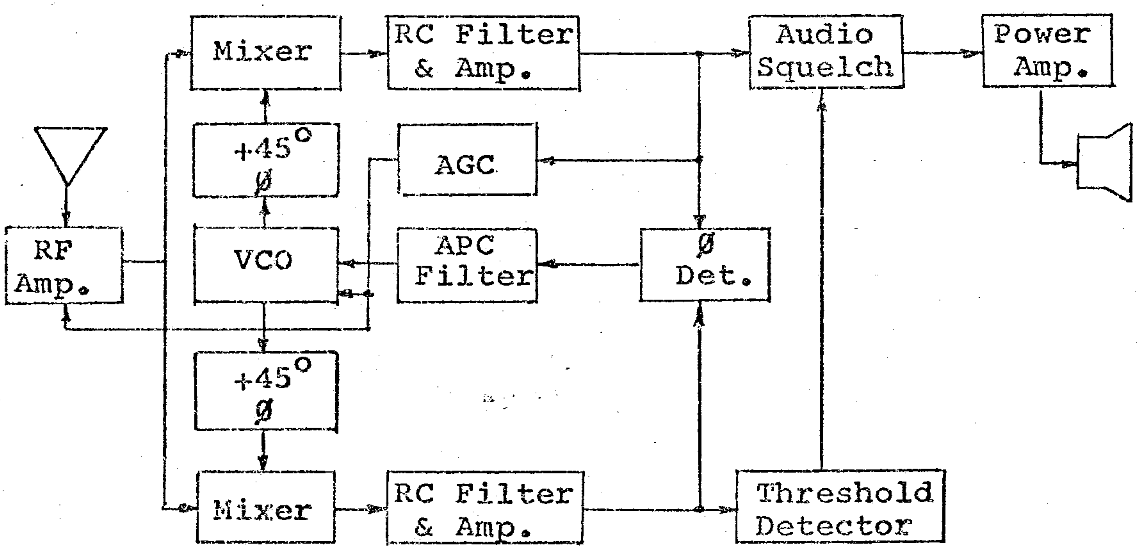


Fig. 7-2. AM radio receiver.

where $\emptyset_1$ = phase shift of network I

$\emptyset_2$ = phase shift of network II.

Figure 7-3 shows the network configuration and the optimum relationships of the parameters must be found for the particular signal band (for AM, 500 KHz to 1700 KHz). By use of this phase shift, one can obtain a maximum deviation of less than one degree over the AM frequency range. The relationship of the parameters (all values in kilo-ohms and pico-farads) are as follow:

Phase Shift I	Phase Shift II
$R_2 = 5R_1$	$R_2 = 5R_1$
$R_3 = R_2/4$	$R_3 = R_2/4$
$C_1 = 98.4/R_1$	$C_1 = 318.2/R_1$
$C_2 = C_1/5$	$C_2 = C_1/5$
$C_3 = 4C_2$	$C_3 = 4C_2$

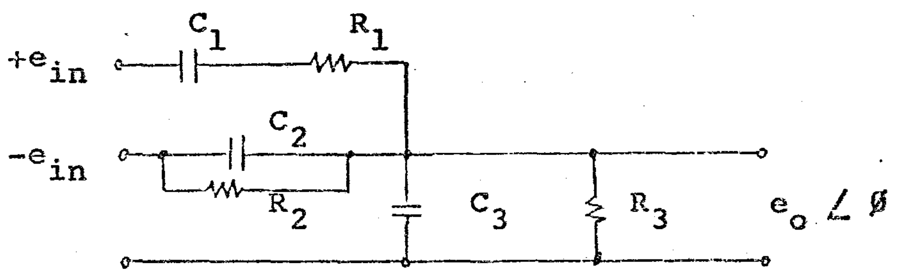


Fig. 7-3. ± 45 phase shift network.

7-B. Phase-Locked Loop Design. - The design of a PLL involves the determination of the type of loop required, selection of the proper bandwidth, and establishment of the desired stability. In this experiment, it is desired that the system have the following specifications:

output frequency	1.0 MHz to 2.0 MHz
frequency steps	100 KHz
lock-up time between channels	2 ms
overshoot	$\angle$ 20 %

The forward and feedback transfer functions are given by (see Fig. 7-4):

$$G(s) = K_M K_F K_V \qquad H(s) = K_N \qquad (7-1)$$

where G(s) and H(s) are came from the servo theory. The phase detector produces a voltage proportional to the phase difference between the signals θ_i and θ_o/N. This voltage upon filtering is used as the control signal for the VCM (voltage controlled multivibrator). Since the VCM produces a frequency proportional to its input voltage, any time variant signal appearing on the control signal will frequency modulate the VCM. The output frequency is $f_o = Nf_i$ during phase-lock, so that the programmable counter divide ratio K_N can be found by:

$$N_{min} = \frac{f_{o\,min}}{f_i} = \frac{1\text{ MHz}}{100\text{ KHz}} = 10 \qquad (7-2)$$

$$N_{max} = \frac{f_{o\,max}}{f_i} = \frac{2\text{ MHz}}{100\text{ KHz}} = 20 \qquad (7-3)$$

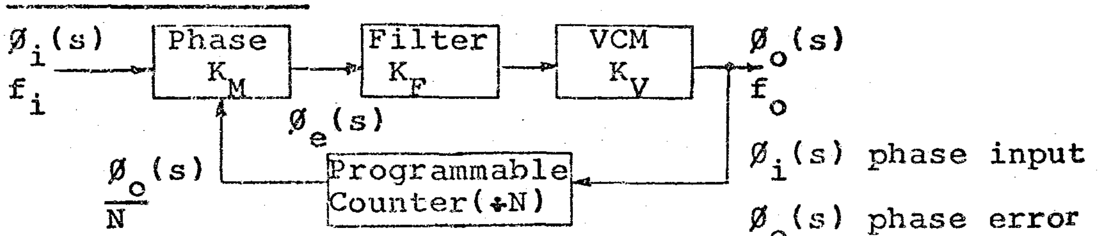


Fig. 7-4. PLL circuit parameters.

$K_N = 1/10$ to $1/20$ (7- 4)

Removal of the programmable counter produces unity gain in the feedback path (N=1). As a result, the output frequency is then equal to that of the input. If the type 2 system is used, the transfer function become

$$G(s)H(s) = K(s+a)/s^2 \qquad (7\text{- }5)$$

where K is the loop gain and a is the constant.

The root locus contour is shown in Figure 7-5. The root locus has two branches beginning at the origin with one asymptote located at 180 degrees. The center of gravity is s=a; however, with only one asymptote there is no intersection at this point. The root locus lies on a circle centered at s=-a and continues on all portions of the negative real axis to the zero. The brackaway point is s=-2a.

The operating range of the VCM must cover 1 MHz to 2 MHz. Selecting the MC4324 (Motorola) VCM control capacitor according to the rule contained on the data sheet yields C=100 pf. The desired operating range must be set at center within the total range of the device. The transfer function of the VCM is given by

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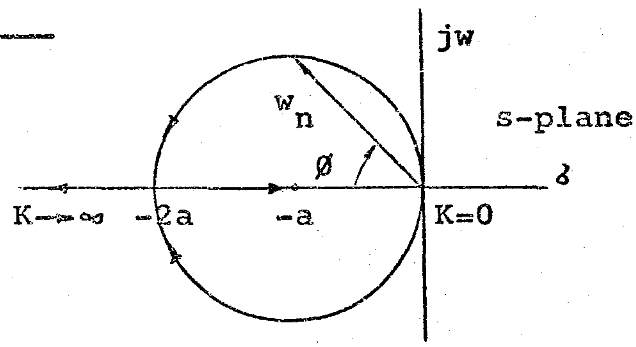

Fig. 7-5. Type 2 Second-Order root locus.

$$K_V = K_o/s \tag{7-6}$$

where K_o is the sensitivity in radians per second per volt. From the data sheet, K_o=11.2X10^6 rad/s/v.

$$K_V = 11.2X10^6/s \quad (rad/s/v) \tag{7-7}$$

The gain constant of the phase detector (MC4344 from Motorola 23) can be found from the data sheet and K_M=0.111v/rad. Thus far, the parameters determined include K_M,Ko,K_V, K_N leaving only K_F as the variable for design. From (7-5), one can relate

$$G(s)H(s) = \frac{K_M K_o K_N K_F}{s} = \frac{K(s + a)}{s^2} \tag{7-8}$$

then $$K_F = \frac{s + a}{s} \tag{7-9}$$

This is the exact form for the perfect integrator (see (3-18)). It can be shown that the active filter as shown in Figure 3-2 will be able to simulate the (7-9). K_F now becomes

$$K_F = \frac{R_2Cs + 1}{R_1Cs} \quad \text{for large A} \tag{7-10}$$

The gain of the active filter is not infinite, a gain correction factor K_c must be applied to K_F in order to experimentally to be K_c =0.5. Then (7-10) becomes

$$K_F = 0.5\left(\frac{R_2Cs + 1}{R_1Cs}\right) \tag{7-11}$$

The loop transfer function is

$$G(s)H(s) = \left(\frac{0.5K_M \; R_2Cs + 1}{R_1Cs}\right)\left(\frac{K_o}{s}\right)\left(\frac{1}{N}\right) \tag{7-12}$$

The PLL circuit Laplace representation is shown in Figure 7-6. The characteristic equation takes the form

$$C.E. = 1 + G(s)H(s) = 0$$

$$= s^2 + \frac{0.5\ K_M K_o R_2 s}{R_1 N} + \frac{0.5\ K_M K_o}{R_1 CN} \qquad (7\text{-}13)$$

One can relate (7-13) by

$$s^2 + 2\zeta w_n s + w_n^2 \qquad (7\text{-}14)$$

Equating coefficients yields

$$\frac{0.5\ K_M K_o}{R_1 CN} = w_n^2 \qquad (7\text{-}15)$$

and

$$\frac{0.5\ K_M K_o R_2}{R_1 N} = 2\zeta w_n \qquad (7\text{-}16)$$

A w_n can be determined by the step response curve shown in the data sheet [23]. From the step response curve it is seen that a damping ratio = 0.8 will produce a peak overshoot less than 20 % and will settle to within 5 % at $w_n t=4.5$. The required lock-up time is 2 ms.

$$w_n = 4.5/0.002 = 2.25X10^3 \text{ rad/sec.} \qquad (7\text{-}17)$$

From (7-15)

$$R_1C_1 = \frac{0.5\ K_M K_o}{w_n^2 N} \qquad (7\text{-}18)$$

$$= \frac{0.5(0.111)(11.2X10^6)}{(2250)^2(20)} = 0.00614$$

(Maximum overshoot occurs at N_{max} which is minimum loop

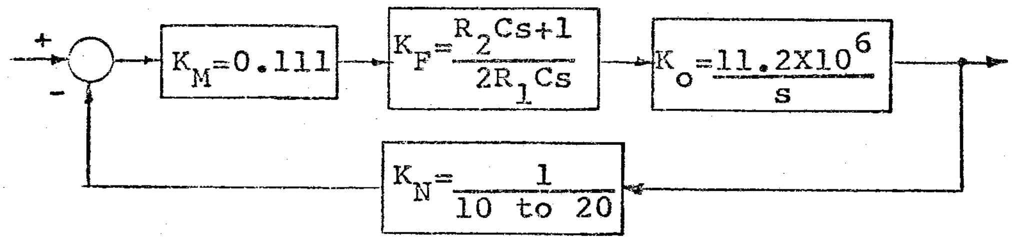

Fig. 7-6. Equivalent block diagram of PLL.

gain) Let C = 0.56 Mf.

Then $R_1 = 0.00614/0.56X10^{-6} = 10.96$ K-ohms

use $R_1 = 11$ k-ohms

R_2 can be found by (7-16)

$$R_2 = \frac{2\zeta w_n R_1 N}{0.5K_M K_o} = \frac{2\zeta}{Cw_n} \qquad (7\text{-}19)$$

$$= \frac{2(0.8)}{(0.56X10^{-6})(2.25X10^{3})} = 1.269 \text{ k-ohms}$$

use $R_2 = 1.3$ k-ohms.

The loop was designed for N=20. The system response for N=10 exhibits a wider bandwidth and larger damping factor, thus reducing both lock-up time and percent overshoot. The -3dB bandwidth of the PLL is given by

$$W_{-3dB} = w_n(1 + 2(\zeta)^2 + (2 + 4\zeta^2 + 4\zeta^4)^{1/2})^{1/2} \qquad (7\text{-}20)$$

$$= 2.25X10^3(1+2(0.8)^2+(2+4(0.8)^2+4(0.8)^4)^{1/2})^{1/2}$$

$$= 4.91 \text{ KHz.}$$

for a type 2 second-order system [6].

The PLL circuit diagram is shown in Figure 7-7 and the transient frequency response of theoretical and experimental results are shown in Figure 7-8.

Experimental results. - The curve N=20 illustrates the frequency response when counter is stepped from 19 to 20 thus producing a change in the output frequency from 1.9 MHz to 2.0 MHz. An overshoot of 18% is obtained and the output frequency is within in 5 KHz (5 mv) of the final value one millisecond after the applied step. The curve N =10 illustrates the output frequency change as the counter is stepped from 11 to 10.

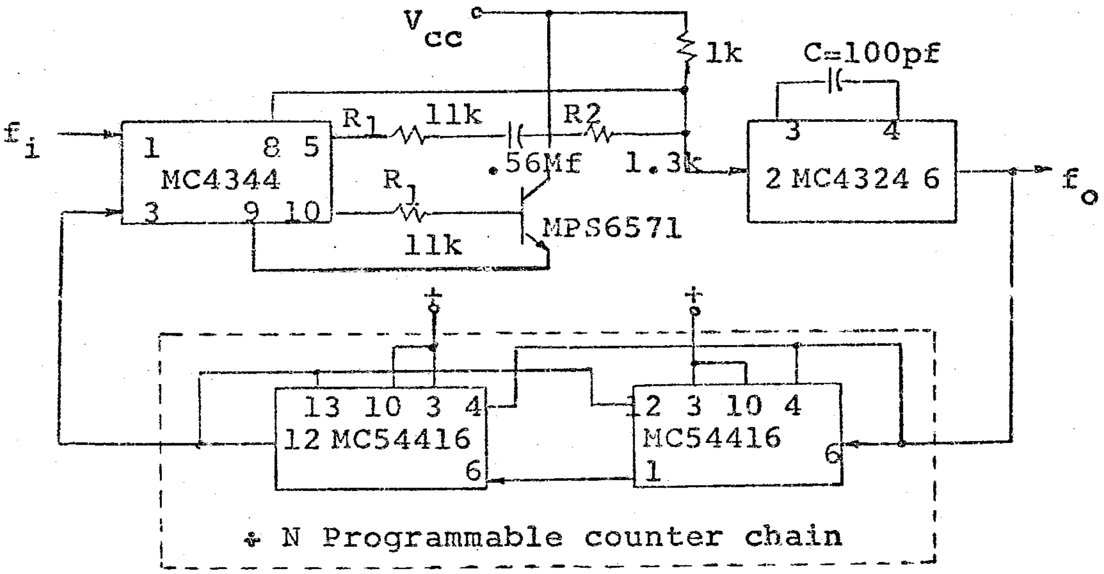

Fig. 7-7. Circuit diagram of Type 2 PLL.

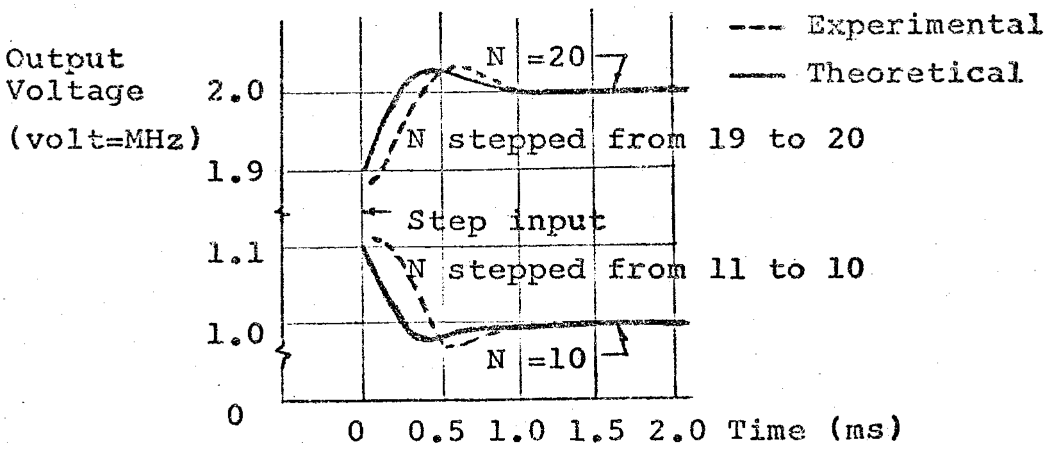

Fig. 7-8. Frequency-time response.

8. CONCLUSION

The phase-locked loop is a practical device for the detection of digital and analog modulated signals. Compared to the purely coherent detector, its performance is lower since phase reference is derived directly from the noisy signal. It does offer the advantage of being able to track a signal that changes in frequency due to instability or doppler shift.

It can be shown through the experiment that the design procedure of PLL is straight forward. However, each design must be an individual attempt to synthesize the optimum networks for the given problem. Only after this procedure has been completed may an evaluation of the improved performance be determined.

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APPENDIX A SYNCHRONOUS DETECTOR AGC SYSTEMS

With the AGC device in Figure A-1, the AGC voltage C(t) is given by

$$C(t) = C(p)\left[a(t) + \left(\frac{20 \log e}{e_g}\right) K_D m_i(t) + 20 \log \cos \phi(t) - K_r\right] \quad (A-1)$$

where $a(t) = 20 \log A(t)$ (dB-volts^2)

$$C(t) = \frac{K_{AGC}}{K_A(1+K_{AGC}Y(s))}$$

$$K_{AGC} = \frac{K_A e_g K_c}{20 \log e}$$

and Y(s) is the AGC loop filter and K_r is the adjusted receiver attenuation with no AGC, dB.

The steady-state mean value of AGC voltage C is related to the input signal strength a by

$$a = K_{rec} + 20 \log \left(\frac{K_D}{e_g}\right) + \left(\frac{1}{C(0)}\right) C + (10 \log e)^2 \quad \text{(dB-volts}^2\text{)} \quad (A-2)$$

where $K_{rec} = K_r + 20 \log(K_D/e_g)$ and C(0) is p = 0.

At the steady-state a(t) becomes a and C(t) becomes C. From (A-2), given C, one can infer a value of a.

The AGC voltage fluctuates with noise; its steady-

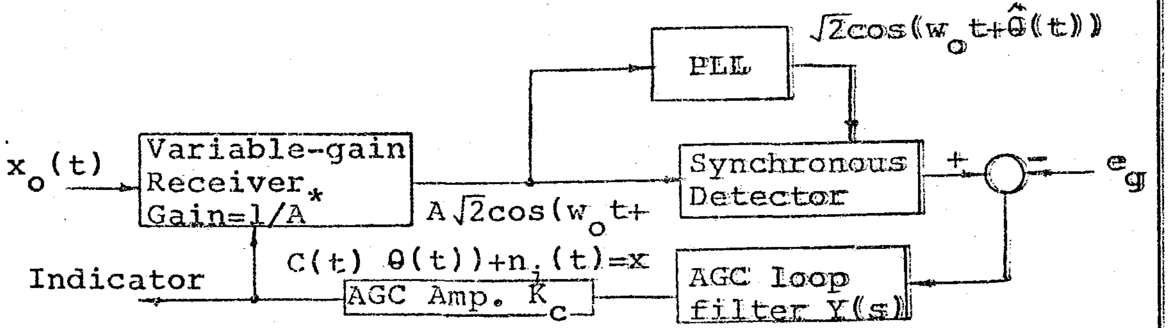

Fig. A-1. A synchronous-detector AGC loop.

state variance is given by Tawsworth.

$$\text{var } C = \left[(20 \log e)^2 \left(\frac{N_o w_c}{A^2}\right) + 2(10 \log e)^2 \delta^4 \frac{w_c}{w_L}\right] C^2(0) \qquad \text{(A- 3)}$$

where N_o = two sided noise spectral density, volts^2/cps

δ^2 = the phase noise variance (3-11).

As the carrier signal level increases, the departure from linear behaiver deviates due to deviation of phase detector and non-logarithmic amplifier characteristic.